PHYSICAL REVIEW D VOLUME 43, NUMBER 7 1 APRIL 1991

# $D$-wave quarkonium production and annihilation decays: Formalism and applications

L. Bergström
*Department of Physics, University of Stockholm, Vanadisvägen 9, S-113 46 Stockholm, Sweden*

H. Grotch and R. W. Robinett
*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802*
(Received 12 November 1990)

We derive a covariant formalism to describe $D$-wave quarkonium production and annihilation decays in the context of nonrelativistic potential models. We apply it to evaluate the decay rates for such processes as $Z^0 \to {}^1D_2+\gamma, {}^3D_J+\gamma$ (where $J=1,2,3$) and other previously uncalculated processes. Despite the appearance of relatively large numerical coefficients, we find no case in which $D$-wave states (followed by radiative decays) make any significant contributions to inclusive ${}^3S_1$ quarkonium production.

## I. INTRODUCTION

The systematic study of heavy-quarkonium systems[1] has provided a wealth of information not only on the properties of the heavy $(c,b)$ quarks themselves but also on QCD and the description of such systems in terms of (QCD-motivated) potential models (with relativistic corrections) has proved very successful. An important feature which can be addressed in such models, in addition to fitting masses and radiative decays, is that of annihilation decays and related processes in quarkonium production. A covariant description for calculating annihilation decay and production amplitudes has been derived by Kühn, Kaplan, and Safiani[2] (KKS) for $S$- and $P$-wave states. It has been extensively used in the calculation of various partonic-level quarkonium production processes [such as $gg \to S(P)g$ (Ref. 3) and $qg \to qS(P), q\bar{q} \to S(P)g$ (Ref. 4)] and subsequent analyses of inclusive high-$p_T$ quarkonium production at collider[5,6] and supercollider[7] energies, in polarized $pp$ collisions,[8] as well as in rarer processes.[9,10] It has also been used to examine the prospects for quarkonium production in $Z^0$ decays,[11,12] aspects of toponium decays,[13] and even in processes involving dark matter.[14,15] A technically useful aspect of the covariant formulation is that it is readily used with popular algebraic-manipulation software designed for standard trace calculations.

While there have been extensive calculations involving $S$- and $P$-wave quarkonium states, $D$-wave states, given the limited experimental data, have received, probably deservedly, less attention. This situation may change as there are hopes of observing charmonium $D$ states [in addition to the $\psi''(3770)$] in a high-statistics exclusive $p\bar{p}$ charmonium production experiment (Fermilab E760)[16] and the $b\bar{b}$ states in $\Upsilon$ radiative decays.[17] Relatively few calculations involving $D$-wave annihilation decays or production prospects exist with the notable exceptions of the simple cases ${}^3D_1 \to e^+e^-$ and ${}^1D_2 \to gg$ (which appear in Ref. 18) and the more recent calculation[19] of the hadronic decay widths of the triplet $D$ states, i.e., $\Gamma({}^3D_J \to ggg)$ where $J=1,2,3$. (There have been several investigations of $D$-wave radiative decays[20,21] and hadronic transitions, i.e., $D \to S\pi\pi$.[22])

The surprisingly large, indeed dominant, contributions of $P$-wave charmonium production (followed by radiative decays) to inclusive $\psi$ production at large transverse momentum in hadronic collisions suggests that the possible role of $D$-wave production be examined as well. The radiative decay rates for many $D$-wave states which ultimately yield an $n\,{}^3S_1$ state (which is then observable via its clean dimuon decay) are thought to be large[20] so that any produced $D$ state would contribute a significant fraction of the time to the observed inclusive $\psi$ signal.

One would naively expect that $D$-wave production will be suppressed relative to $S$-state production by factors such as $|R_D''(0)|^2/(|R_S(0)|^2M^4) \approx 2\times 10^{-4}$ for both $c\bar{c}$ and $b\bar{b}$ quarkonia (where $R_{S,P,D}$ are the quarkonium radial wave functions and we use potential-model predictions for the various values of the wave functions and their derivatives at the origin[23]). Similar arguments, however, would suggest that $P$-wave production would be suppressed by a factor such as $|R_P'(0)|^2/[|R_S(0)|^2M^2] \approx 8\times 10^{-3}$ while in hadronic collisions it is, in fact, the major source of inclusive $\psi$'s due to a different production process. So, large numerical coefficients or different kinematic structures can enhance these effects as evidenced in the case of the $\chi$ contribution to inclusive $\psi$ production mentioned above. For example, in the calculation of Bélanger and Moxhay,[19] relatively large numerical coefficients were found for the ${}^3D_J \to ggg$ decay rates. Such a calculation is not, however, directly generalizable to the calculation of the cross section for $gg \to {}^3D_Jg$ as the decay rate calculation made use of soft-photon theorems and needed to consider only a subset of the relevant amplitudes to obtain the most divergent pieces. A full calculation of this cross section (and the amplitudes for other processes involving $D$ states as well) would benefit from a covariant formulaton.

With these motivating factors, in this article we describe a covariant formulation for annihilation and production amplitudes for quarkonium $D$ states in the nonrelativistic potential-model formalism. We describe the formalism in Sec. II and then apply it to some previously uncalculated processes in Sec. III and briefly state our conclusions and discuss possible further applications.

## II. FORMALISM

We begin by briefly reviewing the quarkonium annihilation formulation of Kühn, Kaplan, and Safiani[2] and then extend it to $D$ states. We write the amplitude for the annihilation interaction of a free fermion-antifermion pair as

$$\mathcal{A}=\bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s)\ , \tag{1}$$

where $f,\bar{f}$ and $s,\bar{s}$ are the particle and antiparticle momenta and spins and $\mathcal{O}$ is the relevant Dirac operator. The amplitude for a bound pair can then be described in a nonrelativistic approximation by

$$A=\left[\frac{1}{m}\right]^{1/2}\int\frac{d\mathbf{k}}{(2\pi)^{3/2}}\psi_{s\bar{s}}(\mathbf{k})\bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s)\ , \tag{2}$$

where

$$f+\bar{f}=P=(M,0),\quad f-\bar{f}=2k=(0,2\mathbf{k})\ , \tag{3}$$

and

$$\sum_{s\bar{s}}\int d\mathbf{k}|\psi_{s\bar{s}}|^2=1\ , \tag{4}$$

$M$ and $m$ are the masses of the bound state and quark, respectively, and we work in the rest frame of the bound state described by the momentum-space wave function $\psi_{s\bar{s}}(\mathbf{k})$.

In Eq. (2) we may consider spin-singlet and -triplet states. This leads to appropriate sums on $s$ and $\bar{s}$ which can be expressed in terms of traces as

$$\sum_{s\bar{s}}\bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s)\langle\tfrac{1}{2},s;\tfrac{1}{2},\bar{s}|S,S_Z\rangle =\frac{1}{\sqrt{E_{\bar{f}}+m}}\frac{1}{\sqrt{E_f+m}}\times\mathrm{Tr}\left[(m-\bar{f}\!\!\!/)\mathcal{O}(f\!\!\!/+m)\frac{1+\gamma_0}{2\sqrt{2}}\Pi_{SS_Z}\right]\ , \tag{5}$$

with

$$\Pi_{00}=-\gamma_5,\quad \Pi_{1,S_Z}=-\not{e}_{(S_Z)}\ . \tag{6}$$

Then, following Ref. 2, we expand $A$ in powers of $k/m$ to the desired order, which in the case of $D$ waves is second order in $k$. We find the result

$$A=\frac{1}{M2\sqrt{2}}\left[\frac{1}{M}\mathrm{Tr}[\not{k}\mathcal{O}_0\not{k}(\not{P}+M)\Pi_{SS_Z}] +\mathrm{Tr}[\{\mathcal{O}_1^{\alpha}k_\alpha,\not{k}\}_+(\not{P}+M)\Pi_{SS_Z}] +\frac{M}{2}\mathrm{Tr}[k_\alpha\mathcal{O}_2^{\alpha\beta}k_\beta(\not{P}+M)\Pi_{SS_Z}]\right]\ , \tag{7}$$

where $\mathcal{O}_0,\mathcal{O}_1^{\alpha},\mathcal{O}_2^{\alpha\beta}$ are the zeroth, first, and second derivatives of $\mathcal{O}$ with respect to $k^\alpha$, respectively (with $k=0$).

For the orbital part, the integral of the $k_\alpha k_\beta$ over $d^3k$ [the analog of Eq. (6) of KKS] is then given by

$$\left[\frac{1}{m}\right]^{1/2}\int\frac{d\mathbf{k}}{(2\pi)^{3/2}}k_\alpha k_\beta\psi_2^{(m)}(\mathbf{k})=be_{\alpha\beta}^{(m)}\ , \tag{8}$$

where

$$b=\frac{1}{\sqrt{m}}\left[\frac{15}{8\pi}\right]^{1/2}\phi''(0)\ , \tag{9}$$

and $e_{\alpha\beta}^{(m)}$ is the spin-2 polarization tensor.

For the $S=0$ case where $J=L=2$, we directly associate $e_{\alpha\beta}^{(m)}$ with $e_{\alpha\beta}^{(Jz)}$ while for the spin-triplet case where $J=1,2$, or 3, using explicit Clebsch-Gordan coefficients, we find the following results for the three cases.

$J=1$:

$$e_{\alpha\beta}^{(m)}e_\rho^{(S_Z)}\langle 21;mS_Z|1J_Z\rangle=-\left[\tfrac{3}{20}\right]^{1/2}[(g_{\alpha\rho}-P_\alpha P_\rho/M^2)e_\beta^{(J_Z)}+(g_{\beta\rho}-P_\beta P_\rho/M^2)e_\alpha^{(J_Z)}-\tfrac{2}{3}(g_{\alpha\beta}-P_\alpha P_\beta/M^2)e_\rho^{(J_Z)}]\ , \tag{10}$$

where $e_\alpha$ is the usual spin-one polarization vector satisfying $e_\alpha P^\alpha=0$.

$J=2$:

$$e_{\alpha\beta}^{(m)}e_\rho^{(S_Z)}\langle 21;mS_Z|2J_Z\rangle=\frac{i}{M\sqrt{6}}(e_{\alpha\sigma}^{(J_Z)}\epsilon_{\tau\beta\rho\sigma'}P^\tau g^{\sigma\sigma'} +e_{\beta\sigma}^{(J_Z)}\epsilon_{\tau\alpha\rho\sigma'}P^\tau g^{\sigma\sigma'})\ , \tag{11}$$

where $e_{\alpha\beta}$ is the symmetric spin-2 polarization tensor which satisfies

$$g^{\alpha\beta}e_{\alpha\beta}=0,\quad P^\alpha e_{\alpha\beta}=0\ , \tag{12}$$

and where the sum over polarizations is given by the familiar expression

$$\sum_{m=-2}^{2}e_{ab}(m)e_{xy}^*(m)=\tfrac{1}{2}(\mathcal{P}_{ax}\mathcal{P}_{by}+\mathcal{P}_{ay}\mathcal{P}_{bx})-\tfrac{1}{3}\mathcal{P}_{ab}\mathcal{P}_{xy}\ , \tag{13}$$

where

$$\mathcal{P}_{ab}=-g_{ab}+\frac{P_aP_b}{M^2}\ . \tag{14}$$

Finally, $J=3$:

$$e^{(m)}_{\alpha\beta}e^{(S_Z)}_{\rho}\langle 21;mS_Z|3J_Z\rangle=e^{(J_Z)}_{\alpha\beta\rho} , \quad (15)$$

where $e_{\alpha\beta\rho}$ is the completely symmetric spin-3 polarization tensor which satisfies

$$g^{\alpha\beta}e_{\alpha\beta\rho}=0, \quad P^{\alpha}e_{\alpha\beta\rho}=0 . \quad (16)$$

This required sum over polarization states is given by (see, e.g., Ref. 24)

$$\sum_{m=-3}^{3} e_{abc}(m)e^{*}_{xyz}(m)=\tfrac{1}{6}\Omega^{(1)}_{abc;xyz}-\tfrac{1}{15}\Omega^{(2)}_{abc;xyz} , \quad (17)$$

where

$$\Omega^{(1)}_{abc;xyz}=\mathcal{P}_{ax}\mathcal{P}_{by}\mathcal{P}_{cz}+\mathcal{P}_{ax}\mathcal{P}_{bz}\mathcal{P}_{cy}+\mathcal{P}_{ay}\mathcal{P}_{bx}\mathcal{P}_{cz}+\mathcal{P}_{ay}\mathcal{P}_{bz}\mathcal{P}_{cx}+\mathcal{P}_{az}\mathcal{P}_{by}\mathcal{P}_{cx}+\mathcal{P}_{az}\mathcal{P}_{bx}\mathcal{P}_{cy} \quad (18)$$

and

$$\Omega^{(2)}_{abc;xyz}=\mathcal{P}_{ab}\mathcal{P}_{cz}\mathcal{P}_{xy}+\mathcal{P}_{ab}\mathcal{P}_{cy}\mathcal{P}_{xz}+\mathcal{P}_{ab}\mathcal{P}_{cx}\mathcal{P}_{yz}+\mathcal{P}_{ac}\mathcal{P}_{bz}\mathcal{P}_{xy}+\mathcal{P}_{ac}\mathcal{P}_{by}\mathcal{P}_{xz}+\mathcal{P}_{ac}\mathcal{P}_{bx}\mathcal{P}_{yz}+\mathcal{P}_{bc}\mathcal{P}_{az}\mathcal{P}_{xy}+\mathcal{P}_{bc}\mathcal{P}_{ay}\mathcal{P}_{xz}+\mathcal{P}_{bc}\mathcal{P}_{ax}\mathcal{P}_{yz} . \quad (19)$$

Finally, for decays to color-neutral final states, the appropriate color factor in Eq. (7) is simply $\sqrt{3}$ (as in Ref. 2). We have used this formalism to check the decay rates for ${}^3D_1\to e^+e^-$ and ${}^1D_2\to gg$ $(\gamma\gamma)$ against existing results[18] and find agreement.

## III. APPLICATIONS

We can now make use of this formalism to evaluate some previously uncalculated decay rates and cross sections involving $D$-wave quarkonium states. The two-body decay rates of the $Z^0$ to quarkonium states plus photon have been calculated some time ago for $S$- and $P$-wave states[11] and we can now extend their results to the case of $D$ waves. Final-state quarkonium states with any quantum number are allowed as the process can go via either the vector or axial-vector $Z^0$ coupling. The calculations were performed using symbolic manipulation programs, with considerable savings in computer time obtained by using various symmetry properties of the amplitudes. The results are mostly easily quoted as ratios to the relevant $S$-wave decay rates and we find

$$\frac{\Gamma(Z^0\to{}^1D_2+\gamma)}{\Gamma(Z^0\to{}^1S_0+\gamma)}=80R , \quad (20)$$

$$\frac{\Gamma(Z^0\to{}^3D_1+\gamma)}{\Gamma(Z^0\to{}^3S_1+\gamma)}=\frac{2R(1+67\mu+123\mu^2+9\mu^3)}{(1+\mu)(1-\mu)^2} , \quad (21)$$

$$\frac{\Gamma(Z^0\to{}^3D_2+\gamma)}{\Gamma(Z^0\to{}^3S_1+\gamma)}=\frac{5R(27+25\mu+202\mu^2+16\mu^3)}{2(1+\mu)(1-\mu)^2} , \quad (22)$$

$$\frac{\Gamma(Z^0\to{}^3D_3+\gamma)}{\Gamma(Z^0\to{}^3S_1+\gamma)}=\frac{16R(3+2\mu)}{1+\mu} , \quad (23)$$

where

$$R=\frac{|R''_D(0)|^2}{|R_S(0)|^2M^4} \quad (24)$$

and

$$\mu=\frac{M^2}{M_Z^2} . \quad (25)$$

(We assume a common quarkonium mass for $S$-, $P$-, and $D$-wave states for simplicity.) The rates for the normalizing processes (see Ref. 11) $Z^0\to{}^1S_0+\gamma$ and $Z^0\to{}^3S_1+\gamma$ are given by

$$\frac{\Gamma(Z^0\to{}^1S_0+\gamma)}{\Gamma(Z^0\to\mu^+\mu^-)}=C_\gamma V\mu(1-\mu) , \quad (26)$$

$$\frac{\Gamma(Z^0-{}^3S_1+\gamma)}{\Gamma(Z^0\to\mu^+\mu^-)}=C_\gamma A\mu(1-\mu^2) , \quad (27)$$

where

$$V=\frac{(1-4|e_Q|\sin^2\theta_W)^2}{1+(1-4\sin^2\theta_W)^2} , \quad (28)$$

$$A=\frac{1}{1+(1-4\sin^2\theta_W)^2} , \quad (29)$$

$$C_\gamma=\frac{24\alpha e_Q^2|R_S(0)|^2}{M^3} , \quad (30)$$

and

$$\Gamma(Z^0\to\mu^+\mu^-)=\frac{M_Z}{12\pi}\left[\frac{G_FM_Z^2}{2\sqrt{2}}\right][1+(1-4\sin^2\theta_W)^2] . \quad (31)$$

Although it is seen that in some cases there are large coefficients enhancing the rates compared to $S$-wave production, given the ratios of the $D$-to-$S$-state wave functions at the origin quoted above, the branching ratios are too small to be experimentally interesting. It is interesting to note that the ${}^3D_1$ and ${}^3D_2$ states, which may be reached from the spin-1 $Z^0$ resonance without centrifugal barrier in the radiative transitions, have a $1-\mu$ enhancement factor in the denominator. [The same enhancement factor is present in all of the $Z^0\to{}^3P_J+\gamma$ $(J=0,1,2)$ decay rates as these can all be realized with no relative angular momentum in the final state.] Still, the smallness of $\mu$ for the known quarkonium states means that this is not a numerically big effect.

Another application is the calculation of the partonic level cross section for the process $qg\to q\,{}^1D_2$ which we can once again compare to the relevant $S$-wave expression (in Ref. 4) to find

$$\frac{d\sigma}{d\hat{t}}(qg\to q\,{}^1D_2)\Big/\frac{d\sigma}{d\hat{t}}(qg\to q\,{}^1S_0)=80R . \quad (32)$$

The similarity with Eq. (20) is easily explained as both processes involve the same expressions for the ${}^1D_2\leftrightarrow\gamma\gamma^*$ and ${}^1S_0\leftrightarrow\gamma\gamma^*$ couplings, in one case with the $\gamma^*$ representing the real $Z^0$ while in the other the virtual $t$-channel gluon. The cross sections for the related reactions $q\bar{q} \to g\,^1D_2$ (or the related $e^+e^- \to \gamma\,^1D_2$) are easily related by crossing (as in Ref. 4).

To conclude, we have set up a general covariant formalism, well suited for calculations using computer algebra, for the coupling of $D$-wave quarkonium states. We have applied this formalism to the previously uncalculated processes of $D$-wave radiative production in $Z^0$ decays.

At least two other areas of interest suggest themselves as applications of this formalism. As mentioned above, the possible contributions of $D$-wave states to inclusive high-$p_T$ $\psi$ and $\Upsilon$ production in hadronic collisions via the subprocesses $gg \to g\,^3D_J$ should be calculated. The predominance of $P$-wave contributions to such processes is understood as being due to the different production mechanisms for $P$ states ($t$-channel gluon-exchange diagrams) and the presence of the three-gluon coupling. $^3D_J$ production would be more similar to $^3S_1$ processes in this regard, albeit suppressed by wave function factors but once again the presence of large coefficients must be checked explicitly. In addition, if $D$-state production in hadronic collisions is only comparable to $S$-wave contributions, and hence much smaller than $P$-wave contributions, then it could still play an important role in inclusive $\psi$ production in $ep$ collisions where the dominant process is assumed to be $\gamma g \to g\,^3S_1$ and $P$ states do not contribute (except via the gluon structure of the photon at higher order.[25]) Finally, predictions of exclusive charmonium production in proton-antiproton collisions ($p\bar{p} \to D$ state) following standard methods (see, e.g., Ref. 26) would be very useful for confrontation with future data. It is this case, in which there are no competing $S$- or $P$-wave contributions, which will likely be the most relevant as the $D$-wave formalism gives all the relevant information. Both of these projects are computationally more extensive than the processes presented here and are currently under study.

## ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under Grants Nos. PHY-8819727-01 (H.G.) and PHY-9001744 (R.R.), by the Texas National Research Laboratory Commission (R.R.), and by the Swedish Natural Science Research Council (L.B.).

[1]W. Kwong, J. L. Rosner, and C. Quigg, Annu. Rev. Nucl. Part. Sci. **37**, 325 (1987).

[2]J. H. Kühn, J. Kaplan, and E. G. O. Safiani, Nucl. Phys. **B157**, 125 (1979).

[3]R. Gastmans, W. Troost, and T. T. Wu, Phys. Lett. B **184**, 257 (1987); Nucl. Phys. **B291**, 731 (1987); B. Humpert, Phys. Lett. B **184**, 105 (1987).

[4]R. Baier and R. Rückl, Z. Phys. C **19**, 251 (1983).

[5]See, e.g., E. W. N. Glover, A. D. Martin, and W. J. Stirling, Z. Phys. C **28**, 473 (1988), and references therein.

[6]B. van Eijk and R. Kinnunen, Z. Phys. C **41**, 489 (1988).

[7]L. Bergström, R. W. Robinett, and L. Weinkauf, Phys. Rev. D **42**, 825 (1990).

[8]M. A. Doncheski and R. W. Robinett, Phys. Lett. B **248**, 188 (1990); R. W. Robinett, Phys. Rev. D **43**, 113 (1991).

[9]B. Humpert and P. Méry, Phys. Lett. B **124**, 265 (1983); Z. Phys. C **20**, 83 (1983); S. Heppelmann, R. W. Robinett, and P. Moxhay, Phys. Lett. B **233**, 245 (1989).

[10]L. Bergström and R. W. Robinett, Phys. Lett. B **238**, 112 (1990).

[11]B. Guberina, J. H. Kühn, R. Peccei, and R. Rückl, Nucl. Phys. **B174**, 317 (1980).

[12]K. J. Abraham, Z. Phys. C **44**, 467 (1989); V. Barger, K. Cheung, and W. Y. Keung, Phys. Rev. D **41**, 1541 (1990).

[13]B. Grzadkowski *et al.*, Nucl. Phys. **B281**, 18 (1987); V. Barger and W. Y. Keung, Wisconsin Report No. MAD/PH/370, 1987 (unpublished).

[14]S. Rudaz, Phys. Rev. Lett. **56**, 2128 (1986).

[15]L. Bergström and H. Snellman, Phys. Rev. D **37**, 3737 (1988).

[16]See, e.g., E. Menichetti, in *Proceedings of the First Workshop on Antimatter Physics at Low Energy,* Batavia, Illinois, 1986, edited by B. E. Bonner and L. S. Pinsky (Fermilab, Batavia, Illinois, 1986), p. 95.

[17]J. L. Rosner, in *Particles and Fields 3,* proceedings of the Banff Summer Institute (CAP) 1988, Banff, Alberta, 1988, edited by A. N. Kamal and F. C. Khanna (World Scientific, Singapore, 1989), p. 395. See also Ref. 1.

[18]V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtain, M. B. Voloshin, and V. I. Zakharov, Phys. Rep. **41C**, 1 (1978).

[19]G. Bélanger and P. Moxhay, Phys. Lett. B **199**, 575 (1987).

[20]W. Kwong and J. L. Rosner, Phys. Rev. D **38**, 279 (1988).

[21]H. Grotch, X. Zhang, and K. J. Sebastian, Phys. Rev. D **35**, 2900 (1987).

[22]P. Moxhay, Phys. Rev. D **37**, 2557 (1988); see also Y.-P. Kuang and T.-M. Yan, *ibid.* **41**, 155 (1990).

[23]P. Moxhay and J. L. Rosner, Phys. Rev. D **28**, 1132 (1983); see also Ref. 20; P. Moxhay (private communication).

[24]L. Durand III, Phys. Lett. **18**, 58 (1967); see also J. Hadamard, Nucl. Phys. **B38**, 350 (1972), for a thorough discussion of the polarization sums for massive spin-$J$ bosons.

[25]R. S. Fletcher, F. Halzen, and R. W. Robinett, Phys. Lett. B **225**, 176 (1989).

[26]V. L. Chernyak, A. A. Oglobin, and I. R. Zhitnitsky, Z. Phys. C **42**, 583 (1989), and extensive references therein.