SU(3)-symmetry breaking and configuration mixing in baryon semileptonic decays

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(Received 17 September 1990)

We discuss small contributions to baryon semileptonic decays which provide corrections to the simplest SU(3)-symmetric formulas. In particular, we consider (1) SU(3)-symmetry breaking resulting from the wave-function deformations due to the *s*-*u*,*d* quark mass difference and (2) the effects of the configuration mixing of ^2S_M and ^2S_S states, originating from QCD hyperfine interactions. Both effects are treated in combination with the form-factor suppression effect which is operative if the three-momentum transfer between the initial and final hadron is nonzero. Our estimates are performed in the framework of the harmonic-oscillator quark model. While the inclusion of the *s*-*u*,*d* quark mass difference leads to an unacceptable g_A/g_V ratio for $\Lambda \to pe^-\nu$ semileptonic decay, the corrections due to configuration mixing do not lead to disagreement with the data. Better measurements of the $\Xi \to ne^-\nu$ and $\Xi \to \Lambda e^-\nu$ decays are needed if a choice is to be made between the standard SU(3)-symmetric parametrization and the pattern of SU(3) breaking resulting from configuration mixing.

I. INTRODUCTION

One of the early successes of the quark model was its ability to describe baryon magnetic moments *better* than the SU(3) parametrization.[1] Since precisely such an SU(3) parametrization is still used for the description of baryon semileptonic decays, its validity might and should be questioned.[2] Although there appears to be no experimental evidence for departures from the exact SU(3)-symmetric description as yet, the question of the relevance of the SU(3) parametrization has become even more important recently. Indeed, at present a lot of attention is being paid to the question of the origin of proton spin,[3] a problem that is closely connected with the questions of the applicability and experimental determination of F and D,[4] the standard two parameters of the SU(3) approach to baryon semileptonic decays. Thus, it is interesting to have some idea as to what level of accuracy the determination of F and D parameters might be considered meaningful and what should be the most important corrections to the naive SU(3) picture.

In this paper we try to address these questions within the framework of the potential quark model. We discuss here two kinds of previously neglected quark-model effects which invalidate the SU(3) parametrization. One of these effects consists of SU(3)-symmetry breaking resulting from the wave-function deformations due to the m_s-m_n ($m_n = m_{u,d}$) quark mass difference; the other is configuration mixing. Both are combined with the form-factor suppression effect (and isospin mixing).[5] We also estimate nonstatic corrections to the predictions of the nonrelativistic quark model. To this end, in the case of wave-function deformation we use Dirac quark spinors as originally proposed in Ref. 6, while in the case of configuration mixing we neglect the wave-function deformation effect and estimate all nonstatic corrections simply by rescaling our predictions so that $(g_A/g_V)_{np}$ agrees with experiment.[5] Although the existence of all effects under consideration should be quite model independent, the precise values of SU(3)-breaking corrections must depend on the (unknown) details of baryon wave functions. Thus, this paper should be regarded as an attempt to find the direction and a rough estimate of the size of such corrections only. It should also be noted that, at present, no model can satisfactorily explain the peculiar nonadditive SU(3)-breaking pattern observed in baryon magnetic moments.[7] Thus, there is no reason to expect our calculations of related semileptonic decays to work to a better than ~10% accuracy level.

II. WAVE-FUNCTION DEFORMATION

The existence of the SU(3)-breaking quark mass terms results in the deformation of baryon wave functions. In excited strange baryons this deformation leads to an effect of "kinematic mixing" first proposed by Isgur and Karl[8] as an explanation of the Σ-Λ splitting in the $J^P = \frac{5}{2}^-$ sector. Because of the presence of quarks of unequal mass, the two oscillators, present in the harmonic-oscillator quark model of baryons, have different frequencies. For the ground-state baryons the relevant wave function is

$$\Psi_{00} = \left[\frac{\alpha_\rho \alpha_\lambda}{\pi} \right]^{3/2} \exp[-\tfrac{1}{2}(\alpha_\rho^2 \boldsymbol{\rho}^2 + \alpha_\lambda^2 \boldsymbol{\lambda}^2)] , \tag{1}$$

where

$$\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}, \quad \boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6} \tag{1a}$$

are the two relative coordinates

$$\alpha_\rho^2 = \sqrt{3Km_\rho}, \quad \alpha_\lambda^2 = \sqrt{3Km_\lambda} \tag{1b}$$

and for equal-mass quarks 1,2 we have

$$m_\rho = m_1 = m_2 = m \; , \quad m_\lambda = 3mm_3/(2m + m_3) \; . \tag{1c}$$

For the $S=0$ sector we then set $m=m_3=m_n$, for the $S=-1$ sector $m=m_n$, $m_3=m_s$ and for $S=-2$ sector $m=m_s$, $m_3=m_n$.

Baryon semileptonic decays are characterized by vector and axial-vector couplings g_V and g_A. It is straightforward to calculate in the quark model the factors which modify standard SU(6) values of these couplings. Such factors result from a possible mismatch between the wave functions of the initial and final baryons (which may differ in the number of strange quarks) as well as the presence of a factor of $\exp(-i\mathbf{q}\cdot\mathbf{r})$ with $\mathbf{q}$ being the three-momentum transfer from the electron neutrino to the hadron system (nonvanishing on account of the difference in masses of the two baryons) and $\mathbf{r}$ being the position of the quark undergoing the weak transition.[5] As a result both the vector and axial-vector couplings are modified by a $\mathbf{q}^2$-dependent form factor which suppresses the two couplings by a factor of order of $\exp[-\mathbf{q}^2/(6\alpha^2)]$ (with $\alpha \equiv \alpha_p$ for $m=m_n$). The values of the two coupling constants, as used nowadays, are extracted from the data as the values of the corresponding form factors at four-momentum transfer equal zero ($q^2=0$). This corresponds to a generally nonzero three-momentum transfer

$$\mathbf{q}^2=(m_f-m_i)^2[1+(m_f-m_i)/(2m_i)]^2 , \tag{2}$$

where m_i (m_f) is the mass of the initial (final) baryon.

Thus, to compare models with experimental data we should evaluate our theoretical formulas at $\mathbf{q}^2$ given in Eq. (2). Since $\mathbf{q}^2$ depends on the masses of initial and final baryons we obtain an SU(3)-breaking multiplicative correction to standard formulas. As seen from Eq. (2) this SU(3) breaking enters g_V in second order only[9] and thus its effect is not big. Still, it is bigger than it was estimated in the bag model by Donoghue and Holstein,[10] who evaluated the wave-function-mismatch suppression factor but neglected the (more important) form-factor effect altogether. Apart from the form-factor correction, the axial-vector couplings are additionally suppressed due to the contribution from the "small" components of quark Dirac spinors. Within the framework of the harmonic-oscillator quark-model corrections of this type were originally considered [in the SU(3)-symmetry case] by Le Yaouanc, Oliver, Pène, and Raynal.[6] Since the expressions for small components of Dirac spinors involve the mass of the quark in question, in reality these (relativistic) corrections are SU(3) breaking as well. Within the harmonic-oscillator model of baryons with Dirac quark spinors (as in Ref. 6) it is straightforward to find the expressions for the vector and axial-vector couplings between the octet members of the $(56,0^+)$ multiplet of SU(6)×O(3). Assuming that the SU(3)-breaking parameter $\delta \equiv (m_s-m_n)/m_n$ is small we obtain the following formulas:

$n \to p$

$$g_V=g_V^{SU(6)}\exp[-\mathbf{q}^2/(6\alpha^2)] ,$$
$$g_A/g_V=(g_A/g_V)^{SU(6)}[1-\alpha^2/(3m_n^2)] \quad (\mathbf{q}^2\simeq 0) ; \tag{3a}$$

$\Sigma^- \to n, \Lambda \to p$

$$g_V=g_V^{SU(6)}\exp[-(1-\delta/6)\mathbf{q}^2/(6\alpha^2)] ,$$
$$g_A/g_V=(g_A/g_V)^{SU(6)}[1-(1-5\delta/6)\alpha^2/(3m_n^2)] \quad (\mathbf{q}^2\neq 0) ; \tag{3b}$$

$\Xi^- \to \Xi^0$

$$g_V=g_V^{SU(6)}\exp[-(1-\delta/6)\mathbf{q}^2/(6\alpha^2)] ,$$
$$g_A/g_V=(g_A/g_V)^{SU(6)}[1-(1+\delta/6)\alpha^2/(3m_n^2)] \quad (\mathbf{q}^2\simeq 0) ; \tag{3c}$$

$\Xi^- \to \Lambda, \Xi^0 \to \Sigma^+, \Xi^- \to \Sigma^0$

$$g_V=g_V^{SU(6)}\exp[1-(1-\delta/4)\mathbf{q}^2/(6\alpha^2)] ,$$
$$g_A/g_V=(g_A/g_V)^{SU(6)}[1-(1-3\delta/4)\alpha^2/(3m_n^2)] \quad (\mathbf{q}^2\neq 0) . \tag{3d}$$

In the above formulas the Λ-Σ^0 mixing is still unaccounted for. The physical states Λ_p, Σ_p^0 are obtained from Λ, Σ^0 by

$$\begin{aligned} |\Lambda_p\rangle &= \cos\theta|\Lambda\rangle+\sin\theta|\Sigma^0\rangle , \\ |\Sigma_p^0\rangle &= -\sin\theta|\Lambda\rangle+\cos\theta|\Sigma^0\rangle . \end{aligned} \tag{4}$$

Estimates of the mixing angle θ give $\sin\theta \simeq 0.02$,[5] which is the value we use in our numerical evaluation below. We fix the value of the parameter $\alpha^2/(3m_n^2)$ by requiring the $(g_A/g_V)_{n\to p}$ ratio of Eq. (3a) to fit the experimental value of 1.258. For $m_n=330$ MeV (and $m_s \simeq 500$ MeV) this amounts to using $\alpha=0.29$ GeV, a reasonable value corresponding to $R_{\text{bag}} \simeq 0.8$ fm. This value of α is only 10% smaller than the one used by Isgur and Karl in their description of ground-state baryons.[11]

With the help of (3) and (4) we can now estimate the combined effects of quark motion and Σ^0-Λ mixing in the presence of an SU(3)-breaking mass term. Comparison of our numerical results with experiment is given in Table I.

From Table I we see that the $(g_A/g_V)_{\Lambda\to p}$ ratio is not

TABLE I. Ratio of axial-vector-to-vector couplings (g_A/g_V) for semileptonic decays. The underlined entry is a fit to experiment.

Process	SU(6)	Model of Sec. II	Experiment
$n \to pe^-\nu$	1.67	<u>1.25</u>	1.25
$\Sigma^- \to ne^-\nu$	−0.33	−0.28	∓(0.36∓0.04)
$\Lambda \to pe^-\nu$	1.00	0.84	0.696∓0.025
$\Xi^- \to \Xi^0 e^-\nu$	−0.33	−0.24	
$\Xi^- \to \Lambda e^-\nu$	0.33	0.29	0.25∓0.05
$\Xi^0 \to \Sigma^+ e^-\nu$	1.67	1.38	

modified by effects under consideration in a satisfactory way. The quark-motion-induced suppression of this ratio is significantly smaller than in the $n \to pe^- \nu$ case. Indeed, for (heavier) strange quarks, the relativistic corrections must be smaller as can be also seen from Eq. (3b) for $\delta \geq 0$. Thus, the mass-term-induced SU(3)-breaking effect works here in the wrong direction. Below we shall therefore turn our attention to the second of the discussed effects.

III. CONFIGURATION MIXING

It has been argued[12] that many violations of SU(6) selection rules have their origin in configuration mixing in the ground-state baryon wave functions. This configuration mixing is a consequence of color hyperfine interactions. In the framework of the harmonic-oscillator quark model it was originally discussed by Isgur and Karl.[11,13] The same authors discussed later its effect on baryon magnetic moments.[14] In this section we shall be concerned with the effects of configuration mixing on vector and axial-vector couplings of ground-state baryons.

We consider mixing between N, Λ, Σ, Ξ states of the $(8_S;56,0^+)$, $(8_{S'};56',0^+)$ and $(1_M, 8_M, 10_M; 70,0^+)$ multiplets of the symmetric quark model. We neglect D-wave impurities, which are also induced by color hyperfine interactions, because calculations indicate they are very small.[13] We shall not give here any details of our calculations since they proceed exactly along the lines of Ref. 11. All our sign conventions are in agreement with Refs. 11 and 13. We obtain the following decompositions of ground-state baryons:

$$\begin{aligned}
|N\rangle &= \cos\phi_N^8(\cos\theta_N|N_S\rangle + \sin\theta_N|N_{S'}\rangle) + \sin\phi_N^8|N_M\rangle \ , \\
|\Lambda\rangle &= \cos\phi_\Lambda^1[\cos\phi_\Lambda^8(\cos\theta_\Lambda|\Lambda_S\rangle + \sin\theta_\Lambda|\Lambda_{S'}\rangle) + \sin\phi_\Lambda^8|\Lambda_M^8\rangle] + \sin\phi_\Lambda^1|\Lambda_M^1\rangle \ , \\
|\Sigma\rangle &= \cos\phi_\Sigma^{10}[\cos\phi_\Sigma^8(\cos\theta_\Sigma|\Sigma_S\rangle + \sin\theta_\Sigma|\Sigma_{S'}\rangle) + \sin\phi_\Sigma^8|\Sigma_M^8\rangle] + \sin\phi_\Sigma^{10}|\Sigma_M^{10}\rangle \ , \\
|\Xi\rangle &= \cos\phi_\Xi^{10}[\cos\phi_\Xi^8(\cos\theta_\Xi|\Xi_S\rangle + \sin\theta_\Xi|\Xi_{S'}\rangle) + \sin\phi_\Xi^8|\Xi_M^8\rangle] + \sin\phi_\Xi^{10}|\Xi_M^{10}\rangle \ ,
\end{aligned} \tag{5}$$

where the mixing angles are computed to be

	$\sin\theta$	$\sin\phi^8$	$\sin\phi^1$	$\sin\phi^{10}$
N	-0.25	-0.22		
Λ	-0.24	-0.16	0.05	
Σ	-0.19	-0.12		0.0
Ξ	-0.19	-0.13		-0.02

(6)

We have performed our estimates independently of those of Ref. 14. Our angles are systematically smaller by $\simeq 20\%$ from those given there but the overall patterns of mixing found in Ref. 14 and in this paper are in full agreement. The actual values of mixing angles must depend on several decisions concerning the treatment of results of Ref. 13, i.e., the way these results are tailored to our needs (omission of D-wave impurities, etc.) and the choice of the strength of hyperfine interactions (see also Ref. 11). Thus, we consider the agreement of our estimates and those of Ref. 11 satisfactory. The discrepancies show the extent to which numerical results of such calculations may be trusted.

Calculation of the values of vector and axial-vector couplings between the states of Eq. (5) is briefly sketched in the Appendix. The presence of the factor $\exp(-i\mathbf{q}\cdot\mathbf{r})$ with (in general) nonzero three-momentum transfer $\mathbf{q}$ results in nonvanishing of the (interference) cross terms between B_S, $B_{S'}$, and B_M. Because of the smallness of $\sin\phi^{10}$ for Σ and Ξ ground states we have neglected the contribution of these admixtures in our numerical results. On the other hand, we kept the contribution from the admixture of $(1_M,70,0^+)$ to the ground-state Λ particle. Using then the compositions of Eqs. (5) and (6) and carrying out the calculations sketched in the Appendix we obtain the results gathered in Table II.

TABLE II. Configuration-mixing-induced factors modifying the SU(6) values of g_A and g_V. $[g_{A,V}=(g_{A,V})^{SU(6)}\times$(modifying factor).]

Process	SU(6) g_V	SU(6) g_A	Modifying factors g_V	Modifying factors g_A
$n\to pe^-\nu$	1	5/3	1.0	0.96
$\Sigma^-\to\Lambda e^-\nu$	0	$\sqrt{2/3}$	$-0.08B_{\Sigma\Lambda}$[a]	$0.95A_{\Sigma\Lambda}+0.17B_{\Sigma\Lambda}$
$\Sigma^-\to ne^-\nu$	-1	1/3	$0.99A_{\Sigma n}+0.29B_{\Sigma n}$	$0.95A_{\Sigma n}+0.32B_{\Sigma n}$
$\Lambda\to pe^-\nu$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$0.997A_{\Lambda p}+0.08B_{\Lambda p}$	$0.94A_{\Lambda p}+0.15B_{\Lambda p}$
$\Xi^-\to\Lambda e^-\nu$	$-\sqrt{3/2}$	$-1/\sqrt{6}$	$0.987A_{\Xi\Lambda}+0.22B_{\Xi\Lambda}$	$0.987A_{\Xi\Lambda}+0.075B_{\Xi\Lambda}$
$\Xi^-\to\Sigma^0e^-\nu$	$-1/\sqrt{2}$	$-5/(3\sqrt{2})$	$A_{\Xi\Sigma}+0.023B_{\Xi\Sigma}$	$0.987A_{\Xi\Sigma}+0.125B_{\Xi\Sigma}$

[a]Term to be added to $g_V^{SU(6)}=0$.

TABLE III. Ratios of axial-vector-to-vector couplings g_A/g_V modified by configuration mixing. The underlined entry constitutes a fit to the experimental number.

Process	$(g_A/g_V)^{SU(6)}$	g_A/g_V modified	+ relativistic effects	Experiment	SU(3) param.
$n \to pe^-\nu$	5/3	1.60	<u>1.258</u>	1.258 ∓ 0.004	$F+D$
$\Sigma^- \to ne^-\nu$	$-1/3$	-0.32	-0.25	$\mp(0.36 \mp 0.04)$	$F-D$
$\Lambda \to pe^-\nu$	1	0.93	0.73	0.696 ∓ 0.025	$F+\frac{D}{3}$
$\Xi^- \to \Lambda e^-\nu$	1/3	0.35	0.28	0.25 ∓ 0.05	$F-\frac{D}{3}$
$\Xi^- \to \Sigma^0 e^-\nu$	5/3	1.70	1.34		$F+D$

In Table II we introduced

$$A_x \equiv \exp[-\mathbf{q}_x^2/(6\alpha^2)] ,$$
$$B_x \equiv \mathbf{q}_x^2/(6\alpha^2)\exp[-\mathbf{q}_x^2/(6\alpha^2)] , \quad (7)$$

where $\mathbf{q}_x$ is the momentum corresponding to the process x in question. No Σ^0-Λ mixing was yet considered in the evaluation of entries of Table II. Estimating the effect of Σ^0-Λ mixing and using $\alpha \simeq 0.29$ GeV, the predictions for (g_A/g_V) modified by configuration mixing are easily obtainable from Table II. They are collected in column 3 of Table III.

We have included relativistic effects on all static results by simply multiplying entries of column 3 with the ratio 1.258/1.60 appropriate to neutron β decay (see Ref. 5). These corrected ratios are given in column 4. It can be seen from Table III that $(g_A/g_V)_{\Lambda\to p}$ is quite acceptable here. The overall agreement with experiment is reasonable. At the same time attempts to describe numbers given in column 4 with the standard F and D SU(3) parameters (as in the last column) fail: obviously configuration mixing does not yield to an SU(3)-invariant description. The values of F and D parameters extracted from columns 4 and 6 of Table III are meaningful only to about 10–15% accuracy level. This strengthens our doubts on the unconcerned use of the SU(3) parametrization and highlights the importance of an accurate experimental determination of the axial-vector-to-vector ratios in semileptonic decays.

ACKNOWLEDGMENTS

I would like to thank Professor G. Karl for discussions and financial assistance during my stay in Guelph.

APPENDIX

The wave functions of the states of the symmetric quark model are taken from Appendix B of Ref. 13. Thus, e.g.,

$$|8_S\rangle \equiv |8;56,0^+\rangle = \frac{1}{\sqrt{2}}(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)\psi_{00} ,$$
$$|8_{S'}\rangle \equiv |8;56',0^+\rangle = \frac{1}{\sqrt{2}}(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)\psi'_{00} , \quad (A1)$$
$$|8_M\rangle \equiv |8;70,0^+\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\chi^\rho\phi^\rho - \chi^\lambda\phi^\lambda)\psi^\lambda_{00} + \frac{1}{\sqrt{2}}(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)\psi^\rho_{00}\right] ,$$

where χ^λ and χ^ρ are spin-wave functions, respectively, symmetric and antisymmetric under the interchange of the first two quarks, $\phi^\lambda(\phi^\rho)$ are the flavor wave functions of similar symmetry properties (for more details, see Appendix B of Ref. 13), ψ_{00} is given in Eq. (1) and ψ^λ_{00}, ψ^ρ_{00}, and ψ'_{00} are spatial wave functions for $(N=2)$ excited states:

$$\psi^\lambda_{00} = \frac{1}{\sqrt{3}}\frac{\alpha^5}{\pi^{3/2}}(\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2)\exp[-\alpha^2(\boldsymbol{\rho}^2+\boldsymbol{\lambda}^2)/2] ,$$
$$\psi^\rho_{00} = \frac{2}{\sqrt{3}}\frac{\alpha^5}{\pi^{3/2}}\boldsymbol{\rho}\cdot\boldsymbol{\lambda}\exp[-\alpha^2(\boldsymbol{\rho}^2+\boldsymbol{\lambda}^2)/2] , \quad (A2)$$
$$\psi'_{00} = \frac{1}{\sqrt{3}}\frac{\alpha^5}{\pi^{3/2}}(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2 - 3\alpha^{-2})\exp[-\alpha^2(\boldsymbol{\rho}^2+\boldsymbol{\lambda}^2)/2] .$$

[In our estimates of the effects due to configuration mixing, we neglect SU(3) breaking in spatial wave functions and thus we use a single α parameter.]

Calculation of the values of vector and axial-vector couplings between the states of Eq. (5) requires the computation of the spin-flavor and spatial matrix elements separately. Because of the symmetry of the wave functions it suffices to consider contributions from the third quark only. The evaluation of the spin-flavor part is straightforward: we evaluate the matrix elements of $\sigma_3\lambda_{d\to u(s\to u)}$ $(\sigma_0\lambda_{d\to u(s\to u)})$ acting on the third quark in between the spin-flavor wave functions as in (A1). The spatial wave functions ψ^λ_{00}, ψ^ρ_{00}, ψ_{00}, and ψ'_{00} are orthogonal. Thus, if the factor $\exp(-i\mathbf{q}\cdot\mathbf{r}_3)$ were equal to 1 (as it happens for $\mathbf{q}=0$) the interference terms between (say) Σ^8_M and N_S would vanish. In general, however, because of SU(3) breaking exhibited by Eq. (2), the value of three-momentum $\mathbf{q}$ is significantly different from zero. Consequently, the interference terms do appear. Calculating the spatial parts of the matrix elements we obtain the diagonal overlaps,

$$I_{00} \equiv \int \psi^*_{00}\psi_{00}\exp(-i\mathbf{q}\cdot\mathbf{r}_3) = A ,$$
$$I_{0'0'} \equiv \int \psi'^*_{00}\psi'_{00}\exp(-i\mathbf{q}\cdot\mathbf{r}_3) = A - \tfrac{2}{3}B ,$$
$$I_{\rho\rho} \equiv \int \psi^{\rho*}_{00}\psi^\rho_{00}\exp(-i\mathbf{q}\cdot\mathbf{r}_3) = A - \tfrac{2}{3}B ,$$
$$I_{\lambda\lambda} \equiv \int \psi^{\lambda*}_{00}\psi^\lambda_{00}\exp(-i\mathbf{q}\cdot\mathbf{r}_3) = A - \tfrac{2}{3}B ,$$

and the off-diagonal overlaps , (A3)

$$I_{00'} \equiv \int \psi_{00}^{*} \psi_{00}' \exp(-i\mathbf{q}\cdot\mathbf{r}_3) = -\frac{1}{\sqrt{3}} B ,$$

$$I_{0\lambda} \equiv \int \psi_{00}^{*} \psi_{00}^{\lambda} \exp(-i\mathbf{q}\cdot\mathbf{r}_3) = \frac{1}{\sqrt{3}} B ,$$

$$I_{0'\lambda} \equiv \int \psi_{00}'^{*} \psi_{00}^{\lambda} \exp(-i\mathbf{q}\cdot\mathbf{r}_3) = \tfrac{2}{3} B ,$$

$$I_{0\rho} = I_{0'\rho} = I_{\rho\lambda} = 0 ,$$

where

$$A \equiv \exp[-\mathbf{q}^2/(6\alpha^2)] ,$$

$$B \equiv \mathbf{q}^2/(6\alpha^2) \exp[-\mathbf{q}^2/(6\alpha^2)]$$

and we have neglected terms of order $[\mathbf{q}^2/(6\alpha^2)]^2 \exp[-\mathbf{q}^2/(6\alpha^2)]$ since $\mathbf{q}^2/(6\alpha^2)$ is of order 0.05 at most.

Using the compositions of Eq. (5) and combining spin-flavor matrix elements with the spatial overlaps given above, one obtains results presented in Table II.

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[1]H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. **154**, 1608 (1967); J. Franklin, *ibid.* **172**, 1807 (1968).

[2]J. F. Donoghue, E. Golowich, and B. Holstein, Phys. Rep. **131**, 319 (1986).

[3]European Muon Collaboration, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988).

[4]F. E. Close and R. G. Roberts, Phys. Rev. Lett. **60**, 1471 (1988); F. E. Close, *ibid.* **64**, 361 (1990); H. Lipkin, Phys. Lett. B **214**, 429 (1988); S. Brodsky, J. Ellis, and M. Karliner, *ibid.* **206**, 309 (1988).

[5]P. Bracken, A. Frenkel, and G. Karl, Phys. Rev. D **24**, 2984 (1981).

[6]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D **15**, 844 (1977).

[7]H. J. Lipkin, Phys. Rev. D **24**, 1437 (1981); Nucl. Phys. **B214**, 136 (1983).

[8]N. Isgur and G. Karl, Phys. Lett. **74B**, 353 (1978); Phys. Rev. D **18**, 4187 (1978).

[9]M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

[10]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **25**, 206 (1982); J. F. Donoghue, B. R. Holstein, and S. Klimt, *ibid.* **35**, 934 (1987).

[11]N. Isgur and G. Karl, Phys. Rev. D **20**, 1191 (1979).

[12]N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978).

[13]N. Isgur and G. Karl, Phys. Rev. D **19**, 2653 (1979).

[14]N. Isgur and G. Karl, Phys. Rev. D **21**, 3175 (1980).