Limits on particles of small electric charge

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We consider experimental limits on particles of small (not necessarily rational) electric charge. Such particles are possible within the standard model, and may be a natural consequence of extensions of the standard model incorporating an extra U(1) gauge group associated with a "mirror universe" sector. For both these cases we examine the limits from low-energy quantum electrodynamics corrections, direct accelerator searches, stellar astrophysics, constraints on big-bang nucleosynthesis, and relic cosmological densities. The combined results exclude significant regions of the charge-mass parameter plane.

I. INTRODUCTION

The question of whether the electric charges of elementary particles are quantized as integral multiples of some standard unit of electric charge is long standing and of fundamental interest. To the present limit of experimental accuracy the charges of the elementary particles are consistent with being integral multiples of a standard unit whose value is (minus) one-third the charge of the electron. For example, the upper limit on the electron-proton charge difference is $(0.8\pm 0.8)\times 10^{-21}e$.[1]

This apparent quantization of electric charge has been a mystery from the point of view of the quantum theory of electromagnetism since its inception in the early part of this century. The gauge group for electrodynamics is an Abelian phase invariance. Because the Abelian theory has no nontrivial commutation relations between its generators [indeed U(1) has only generator] there is no algebraic quantization of the charge eigenvalues. It was the absence of an algebraic explanation of charge quantization that led Dirac to explore possible topological explanations in his seminal work[2] on magnetic monopoles. However, with the continuing absence of experimental evidence for monopoles, there is presently no explanation for charge quantization in the quantum theory of electrodynamics.

This situation is not improved by the embedding of electrodynamics within the standard model of the electroweak interactions based on the $SU(2)_L\times U(1)_Y$ gauge group. In this theory, we still have an Abelian factor in the gauge group, the $U(1)_Y$ of weak hypercharge. The electrical charges of elementary particles are now related to $SU(2)_L$ and $U(1)_Y$ eigenvalues

$$Q_{\rm em}=T_{3_L}+\frac{Y}{2} \qquad (1.1)$$

and, in the absence of quantization of the weak hypercharge Y, there is no quantization of electric charge. Indeed it is easy to introduce, into the standard model, particles of arbitrary charge and mass. One need only add a Dirac fermion that is an $SU(2)_L$ singlet and of the desired weak hypercharge. Such a Dirac fermion has a gauge-invariant-mass term, and will contribute to neither local nor global anomalies due to the vectorlike nature of its coupling. So such new fermions of arbitrary mass and electric charge may be introduced into the standard model (and scalars of arbitrary mass and charge likewise).

While some extensions of the standard model [superstrings,[3] grand unified theories[4] (GUT's), . . .] provide mechanisms for enforcing charge quantization, other possible extensions suggest the existence of particles of small, unquantized charge. Holdom[5] has recently emphasized that such a possibility exists in theories with a "mirror" sector, where the mirror symmetry is slightly broken. In such theories, one introduces a mirror sector of particles and interactions resembling our own; in particular one arranges that at low energies there is a new unbroken $U(1)'$ in the mirror sector and matter charged under this $U(1)'$.

We do not observe two U(1) interactions in our low-energy world, so clearly ordinary particles do not carry the charge of the second $U(1)'$. However, if, at some arbitrarily large mass scale, there are particles carrying both charges (call them Ψ and Ψ'), then the gauge boson of the first U(1) ("photon") can turn into a virtual pair of these particles and then into the gauge boson of the second $U(1)'$ ("paraphoton"). This gives an effective interaction between the two gauge bosons, and therefore between particles carrying only the charge of the U(1) (call these ψ) and particles carrying only the charge of the $U(1)'$ (paratons $\equiv f$). This can be described by giving the f a small effective U(1) charge. Intuitively, virtual pairs of Ψ and Ψ' formed around a ψ or f give it a small interaction with the gauge boson of the other U(1). It turns out that one can always arrange to describe the ψ-f interaction by only giving one particle a charge:[5]

$$\delta=\frac{e'^2}{6\pi^2}\ln\left[\frac{M}{M'}\right], \qquad (1.2)$$

where M and M' are the masses of the Ψ and Ψ'. If $M, M' \geq$ a few hundred GeV (which is likely), the standard-model U(1) that the para-U(1) mixes with will be hypercharge, so the fractional electric charge of the paraton will be

$$\epsilon = \frac{\delta}{2} = \frac{\alpha'}{3\pi} \ln \left[\frac{M}{M'} \right] . \quad (1.3)$$

In this scenario the particles that appear to us to have a small induced electric charge do so because of a small rotation in the charge space of their paracharge. They will have interactions through the paraphotons of essentially normal electromagnetic strength. The paraphoton interaction will affect the physics and astrophysics of these particles, giving a somewhat different picture of constraints on their existence.

In this paper we consider the experimental limits on particles of mass μ, and of small (not necessarily rational) charge ϵ, both in the minimal case of standard-model electromagnetism, and when there is a second unbroken U(1) paraphoton present. In Sec. II we examine the constraints arising from direct searches at accelerators. In Sec. III we note the limits inherent in the experimental validity of standard quantum electrodynamics, getting a constraint from the agreement of theory and experiment for the Lamb shift. Section IV discusses constraints arising from cosmology, both from the validity of big-bang nucleosynthesis and the requirement that relic paratons not overclose the Universe. Section V discusses astrophysical limits coming from the standard understanding of red giants and white dwarfs. In Sec. VI we present our conclusions; more details may be found in Ref. 6.

As this manuscript was in preparation we received a paper by Dobroliubov and Ignatiev[7] which examines constraints on particles of small charge in the case of models without extra U(1) paraphotons; in the case where their results overlap those presented here, they are in agreement.

II. ACCELERATORS

The most obvious place to look for particles with large fractional charge ($\epsilon > 10^{-2}$) is in high-energy experiments. Such particles could have been seen in free quark searches, at the anomalous single photon (ASP) detector at the SLAC storage ring PEP and in beam-dump experiments.

There have been a number of accelerator searches for the production of free quark pairs.[8-12] The most stringent limits come from PEP,[11] which has $\mu > 14$ GeV for $0.2 \leq \epsilon \leq 0.8$ and from KEK TRISTAN,[10] whose limit is $\mu > 26$ GeV for $\epsilon = \frac{2}{3}$.

One could hope that the width of the Z from CERN LEP would give constraints on heavier paratons than the free quark searches. However, since the paraton only carries hypercharge, its coupling to the Z is suppressed by $\epsilon \tan\theta_W$. The decay rate is therefore (neglecting the paraton mass)

$$\Gamma(Z \to f\bar{f}) = \frac{\tan^2\theta_W \epsilon^2 \alpha M_Z}{3} = \epsilon^2 \times 6.79 \times 10^{-2} \text{ GeV} \quad (2.1)$$

TABLE I. Values of the paraton mass μ excluded by the ASP experiment.

Mass (GeV)	Charge
$\mu < 1$	$\epsilon = 0.08$
$\mu < 5$	$\epsilon = 0.08$
$\mu < 10$	$\epsilon = 0.09$
$\mu < 13$	$\epsilon = 0.20$

and the width from the ALEPH Collaboration is $\Gamma_{\text{expt}} = 2.68 \pm 0.15$ GeV.[13] Subtracting the theoretical width ($\Gamma_{\text{theo}} = 2.487 \pm 0.027$ GeV with three neutrinos[13]) allows $\Delta\Gamma = 0.307$ GeV for other particles. This rules out $\epsilon > 2.3$, which is useless.

The ASP detector[14,15] was designed to look for events of the form $e^+e^- \to \gamma +$ weakly interacting particles at PEP. The unseen particles presumably include three flavors of neutrino and any other light " undetectables" one cares to believe in. After subtracting the neutrinos, the limit on the cross section for the production of other weakly interacting particles (assumed in this case to be paratons) is $\sigma_f \leq 0.049$ pb. Using Bonneau and Martin's approximation to the cross section for $e^+e^- \to \gamma +$ other things,[16] which neglects Z exchange and photon emission by the paratons, one gets the results of Table I.

Beam-dump experiments have often been used to constrain the parameters of weakly interacting particles (see Ref. 17 for a list of some of these limits). In the case of paratons, this calculation was done by Golowich and Robinett.[17] Their limit calculated from vector-meson decay and direct Drell-Yan production is approximately $\mu > 10\epsilon$ GeV for $0.1 > \epsilon > 0.02$. (See Ref. 17 for an accurate graph.)

Trident processes[18] provide one of the best accelerator experiment limits on low-mass particles with small electric charges. Paraton-antiparaton pairs can be trident produced by an electron in a beam dump, and the paratons can be detected if they trident produce an electron-positron pair in a detector. (Trident processes are more important than real photoproduction or bremmstrahlung because the presence of electrons allows processes that are first order in the paraton charge.) An electron beam-dump experiment[19] at SLAC has searched for new neutrinolike particles. Using trident production cross sections based on Ref. 18, we estimate that this experiment conservatively excludes paratons with the charges and masses of Table II.

TABLE II. Values of the paraton mass μ excluded by the SLAC electron beam-dump experiment.

Mass (GeV)	Charge
$\mu < 0.2$	$\epsilon = 0.0003$
$\mu < 1$	$\epsilon = 0.0006$
$\mu < 2$	$\epsilon = 0.001$
$\mu < 10$	$\epsilon = 0.003$
$\mu < 100$	$\epsilon = 0.01$
$\mu < 10^3$	$\epsilon = 0.03$

III. LAMB SHIFT

Another way of setting experimental limits on paratons is to require that their contribution to $g-2$ and the Lamb shift not disrupt the present agreement between theory and experiment. The Lamb shift gives an interesting constraint, but as noted by Golowich and Robinett[17] the $g-2$ limit is within the region ruled out by accelerator experiments.

The Lamb shift separates the $2P_{1/2}$ and $2S$ states of the hydrogen atom by approximately 1063 MHz, and is due to QED corrections to the vertex and photon propagator. To lowest order, the paratons only contribute to vacuum polarization, and, in the low-momentum-transfer limit, introduce a correction to the photon propagator of[20]

$$\frac{ig_{\mu\nu}}{\mathbf{q}^2}\frac{2\alpha\epsilon^2}{\pi}\frac{\mathbf{q}^2}{30\mu^2} . \tag{3.1}$$

This is just $(\epsilon^2 m_e^2)/\mu^2 \times$(the electron term), so the paraton contribution to the Lamb shift is $\Delta E_f = \epsilon^2 m_e^2/\mu^2 \times 27.13$ MHz where 27.13 MHz is the vacuum-polarization contribution from electrons. The maximum difference between calculations and measurements of the Lamb shift is approximately 0.09 MHz,[20] so this implies

$$\mu > 9\times 10^{-3}\epsilon \text{ GeV} \tag{3.2}$$

which differs from Ref. 7 by a factor of 2. This is only applicable if $\mu^2 \gg \mathbf{q}^2$ because it was derived on the assumption that $\ln(1+x)=x$ (where $x \propto \mathbf{q}^2/\mu^2$). Bethe *et al.*[22] have calculated that the average momentum transfer between a $2S$-state electron and the proton is 226 eV, so this limit probably applies for $\mu > 1$ keV, or $\epsilon > 10^{-4}$.

IV. COSMOLOGY

Cosmological arguments provide two interesting constraints on the parameters of the neutrino:[23] a limit on the number of light neutrino flavors, and a limit on the mass. Both of these can be transposed into constraints on the paraton.

Big-bang nucleosynthesis calculations predict the observed abundances of the light elements (^{4}He, ^{3}He, D, Li) in the Universe today. These calculations depend, among other things, on the number of light neutrino flavors (or equivalently, on the energy density) at weak-interaction freeze-out ($T\sim 1$ MeV). One can therefore get an upper bound on N_ν by requiring that the predicted abundances not disagree with observation. If the paratons were relativistic at $T\sim 1$ MeV, they would count as "too many neutrinos," which gives the first cosmological limit on paratons.

The second constraint on neutrinos and paratons comes from requiring that the relic density in the Universe today not exceed the critical density ρ_c that would make the Universe flat.

Both these limits are derived on the assumption that the paratons are in thermal equilibrium with the rest of the matter in the Universe. For sufficiently small ϵ, this will clearly not be the case, so for each limit there will be some value of ϵ below which the paratons conceivably could exist.

A. Limits on paratons from nucleosynthesis

If there were less than 4.6 Majorana neutrinos at nucleosynthesis,[24] the predicted abundances of light elements are compatible with observation. This allows 1.6 extra two-component neutrinos, or their equivalent in other relativistic particles at $T\sim 1$ MeV.

In the model without a paraphoton, this rules out paratons with $\mu < 1$ MeV (the paraton is a Dirac fermion and so counts as two neutrinos), providing they are in thermal equilibrium with the photons at nucleosynthesis. The implications of needing to be in equilibrium will be discussed later. (A strong bound, 0.4 extra Majorana neutrinos at nucleosynthesis, has recently been calculated,[25] so our calculation based on not more than 1.6 extra neutrinos is clearly conservative.)

In the model with a paraphoton, the mass limit is considerably more stringent. The difference here is that the paraphotons contribute $\frac{8}{7}$ of a neutrino to the energy density if they are at the same temperature as the photons. Moreover, the paratons will annihilate principally to paraphotons, which raises the paraphoton temperature with respect to that of the photons. This means that a number of ordinary particle species will need to annihilate into the photon gas between paraton annihilation and nucleosynthesis. If $g_{\text{eff}}(p,T)$ is the effective number of degrees of freedom that particle p contributes to the energy density at a photon temperature T, then[23]

$$g_{\text{eff}}(\gamma',T_a)=2\left[\frac{T_{\gamma'}}{T_a}\right]^4 , \tag{4.1}$$

where T_a is the photon temperature just after the paratons annihilate and $T_{\gamma'}$ (the paraphoton temperature) is determined from entropy conservation to be

$$T_{\gamma'}=\left[\frac{2+\frac{7\times 4}{8}}{2}\right]^{1/3} T_a . \tag{4.2}$$

This gives $g_{\text{eff}}(\gamma',T_a)=7.7 \gg g_{\text{eff}}(1.6\nu, 1\text{ MeV})=2.8$.

If at nucleosynthesis

$$g_{\text{eff}}(\gamma',T_n)=2\left[\frac{T_{\gamma'}}{T_n}\right]^4 \le 2.8 \tag{4.3}$$

then the photon temperature must increase with respect to that of the paraphotons by $[g_{\text{eff}}(\gamma',T_a)/g_{\text{eff}}(\gamma',T_n)]^{1/4}$. But by entropy conservation this is just

$$\left[\frac{7.7}{2.8}\right]^{1/4} = \left[\frac{n_{\text{eff}}(T_a)}{n_{\text{eff}}(T_n)}\right]^{1/3} , \tag{4.4}$$

where $n_{\text{eff}}(T_a)$ $[n_{\text{eff}}(T_n)]$ is the effective number of degrees of freedom of the gas of particles in equilibrium with the photons just after the paratons annihilate [at nucleosynthesis]. $n_{\text{eff}}(T_n)=2+\frac{7}{2}+3\times\frac{7}{4}$ (for photons, electrons, and neutrinos) so $n_{\text{eff}}(T_a)=\frac{93}{4}$ which implies

$T_a > T_c$, where T_c is the quark-hadron transition temperature, 200 MeV $< T_c <$ 400 MeV. So nucleosynthesis gives

$$\mu > 1 \text{ MeV} \quad (\text{without } \gamma') , \tag{4.5}$$

$$\mu > 200 \text{ MeV} \quad (\text{with } \gamma') . \tag{4.6}$$

If only 0.4 extra neutrinos are allowed at nucleosynthesis, the lower bound on μ (with γ') increases to the charmed-quark mass, or $\mu > 1.6$ GeV.

This lower bound on μ applies if the paratons are in equilibrium with the electrons and photons at $T\sim$a few MeV. However, paraton cross sections decrease as the temperature rises ($\sigma \sim 1/T^2$), so the paraton interaction rate [see (4.8) and (4.10)] increases more slowly with temperature than the expansion rate of the Universe [$H\sim T^2$; see (4.18)]. This suggests that paratons will be in equilibrium at low temperatures but not at high ones. One can calculate a value of ϵ below which the paratons will not be in equilibrium at nucleosynthesis by assuming that the paratons come into thermal equilibrium when $\Gamma = H$, and that this must happen before T=5 MeV for the mass limit to apply.

At $T\sim$a few MeV, the only charged particle available for the paratons to interact with is the electron (collisions with photons are unimportant), so the interaction rate is

$$\Gamma \sim n_e \sigma(fe \to fe)\beta + \frac{n_f}{4}\sigma(f\bar{f} \to e^- e^+)\beta , \tag{4.7}$$

where n_p is the number density for particles p and their antiparticles. Assuming that the electrons and paratons are relativistic (so $n \sim T^3$), the interaction rate for annihilations is[23]

$$\Gamma_{\text{ann}} \sim \frac{\alpha^2 \epsilon^2}{2} T . \tag{4.8}$$

$\sigma(fe \to fe)$ is infrared divergent, so must be dealt with more carefully. A particle is defined to be in equilibrium if $\Gamma \gg H$ because this means that it can change its energy as fast as the Universe is cooling. However, in this argument, it is assumed that the energy exchanged in an interaction is of order the energy of the particles, and it is precisely because this is not true that the scattering cross section diverges. For a fixed momentum transfer $|\Delta \mathbf{p}|$, it will take $\sim |\Delta \mathbf{p}|^2/|\mathbf{p}|^2$ interactions (squared because it is random process) to change the momentum by an order of magnitude. One therefore needs

$$\int \frac{|\Delta \mathbf{p}|^2}{|\mathbf{p}|^2} \frac{d\Gamma_{\text{scat}}}{d\Omega} d\Omega \gg H \tag{4.9}$$

to keep the particle in equilibrium. Taking the lower bound on $|\Delta \mathbf{p}|$ to be eT ($\sim$effective mass of a photon in a thermal bath) gives a scattering interaction rate of

$$\Gamma_{\text{scat}} = 2.6 \epsilon^2 \alpha^2 T . \tag{4.10}$$

An upper bound on can therefore be calculated by setting $H = \Gamma_{\text{ann}} + \Gamma_{\text{scat}}$ at T=5 MeV, which gives

$$\epsilon < 4 \times 10^{-9} . \tag{4.11}$$

B. Limits from $\Omega < 1$

The second cosmological limit on paratons comes from requiring that they not overclose the Universe, i.e., that $\Omega(\equiv \rho_f/\rho_c$, where ρ_f is the paraton energy density) be less than 1. In the hot very early Universe, the paratons are assumed to be in thermal equilibrium with ordinary particles for $T > \mu$ (this assumption will be discussed later). As the temperature drops to $\le \mu$, they become nonrelativistic and their equilibrium number density will drop very fast. Numerical calculations for heavy neutrinos[26] and charged particles[27] show that when $T\sim$(the particle mass)/20, the expansion of the Universe has sufficiently diluted the particle number density that they freeze-out of equilibrium and the number per comoving volume stays approximately constant until today. Presumably paratons behave in the same way.

The more strongly interacting paratons are, the more efficiently they will find each other to annihilate, so the relic density today will be lower. Nonrelativistic paraton annihilation cross sections go as $\sim \epsilon^2/\mu^2$, so they become more "weakly interacting" as ϵ gets smaller or as μ gets larger. Requiring $\rho_f < \rho_c$ therefore gives a lower bound on ϵ and an upper bound on μ. The same argument for neutrinos gives a lower bound on m_ν because (for $E_\nu < M_W$) the cross sections go as $\sim E_\nu^2/M_W^4$.

Using Lee and Weinberg's[26] analytic approximation to the relic density today, the present mass density of paratons should be

$$\rho_f = \frac{3.2 \times 10^{-39}}{\sigma\beta\sqrt{g_{\text{eff}}(T_f)}} \frac{1}{x_f^2(1+1/x_f)} \text{ g/cm}^3 , \tag{4.12}$$

where $\sigma\beta$ is in GeV^{-2} and $x_f = T_f/\mu$ is the freeze-out temperature in units of the paraton mass:

$$x_f = \left[41 + \ln \left[\frac{\mu\sigma\beta}{\sqrt{g_{\text{eff}}}} \right] \right]^{-1} , \tag{4.13}$$

where $\mu\sigma\beta$ is in GeV^{-1}. Paratons will annihilate to two paraphotons in the model where these exist; otherwise they will annihilate to charged particle pairs. The nonrelativistic cross section for $f\bar{f} \to \gamma'\gamma'$ is[28]

$$\sigma = \frac{\pi \alpha'^2}{\beta \mu^2} \tag{4.14}$$

so requiring that the paraton density today$=\rho_f$ be less than $\rho_c = 1.88 \times 10^{-29} h^2$ g/cm^3 [$0.4 < h < 1$ comes from the uncertainty in the Hubble parameter today: $H = 100h$ (km/sec)/Mpc] gives a rough limit $\mu < 10^5 \alpha'$ GeV, in agreement with Ref. 29. Assuming $\alpha' < \frac{1}{10}$, this is

$$\mu < 10^4 \text{ GeV} \quad (\text{with } \gamma') . \tag{4.15}$$

If there is no paraphoton, the paratons do not interact as strongly, which gives a higher relic density today. The upper bound on μ from $\rho_f < \rho_c$ should therefore be lower than (4.15). A paraton with intrinsic fractional hypercharge should annihilate to a pair of charged particles, providing $\mu > m_e$, and since nucleosynthesis rules out $\mu < 1$ MeV, this is a reasonable assumption. The cross section for a pair of nonrelativistic particles to annihilate to a pair of relativistic ones is[27]

$$\frac{\pi\epsilon^2\alpha^2}{\mu^2\beta}N(T_a)\ , \tag{4.16}$$

where $N(T_a)$ is the effective number of relativistic charged-particle species present when the paratons annihilate. Requiring $\rho_f<\rho_c$ therefore gives a rough upper bound of

$$\mu<2\times10^3\epsilon\ \text{GeV}\ . \tag{4.17}$$

Using the correct values for x_f and g_{eff} charges this by less than a factor of 2.

The limits (4.15) and (4.17) assume that the paratons are in thermal equilibrium with ordinary matter at $T>\mu$, so will only apply if $\Gamma>H$, which gives [see (4.8) and (4.10)]

$$3N(T)\epsilon^2\alpha^2T>\frac{1.66}{m_{\text{PL}}}\sqrt{g_{\text{eff}}}T^2\ . \tag{4.18}$$

At temperatures satisfying (4.18) paratons with $\mu\ll T$ will be in equilibrium. For $\mu\gg T$, the paratons are presumably not in equilibrium when they annihilate, so there is a tentative upper bound to the masses ruled out by the $\rho<\rho_c$ limit; i.e., paratons may be allowed if they are sufficiently heavy. The dividing line is $\mu\sim T$, which gives a lower bound on the paraton mass of

$$\mu>\frac{\epsilon^2N(T)}{\sqrt{g_{\text{eff}}(T)}}1.2\times10^{15}\ \text{GeV} \tag{4.19}$$

or roughly

$$\mu>10^{15}\epsilon^2\ \text{GeV}\ . \tag{4.20}$$

The limits (4.20) and (4.17) are very similar to those calculated by Dobroliubov and Ignatiev.

V. ASTROPHYSICS

Unless ϵ is very small, a light paraton would remove energy from red-giant cores more rapidly than it is produced by nuclear burning, and would cool white dwarfs faster than is observed. The standard "undetectable particle" limit from red giants (Ref. 30 and references in 31) can therefore be transposed into a limit on the paraton, and one can require that the time scale on which paratons cool a hot white dwarf be comparable to that of photon cooling.

A. Red giants

The principal method of paraton production in stars will be plasmon decay: a photon in a plasma (plasmon) acquires an effective "mass" from its constant interactions with the electron gas, so can decay without violating gauge invariance or conservation of energy and momentum.[32] The decay rate of a massive vector boson into a paraton pair is

$$\Gamma=\frac{\epsilon^2\alpha m_\gamma}{3}\frac{\beta(3-\beta^2)}{2}\ , \tag{5.1}$$

where β is the velocity of the outgoing paraton. This is very large; essentially a plasmon will decay if it can. This implies $\mu>m_\gamma/2$ down to very small values of ϵ, provided that the paratons escape from the star. (If the charge is large enough that they are trapped inside the star, this limit no longer applies.)

The plasmon mass m_γ depends on the energy and number density of electrons and positrons in a complicated way.[32] However, in the limit of a nonrelativistic electron gas, $T\ll m_e$,[33]

$$m_\gamma=\left[\frac{4\pi\alpha n_e}{m_e}\right]^{1/2}=2\ \text{keV}\ , \tag{5.2}$$

where the numerical value has been calculated for $\rho=10^4$ g/cm^3. [The red giant is very naively assumed to have a homogeneous helium core of density $\rho=10^4$ g/cm^3, temperature $T=10^8$ K (8.6 keV), mass $0.5M_\odot$, radius $r=3\times10^9$ cm, and energy generation rate $R_n=10^6$ ergs/(cm^3 sec); this follows Ref. 31.] This gives a plasmon lifetime in red giants of $\tau=\epsilon^{-2}\times10^{-16}$ sec, which implies

$$\mu>1\ \text{keV} \tag{5.3}$$

(unless ϵ is small).

If ϵ is very small, the plasmons will decay very slowly and the star will not lose to much energy to paratons. The upper bound on ϵ is estimated by requiring that the rate of energy loss per unit volume to paratons,

$$\frac{d^2E}{dV\,dt}=\int_0^\infty\frac{\omega}{(2\pi)^3}\frac{3\,d^3k}{e^{\omega/T}-1}\frac{m_\gamma}{\omega}\Gamma$$
$$\simeq\epsilon^2\times1.6\times10^{34}\ \text{ergs/(cm}^3\text{sec)}\ , \tag{5.4}$$

(where $\omega=\sqrt{m_\gamma^2+\mathbf{k}^2}$ is the plasmon energy, $\mathbf{k}$ is its momentum and m_γ/ω is the time dilation factor in the decay rate) be less than the nuclear energy generation rate per unit volume. This implies that

$$\epsilon<10^{-14} \tag{5.5}$$

is allowed. (This is an order of magnitude smaller than the limit calculated by Bernstein, Ruderman and Feinberg[30] and so is perhaps too optimistic.)

The lower bound on the mass assumes that the paratons escape the star when they are produced, which will not be the case for large ϵ. It should therefore be cut off when this no longer happens. (The paraphotons are trapped if the paratons are, so do not provide a mechanism for the paraton energy to escape.) This is a conservative upper bound on ϵ, but since the region of μ-ϵ space in question is ruled out by nucleosynthesis, it is not important.

The principal interactions that should interfere with a paraton's escape from the star are scattering off electrons and helium nuclei (assuming that all the protons are in helium). The paraton mean free path is therefore

$$\bar{l}_f=\frac{1}{n_e\sigma_e+n_{\text{He}}\sigma_{\text{He}}}\ , \tag{5.6}$$

where σ_e is the cross section for paraton-electron scattering, and $\sigma_{\rm He}$ is the cross section for paraton-helium scattering. These are calculated using a screened Coulomb potential, which gives

$$\bar{l}_f \approx \frac{1}{\epsilon^2\mu^2} \times 3\times 10^{-19}\ \text{cm} \tag{5.7}$$

with μ in GeV.

If the paratons are assumed to be trapped if the core radius is greater than 10 mean free paths, then the lower bound on ϵ is

$$\begin{aligned} &\epsilon \geq 10^{-8} \quad \text{for } \mu = 1\ \text{keV}\ , \\ &\epsilon \geq 10^{-5} \quad \text{for } \mu = 1\ \text{eV}\ . \end{aligned} \tag{5.8}$$

B. White dwarfs

At first sight it is strange that a dying star should give useful limits on paratons. However, the "mass" of a plasmon in a degenerate electron gas is

$$m_\gamma = \left[\frac{4\pi\alpha n_e}{\varepsilon_F}\right]^{1/2}, \tag{5.9}$$

where ε_F is the Fermi energy (including rest mass) and n_e is the electron number density. In a pure carbon white dwarf with M=one solar mass and a radius R of 5000 km, one has $m_\gamma \simeq 40$ keV, which could increase the lower bound on the paraton mass to $\mu > 20$ keV: the present theory of white dwarf cooling seems to agree with the observed luminosity distribution of the white-dwarf population,[34] so white dwarfs cannot be losing substantial amounts of energy to paratons.

Assuming that the plasmons can be treated as massive bosons, their number density will be

$$n_\gamma = 3e^{-m_\gamma/T}\left[\frac{m_\gamma T}{2\pi}\right]^{3/2}. \tag{5.10}$$

Since m_γ is greater than and independent of the temperature, the plasmon number density will be exponentially suppressed as the star cools. Any limits on paratons from plasmon decay must therefore come from the early stages of white-dwarf cooling.

The rate at which the star loses energy to paratons can be approximated as

$$L_{f\bar{f}} \simeq n_\gamma \Gamma m_\gamma V \tag{5.11}$$

because the star is assumed to be a sphere of uniform density and temperature. Setting this equal to the rate of energy loss of the ions gives[34]

$$\tau = \frac{1}{\alpha\epsilon^2}\left[\frac{\pi T_f}{2}\right]^{1/2} \frac{9M}{2m_{\rm C} m_\gamma^{9/2} R^3} e^{m_\gamma/T_f}\ , \tag{5.12}$$

where τ is the time it takes to cool to T_f and $m_{\rm C}$ is the mass of a carbon atom. Using $T_f = 1.5$ keV (which corresponds to $L_f = 10^{-2} L_\odot$), this gives $\tau \simeq \epsilon^{-2} \times 5 \times 10^{-5}$ sec. The observations require $\tau \simeq 3.4\times 10^{15}$ sec,[34] which implies that the paraton mass must be greater than $m_\gamma/2$, or that $\epsilon > 10^{-10}$.

In this calculation, the cooling time increases exponentially with the plasmon mass, so m_γ has been taken as large as possible. This gives the weakest limit on ϵ, but perhaps makes the mass limit too high. It is safer to use a smaller plasmon mass for the limit on μ, which means the white dwarf limit is

$$\text{for } \epsilon > 10^{-10}\ \ \mu > 10\ \text{keV}\ . \tag{5.13}$$

This calculation has been done more carefully by Dobroliubov and Ignatiev,[7] who get $\mu > 25$ keV for $\epsilon > 10^{-13}$.

As in the case of red giants, paratons with large ϵ will be trapped in the star. They are unlikely to interact with the electrons, because these are degenerate, but will scatter off the carbon ions. Using a screened Coulomb potential with a screening length of[33]

$$a = \left[\frac{\varepsilon_F}{24\pi n_e \alpha}\right]^{1/2} \tag{5.14}$$

gives

$$\bar{l}_f \simeq \frac{3.4\times 10^{-14}}{\epsilon^2\mu^2}\ \text{cm}\ . \tag{5.15}$$

Assuming again that the paratons are trapped if their mean free path is less than 1/10 of the white dwarf radius, one has that paratons are allowed for

$$\begin{aligned} &\mu = 10\ \text{keV}, \quad \epsilon > 10^{-6}\ , \\ &\mu = 1\ \text{eV}, \quad \epsilon > 10^{-2}\ . \end{aligned} \tag{5.16}$$

Dobroliubov and Ignatiev[7] improved on the astrophysical limits set here using white dwarfs and energy transport in red giants. They rule out our red giant region (but up to 25 MeV) using white dwarfs, and then use the red giants to constrain paratons under our lower bound for nucleosynthesis.

C. Dark matter

Our Galaxy is assumed to be surrounded by a halo of nonluminous matter[35] with a local density of approximately 0.3 GeV/cm^3. If the halo is composed of (nonbaryonic) dark-matter particles, then there are experimental bounds on the mass and cross section on nuclei of these particles from their nonobservation in dark-matter detection experiments.[36,37] These constraints have been calculated for heavy neutrinos and similar particles that are weakly interacting because they exchange a massive gauge boson.

Since electrically charged particles are not "dark," and interact via a long-range force, these limits must be transposed to paratons with some care. The dark-matter constraints will apply if the paraton could not have cooled out of the halo within the age of the Universe, i.e.,[38] if

$$\epsilon^2 < 7\times 10^{-11}\frac{\mu}{n_e}\ , \tag{5.17}$$

where μ is in GeV, and n_e is the free electron number density in cm^{-3}. (If all the electrons are free, $n_e \sim 1$.) For large values of ϵ, the paraton density in the solar system should be at least $(\Omega_f/\Omega_b)\times$(baryon density of the

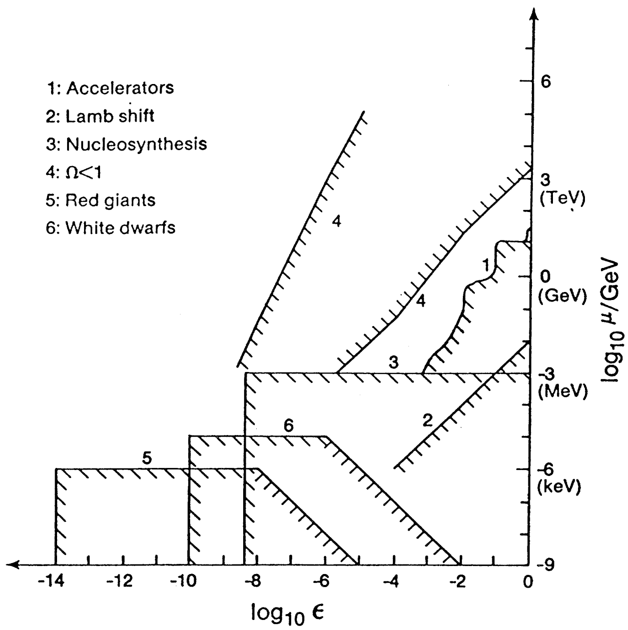

FIG. 1. Region of the mass-charge (μ-ϵ) plane ruled out for the model without a paraphoton.

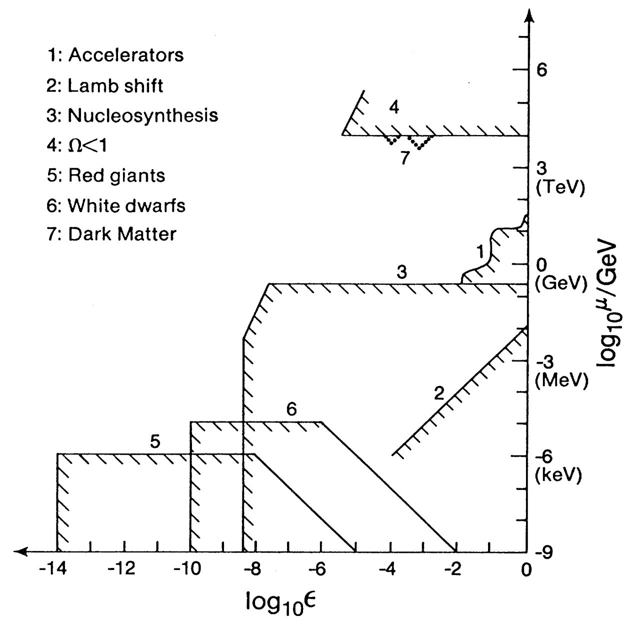

FIG. 2. Region of the mass-charge (μ-ϵ) plane ruled out for the model with a paraphoton.

local interstellar medium), which would not be detectable. Ω_b (Ω_f) is the baryon (paraton) relic density divided by the critical density ρ_c.

ϵ must also be small enough that the paratons reach the detectors, but large enough to produce a detectable signal. Since charged particles lose energy scattering off electrons (long-range force) they travel considerably less far in matter than ordinary weakly interacting particles with the same cross section on nuclei. Using Holdom's approximation for the range of a paraton,[29] and assuming that the paraton density is $\Omega_f \times 0.3$ GeV/cm^3, gives the "teeth" in Fig. 2. (These are very rough estimates of what the present dark-matter detectors could rule out.) The small ϵ tooth is the underground germanium detector of Caldwell *et al.*,[36] and the other is the balloon experiment of Rich, Rocchio, and Spiro;[37] in the second case, the lower bound on ϵ comes from (5.17) These dark-matter direct detection experiments are clearly interesting to pursue as they access a region of parameter space not excluded by other considerations.

VI. CONCLUSION

One can get useful constraints on paratons from accelerator experiments, stellar evolution, and cosmological arguments. The limits for the model without paraphotons are plotted in Fig. 1; those for the model with paraphotons are in Fig. 2.

It is unfortunate that these calculations leave a central region where paratons are allowed, but there do not seem to be any simple arguments to rule this area out. Searches for anomalous nuclei, proton decay and galactic γ rays provide an upper limit on the number density of paratons with parameters in the central region, but one must be able to calculate "gold-plated" lower bounds to use these experiments to rule paratons out. For a given mass and charge, the relic abundance today can be calculated from (4.12), but this number density is averaged over the whole Universe. The local density will depend on how the paratons get through galaxy formation and the magnetic field and atmosphere of the Earth, which makes it very difficult to calculate.

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