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Relativistic effects in $\gamma\gamma$ decays of P-wave positronium and $q\bar{q}$ systems

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Theories of the two-photon couplings of P-wave positronium and $q\bar{q}$ systems are reviewed and a relationship is found between field-theoretic calculations of $q\bar{q}\rightarrow\gamma\gamma$ and the vector-meson-dominance model for both heavy- and light-quark systems. Relativistic corrections to the two-photon decay rates are evaluated and are found to be very important for the light-quark system. We find that the widely quoted ratio of 15/4 between the $\gamma\gamma$ widths of heavy-quark scalar and tensor mesons is significantly modified in the light-$q\bar{q}$ sector. We find however that the helicity-zero amplitude for the $\gamma\gamma$ coupling of tensor mesons, which vanishes in the nonrelativistic limit, remains small for light quarks, in accord with experimental observation.

I. INTRODUCTION

The two-photon couplings of $q\bar{q}$ systems can play a very important role in the interpretation of hadron spectroscopy. The cases of scalar and tensor resonances are of particular interest because the photons couple directly to the charged quarks in the resonances, so that experiments may provide us with clear signatures of their flavor content. Both improved data and a deeper theoretical understanding of $\gamma\gamma$ couplings could be crucial in establishing or refuting candidate gluonic mesons. In this paper we address some theoretical aspects of two-photon couplings and indicate some incomplete or inconsistent features of existing models.

We shall concentrate on two particular approaches in the quark model, first a field-theoretic calculation based on the $\gamma\gamma$ decays of P-wave positronium and second a vector-meson-dominance (VMD) model, and show how these superficially very different approaches can be related.

The field-theoretic calculation generalizes the work of Alekseev[1] and Tumanov,[2] who derived the $\gamma\gamma$ decay widths of positronium in the nonrelativistic limit of QED. In particular they have shown that the ratio of the $\gamma\gamma$ widths of the $J=0^{++}$ and $J=2^{++}$ $^3P_0$ and $^3P_2$ states is 15/4 in this limit, which is a widely quoted result for the $\gamma\gamma$ couplings of $q\bar{q}$ resonances. Insofar as the total hadronic widths of P-wave charmonium can be modeled by transitions to gg intermediate states, the $\Gamma_{\rm total}$ of 2^{++} and 0^{++} charmonium χ states should satisfy this relation as well.[3] The ratio of these hadronic widths is in fact consistent with this value, but the $\gamma\gamma$ couplings of charmonium P-wave levels are not yet accurately measured. Recent experimental work has motivated the suggestion[4] that such a relationship might not be well satisfied in the *light-quark* sector. If one extends these positronium calculations to $q\bar{q}$ resonances, relativistic effects should be incorporated; this is one motivation for the present paper.

In the VMD model,[5] a vector meson replaces one of the final photons and the $\gamma\gamma$ decay is then treated as a trivial modification of a radiative transition between two $q\bar{q}$ states, the initial P-wave $q\bar{q}$ meson and a final $q\bar{q}$ vector state. Because two real photons are identical particles, special care is required in applying the VMD model to $\gamma\gamma$ final states; this has not always been handled consistently in the literature.

We shall show how the nonrelativistic ratio of 15/4 between scalar and tensor $\gamma\gamma$ widths arises in field theory and how the transition operators in the VMD model can be derived from the positronium calculations of Alekseev and Tumanov. A clear connection between these two approaches is thereby established, and this connection will show that relativistic effects are especially important in the light-quark systems and how the nonrelativistic ratio of 15/4 is modified.

In the next section we shall investigate relativistic effects by extending the original nonrelativistic positronium calculation of Alekseev[1] to order $(v/c)^2$. Our calculation will give correct results to $O(v^2/c^2)$ for the decay rate of a pure $q\bar{q}$ state to $\gamma\gamma$. The physical mesons actually have $q\bar{q}g$ components with amplitude $O(\sqrt{\alpha_s})$ [$O(v/c)$ in the nonrelativistic limit], and these give additional $O(\alpha_s)$ [$O(v^2/c^2)$] corrections to the $\gamma\gamma$ decay rate. The effect of these higher Fock-space components is not considered in this paper. A numerical estimate of the sensitivity of the decay rates to relativistic effects is given in a harmonic-oscillator model. The two-photon coupling in the VMD model and its relation to the field-theoretic approach is discussed in Sec. III. In Sec. IV the helicity structure of the relativistic correction will be analyzed, and we shall show that the contribution of the helicity-zero amplitude in the tensor decay, which is zero in the nonrelativistic limit, remains small even for light quarks.

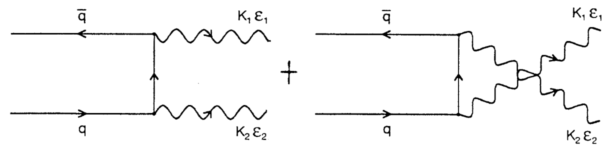

FIG. 1. Assumed two-photon coupling of a $q\bar{q}$ system.

II. FIELD-THEORETIC APPROACH

We assume that the two-photon decay amplitude is well described by the Feynman diagrams in Fig. 1. The transition matrix element can be written as

$$M_{fi}=\int \frac{d^3p}{\sqrt{(2\pi)^3}}\left[\frac{m_q^2}{E_{\bar{q}}E_q}\right]^{1/2}\bar{v}(\mathbf{p}_{\bar{q}},s_{\bar{q}})\left[(-ie_q\not{\epsilon}_2^*)\frac{i}{\not{p}_q-\not{k}_1-m_q}(-ie_q\not{\epsilon}_1^*)+(-ie_q\not{\epsilon}_1^*)\frac{i}{\not{p}_q-\not{k}_2-m_q}(-ie_q\not{\epsilon}_2^*)\right]u(\mathbf{p}_q,s_q)\psi(\mathbf{p}) \tag{2.1}$$

and the $\gamma\gamma$ partial width of the $q\bar{q}$ initial state is

$$\Gamma(q\bar{q}\to\gamma\gamma)=\int d\Omega_k \sum_{\lambda_1,\lambda_2}\frac{1}{64\pi^2}|M_{fi}|^2 . \tag{2.2}$$

Following the procedure of Alekseev, we rewrite (2.1) in terms of Pauli matrices and nonrelativistic Pauli spinors χ; the spinors in Eq. (2.1) can be written as

$$u(\mathbf{p}_q,s_q)=\left[\frac{E_q+m_q}{2m_q}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{p}_q}{E_q+m_q}\end{bmatrix}\chi \tag{2.3}$$

and

$$\bar{v}(\mathbf{p}_{\bar{q}},s_{\bar{q}})=\left[\frac{E_{\bar{q}}+m_q}{2m_q}\right]^{1/2}\chi^\dagger i\sigma_2\left[\frac{-\sigma\cdot\mathbf{p}_{\bar{q}}}{E_{\bar{q}}+m_q},1\right] . \tag{2.4}$$

In the $q\bar{q}$ rest frame we have $E_q=E_{\bar{q}}=(\mathbf{p}_q^2+m_q^2)^{1/2}$, $\mathbf{p}_q=-\mathbf{p}_{\bar{q}}$, $\mathbf{p}=\mathbf{p}_q-\mathbf{p}_{\bar{q}}$, $k_1^0=k_2^0$, and $\mathbf{k}_1=-\mathbf{k}_2$, so that (2.1) becomes

$$M_{fi}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q}\chi^\dagger\sigma_2\left[\frac{E_qk_1^0}{(E_qk_1^0)^2-(\mathbf{k}_1\cdot\mathbf{p})^2}F^S+\frac{\mathbf{k}_1\cdot\mathbf{p}}{(E_qk_1^0)^2-(\mathbf{k}_1\cdot\mathbf{p})^2}F^A\right]\chi\psi(\mathbf{p}) , \tag{2.5}$$

where

$$F^S=i\mathbf{k}_1\cdot(\epsilon_1^*\times\epsilon_2^*)-(\sigma\cdot\epsilon_1^*\epsilon_2^*\cdot\mathbf{p}+\sigma\cdot\epsilon_2^*\epsilon_1^*\cdot\mathbf{p})+\left[\frac{1}{m_q(E_q+m_q)}[2\sigma\cdot\mathbf{p}\epsilon_1^*\cdot\mathbf{p}\epsilon_2^*\cdot\mathbf{p}-\mathbf{p}^2(\sigma\cdot\epsilon_1^*\epsilon_2^*\cdot\mathbf{p}+\sigma\cdot\epsilon_2^*\epsilon_1^*\cdot\mathbf{p})]\right] \tag{2.6}$$

is symmetric under exchange of ϵ_1^* and ϵ_2^*, and

$$F^A=-\sigma\cdot\mathbf{k}_1\epsilon_1^*\cdot\epsilon_2^*+\left[\frac{1}{m_q(E_q+m_q)}\epsilon_1^*\cdot\epsilon_2^*(\sigma\cdot\mathbf{p}\,\mathbf{k}_1\cdot\mathbf{p}-\mathbf{p}^2\sigma\cdot\mathbf{k}_1)\right] \tag{2.7}$$

is antisymmetric under $\epsilon_1^*,\epsilon_2^*$ exchange. The terms in large parentheses in (2.6) and (2.7) were neglected in Alekseev's calculation; positronium is sufficiently nonrelativistic for this to be a good approximation. This is not necessarily the case in the $q\bar{q}$ system, however, especially for light quarks; the magnitude of the photon momentum $|\mathbf{k}_1|$ equals $M_{q\bar{q}}/2$, and E_q and $|\mathbf{k}_1|$ are comparable, so we should keep terms to higher order in (v/c). For simplicity we call the expression in large parentheses in (2.5) F; expanding the denominators in this expression to $O((v/c)^2)$, we find the result

$$F=-\left[1+\frac{(\mathbf{k}_1\cdot\mathbf{p})^2}{(E_q k_1^0)^2}\right]\left[\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1^*\boldsymbol{\epsilon}_2^*\cdot\mathbf{p}+\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2^*\boldsymbol{\epsilon}_1^*\cdot\mathbf{p}-i\mathbf{k}\cdot(\boldsymbol{\epsilon}_1^*\times\boldsymbol{\epsilon}_2^*)+\frac{1}{(k_1^0)^2}\boldsymbol{\sigma}\cdot\mathbf{k}_1\boldsymbol{\epsilon}_1^*\cdot\boldsymbol{\epsilon}_2^*\mathbf{k}_1\cdot\mathbf{p}\right]$$
$$+\frac{1}{m_q(E_q+m_q)}\left[2\boldsymbol{\sigma}\cdot\mathbf{p}\,\boldsymbol{\epsilon}_1^*\cdot\mathbf{p}\,\boldsymbol{\epsilon}_2^*\cdot\mathbf{p}-\mathbf{p}^2(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1^*\boldsymbol{\epsilon}_2^*\cdot\mathbf{p}+\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2^*\boldsymbol{\epsilon}_1^*\cdot\mathbf{p})+\frac{1}{(k_1^0)^2}\boldsymbol{\epsilon}_1^*\cdot\boldsymbol{\epsilon}_2^*(\boldsymbol{\sigma}\cdot\mathbf{p}\,\mathbf{k}_1\cdot\mathbf{p}-\mathbf{p}^2\boldsymbol{\sigma}\cdot\mathbf{k}_1)\right], \tag{2.8}$$

and the corresponding transition matrix element is

$$M_{fi}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^2k_1^0}\chi^\dagger\sigma_2F\chi\psi(\mathbf{p})\ . \tag{2.9}$$

Because σ_2 is an antisymmetric matrix the term $\mathbf{k}_1\cdot(\boldsymbol{\epsilon}_1^*\times\boldsymbol{\epsilon}_2^*)$ in (2.6) only contributes to the $\gamma\gamma$ decay of pseudoscalar states (which are $S=0$ and $L=$even), and the remaining terms containing $\boldsymbol{\sigma}$ contribute to the $\gamma\gamma$ decays of scalar and tensor states ($S=1$ and $L=$odd), as was shown by Alekseev. On summing over polarizations and integrating over the $d\Omega_k$ of the two outgoing photons, the total width to order $(v/c)^2$ for P-wave $q\bar{q}$ systems becomes

$$\Gamma=\frac{2\pi\alpha^2}{15(k_1^0)^2}\sum_{\mu,\nu}\{11B^{0*}_{\mu,-\mu}B^0_{\nu,-\nu}+6B^{0*}_{\mu,\nu}(B^0_{-\mu,-\nu}+B^0_{-\nu,-\mu})+2\,\mathrm{Re}[11B^{0*}_{\mu,-\mu}B^1_{\nu,-\nu}+6B^{0*}_{\mu,\nu}(B^1_{-\mu,-\nu}+B^1_{-\nu,-\mu})]$$
$$+\tfrac{2}{7}\mathrm{Re}[15B^{0*}_{\mu,-\mu}B^{21}_{\nu,-\nu}+10B^{0*}_{\mu,\nu}(B^{21}_{-\mu,-\nu}+B^{21}_{-\nu,-\mu})+16B^{0*}_{\mu,-\mu}B^{22}_{\nu,-\nu}-4B^{0*}_{\mu,\nu}(B^{22}_{-\mu,-\nu}+B^{22}_{-\nu,-\mu})]\}, \tag{2.10}$$

where

$$B^0_{\mu,\nu}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^2}\chi^\dagger\sigma_2\sigma_\mu p_\nu\chi\psi(\mathbf{p})\ , \tag{2.11}$$

$$B^1_{\mu,\nu}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{1}{E_q^2(E_q+m_q)}\chi^\dagger\sigma_2(\mathbf{p}^2\sigma_\mu p_\nu-\boldsymbol{\sigma}\cdot\mathbf{p}\,p_\mu p_\nu)\chi\psi(\mathbf{p})\ , \tag{2.12}$$

$$B^{21}_{\mu,\nu}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q\mathbf{p}^2}{E_q^4}\chi^\dagger\sigma_2\sigma_\mu p_\nu\chi\psi(\mathbf{p})\ , \tag{2.13}$$

$$B^{22}_{\mu,\nu}=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^4}\chi^\dagger\sigma_2\boldsymbol{\sigma}\cdot\mathbf{p}\,p_\mu p_\nu\chi\psi(\mathbf{p})\ , \tag{2.14}$$

and the matrices $\sigma_2\sigma_\mu$ ($\mu=1,0,-1$) are

$$\sigma_2\sigma_\mu=\left\{-i\sqrt{2}\begin{bmatrix}0&0\\0&1\end{bmatrix},i\begin{bmatrix}0&1\\1&0\end{bmatrix},i\sqrt{2}\begin{bmatrix}1&0\\0&0\end{bmatrix}\right\}. \tag{2.15}$$

The P-wave $q\bar{q}$ spatial wave function is of the form

$$\psi(\mathbf{p})=Y_{1m}(\hat{\boldsymbol{\Omega}}_{\mathrm{p}})\phi(p)\ , \tag{2.16}$$

where $\phi(p)$ is the radial wave function of the $q\bar{q}$ system, normalized to $\int_0^\infty p^2|\phi(p)|^2dp=1$. The total $q\bar{q}$ wave function for a ^3P_J $q\bar{q}$ initial state is given by

$$\Psi_{JM}=\sum_m\langle 1\,M-m,\,1\,m|J\,M\rangle\varphi_{1\,M-m}Y_{1m}(\hat{\boldsymbol{\Omega}}_{\mathrm{p}})\phi(p)\ . \tag{2.17}$$

The spin wave function $\varphi_{1\,M-m}$ in (2.17) is coupled through $\chi^\dagger$ and χ in Eqs. (2.11)–(2.14). We may then evaluate the tensors $B_{\mu,\nu}$ in (2.10) for a Ψ_{JM} state:

$$B^0_{\mu,\nu}|_{JM}=\sum_{m,\lambda}\langle 1\,m,1\,M-m|J\,M\rangle\langle\tfrac{1}{2}\lambda,\tfrac{1}{2}m-\lambda|1\,m\rangle\chi^\dagger_\lambda\sigma_2\sigma_\mu\chi_{m-\lambda}\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^2}p_\nu Y_{1\,M-m}\phi(p)\ . \tag{2.18}$$

In general it can be shown that

$$B_{\mu,\nu}|_{JM}=-i\langle 1\,-\nu,\,1\,-\mu|J\,M\rangle(-1)^{\delta_{\nu,1}+\delta_{\mu,1}}\sqrt{2}\|B\|_J\ , \tag{2.19}$$

where

$$\|B^0\|_J=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^2}p_\nu Y_{1-\nu}\phi(p)\ , \tag{2.20}$$

$$\|B^1_{\mu,\nu}\|_J=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{\mathbf{p}^2}{E_q^2(E_q+m_q)}p_\nu Y_{1-\nu}\phi(p)-\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{1}{E_q^2(E_q+m_q)}[p_\lambda Y_{1\gamma}]_J[p_\mu p_\nu]_J\phi(p)\ , \tag{2.21}$$

$$\|B^{21}\|_J=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q\mathbf{p}^2}{E_q^4}p_\nu Y_{1-\nu}\phi(p)\ , \tag{2.22}$$

and

$$\|B^{22}\|_J=\int\frac{d^3p}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^4}[p_\lambda Y_{1\gamma}]_J[p_\mu p_\nu]_J\phi(p) \tag{2.23}$$

are reduced matrix elements and are independent of μ and ν. The square brackets in (2.21) and (2.23) define a Clebsch-Gordan product of two vectors, $[p_\mu p_\nu]_{JM}\equiv\sum_{\mu,\nu}\langle 1\mu,1\nu|JM\rangle p_\mu p_\nu$. The result in (2.21) and (2.23) is independent of M so we have suppressed that label. Using these results, we find

$$\sum_\mu B_{\mu,-\mu}|_{JM}=-\sum_\mu\langle 1\mu 1-\mu|JM\rangle(-1)^\mu i\sqrt{2}\|B\|_J=-i\delta_{J,0}\delta_{M,0}\sqrt{6}\|B\|_J \tag{2.24}$$

and

$$\begin{aligned}\sum_{\mu,\nu}B^*_{\mu,\nu}|_{JM}(B_{-\mu,-\nu}|_{JM}+B_{-\nu,-\mu}|_{JM})&=\sum_{\mu,\nu}\langle 1\mu 1\nu|JM\rangle^2[1+(-1)^J]2\|B\|_J^2\\&=[1+(-1)^J]2\|B\|_J^2\ .\end{aligned} \tag{2.25}$$

On substituting (2.24) and (2.25) into (2.10), we find for the $\gamma\gamma$ width

$$\Gamma=\frac{4\pi\alpha^2}{15(k_1^0)^2}\times\begin{cases}45[\|B^0\|_J^2+2\|B^0\|_J(\|B^1\|_J+\frac{13}{63}\|B^{21}\|_J+\frac{8}{63}\|B^{22}\|_J)]\quad(J=0)\ ,\\12[\|B^0\|_J^2+2\|B^0\|_J(\|B^1\|_J+\frac{5}{21}\|B^{21}\|-\frac{2}{21}\|B^{22}\|)]\quad(J=2)\ .\end{cases} \tag{2.26}$$

This calculation also gives a zero decay width for the $J=1$ state, which follows from the transformation law (2.19) for the $\{B_{\mu,\nu}\}$. This of course follows from the proof by Landau[6] and Yang[7] that there is no $J=1$ state for two physical (transverse) photons.

The reduced matrix elements in (2.26) are explicitly

$$\|B^0\|_{J=0}=\|B^0\|_{J=2}=2\left[\frac{\pi}{3}\right]^{1/2}\int_0^\infty\frac{dp}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^2}p^3\phi(p)\ , \tag{2.27}$$

$$\|B^1\|=\begin{cases}0\quad(J=0)\ ,\\\frac{6}{5}\left[\frac{\pi}{3}\right]^{1/2}\int_0^\infty\frac{dp}{\sqrt{(2\pi)^3}}\frac{1}{E_p^2(E_q+m_q)}p^5\phi(p)\quad(J=2)\ ,\end{cases} \tag{2.28}$$

and

$$\|B^{21}\|_{J=0}=\|B^{21}\|_{J=2}=\|B^{22}\|_{J=0}=\tfrac{5}{2}\|B^{22}\|_{J=2}=2\left[\frac{\pi}{3}\right]^{1/2}\int_0^\infty\frac{dp}{\sqrt{(2\pi)^3}}\frac{m_q}{E_q^4}p^5\phi(p)\ . \tag{2.29}$$

so that the ratios of the widths with and without relativistic corrections [keeping only $O(v^2/c^2)$ corrections] are

$$\frac{\Gamma^{\rm rel}_{J=0}}{\Gamma^{\rm nr}_{J=0}}=1+\frac{2}{3}\frac{\int_0^\infty dp\frac{m_q}{E_q^4}p^5\phi(p)}{\int_0^\infty dp\frac{m_q}{E_q^2}p^3\phi(p)} \tag{2.30}$$

and

$$\frac{\Gamma^{\rm rel}_{J=2}}{\Gamma^{\rm nr}_{J=2}}=1+\frac{6}{5}\frac{\int_0^\infty dp\frac{1}{E_p^2(E_q+m_q)}p^5\phi(p)}{\int_0^\infty dp\frac{m_q}{E_q^2}p^3\phi(p)}+\frac{2}{5}\frac{\int_0^\infty dp\frac{m_q}{E_q^4}p^5\phi(p)}{\int_0^\infty dp\frac{m_q}{E_q^2}p^3\phi(p)}\ . \tag{2.31}$$

For completeness we note that our absolute nonrelativistic rate $\Gamma^{\rm nr}(2^{++}\to\gamma\gamma)$, which is given by

$$\Gamma^{\rm nr}(2^{++}\to\gamma\gamma)=\frac{8}{5\pi}\frac{\alpha^2}{k_0^2m_q^2}\left|\int_0^\infty dp\, p^3\phi(p)\frac{m_q^2}{E_q^2}\right|^2|\langle 2^{++}|e_q^2|0\rangle|^2 , \tag{2.32}$$

equals the result given by Godfrey and Isgur[8] in their Table VII(b), except for a factor of $(k_0/m_q)^2$, which is unity in the nonrelativistic limit. There is also a difference in the absolute rates due to their insertion of a $(M_R/\tilde{M}_R)^3$ "mock-meson" factor.

In a harmonic-oscillator model, $\phi(p)$ is

$$\phi(p)=Npe^{-p^2/2\beta_{\rm SHO}^2} , \tag{2.33}$$

where N is the normalization constant, and $\beta_{\rm SHO}$ is the momentum scale determined by the quark mass and the spring constant of the simple-harmonic-oscillator (SHO) $q\bar{q}$ potential. In Fig. 2 we show the ratios (2.30) and (2.31) as a function of $\beta_{\rm SHO}^2/m_q^2$. Assuming that m_q is the "constituent" quark mass implies $\beta_{\rm SHO}/m_q\approx 1$ for light quarks. Note that the relativistic correction is significant for $\gamma\gamma$ decays of both $J=0^{++}$ and $J=2^{++}$ states, and even becomes dominant for the $J=2^{++}$ state as the quark mass decreases. The ratio of the relativistic 0^{++} and 2^{++} partial widths is shown in Fig. 3. As expected, this ratio is very close to 15/4 for heavy quarks, where $\beta_{\rm SHO}^2/m_q^2$ is small, and falls to approximately 2.0 for light quarks, if the parameters of Hayne and Isgur[9] are used. Thus, we find a reduction of the ratio $\Gamma(0^{++}\to\gamma\gamma)/\Gamma(2^{++}\to\gamma\gamma)$ by about a factor of 2 for light quarks if the relativistic effects are included.

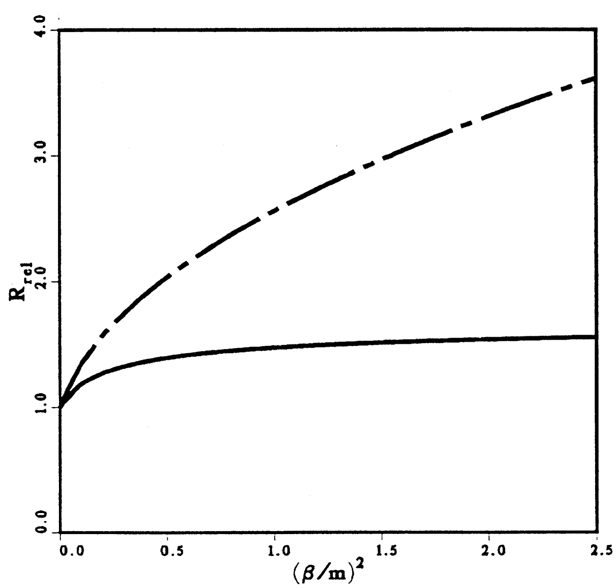

FIG. 2. The ratio $R_{\rm rel}=\Gamma_J^{\rm rel}/\Gamma_J^{\rm nr}$ versus $\beta_{\rm SHO}^2/m_q^2$, where $\Gamma_J^{\rm rel}$ ($\Gamma_J^{\rm nr}$) is the decay width with (without) relativistic corrections. The dashed line represents the tensor-meson partial width ratio and the solid line the scalar ratio.

III. THE VMD MODEL AND ITS RELATION TO THE FIELD-THEORETIC APPROACH

An alternative approach is to consider the sequential decay $S,T\to V\gamma\to\gamma\gamma$, where S, T, and V refer to scalar, tensor, and vector mesons, respectively, and the vector meson is transversely polarized ($J_z=\pm 1$) and subsequently transforms into a real photon, as shown in Fig. 4.

For our initial example, we restrict the vector meson to a $^3S_1$ state as distinct from $^3D_1$, the motivation being that in the nonrelativistic limit the vanishing $^3D_1$ wave function at contact suppresses the $V\to\gamma$ transition. The most general single-quark radiative transition operator then has the form

$$H^{\rm em}={\bf j}(1)^{\rm em}\cdot\epsilon_1^* + {\bf j}(2)^{\rm em}\cdot\epsilon_2^* , \tag{3.1}$$

where the ${\bf j}(i)^{\rm em}$ may be written[10] in terms of its spin and orbital properties as

$$j(i)_\mp^{\rm em}=AL(i)_\pm+B\sigma(i)_\pm \pm C\sigma_z L_\pm . \tag{3.2}$$

In the nonrelativistic limit $C=0$ (we retain it here for later use and for comparison with other calculations), and the A term corresponds to the electric and B the magnetic multipole transition. The wave function for the vector-meson state is

$$|\Psi_v(i)\rangle=|\phi_{S=1,M_S}\rangle|\psi_{L=0}({\bf r})\rangle , \tag{3.3}$$

where $|\phi_{S=1,M_S}\rangle$ is a spin-1 $q\bar{q}$ wave function and $|\psi_{L=0}\rangle$ is the S-wave spatial wave function. The flavor wave function is implicit and is a trivial modification.

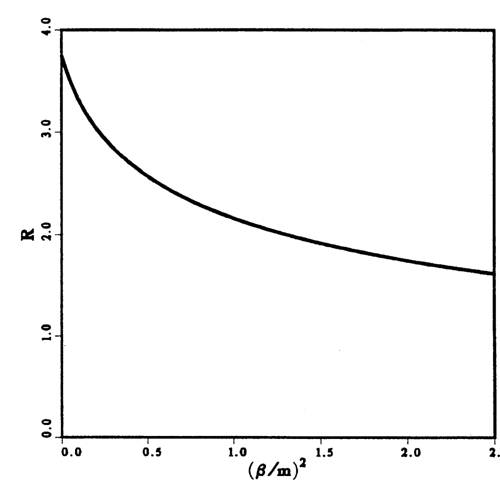

FIG. 3. The ratio $R=\Gamma_{J=0}^{\rm rel}/\Gamma_{J=2}^{\rm rel}$ (15/4 in the nonrelativistic limit).

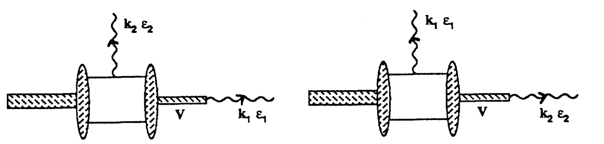

FIG. 4. A two-photon decay in the vector-meson-dominance model.

In terms of helicity amplitudes the decay width is

$$\Gamma(J^{++}\to\gamma\gamma)\propto\frac{1}{2J+1}(|A_J^{\lambda=2}|^2+|A_J^{\lambda=0}|^2)\ , \quad (3.4)$$

where, in the VMD model,

$$A_J^{\lambda(J^{++})=\lambda(\gamma)+\lambda(v)}=\langle\Psi_v(2)|\mathbf{j}(1)^{\rm em}\cdot\boldsymbol{\epsilon}_1^*|\Psi_{J^{++}}\rangle + \langle\Psi_v(1)|\mathbf{j}(2)^{\rm em}\cdot\boldsymbol{\epsilon}_2^*|\Psi_{J^{++}}\rangle\ . \quad (3.5)$$

The radiated γ can have $J_z=\pm1$; we need only consider vector mesons with $J_z=\pm1$ because the $J_z=0$ state cannot convert into a real photon. The helicity-zero amplitude arises either from $(J_z^\gamma=+1,J_z^V=-1)$ or from $(J_z^\gamma=-1,J_z^V=+1)$, and it is important to include both terms in order to symmetrize the final $\gamma\gamma$ state correctly. Thus we obtain

$$A_J^{\lambda=2}=2(A+C) \quad (3.6)$$

and

$$A_J^{\lambda=0}=(A-C)\langle 1\,1,1\,-1|J\,0\rangle[1+(-1)^J] +2B\langle 1\,0,1\,0|J\,0\rangle\ ; \quad (3.7)$$

$A_J^{\lambda=1}$ vanishes trivially.

This leads to

$$A^{\lambda=0}=\begin{cases}\frac{2}{\sqrt{6}}(A-C+2B) & (J=2)\ ,\\ 0 & (J=1)\ ,\\ \frac{2}{\sqrt{3}}(A-C-B) & (J=0)\ .\end{cases} \quad (3.8)$$

The ratio of the scalar and tensor decay widths has the value 15/4 if we impose $C=0$, as expected in a nonrelativistic model, and if in addition

$$A=-2B\ . \quad (3.9)$$

An interesting consequence of this constraint is that the helicity-zero decay amplitude for the $J=2$ state vanishes, in agreement with the conclusion of Krammer and Kraseman.[11] This shows that the ratio 15/4 and the vanishing of the helicity-zero amplitude are closely related, and that the VMD model leads rather naturally to this connection.

Furthermore, our calculation shows that the amplitude for $J=1$ to $\gamma\gamma$ vanishes as required by Yang's theorem. The physical reason for this in the VMD model is the symmetrization of the state of two transversely polarized photons, as these are identical bosons. It should be noted that this was not done consistently in an earlier VMD calculation,[5] which omitted the $1+(-1)^J$ factor in (3.7) and forced the amplitude for $J=1$ to vanish by imposing the constraint $A=C$. Some results of earlier VMD model calculations may therefore require revision due to this incorrect constraint.

Equation (3.9) is consistent with the nonrelativistic part of the transition operator in (2.8). In order to establish the relationship between field theory and the single-quark-transition (VMD) model, the two-photon decay process is divided into two steps, first, a radiative transition from the initial scalar or tensor meson to an intermediate, virtual vector meson, and second the conversion of the vector meson into a final photon. The nonrelativistic transition operator from a $^3S_1$ vector meson to a photon is of the form

$$H^{V\gamma}\propto\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\ . \quad (3.10)$$

To show the relation of the field-theoretic transition operator (2.8) in the nonrelativistic limit to the radiative transition ($H^{\rm em}$) and vector-dominance ($H^{V\gamma}$) operators we rewrite (2.8) as

$$F_{\rm NR}=-(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1^*\boldsymbol{\epsilon}_2^*\cdot\mathbf{p}+\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2^*\boldsymbol{\epsilon}_1^*\cdot p+\boldsymbol{\epsilon}_1^*\cdot\boldsymbol{\epsilon}_2^*\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\,\hat{\mathbf{k}}\cdot\mathbf{p}) \equiv H(1)^{\rm em}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2^*+H(2)^{\rm em}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1^*\ . \quad (3.11)$$

The operator $H(i)^{\rm em}$ in (3.11) is

$$H^{\rm em}=\mathbf{j}^{\rm em}\cdot\boldsymbol{\epsilon}^*,\quad j_+^{\rm em}=-(p_+-\tfrac{1}{2}p_z\sigma_+)\ . \quad (3.12)$$

Comparing (3.12) and (3.2), we see immediately that we recover the results of (3.9), namely, that $A=-2B, C=0$, because the reduced matrix elements for p_+ and p_z are identical. Thus we have related the radiative transition amplitude in a quark model supplemented by VMD to the explicit field-theoretic result in the nonrelativistic limit. In particular, this highlights the fact that the ratio 15/4 of scalar and tensor $\gamma\gamma$ widths requires $C=0$ and is a nonrelativistic result.

The incorporation of relativistic effects in the VMD model is rather more complicated. Intermediate vector mesons in a $^3D_1$ state are allowed by the transition operator (3.2), in addition to the previously assumed $^3S_1$ vectors. When one projects the full expression (2.8) onto the VMD model, one finds that the $^3D_1$ component is present at $O(v^2/c^2)$. Taking our earlier nonrelativistic example (3.11) as a guide, we rewrite the full transition operator (2.8) as

$$F=H(1)_S^{\rm em}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2^*+H(2)_S^{\rm em}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_1^*+H(1)_{SD}^{\rm em}\cdot\boldsymbol{\sigma}\cdot\mathbf{p}\,\mathbf{p}\cdot\boldsymbol{\epsilon}_2^* +H(2)_{SD}^{\rm em}\cdot\boldsymbol{\sigma}\cdot\mathbf{p}\,\mathbf{p}\cdot\boldsymbol{\epsilon}_1^*\ , \quad (3.13)$$

where $H_S^{\rm em}$ has the general form

$$H_S^{\rm em}=\mathbf{j}_S^{\rm em}\cdot\boldsymbol{\epsilon}^*,\quad j_{S+}^{\rm em}=AL_++BS_++CS_zL_+ \quad (3.14)$$

for transitions to the $^3S_1$ vector meson, where the reduced matrix elements A, B, and C are explicitly

$$A=-\left\langle\Psi_v\left|p_+\left[1+\frac{1}{m_q(E_q+m_q)}(p^2-\tfrac{1}{2}p_z^2)+\frac{1}{E_q^2}p_z^2\right]\right|\Psi_{J++}\right\rangle , \quad (3.15)$$

$$B=\left\langle\Psi_v\left|p_z\left[1+\frac{1}{m_q(E_q+m_q)}(p^2-p_z^2)+\frac{1}{E_q^2}p_z^2\right]\right|\Psi_{J++}\right\rangle , \quad (3.16)$$

$$C=-\left\langle\Psi_v\left|p_+\frac{p_z^2}{m_q(E_q+m_q)}\right|\Psi_{J++}\right\rangle . \quad (3.17)$$

At $O(v^2/c^2)$ there is another contribution to the $V\to\gamma$ transition, namely the $\sigma\cdot\mathbf{p}\,\mathbf{p}\cdot\epsilon^*$ piece in Eq. (3.13). The $S,T\to V\gamma$ transition in this case involves only the electric dipole transition operator $H_{SD}^{\rm em}$:

$$H_{SD}^{\rm em}=\mathbf{j}_{SD}^{\rm em}\cdot\epsilon^*;\, j_{SD+}^{\rm em}=A'L_+ , \quad (3.18)$$

where the reduced matrix element is

$$A'=\left\langle\Psi_{J++}\left\|p_+\frac{1}{m_q(E_q+m_q)}\right\|\Psi_v\right\rangle . \quad (3.19)$$

The full wave functions for the $^3S_1$ and $^3D_1$ vector mesons may be written as

$$|\Psi_v\rangle=\sum_{M_s}\langle L\; M-M_s,1\; M_s|1\; M\rangle|\phi_{S=1,M_s}\rangle|\psi_{L,M-M_S}\rangle , \quad (3.20)$$

where the spatial wave function $|\psi_{L,M}\rangle$ is

$$|\psi_{L,M}\rangle=Y_{L,M}(\hat{\Omega}_{\mathbf{p}})p^L f_{N,L}(p) , \quad (3.21)$$

with the normalization $\int_0^\infty p^{2L+2}f(p)^2dp=1$, and the orbital angular momentum L can be 0 or 2. The matrix elements for the vector-meson-to-photon couplings are

$$G_S=\langle\gamma|\sigma\cdot\epsilon^*|\Psi_v\rangle=\sqrt{2}\int\frac{d^3p}{\sqrt{(2\pi)^3}}f_{N,L}(p)\delta_{L,0}\delta_{M,\lambda} \quad (3.22)$$

and

$$G(L)_{SD}=\langle\gamma|\sigma\cdot\mathbf{p}\,\mathbf{p}\cdot\epsilon^*|\Psi_v\rangle=\sqrt{2}\int\frac{d^3p}{\sqrt{(2\pi)^3}}p^L f_{N,L}(p)([p_\mu Y_{L\,M'}]_\lambda^{J=1})^* p_\lambda\delta_{M,\lambda} , \quad (3.23)$$

where λ labels the photon polarization. The resulting selection rule for the operator $\sigma\cdot\epsilon^*$ is that only the transition from the S-wave vector meson to the photon is allowed, whereas the $\sigma\cdot\mathbf{p}\,\mathbf{p}\cdot\epsilon^*$ terms allow the transition from both S- and D-wave vector mesons. The quantum number M of the intermediate vector meson is determined by the polarization of the photon.

The complete helicity amplitude may be written as

$$A_\lambda^J=\sum_{i\neq j}\left[\langle\gamma|\sigma\cdot\epsilon_j^*|\Psi(j)_v\rangle\langle\Psi(j)_v|\mathbf{j}(i)_S^{\rm em}\cdot\epsilon_i^*|\Psi_{J++}\rangle+\sum_{S,D}\langle\gamma|\sigma\cdot\mathbf{p}\,\mathbf{p}\cdot\epsilon_j^*|\Psi(j)_v\rangle\langle\Psi(j)_v|\mathbf{j}(i)_{SD}^{\rm em}\cdot\epsilon_i^*|\Psi_{J++}\rangle\right] . \quad (3.24)$$

Following the same procedure as in (3.5) to (3.8), (3.24) leads to

$$A_{\lambda=2}^{J=2}=2(A=C)G_S+\sum_{L=0,2}6\,W(2\,1\,1\,L;\,1\,1)A'G(L)_{SD} , \quad (3.25)$$

$$A_{\lambda=0}^{J=2}=\frac{2}{\sqrt{6}}\left[(A-C+2B)G_S+\sum_{L=0,2}3\,W(2\,1\,1\,L;\,1\,1)A'G(L)_{SD}\right] \quad (3.26)$$

and

$$A_{\lambda=0}^{J=0}=\frac{2}{\sqrt{3}}\left[(A-C-B)G_S+\sum_{L=0,2}A'G(L)_{SD}\right] , \quad (3.27)$$

where the $\{W(2\,1\,1\,L;\,1\,1)\}$ are SO(3) Racah coefficients. In the VMD model G_S and $G(L)_{SD}$ could be treated as effective coupling parameters, but here we can identify these amplitudes explicitly with results of the field-theory calculation by expressing the helicity amplitudes in terms of $q\bar{q}$ wave-function integrals. We find that (3.25), (3.26), and (3.27) become (with all integrals implicitly over $[0,\infty]$)

$$A^{J=2}_{\lambda=2}=\frac{4}{(2\pi)^{5/2}\sqrt{3}}\left\{\left[\int dp\, p^3\phi(p)f_{N,L=0}(p)\left[1+\frac{p^2}{m_q(E_q+m_q)}+\frac{p^2}{5E_q^2}\right]\right]\left[\int dp\, p^2 f_{N,L=0}(p)\right]\right.$$
$$-\frac{1}{3}\left[\int dp\frac{p^3\phi(p)f_{N,L=0}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^4 f_{N,L=0}(p)\right]$$
$$\left.-\frac{1}{15}\left[\int dp\frac{p^5\phi(p)f_{N,L=2}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^6 f_{N,L=2}(p)\right]\right\}, \tag{3.28}$$

$$A^{J=2}_{\lambda=0}=\frac{4\sqrt{2}}{15(2\pi)^{5/2}}\left\{\left[\int dp\, p^3\phi(p)f_{N,L=0}(p)\left[\frac{p^2}{m_q(E_q+m_q)}+\frac{p^2}{E_q^2}\right]\right]\left[\int dp\, p^2 f_{N,L=0}(p)\right]\right.$$
$$-\frac{5}{6}\left[\int dp\frac{p^3\phi(p)f_{N,L=0}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^4 f_{N,L=0}(p)\right]$$
$$\left.-\frac{1}{6}\left[\int dp\frac{p^5\phi(p)f_{N,L=2}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^6 f_{N,L=2}(p)\right]\right\} \tag{3.29}$$

and

$$A^{J=0}_{\lambda=0}=\frac{4}{3(2\pi)^{5/2}}\left\{\left[\int dp\, p^3\phi(p)f_{N,L=0}(p)\left[\frac{3}{2}+\frac{p^2}{m_q(E_q+m_q)}+\frac{p^2}{2E_q^2}\right]\right]\left[\int dp\, p^2 f_{N,L=0}(p)\right]\right.$$
$$-\frac{1}{3}\left[\int dp\frac{p^3\phi(p)f_{N,L=0}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^4 f_{N,L=0}(p)\right]$$
$$\left.-\frac{2}{3}\left[\int dp\frac{p^5\phi(p)f_{N,L=2}(p)}{m_q(E_q+m_q)}\right]\left[\int dp\, p^6 f_{N,L=2}(p)\right]\right\}. \tag{3.30}$$

In the VMD model, $f_{N,L}(p)$ is the radial wave function of the intermediate vector mesons ρ^0 and ω, and the quantum number N in the potential model is 0 for $L=0$ and 2 for $L=2$. In a more complete "generalized VMD" scheme, one could sum over all quantum numbers N and use the completeness relation

$$\sum_N p^L f_{N,L}(p)p'^L f_{N,L}(p')=\frac{\delta(p-p')}{p^2}(2\pi)^{3/2}. \tag{3.31}$$

Using (3.28), (3.29), and (3.30), one may show that the ratio of the scalar-to-tensor decay widths to order $(v/c)^2$ is equal to (2.30). Of course this is expected, and primarily serves as an algebra check and again establishes the connection between the VMD model and field-theoretic approach. The results of the two approaches are thus found to be consistent to $O(v^2/c^2)$.

IV. THE HELICITY STRUCTURE OF THE RELATIVISTIC CORRECTIONS

The VMD calculation has shown that the ratio of the scalar to tensor widths is consistent with (2.30) to order $(v/c)^2$, and has also shown that the $\lambda=0, J=2$ amplitude vanishes in the nonrelativistic limit. As the relativistic corrections significantly modify the 15/4 width ratio, it is important to investigate the size of the relativistic contributions to the helicity-zero decay amplitude of the tensor state. Experimental data[12] suggests that helicity-zero amplitude in the tensor decay is in fact very small.

We can extract the various amplitudes directly from (2.5); taking $\hat{\mathbf{k}}=\hat{\mathbf{z}}$, the helicity amplitudes are

$$A^{J=2}_{\lambda=2}=\left[\frac{3}{2\pi}\right]^{1/2}\int\frac{dp\,d\Omega}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^0}\frac{\sin^2\theta}{1-\beta^2\cos^2\theta}\left[1+\frac{p^2}{m_q(E_q+m_q)}(1-\tfrac{1}{2}\sin^2\theta)\right]\phi(p), \tag{4.1}$$

$$A^{J=2}_{\lambda=0}=\frac{1}{2\sqrt{\pi}}\int\frac{dp\,d\Omega}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^2}\frac{1}{1-\beta^2\cos^2\theta}(3\cos^2\theta-1)\phi(p) \tag{4.2}$$

and

$$A^{J=0}_{\lambda=0}=\frac{1}{\sqrt{2\pi}}\int\frac{dp\,d\Omega}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^0}\frac{1}{1-\beta^2\cos^2\theta}\phi(p), \tag{4.3}$$

where $\beta=p/E_q=v/c$. The angular integrals can be carried out exactly, and we find

$$A^{J=2}_{\lambda=2}=\sqrt{6\pi}\int_0^\infty \frac{dp}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^0}\phi(p)\left\{\frac{2}{\beta^2}+\frac{\beta^2-1}{\beta^3}\ln\left[\frac{1+\beta}{1-\beta}\right]+\frac{p^2}{m_q(E_q+m_q)}\left[\frac{1}{3\beta^2}+\frac{1}{\beta^4}+\frac{\beta^4-1}{2\beta^5}\ln\left[\frac{1+\beta}{1-\beta}\right]\right]\right\}, \tag{4.4}$$

$$A^{J=2}_{\lambda=0}=\sqrt{\pi}\int_0^\infty \frac{dp}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^0}\phi(p)\left[-\frac{6}{\beta^2}+\frac{3-\beta^2}{\beta^3}\ln\left[\frac{1+\beta}{1-\beta}\right]\right], \tag{4.5}$$

and

$$A^{J=0}_{\lambda=0}=\sqrt{2\pi}\int_0^\infty \frac{dp}{\sqrt{(2\pi)^3}}\frac{m_q p^3}{E_q^2 k^0}\phi(p)\frac{1}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right]. \tag{4.6}$$

[Note that only $A^{J=2}_{\lambda=2}$ is nonzero in the $m_q=0$ limit, in agreement with the result of massless QCD. Our general expression [Eqs. (4.4)–(4.6)] shows however that the approach to this limit is very gradual (see Fig. 3, for example), so comparison of $m_q=0$ results to experiment must be treated with caution.]

On expanding these amplitudes to $O(\beta^2)$, (4.4)–(4.6) give the same expression for the decay width we found in Sec. II. Furthermore, these expressions are consistent with the helicity amplitudes obtained in Sec. III. This angular decomposition shows the source of the $A^{J=2}_{\lambda=0}=0$ nonrelativistic selection rule and its violation at $O(v^2/c^2)$. In the nonrelativistic limit $\beta\to 0$, the vanishing $J=2$, helicity-zero amplitude follows from the angular integration in (4.2); when $\beta>0$ this cancellation no longer occurs. Note however that this amplitude remains relatively small; specifically, at order $(v/c)^2$, the ratio between helicity-0 and -2 amplitudes for the tensor ($J=2$) state is

$$\frac{A^{J=2}_{\lambda=0}}{A^{J=2}_{\lambda=2}}=\frac{1}{5}\left[\frac{2}{3}\right]^{1/2}\frac{\int_0^\infty dp\frac{p^5}{E_q^4}\phi(p)}{\int_0^\infty dp\frac{p^3}{E_q^2}\left[1+\frac{3}{5}\frac{p^2}{m_q(E_q+m_q)}+\frac{1}{5}\frac{p^2}{E_q^2}\right]\phi(p)}, \tag{4.7}$$

and the corresponding partial width ratio [the square of (4.7)] is shown in Fig. 5 for harmonic-oscillator wave functions (2.33). Note that this ratio is quite small even for light-quark masses, so that the $\lambda=2$ amplitude dominates the $\gamma\gamma$ width of the tensor state.

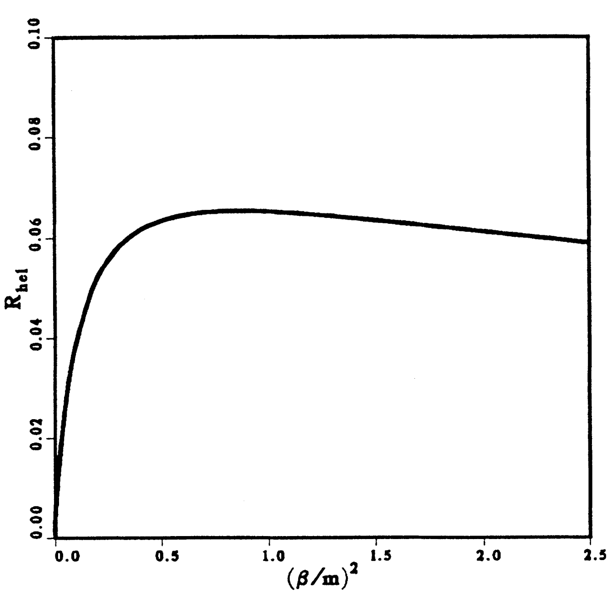

FIG. 5. The ratio $R_{hel}=\Gamma^{\lambda=0}_{J=2}/\Gamma^{\lambda=2}_{J=2}$ of helicity-zero to helicity-two amplitudes for $q\bar{q}(2^{++})\to\gamma\gamma$.

V. SUMMARY

These calculations have shown that relativistic effects are indeed important in quark model calculations of the $\gamma\gamma$ decays of scalar and tensor mesons. In the nonrelativistic limit we showed how the field theory $q\bar{q}\to\gamma\gamma$ amplitude and the quark model with VMD are related, which establishes the VMD model on a more fundamental level. We have also found that the familiar nonrelativistic ratio 15/4 of scalar-to-tensor meson $\gamma\gamma$ widths is changed markedly by relativistic corrections in light-quark systems. We have also shown that, at least in this example, both $^3S_1$ and $^3D_1$ vector meson contributions appear at $O(v^2/c^2)$ in the VMD model of $\gamma\gamma$ decays when applied to light-quark systems.

It is not immediately evident why relativistic effects dominate the $\gamma\gamma$ total width of the tensor state [recall that relativistic corrections play a relatively minor role in $N^*\to N\gamma$ (Ref. 13) and in the helicity-zero selection rule for $T\to\gamma\gamma$]. A better understanding of this result may reveal why nonrelativistic calculations of the $\gamma\gamma$ couplings of radially excited mesons appear to be overestimated in the quark model (specifically, the model predicts significant $\gamma\gamma$ widths, whereas there are no radially excited states observed in the data[14]). In a nonrelativistic calculation the partial width of a P-wave $q\bar{q}$ state to $\gamma\gamma$ is proportional to the square of the derivative of the wave function at the origin. This is very sensitive to the large-p behavior of the wave function in momentum space, and relativistic effects may be important in these decays for this reason. Matrix elements of other processes, for example electroweak transitions which nonrelativistically involve the wave function at the origin, may be similarly sensitive to relativistic corrections.

Our present work is largely pedagogical since there remain $O(\alpha_s)$ QCD wave-function-mixing effects, including the contribution to the decay from the $q\bar{q}g$ component of the initial meson, which enter at the same order in v/c as the relativistic effects considered here. These corrections will affect the 2^{++} decay rate due to mixing with the $^3F_2$ configuration, and both 0^{++} and 2^{++} will mix with their SHO basis radially excited counterparts. These relativistic and QCD mixing effects have not yet been thoroughly investigated in studies of the $\gamma\gamma$ decays of radially excited mesons. A complete study should be carried out within the framework of a quark model which gives a good description of meson spectroscopy, for example the model of Godfrey and Isgur.[8] As noted in Ref. 12, the gauge-invariant electromagnetic transition operator depends on the binding potential between the quarks at this order in v/c, and this complication has also not yet been investigated in $q\bar{q}\to\gamma\gamma$.

Finally, we have found a connection between the field-theoretic and VMD model descriptions of the decay $q\bar{q}\to\gamma\gamma$ and the Landau-Yang theorem [through the vanishing of the amplitude for $q\bar{q}(1^{++})\to\gamma\gamma$]. We have shown that symmetries in the VMD approach automatically satisfy the Landau-Yang theorem without requiring the constraint $A=C$, as was suggested previously; indeed, for heavy quarks, consistency of the field-theoretic and VMD approaches implies a different result: $A=-2B$ and $C=0$. In consequence, a previous calculation[15] of $\chi\to\gamma\psi$ which unnecessarily imposed the constraint $A=C$ for the (heavy-quark) charmonium system in order to force $\chi_1\not\to\gamma\gamma$ now merits reconsideration. (We note in passing that the relation $A=C$ may still be correct for relating pion production amplitudes[5] of the form $M\to\pi\rho$ and $M\to\pi\gamma$, although this should be regarded as an open question.)

Note added. After completing this work we learned that Ref. 16 has also considered the quark-mass dependence of the ratio $\Gamma(0^{++}\to\gamma\gamma)/\Gamma(2^{++}\to\gamma\gamma)$ using a nonrelativistic reduction of the Bethe-Salpeter equation. They also found important relativistic effects which suppress the overall decay rate, and conclude the $\Gamma(0^{++}\to\gamma\gamma)/\Gamma(2^{++}\to\gamma\gamma)$ is reduced from 15/4 to about 2 for light quarks, consistent with our results.

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[1]A. I. Alekseev, Zh. Eksp. Teor. Fiz. **34**, 1195 (1958) [Sov Phys. JETP 7, 826 (1958)].

[2]K. A. Tumanov, Zh. Eksp. Teor. Fiz. **25**, 385 (1953).

[3]R. Barbieri, R. Gatto, and R. Kogerler, Phys. Lett. **60B**, 183 (1976).

[4]M. Feindt, DESY Report No. 89-142 (unpublished).

[5]J. Babcock and J. Rosner, Phys. Rev. D **14**, 1286 (1976).

[6]L. D. Landau, Dokl. Akad. Nauk. SSSR **60**, 213 (1948).

[7]C. N. Yang, Phys. Rev. **77**, 242 (1950).

[8]S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[9]C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).

[10]F. E. Close, H. Osborn, and A. M. Thomson, Nucl. Phys. **B77**, 281 (1974).

[11]M. Krammer and H. Kraseman, Phys. Lett. **73B**, 58 (1978).

[12]CELLO Collaboration, DESY Report No. 89-177 (unpublished).

[13]F. E. Close and Z. P. Li, Phys. Rev. D **42**, 2194 (1990).

[14]T. Barnes, in *Photon-Photon Collisions,* Proceedings of the VIIth International Workshop, Paris, France, 1986, edited by A. Courau and P. Kessler (World Scientific, Singapore, 1987), pp. 25–50.

[15]G. Karl, S. Meshkov, and J. Rosner, Phys. Rev. D **13**, 1203 (1976).

[16]L. Bergström, G. Hulth, and H. Snellman, Z. Phys. C **16**, 263 (1983).