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Mass scales and symmetry breaking in SU(15) grand unification

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We carefully analyze the recently suggested grand unified model based on the gauge group SU(15). We discuss various bounds on the scales imposed by rare processes. We point out that baryon number is part of the gauge symmetry of the model. We propose changes that get rid of a massless gauge boson corresponding to the unbroken baryon-number symmetry present in the original model.

I. INTRODUCTION

Recently, Frampton and Lee[1](FL) have proposed a grand unification model based on the gauge group SU(15). All known left-handed fermions of a single generation transform like the fundamental representation of this group. Thus, for example,

$$\mathbf{15}=(u_1u_2u_3d_1d_2d_3\,\vdots\,\hat{u}_1\hat{u}_2\hat{u}_3\hat{d}_1\hat{d}_2\hat{d}_3\,\vdots\,e^+\nu_e e^-)_L \qquad (1)$$

for the first generation, and similarly for the higher ones. The subscripts 1, 2, and 3 represent the color indices and we have used the caret to denote antiparticles. The vertical dotted lines put at various places are meant for convenience of the discussion.

It is obvious that all gauge bosons of this model have well-defined values of baryon number (B) and lepton number (L). Thus, in the unbroken theory, exchange of gauge bosons cannot break B or L. Proton decay is therefore absent in the gauge sector.[1] This is an interesting property, which was present in partially unified models such as the Pati-Salam model[2] based on the gauge group $SU(2)_L\times SU(2)_R\times SU(4)_C$, but not in popular unified models such as those based on the gauge groups SU(5), SO(1), or E_6. However, in the literature there has been some discussion[3,4] about grand unified models based on the gauge group SU(16), which shares this property.

For further discussions, one needs to know how the grand unified symmetry group SU(15) breaks down to the manifest symmetry of the low-energy world. FL considered[1] the following chain on which we base all the analysis of the present article:

$$\begin{aligned}SU(15)&\xrightarrow{M_G} SU(12)_q\times SU(3)_l\\ &\xrightarrow{M_B} SU(6)_L\times SU(6)_R\times U(1)_h\times SU(3)_l\\ &\xrightarrow{M_A} SU(3)_c\times SU(2)_L\times U(1)_Y\\ &\xrightarrow{M_W} SU(3)_c\times U(1)_Q\ .\end{aligned} \qquad (2)$$

The fundamental representation of the subgroups $SU(12)_q$ and $SU(3)_l$ are given by the first twelve and the last three entries of Eq. (1). Thus, only quarks and antiquarks transform under $SU(12)_q$ whereas only leptons and antileptons transform under $SU(3)_l$. In the second stage of symmetry breaking, the $SU(12)_{12q}$ subgroup breaks into $SU(6)_L\times SU(6)_R\times U(1)_h$, under which the left-handed quarks and antiquarks transform as $(\mathbf{6},\mathbf{1},h)$ and $(\mathbf{1},\bar{\mathbf{6}},-h)$, respectively. Finally, at the third stage, one obtains the gauge group of the standard model, which is followed by Weinberg-Salam symmetry breaking.

Based on this symmetry-breaking scheme, FL claim[1] the following results.

(1) The grand unification scale M_G can be as low as 10^6-10^7 GeV.

(2) The intermediate scale M_A, defined in Eq. (2), is low enough to be within the reach of the Superconducting Super Collider (SSC).

(3) The first two stages of symmetry breaking can be performed by vacuum expectation values (VEV's) of two adjoint **224** and one fundamental **15** representation of Higgs bosons. The symmetry breaking at the scale M_A can be realized by a number of **15** multiplets.

(4) In this scheme of symmetry breaking, baryon number is not violated even in the Higgs sector. In other words, baryon number remains as a true symmetry of the model even after all symmetry breaking.

(5) One can obtain exactly the experimentally obtained values of the low-energy parameters $\sin^2\theta_W$, α, and α_s at the scale M_W.

The last claim is true, and is not surprising since by adjusting the three unknown mass scales M_G, M_B, and M_A, one should be able to fit three low-energy parameters. The validity of the other claims is what we discuss in this article.

II. EVOLUTION OF GAUGE COUPLING CONSTANTS

We first examine the evolution of the gauge coupling constants. At the SU(15) level, we normalize the generators T^a by the relation

$$\mathrm{Tr}(T^aT^b)=\tfrac{1}{2}\delta^{ab} \qquad (3)$$

in the fundamental representation. Introducing $\alpha_G=g_G^2/4\pi$ for a subgroup G, we can write down the one-loop renormalization-group (RG) equation for the gauge coupling constant as

$$\frac{\partial \alpha_G}{\partial \ln M} = -\frac{B_G}{2\pi}\alpha_G^2 , \tag{4}$$

where M is an arbitrary mass scale. The solution is

$$\alpha_G^{-1}(M_2)=\alpha_G^{-1}(M_1)-\frac{B_G}{2\pi}\ln\frac{M_1}{M_2} . \tag{5}$$

In order to use this solution to find expressions for the coupling constants at the scale M_W in terms of the unified coupling constant, one needs the matching condition at various symmetry-breaking scales. At the scale M_G, this is simply

$$\alpha_{12q}(M_G)=\alpha_{3l}(M_G)=\alpha_U(M_G) , \tag{6}$$

with an obvious notation. At M_B, similarly, we have

$$\alpha_{6L}(M_B)=\alpha_{6R}(M_B)=\alpha_{1h}(M_B)=\alpha_{12q}(M_B) . \tag{7}$$

To obtain the matching condition at M_A, we need to know how the standard model gauge group is embedded in the grand unification group. In $SU(6)_L$, there is a generator

$$\Lambda_{6L}=\frac{1}{2\sqrt{3}}\text{diag}(1,1,1,-1,-1,-1\,\vdots\,\mathbf{0}_{(6)}\,\vdots\,\mathbf{0}_{(3)}) \tag{8}$$

where $\mathbf{0}_{(6)}$, for example, denotes six zeros along the diagonal. This generator combines with the generator

$$\mathcal{Y}_{3l}=\tfrac{1}{2}\text{diag}(\mathbf{0}_{(6)}\,\vdots\,\mathbf{0}_{(6)}\,\vdots\,0,1,-1) \tag{9}$$

of $SU(3)_l$ to give the diagonal generator of $SU(2)_L$:

$$I_{2L}^3=\mathcal{Y}_{3l}+\sqrt{3}\Lambda_{6L} . \tag{10}$$

Therefore,

$$\alpha_{2L}^{-1}(M_A)=\alpha_{3l}^{-1}(M_A)+3\alpha_{6L}^{-1}(M_A) . \tag{11}$$

Similarly, if we consider the $U(1)_h$ generator

$$h=\frac{1}{2\sqrt{6}}\text{diag}(\mathbf{1}_{(6)}\,\vdots\,-\mathbf{1}_{(6)}\,\vdots\,\mathbf{0}_{(3)}) \tag{12}$$

and a diagonal $SU(3)_l$ generator

$$\mathcal{Y}'_{3l}=\frac{1}{2\sqrt{3}}\text{diag}(\mathbf{0}_{(6)}\,\vdots\,\mathbf{0}_{(6)}\,\vdots\,2,-1,-1) , \tag{13}$$

we can write

$$\tilde{Y}=\frac{1}{\sqrt{5}}(3\mathcal{Y}'_{3l}+\sqrt{2}h-3\Lambda_{6R}) , \tag{14}$$

where Λ_{6R} is similar to Λ_{6L} except that the nontrivial elements are in the second block of six elements. The $\tilde{Y}$ defined above is the normalized weak hypercharge, related to the electric charge by the relation

$$Q=I_{2L}^3+\sqrt{\tfrac{5}{3}}\tilde{Y} . \tag{15}$$

From Eq. (14), we obtain the matching condition

$$\alpha_{\tilde{Y}}^{-1}(M_A)=\tfrac{1}{5}[9\alpha_{3l}^{-1}(M_A)+2\alpha_{1h}^{-1}(M_A)+9\alpha_{6R}^{-1}(M_A)] . \tag{16}$$

Similarly, by looking at the color generators, we get

$$\alpha_{3c}^{-1}(M_A)=2\alpha_{6L}^{-1}(M_A)+2\alpha_{6R}^{-1}(M_A) . \tag{17}$$

Notice that, with their usual definitions, the standard-model generators are not normalized according to Eq. (3).[5]

Using the matching conditions and the solution of the renormalization-group equations, we can deduce the following relations for the coupling constants at scale M_W:

$$\begin{aligned}
\alpha_{3c}^{-1}(M_W)&=4\alpha_U^{-1}-\frac{\ln 10}{2\pi}[4B_{12q}(n_G-n_B)+4B_6(n_B-n_A)+B_{3c}(n_A-n_W)] ,\\
\alpha_{2L}^{-1}(M_W)&=4\alpha_U^{-1}-\frac{\ln 10}{2\pi}[3B_{12q}(n_G-n_B)+B_{3l}(n_G-n_A)+3B_6(n_B-n_A)+B_{2L}(n_A-n_W)] ,\\
\alpha_{\tilde{Y}}^{-1}(M_W)&=4\alpha_U^{-1}-\frac{\ln 10}{2\pi}[\tfrac{11}{5}B_{12q}(n_G-n_B)+\tfrac{9}{5}B_{3l}(n_G-n_A)+(\tfrac{9}{5}B_6+\tfrac{2}{5}B_{1h})(n_B-n_A)+B_{\tilde{Y}}(n_A-n_W)] ,
\end{aligned} \tag{18}$$

where

$$n\equiv\log_{10}(M/1\text{ GeV}) \tag{19}$$

for any mass scale M. In writing these relations, we have assumed that $B_{6L}=B_{6R}\equiv B_6$ which is certainly true if we neglect the contribution of scalar fields in RG equations. Neglecting scalar contribution, one obtains for N_f number of fermion families the following B coefficients:

$$\begin{aligned}
&B_{12q}=44-\tfrac{1}{3}N_f , \quad B_6=22-\tfrac{1}{3}N_f ,\\
&B_{3l}=11-\tfrac{1}{3}N_f , \quad B_{1h}=-\tfrac{1}{3}N_f ,\\
&B_{3c}=11-\tfrac{4}{3}N_f , \quad B_{2L}=\tfrac{22}{3}-\tfrac{4}{3}N_f , \quad B_{\tilde{Y}}=-\tfrac{4}{3}N_f .
\end{aligned} \tag{20}$$

One can put these back in Eq. (18) and use the relations

$$\begin{aligned}
\sin^2\theta_W&=\tfrac{3}{8}-\tfrac{5}{8}\alpha(\alpha_{\tilde{Y}}^{-1}-\alpha_{2L}^{-1}) ,\\
\alpha/\alpha_{3c}&=\tfrac{3}{8}-\alpha(\tfrac{5}{8}\alpha_{\tilde{Y}}^{-1}+\tfrac{3}{8}\alpha_{2L}^{-1}-\alpha_{3c}^{-1}) ,
\end{aligned} \tag{21}$$

which are valid for all quantities defined at the scale M_W. Assuming all fermions are lighter than the weak scale, it is easily seen that the quantity N_f cancels out and we get the following relations connecting different mass scales:

$$\begin{aligned}
&264n_G-88n_B-\tfrac{308}{3}n_A-\tfrac{220}{3}n_W=\frac{0.375-\sin^2\theta_W(M_W)}{1.789\times10^{-4}} ,\\
&792n_G-440n_B-220n_A-132n_W\\
&\qquad=\frac{0.375-[\alpha(M_W)/\alpha_{3c}(M_W)]}{1.789\times10^{-4}} ,
\end{aligned} \tag{22}$$

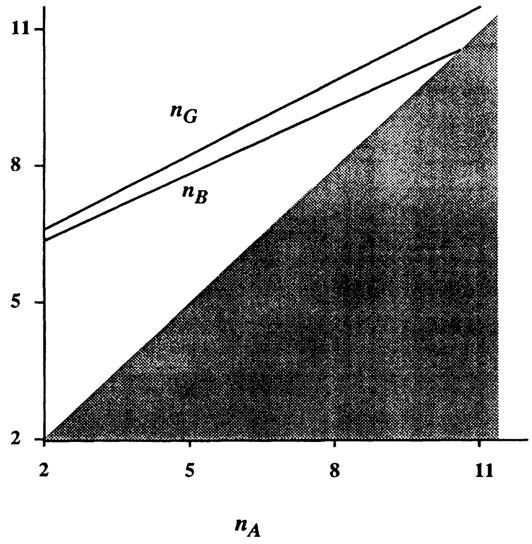

FIG. 1. Values of different mass scales. The quantity n is defined in Eq. (19). The shaded area is not acceptable since by definition $M_G \geq M_B \geq M_A$.

where we have used the value $\alpha^{-1}(M_W)=128$. Using $n_W = \log_{10}81$, we have plotted in Fig. 1 the solutions for these equations for $\sin^2\theta_W(M_W)=0.228$ and $\alpha_{3c}^{-1}=9.35$.

The various scales in Fig. 1 are consistent with the values given by FL.[1] Thus, the unification scale M_G can be as low as of order 10^7 GeV and the scale M_A can be accessible to SSC energies. However, that is not a necessity so far as the renormalization-group analysis alone is concerned. The only constraint seems to be coming from the fact that the definitions of the various scales imply $n_B \geq n_A$, which gives $n_A \leq 10.6$. Recalling Eq. (19), this means that $M_A \leq 4\times10^{10}$ GeV. The corresponding maximum value of M_G is about 2×10^{11} GeV. This conclusion, of course, can be modified once we include the Higgs-boson contributions in the RG equations. In addition, there can be other, phenomenological constraints on the scales, which would we described later.

III. CONSTRAINTS ON THE MASS SCALES FROM VARIOUS PROCESSES

Since the renormalization-group analysis can neither predict the mass scales nor put any bound which can be used as guidelines for forthcoming experiments, let us examine what constraints are put by various particle processes.

The most important one in this respect is the K^0-$\hat{K}^0$ transition amplitude. At the quark level, this arises from the process $d\hat{s} \to s\hat{d}$. Within the standard model, this process arises at the one-loop level and the amplitude is suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism. In the SU(15) model of our present interest, there are additional diagrams which mediate such a transition at the tree level, as shown in Fig. 2. The gauge boson that mediates this process belongs to the $SU(12)_q$ subgroup of SU(15), but is outside the $SU(6)_L \times SU(6)_R$ subgroup shown in Eq. (2). Therefore, the masses of these gauge bosons are given by M_B. We can easily write down the amplitude for the process shown in Fig. 2:

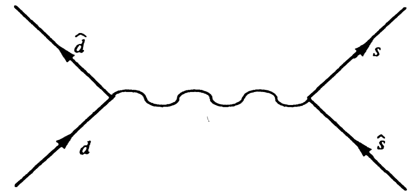

FIG. 2. K^0-$\hat{K}^0$ transition mediated by gauge bosons.

$$\mathcal{A}(K^0\text{-}\hat{K}^0) \simeq \frac{g^2}{M_B^2}(\bar{d}_L^{\beta}\gamma_\mu \hat{d}_L^{\alpha})(\bar{\hat{s}}_L^{\alpha}\gamma^\mu s_L^{\beta})$$

$$= -\frac{g^2}{M_B^2}(\bar{d}_L^{\beta}\gamma^\mu s_L^{\beta})(\bar{d}_R^{\alpha}\gamma_\mu s_R^{\alpha}) , \qquad (23)$$

where α and β are summed color indices. In deriving the last step, we have used Fierz transformation and the definition of an antiparticle. We now define the matrix element

$$\langle K^0 | (\bar{d}_L^{\beta}\gamma^\mu s_L^{\beta})(\bar{d}_R^{\alpha}\gamma_\mu s_R^{\alpha}) | \hat{K}^0 \rangle \equiv b f_K^2 m_K , \qquad (24)$$

where b is a quantity which can be calculated only with a proper understanding of hadronic physics. In absence of that, we take it as a parameter whose estimate is given later. The contribution of the operator in Eq. (24) to the K_L-K_S mass difference is given by

$$\Delta m_K = \frac{g^2 b f_K^2 m_K}{M_B^2} . \qquad (25)$$

Since the standard-model contribution gives roughly the right magnitude and right sign for the experimentally measured value of $\Delta m_K = 3.5\times10^{-15}$ GeV, we can say that the magnitude of the contribution from Eq. (25) is less than the experimental value. This gives

$$M_B \gtrsim \tfrac{1}{2} b^{1/2} \times 10^6 \text{ GeV} , \qquad (26)$$

where we have used $f_K = 114$ MeV and $m_K = 493$ MeV, and have put $g^2 \simeq \frac{1}{10}$ as a rough estimate. Putting the vacuum-insertion estimate[6] which gives $b=2.6$, we thus predict $M_B \geq 0.8\times10^6$ GeV. This certainly does not rule out any allowed values of the scales shown in Fig. 1. However, it has to be borne in mind that this is a very lenient constraint from the process. If the contribution of the standard model can be known to a very good accuracy, one can probably put a more stringent and useful constraint on the scale M_B. Also, once one considers the contribution of the Higgs bosons in the RG equations, the predictions of the scales changes somewhat from the values in Fig. 1, and that might rule out some part of the parameter space from consideration of the K_L-K_S mass difference.

Flavor-changing processes such as $K_L \to \mu^+ e^-$ are mediated by the gauge bosons at the scale M_G and are therefore very suppressed in this model. Observation of such processes close to the present bounds can rule out this model.

Among the gauge bosons at the scale M_A, there are the broken $SU(3)_l$ gauge bosons which can mediate interesting processes such as the muonium-antimuonium transition $\mu^+e^- \rightarrow \mu^- e^+$, as shown in Fig. 3. The strength of this process in this model will thus be given by $G_{\rm eff} \simeq g^2/M_A^2$. The experimental bound on the process is very weak, $G_{\rm eff} \leq 7G_F$. Improvement of this bound will help constrain the scale M_A. The other interesting symmetry which breaks at the scale M_A is $U(1)_h$. The nature of this symmetry breaking is discussed in the next section.

IV. THE QUESTION OF BARYON-NUMBER CONSERVATION

Looking at the generator h in the fundamental representation, as given in Eq. (12), it is immediately obvious that h is very simply related to baryon number:

$$B=\sqrt{\tfrac{8}{3}}h \ . \tag{27}$$

This poses tough questions for the model of FL,[1] which we discuss in this section.

First, it must be remembered that if only the known fermions are present in a model, baryon number is an anomalous symmetry. This makes the model inconsistent since baryon number is part of the gauge symmetry here. Therefore, one must add more fermions in order for the model to work. Mirror fermions can certainly cancel the anomalies. Introduction of extra fermions would not affect Eq. (22) and therefore the predictions about the mass scales as long as all the fermions are lighter than the weak scale.

Secondly, FL claim that B is unbroken in their model. As we see, B is part of the gauge symmetry. Thus, if it remains unbroken in the entire chain of symmetry breaking, $U(1)_B$ [or equivalently $U(1)_h$] will be part of the unbroken-symmetry group of the real world. Certainly, this is not the case. Hence $U(1)_B$ must be broken in order to obtain a realistic theory. With this in mind, we want to discuss the symmetry breaking of the SU(15) model at some length.

As mentioned before, FL claimed that the symmetry breaking from the grand group to the standard-model group can be done by VEV's in a number of **15** and **224** multiplets of Higgs bosons. Particles in the **15** must have the quantum numbers same as the fermions in Eq. (1). Thus, the only particle which can develop a VEV is the uncharged one which has quantum numbers of the ν_e. A

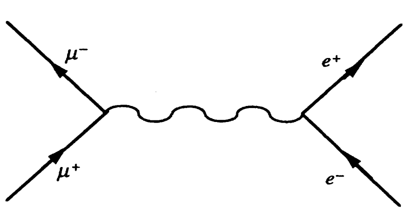

FIG. 3. Muonium-antimuonium transition mediated by gauge bosons.

VEV of this component breaks $SU(2)_L$, and therefore cannot be larger than of order M_W. Moreover, even if a **15** of Higgs boson develops a VEV at or below the scale M_W, that cannot break $U(1)_B$ since the ν_e component does not carry any baryon number.

Turning now to the adjoint **224**, it is well known that the VEV's of the adjoint cannot reduce the rank of a group. Therefore, a **224** can perform the symmetry breaking at the scale M_B, but not any other stage of breaking shown in Eq. (2). The smallest representation that can perform the first stage of symmetry breaking is the **455**-dimensional rank-3 tensor $\Phi^{[ijk]}$ (the square brackets denote antisymmetrization), which decomposes as

$$\mathbf{455}=(\mathbf{220},\mathbf{1})+(\mathbf{66},\mathbf{3})+(\mathbf{12},\overline{\mathbf{3}})+(\mathbf{1},\mathbf{1}) \ . \tag{28}$$

Obviously, a VEV in the last component breaks the symmetry down to $SU(12)_q \times SU(3)_l$.

We now examine what happens to $U(1)_B$ at the weak scale. FL argues[1] that the symmetry breaking can be performed by VEV's of **120**, with the option of adding **105** representations. The **120** decomposes under $SU(12)_q \times SU(3)_l$ as follows:

$$\mathbf{120}=(\mathbf{78},\mathbf{1})+(\mathbf{12},\mathbf{3})+(\mathbf{1},\mathbf{6}) \ . \tag{29}$$

Let us first see how the $(\mathbf{78},\mathbf{1})$ submultiplet decomposes under $SU(6)_L \times SU(6)_R \times SU(3)_l \times U(1)_B$:

$$(\mathbf{78},\mathbf{1})=[6,\overline{6},1]_0+[21,1,1]_{2/3}+[1,\overline{21},1]_{-2/3} \ , \tag{30}$$

where we have put the unnormalized baryon numbers which correspond to $B=\frac{1}{3}$ for a quark. VEV's in the last two submultiplets can break B. However, a sextet of $SU(6)_L$ or $SU(6)_R$ contains only color triplets. The **21** of SU(6), being the symmetric combination of two sextets, contains nothing but the **6** and $\overline{\mathbf{3}}$ representations of $SU(3)_c$. Thus, any VEV here would break $SU(3)_c$. The second submultiplet in Eq. (29) contains only color triplets and antitriplets, as is obvious from the fact that in Eq. (1), the first twelve components constitute a **12** representation of $SU(12)_q$. Thus the only color singlets in the decomposition of Eq. (29) are in the $(\mathbf{1},\mathbf{6})$ submultiplet. But any VEV in this part cannot break $U(1)_B$ since $U(1)_B$ is part of the $SU(12)_q$ subgroup.

One can similarly argue that even the **105** multiplet cannot break $U(1)_B$. Thus, FL correctly argued[1] that in their scheme of symmetry breaking, baryon number remains unbroken. However, this is definitely a problem for the model since $U(1)_B$ remains as an unbroken *gauge* symmetry. One must therefore introduce other Higgs multiplets to break symmetry. In that case, one will obtain baryon-number-violating processes.

It is, of course, possible in principle that some part of a gauge symmetry is broken, but the fermions do not know about it.[7] If such is the case of $U(1)_B$ in this model, then proton decay is still prohibited in absence of scalars lighter than the proton. However, in order for this to happen, all Higgs bosons that the fermions couple to must carry zero baryon number. It is not easy to meet this constraint. Indeed, in similar models based on the

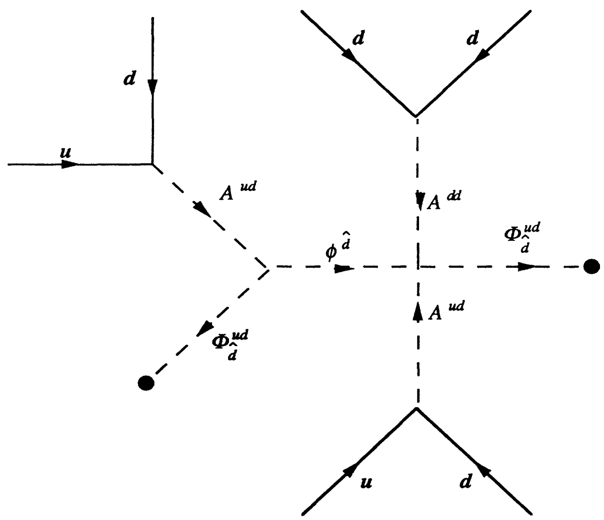

FIG. 4. Tree-level diagram for n-$\bar{n}$ oscillation generated when $\Phi^{ud}_{\hat{d}}$ develops a nonzero VEV. Here ϕ denotes the fundamental representation, and A denotes rank-2 tensorial representations.

group SU(16), baryon-number-violating processes such as proton decay and neutron-antineutron oscillation are induced by gauge symmetry breaking.[3,4]

Rather than going into a detailed discussion of this topic, let us just give two examples of Higgs representations whose VEV can break $U(1)_B$. We already argued that the representations with one or two SU(15) indices do not work. One possibility is to use the $\Phi^{[ijk]}$ mentioned earlier. In the (220,1) submultiplet of Eq. (28), there is a singlet of the standard-model gauge group which carries one unit of baryon number. A VEV of this component can break $U(1)_B$. Alternatively, we can introduce the multiplet $\Phi_k^{[ij]}$, whose VEV of the component $\Phi^{ud}_{\hat{d}}$ can break $U(1)_B$ without breaking the standard-model gauge group. In both instances, the symmetry breaking introduces n-$\bar{n}$ oscillations, for example. In the case of the $\Phi_k^{[ij]}$ multiplet, this occurs through the diagram of Fig. 4. A similar diagram for the other case can be trivially constructed. Taking the naive expectation that the Higgs-boson masses as well as the trilinear coupling constant are all of order M_G, we obtain the effective amplitude to be of order M_A^2/M_G^7, which should be less than $\sim 10^{-26}$ GeV^{-5} in order to meet the experimental bounds on the process.[8] This is satisfied by all values of the scales allowed by renormalization-group calculations. However, if the Higgs-boson masses are lower than M_G or if they introduce proton decay, the bounds can be much more stringent.

V. CONCLUSIONS

We have analyzed the grand unification model based on the group SU(15). The model is interesting because the RG analysis admits of very low unification scale, as pointed out by FL.[1] Figure 1 of our paper suggests that the unification scale M_G can be as low as 6×10^6 GeV as suggested by FL,[1] and at any rate is lower than 2×10^{11} GeV. This is considerable lower than the unification scales in more popular unification models such as those based on SO(10) or E_6. This is definitely an exciting possibility that can arise in this model.

However, the Higgs-boson sector of the original model has to be modified to make the model viable. We have shown that the Higgs-boson representations suggested[1] by FL are not sufficient to break the gauge group to $SU(3)_c\times U(1)_Q$. One needs Higgs multiplets different from the ones introduced[1] by FL in order to have a realistic symmetry-breaking pattern. On the one hand, introduction of these multiplets might substantially modify the RG equations. On the other hand, the symmetry-breaking mechanism introduces baryon-number-violating processes. We emphasized that this is necessary since baryon number is part of the gauge group. The precise nature of these processes will be studied elsewhere.

Note added in proof. U. Sarkar, A. Mann, and T. G. Steele [Report No. PRL-TH-90-18, 1990 (unpublished)] have independently discussed similar issues about baryon-number violation in the model.

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[1] P. H. Frampton and B-H. Lee, Phys. Rev. Lett. **64**, 619 (1990).

[2] J. C. Pati and A. Salam, Phys. Rev. D **8**, 1240 (1973).

[3] J. C. Pati, A. Salam, and J. Strathdee, Nuovo Cimento **26A**, 77 (1975); Nucl. Phys. **B185**, 445 (1981).

[4] R. N. Mohapatra and M. Popović, Phys. Rev. D **25**, 3012 (1982); A. Raychaudhuri and U. Sarkar, *ibid.* **26**, 3212 (1982).

[5] Equivalently, one can use the same normalization for all generators but adjust the coupling constants for different subgroups. This was the approach taken in Ref. 1.

[6] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1984).

[7] Let us give an example of this phenomenon. In electroweak models based on the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, if one uses only the scalar multiplets $\Phi(2,2,0)$, $\chi_R(1,2,1)$, and $\chi_L(2,1,1)$, one can break the gauge symmetry down to $U(1)_Q$ as desired. However, all processes involving only the fermions conserve $B-L$. For a detailed description of this model, see, e.g., G. Senjanović, Nucl. Phys. **B153**, 334 (1979).

[8] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).