Numerical study of cosmic no-hair conjecture: Formalism and linear analysis

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Three-dimensional numerical relativity is applied to an investigation of the so-called "cosmic no-hair conjecture," which has been proposed to explain the homogeneity and isotropy of the present Universe via the inflationary scenario. We present a general formalism to study this cosmological problem using numerical relativity. Following this formalism, we find general solutions of the linearized Einstein equation in the de Sitter background and discuss a homogenization mechanism in the linear regime.

I. INTRODUCTION

The standard model of the Universe, i.e., the big-bang scenario, naturally explains three important observational data of the Universe: Hubble's expansion law, the 3-K cosmic-microwave-background radiation, and abundances of light elements.[1] There are, however, several unanswered theoretical questions in the framework of the standard model. For example, we note the following.

(1) Why is our Universe so homogeneous in the global mean?

(2) Why does the cosmic background radiation from causally disconnected points have the same temperature?

(3) Why is the Universe so flat?

One may say that these facts are due to initial conditions. From a scientific point of view, however, we are not satisfied with this answer.[2] We would like to believe that irrespective of initial conditions the observed Universe should be as it is now. The inflationary model of the Universe may provide such a possibility. When vacuum energy, which is effectively equivalent to a cosmological constant Λ, becomes dominant, the Universe expands exponentially as $e^{H_0 t}$ ($H_0 \equiv \sqrt{\Lambda/3}$). If exponential expansion continues longer than $\sim 60 H_0^{-1}$, then we may solve the above homogeneity, horizon, and flatness problems.

When we discuss such problems, even for the explanation of the homogeneity and isotropy of our Universe, we usually start from homogeneous and isotropic conditions in considering the inflationary universe model. This is, however, in a sense, a tautology. Since the present isotropy and homogeneity cannot be found from any initially chaotic cosmological model with an ordinary matter fluid,[3] we have to show whether or not the inflationary stage really begins under *general* initial conditions. In connection with this subject, there is the so-called "cosmic no-hair conjecture": If a positive cosmological constant exists, all space-time approaches the de Sitter universe; hence, inflation is a unique attractor.[4] If this conjecture is true, inflation is a natural phenomenon in the Universe, because the vacuum energy existing before a cosmological phase transition, which we expect from particle physics, plays the role of Λ. Hence we can understand why our Universe acquired global isotropy and homogeneity.

The cosmic no-hair conjecture, however, is not perfectly correct. The Kerr–Newman–de Sitter space-time is one simple counterexample. One may also set N black holes in a de Sitter universe as initial data,[5] which will not attain homogeneity and isotropy.

On the other hand, if we restrict consideration to homogeneous space-times, then we can prove the cosmic no-hair theorem[6]: "All Bianchi models except for IX always approach isotropic de Sitter space-time within one Hubble expansion time H_0^{-1}." Even for type IX, the quantum cosmological approach may explain the present small anisotropy.[7] We expect that the cosmic no-hair conjecture is true under certain conditions. Hence it is important to know what additional conditions are needed for an initially inhomogeneous and anisotropic universe to approach the de Sitter universe.

We know the basic answer for homogeneous space-time from the above-mentioned cosmic no-hair theorem. So work in the inhomogeneous case is very important. To study inhomogeneous space-time, there are several approaches (both analytic and numerical). Starobinskii discussed the stability of de Sitter space-time against general perturbations, showing it is an attractor.[8] Sato *et al.* investigated a spherically symmetric space-time with two different cosmological constants, which is a typical example of inhomogeneous space-time, and found counterexamples to the conjecture.[9] Another analytic approach toward the cosmic no-hair conjecture is the so-called "inverse-scattering method," by which we can construct new solutions in nonlinear systems from known solutions. We can also apply the same method to Einstein gravity with two commuting Killing vectors.[10] Applying it to the de Sitter background, we find that initially nonlinear localized perturbations on de Sitter space-time become homogenized locally but not globally.[11]

A numerical approach is certainly needed as well, because we can never find all interesting solutions of the nonlinear Einstein equations analytically. So far, there are a few numerical works on universes with some symmetry. Kurki-Suonio *et al.*[12] studied the plane-symmetric case with an inflaton scalar field, but the initial inhomogeneity of space-time is too small to conclude whether or not the cosmic no-hair conjecture is true, or to find what additional conditions are necessary for it. Holcomb, Park, and Vishniac[13] investigated the spherically symmetric and asymptotically Friedmann case with an inflaton field and showed that an event horizon appears for the universe with initially large inhomogeneities of the scale smaller than the cosmological horizon. For the spherically symmetric closed universe, Goldwirth and Piran found that the large initial inhomogeneity prevents inflaton when its spatial scale is less than several horizon sizes.[14]

Although a spherically symmetric case might have a generic feature for inhomogeneous space-times with an inflaton field, they are, of course, restricted to a highly symmetric space. In order to conclude whether or not inflation is a natural phenomenon, however, we should consider more general space-time, three-dimensional space without any symmetry. Another reason why we proceed in our analysis to the nonspherically symmetric case is that gravitational waves, as well as matter fluid including an inflaton scalar field, can be responsible for inhomogeneity. We know that, for the case with asymptotically flat space, not only the matter field, but also nonlinear pure gravitational waves can collapse into a black hole if the nonlinearity is sufficiently large,[15] while cosmic expansion due to the cosmological constant has a homogenization mechanism both for the gravitational waves and for matter fields. The initial inhomogeneity in the early Universe might be due to gravitational waves as well as inhomogeneous matter distributions. Hence we should investigate whether inflation takes place and the space-time is homogenized not only for inhomogeneous matter fields, but also inhomogeneous nonlinear gravitational waves. The above spherically symmetric case, however, can never possess such gravitational effects. Again, we have to proceed to a more general case, at least to the nonspherically symmetric case.

Numerical study seems to be only the way to study the evolution of such a general three-dimensional space without any symmetry. Whether a numerical study of the cosmic no-hair conjecture can be performed depends upon computational power. At present, there exists a supercomputer with 3-GFLOPS speed and 500-Mbyte memory. Using such a machine, Nakamura and Oohara constructed a three-dimensional numerical code with a $(100)^3$ grid, by which one can solve the vacuum Einstein equations without any symmetry as a Cauchy's initial-value problem.[16] Time evolution of a localized wave packet[17] (i.e., Teukolsky waves) was used for a test of their code, and the results for weak gravitational waves agreed with the analytic solutions within a few percent accuracy.

We plan to use and extend their code to study the cosmic no-hair conjecture. This is the first one of a series of papers. We will initially study the effects of a gravitational field in the present problem rather than those of matter fluid. This work may be regarded as complementary of the above spherically symmetric one. Hence we consider the Einstein equations only with a cosmological constant, with the inhomogeneity produced just by gravitational waves. The main changes in code are to add a cosmological constant and treat asymptotically nonflat space-times.

The present paper will discuss only our formulation and general solutions of the linearized equations in a de Sitter background, and is organized as follows. In Sec. II we formulate our problem; then in Sec. III we give analytic solutions to the linearized gravitational waves in de Sitter space-time. Our conclusion and remarks are given in Sec. IV. In forthcoming papers we will discuss how to set up initial data, how to find an apparent horizon, and how to select a time slice, and the results of evolution of three-dimensional initial data.

In this paper we adopt units in which $G=1$. Our conventions for the Riemann and Ricci tensors are

$$\nabla_{[\mu}\nabla_{\nu]}\omega_\rho=\tfrac{1}{2}R_{\mu\nu\rho}{}^{\sigma}\omega_\sigma ,$$

$$R_{\mu\nu}=R_{\mu\rho\nu}{}^{\rho} .$$

II. CONFORMAL TRANSFORMATION AND BASIC EQUATIONS

We consider a space-time governed by the vacuum Einstein equations with a cosmological constant Λ as

$$G_{\mu\nu}=-\Lambda g_{\mu\nu} . \tag{2.1}$$

Instead of the original metric $g_{\mu\nu}$, we shall solve for the conformal metric $\tilde{g}_{\mu\nu}$, which is related to $g_{\mu\nu}$ by

$$g_{\mu\nu}=\Omega^2\tilde{g}_{\mu\nu} , \tag{2.2}$$

where Ω will be fixed later. Piran first introduced a similar approach,[18] in which three-dimensional conformal factor is factorized out (see below for the comparison with ours and his approach). The advantage of solving $\tilde{g}_{\mu\nu}$ instead of $g_{\mu\nu}$ will be discussed later. The Einstein equations of the space-time described by $\tilde{g}_{\mu\nu}$ can be written as

$$\tilde{G}_{\mu\nu}=-\Lambda\Omega^2\tilde{g}_{\mu\nu}+8\pi\tilde{T}_{\mu\nu} , \tag{2.3}$$

where

$$\tilde{T}_{\mu\nu} \equiv \frac{1}{4\pi}\left[\frac{1}{\Omega}(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Omega - \tilde{g}_{\mu\nu}\tilde{\nabla}_\rho\tilde{\nabla}^\rho\Omega) - \frac{2}{\Omega^2}\left[\tilde{\nabla}_\mu\Omega\tilde{\nabla}_\nu\Omega - \frac{1}{4}\tilde{g}_{\mu\nu}\tilde{\nabla}_\rho\Omega\tilde{\nabla}^\rho\Omega\right]\right], \quad (2.4)$$

and $\tilde{G}_{\mu\nu}$ and $\tilde{\nabla}_\mu$ are the Einstein tensor and covariant derivative with respect to $\tilde{g}_{\mu\nu}$.

We proceed to the well-known procedure of 3+1 decomposition of the Einstein equations. Now the external source of the gravitational field is the residual term $\tilde{T}_{\mu\nu}$ due to the conformal transformation (2.2).

Since we will work in the fictitious space-time with metric $\tilde{g}_{\mu\nu}$, the unit normal vector of our three-dimensional hypersurface $\tilde{n}^\mu$ should be defined by

$$\tilde{g}_{\mu\nu}\tilde{n}^\mu\tilde{n}^\nu = -1 . \quad (2.5)$$

We define the projection operator onto the three-dimensional hypersurface and its extrinsic curvature $\tilde{K}_{\mu\nu}$ by

$$\tilde{\gamma}_\mu^\nu = \delta_\mu^\nu + \tilde{n}_\mu\tilde{n}^\nu , \qquad \tilde{K}_{\mu\nu} = -\tfrac{1}{2}\mathcal{L}_{\tilde{n}}\tilde{\gamma}_{\mu\nu} , \quad (2.6)$$

where $\mathcal{L}_{\tilde{n}}$ is the Lie derivative along $\tilde{n}^\mu$. We decompose $\tilde{T}_{\mu\nu}$ into the energy density $\tilde{\rho}_H$, the momentum density $\tilde{J}^\mu$, and the stress tensor $\tilde{S}_{\mu\nu}$, as

$$\tilde{T}_{\mu\nu} = \tilde{n}_\mu\tilde{n}_\nu\tilde{\rho}_H + \tilde{n}_\mu\tilde{J}_\nu + \tilde{n}_\nu\tilde{J}_\mu + \tilde{\gamma}_\mu^\alpha\tilde{\gamma}_\nu^\beta\tilde{S}_{\alpha\beta} , \quad (2.7)$$

where

$$\tilde{\rho}_H \equiv \tilde{n}^\mu\tilde{n}^\nu\tilde{T}_{\mu\nu} = \frac{1}{8\pi}(2\Omega^{-1}\tilde{D}_\rho\tilde{D}^\rho\Omega + 2\Omega^{-1}\chi\tilde{K} - \Omega^{-2}\tilde{D}_\rho\Omega\tilde{D}^\rho\Omega - 3\Omega^{-2}\chi^2) , \quad (2.8)$$

$$\tilde{J}_\mu \equiv -\tilde{n}^\alpha\tilde{\gamma}_\mu^\beta\tilde{T}_{\alpha\beta} = -\frac{1}{8\pi}(2\Omega^{-1}\tilde{D}_\mu\chi + 2\Omega^{-1}\tilde{K}_\mu^\rho\tilde{D}_\rho\Omega - 4\Omega^{-2}\chi\tilde{D}_\mu\Omega) , \quad (2.9)$$

$$\tilde{S}_{\mu\nu} \equiv \tilde{\gamma}_\mu^\rho\tilde{\gamma}_\nu^\sigma\tilde{T}_{\rho\sigma} = \frac{1}{8\pi}[2\Omega^{-1}\tilde{D}_\mu\tilde{D}_\nu\Omega + 2\Omega^{-1}\chi\tilde{K}_{\mu\nu} - 4\Omega^{-2}(\tilde{D}_\mu\Omega)(\tilde{D}_\nu\Omega) - \tilde{\gamma}_{\mu\nu}(2\Omega^{-1}\tilde{D}_\rho\tilde{D}^\rho\Omega + 2\Omega^{-1}\chi\tilde{K} - 2\Omega^{-1}\tilde{\alpha}^{-1}(\mathcal{L}_\eta - \mathcal{L}_{\tilde{\beta}})\chi + 2\Omega^{-1}\tilde{D}_\rho(\ln\tilde{\alpha})\tilde{D}^\rho\Omega - \Omega^{-2}(\tilde{D}_\rho\Omega)(\tilde{D}^\rho\Omega) + \Omega^{-2}\chi^2)] , \quad (2.10)$$

with

$$\tilde{K} \equiv \tilde{\gamma}^{\mu\nu}\tilde{K}_{\mu\nu} , \quad (2.11)$$

and

$$\chi \equiv \mathcal{L}_{\tilde{n}}\Omega = \tilde{\alpha}^{-1}(\mathcal{L}_\eta - \mathcal{L}_{\tilde{\beta}})\Omega , \quad (2.12)$$

where $\mathcal{L}_\eta$ and $\mathcal{L}_{\tilde{\beta}}$ are the Lie derivatives along the time coordinate basis vector $\partial/\partial\eta$ and along the shift vector $\tilde{\beta}^\mu$, respectively, $\tilde{\alpha}$ is the lapse function, and $\tilde{D}_\mu$ is the covariant derivative with respect to $\tilde{\gamma}_{\mu\nu}$.

The constraint equations can then be written as

$$^{(3)}\tilde{R} - \tilde{K}_{\mu\nu}\tilde{K}^{\mu\nu} + \tilde{K}^2 = 16\pi\tilde{\rho}_H + 2\Lambda\Omega^2 , \quad (2.13)$$

$$\tilde{D}_\nu(\tilde{K}^{\mu\nu} - \tilde{\gamma}^{\mu\nu}\tilde{K}) = 8\pi\tilde{J}^\mu , \quad (2.14)$$

where $^{(3)}\tilde{R}$ is the scalar curvature of the three-dimensional hypersurface. The evolution equations are expressed into the usual form

$$\mathcal{L}_\eta\tilde{\gamma}_{\mu\nu} = -2\tilde{\alpha}\tilde{K}_{\mu\nu} + \mathcal{L}_{\tilde{\beta}}\tilde{\gamma}_{\mu\nu} , \quad (2.15)$$

$$\mathcal{L}_\eta\tilde{K}_{\mu\nu} = -\tilde{D}_\mu\tilde{D}_\nu\tilde{\alpha} + \tilde{\alpha}[^{(3)}\tilde{R}_{\mu\nu} + \tilde{K}\tilde{K}_{\mu\nu} - 2\tilde{K}_\mu^\rho\tilde{K}_{\rho\nu} - \Lambda\Omega^2\tilde{\gamma}_{\mu\nu} - 8\pi(\tilde{S}_{\mu\nu} - \tfrac{1}{2}\tilde{\gamma}_{\mu\nu}\tilde{S}_\rho^\rho) - 4\pi\tilde{\gamma}_{\mu\nu}\tilde{\rho}_H] + \mathcal{L}_{\tilde{\beta}}\tilde{K}_{\mu\nu} , \quad (2.16)$$

where $^{(3)}\tilde{R}_{\mu\nu}$ is the Ricci tensor of our three-dimensional hypersurface.

In order to solve the above equations numerically, we also need boundary conditions. Since periodic boundary conditions are not trivial except for three-torus topology, we consider only three-dimensional hypersurfaces, which become the expanding Friedmann-Robertson-Walker (FRW) or de Sitter universes in spatially asymptotic regions. Then we can regard the asymptotic FRW space-time as the "background" metric and fix Ω by the asymptotic condition $\Omega \to a(\eta)$, where $a(\eta)$ is the scale factor of the background FRW universe. Although Ω is so far completely free except for the asymptotic value, we have two simple choices: One is just to set $\Omega = a(\eta)$, and the other is to use the condition $\chi/\Omega^2 = \sqrt{\Lambda/3}$ or, equivalently,

$$\frac{\partial\Omega}{\partial\eta} = \sqrt{\Lambda/3}\tilde{\alpha}\Omega^2 + \tilde{\beta}^\rho\tilde{D}_\rho\Omega . \quad (2.17)$$

Since this condition turns out to be $(d\Omega/d\eta)/\Omega^2 \equiv (d\Omega/dt)/\Omega = \sqrt{\Lambda/3}$ for the homogeneous case or in the asymptotic region, with t the cosmic time, this Ω also naively corresponds to the scale factor of the FRW background.

If we choose the former case, the spatial derivative of Ω vanishes, simplifying the energy-momentum tensor

$\tilde{T}_{\mu\nu}$. The basic equations, hence, are also simple. In particular, the initial-value equations (Hamiltonian and momentum constraints) with $\tilde{K}=0$ are exactly the same as in the vacuum case. The merit of the latter case is that the equation determining Ω is given as a covariant form. Both conditions become the same when $\tilde{\alpha}=1$ and $\tilde{\beta}_i=0$ are imposed. Which of these two, or even a different choice, is better depends on the particular problem.

Now the advantage of our method is clear. We can impose the ordinary asymptotic flatness condition for the fictitious space-time if the asymptotic FRW space-time is spatially flat. Even for spatially nonflat space-times, we can impose static boundary conditions.

As for time slicing, since we treat circumstances in which gravitational collapse may occur, in general it is not appropriate for a numerical study to set the lapse function $\tilde{\alpha}$ to be unity.[19] Rather, it seems better to assume the maximal time slicing condition $\tilde{K}=0$ for the fictitious space-time.[20] Here note that this condition corresponds to the constant mean curvature slicing $K=-\sqrt{3\Lambda}$ in the original space-time [see Eq. (B6) in Appendix B].[21] Then the lapse function $\tilde{\alpha}$ is determined by

$$\tilde{D}_m\tilde{D}^m\tilde{\alpha}-\tilde{\alpha}\tilde{K}_l^m\tilde{K}_m^l = -\Omega^{-1}\tilde{\alpha}\tilde{D}_m\tilde{D}^m\Omega-3\Omega^{-1}(\tilde{D}_m\tilde{\alpha})(\tilde{D}^m\Omega) -\Omega^{-2}\tilde{\alpha}(\tilde{D}_m\Omega)(\tilde{D}^m\Omega) \ . \tag{2.18}$$

Instead of factorization of the four-dimensional conformal factor, Piran defined the conformal factor by

$$\gamma_{ij}=R^2(t,x^i)\tilde{\gamma}_{ij} \ , \tag{2.19}$$

for three-dimensional metric γ_{ij}, where $\det\tilde{\gamma}_{ij}$ is set to unity or an appropriate function with respect to the spatial coordinate. If the lapse α is redefined as $\alpha=R\tilde{\alpha}$ in his approach, we find the same factorization as ours with $\Omega=R$. The difference between his and ours is that we first fix Ω in order to find simpler basic equations, then give gauge conditions in $\tilde{g}$-space, while in Piran's formalism R is determined by the basic equation, since $\det\tilde{\gamma}_{ij}$ is first fixed. There is no freedom to choose an appropriate function for the conformal factor R in his approach. Of course, in some specific cases both approaches become the same, but our case has more freedom.

III. GENERAL SOLUTION OF LINEARIZED PURE GRAVITATIONAL WAVES IN DE SITTER SPACE-TIME

In order to see how cosmic expansion homogenizes inhomogeneous gravitational waves, we analyze in this paper the linearized equations of (2.13)–(2.16). It may also give us some insight for nonlinear gravitational waves as will be discussed in Sec. IV. In the following section we denote fictitious conformally transformed geometrical quantities by those without a tilde. The solutions correspond to linearized gravitational waves in a de Sitter universe.

We adopt the gauge condition that the lapse function is unity and the shift vector vanishes. For a de Sitter universe the conformal factor becomes as

$$\Omega=-1/H\eta \ , \tag{3.1}$$

with

$$H=\sqrt{\Lambda/3} \ , \tag{3.2}$$

where η is the conformal time belonging to the interval $[-\infty,0]$. The metric tensor of our three-dimensional hypersurface γ_{ij} is expressed as

$$\gamma_{ij}={}^{(B)}\gamma_{ij}+h_{ij} \quad (i,j=1,2,3) \ , \tag{3.3}$$

where ${}^{(B)}\gamma_{ij}$ is the background flat metric. We next introduce the variable

$$k_{ij}=(H\eta)^{-1}K_{ij} \ , \tag{3.4}$$

which is a quantity of $O(h)$. Then the linearized Einstein equations become

$$D_jk^{ij}=0 \ , \tag{3.5}$$

$$\frac{\partial h_{ij}}{\partial\eta}=-2H\eta k_{ij} \ , \tag{3.6}$$

$$\frac{\partial k_{ij}}{\partial\eta}=-\frac{1}{2H\eta}\Delta h_{ij}+\frac{1}{\eta}k_{ij} \ , \tag{3.7}$$

and the gauge-fixing conditions, which are consistent with the above equations, are

$$D_jh^{ij}=h_m^m=k_m^m=0 \ . \tag{3.8}$$

The Hamiltonian constraint becomes trivial if Eq. (3.8) is satisfied. Here note that the time derivative of Eq. (3.7) becomes

$$\frac{\partial^2k_{ij}}{\partial\eta^2}-\Delta k_{ij}=0 \ , \tag{3.9}$$

which is the same as that for K_{ij} in Minkowski space-time with the same coordinate conditions.[22] Expanding h_{ij} and k_{ij} with the tensor harmonics defined by Zerilli[23] (see Appendix A) as

$$h_{ij}=\sum_{L,M}[a_{LM}(\theta,\phi)A_{LM}^h(r,\eta)+b_{LM}(\theta,\phi)B_{LM}^h(r,\eta) +g_{LM}(\theta,\phi)G_{LM}^h(r,\eta)+f_{LM}(\theta,\phi)F_{LM}^h(r,\eta) +c_{LM}(\theta,\phi)C_{LM}^h(r,\eta)+d_{LM}(\theta,\phi)D_{LM}^h(r,\eta)] \ , \tag{3.10}$$

$$k_{ij}=\sum_{L,M}[a_{LM}(\theta,\phi)A_{LM}^k(r,\eta)+b_{LM}(\theta,\phi)B_{LM}^k(r,\eta) +g_{LM}(\theta,\phi)G_{LM}^k(r,\eta)+f_{LM}(\theta,\phi)F_{LM}^k(r,\eta) +c_{LM}(\theta,\phi)C_{LM}^k(r,\eta)+d_{LM}(\theta,\phi)D_{LM}^k(r,\eta)] \ , \tag{3.11}$$

we obtain the following solutions for the even-parity mode:

$$A^k_{LM}(r,\eta)=r^{L-2}\left[\frac{1}{r}\frac{d}{dr}\right]^L\frac{P_{LM}(\eta-r)+Q_{LM}(\eta+r)}{r}\ , \tag{3.12}$$

$$G^k_{LM}=-\frac{r^2}{2}A^k_{LM}\ , \tag{3.13}$$

$$B^k_{LM}=\frac{1}{\lambda_L}\frac{1}{r}\frac{\partial}{\partial r}r^3A^k_{LM}\ , \tag{3.14}$$

and

$$F^k_{LM}=\frac{1}{\lambda_L-2}\left[G^k_{LM}+\frac{\partial}{\partial r}r\frac{1}{\lambda_L}\frac{\partial}{\partial r}r^3A^k_{LM}\right]\ . \tag{3.15}$$

For the odd-parity mode we find

$$C^k_{LM}(r,\eta)=r^L\left[\frac{1}{r}\frac{d}{dr}\right]^L\frac{R_{LM}(\eta-r)+S_{LM}(\eta+r)}{r}\ , \tag{3.16}$$

$$D^k_{LM}=\frac{1}{2-\lambda_L}\frac{\partial}{\partial r}r^2C^k_{LM}\ , \tag{3.17}$$

and

$$\lambda_L=L(L+1)\ .$$

Here P_{LM}, Q_{LM}, R_{LM}, and S_{LM} are arbitrary functions constrained by the relations. Since A^k_{LM} and C^k_{LM} should be regular at the origin $r=0$, we find $Q_{LM}(x)=-P_{LM}(x)$ and $S_{LM}(x)=-R_{LM}(x)$.

Integrating Eq. (3.6) for the coefficients of the even-parity modes A^h_{LM}, B^h_{LM}, G^h_{LM}, and F^h_{LM} and the odd-parity modes C^h_{LM} and D^h_{LM} of the metric perturbation, we obtain metric perturbations as

$$\{\Pi^h_{LM}(r,\eta)\}=-\int_{\eta_0}^{\eta}2H\eta\{\Pi^k_{LM}(r,\eta)\}d\eta+\{\Pi^{(0)h}_{LM}(r)\}\ , \tag{3.18}$$

where we denote the expansion coefficients of h_{ij} and k_{ij}, $\{A^h_{LM},B^h_{LM},\dots\}$ and $\{A^k_{LM},B^k_{LM},\dots\}$, and $\{\Pi^h_{LM}\}$ and $\{\Pi^k_{LM}\}$, respectively, and $\{\Pi^{(0)h}_{LM}\}$ are the initial data on the three-dimensional hypersurface labeled by $\eta=\eta_0$ which satisfy the gauge-fixing condition (3.8). From $h^m_m=0$, we have

$$A^{(0)h}_{LM}+\frac{2G^{(0)h}_{LM}}{r^2}=0\ . \tag{3.19}$$

From the transversality condition $D_jh^{ij}=0$, we obtain

$$\frac{d}{dr}A^{(0)h}_{LM}+\frac{3}{r}A^{(0)h}_{LM}=\lambda_L\frac{B^{(0)h}_{LM}}{r^2}\ , \tag{3.20}$$

$$\frac{d}{dr}B^{(0)h}_{LM}+\frac{2}{r}B^{(0)h}_{LM}+\frac{G^{(0)h}_{LM}}{r^2}+(2-\lambda_L)\frac{F^{(0)h}_{LM}}{r^2}=0\ , \tag{3.21}$$

and

$$\frac{d}{dr}C^{(0)h}_{LM}+\frac{2}{r}C^{(0)h}_{LM}+(2-\lambda_L)\frac{D^{(0)h}_{LM}}{r^2}=0\ . \tag{3.22}$$

$\{\Pi^k_{LM}\}$ contains two arbitrary functions P_{LM} and R_{LM}, while $\{\Pi^h_{LM}\}$ have six arbitrary initial functions $\{\Pi^{(0)h}_{LM}\}$ with the four relations (3.19)–(3.22). Hence we have four arbitrary functions which correspond to the degrees of freedom of gravitational waves: two for the even-parity mode, P_{LM} and $A^{(0)h}_{LM}$, and two for the odd-parity mode, R_{LM} and $C^{(0)h}_{LM}$. (Remember that we have adopted the first-order 3+1 formalism.)

Note that if we wish to impose the boundary condition of asymptotically approaching de Sitter space-time, we merely demand the asymptotic flatness of the three-space described by γ_{ij}.

In order to observe the behavior of the above solution, we consider the even-parity mode of $L=2$ and $M=\pm2$ with P_{LM} and Q_{LM} as

$$P_{LM}(\eta-r)=-\frac{A}{2r_0}(r-\eta')\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]\ ,$$
$$Q_{LM}(\eta+r)=-\frac{A}{2r_0}(r+\eta')\exp\left[-\frac{(r+\eta')^2}{2r_0^2}\right]\ , \tag{3.23}$$

where A and r_0 express the amplitude and scale length of the gravitational waves, respectively, and $\eta'=\eta+H^{-1}$. We obtain $\{\Pi^k_{LM}\}$ as

$$A^k_{LM}=-\frac{2}{r^2}G^k_{LM}$$
$$=-\frac{A}{2r_0^5r^5}\left[[r^2(r-\eta')^3+3r_0^2r(r-\eta')^2-3r_0^2(r^2-r_0^2)(r-\eta')-3r_0^4r]\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]+(\eta'\to-\eta')\right]\ , \tag{3.24}$$

$$B^k_{LM}=\frac{A}{12r_0^7r^4}\Bigg[[r^3(r-\eta')^4+3r_0^2r^2(r-\eta')^3+6r_0^2r(r_0^2-r^2)(r-\eta')^2$$
$$+3r_0^4(2r_0^2-3r^2)(r-\eta')+3r_0^4r(r^2-2r_0^2)]\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]+(\eta'\to-\eta')\Bigg]\ , \tag{3.25}$$

and

$$F_{LM}^{k}=-\frac{A}{48r_0^9r^3}\left[[r^4(r-\eta')^5+2r_0^2r^3(r-\eta')^4+r_0^2r^2(3r_0^2-10r^2)(r-\eta')^3+3r_0^4r(r_0^2-4r^2)(r-\eta')^2+3r_0^4(r_0^4-3r_0^2r^2+5r^4)(r-\eta')+3r_0^6r(2r^2-r_0^2)]\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]+(\eta'\to-\eta')\right] . \quad (3.26)$$

Then, setting $\{\Pi_{LM}^h=0\}$ at $\eta=-\infty$, we obtain $\{\Pi_{LM}^h\}$ as

$$A_{LM}^h=-\frac{2}{r^2}G_{LM}^h$$
$$=-\frac{HA}{r_0^3r^5}\left[[r^2(r-\eta')^3-r(r^2-3r_0^2-H^{-1}r)(r-\eta')^2-3r_0^2(r^2-r_0^2-H^{-1}r)(r-\eta')+r_0^2(r^3-H^{-1}r^2+3H^{-1}r_0^2)]\times\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]-3\sqrt{\pi/2}r_0^5\operatorname{erf}\left[\frac{r-\eta'}{\sqrt{2}r_0}\right]+(\eta'\to-\eta' \text{ and } H^{-1}\to-H^{-1})\right] , \quad (3.27)$$

$$B_{LM}^h=\frac{HA}{6r_0^5r^4}\left[[r^3(r-\eta')^4-r^2(r^2-3r_0^2-H^{-1}r)(r-\eta')^3-r_0^2r(5r^2-6r_0^2-3H^{-1}r)(r-\eta')^2+3r_0^2(r^4-2r_0^2r^2+2r_0^4-H^{-1}r^3-2H^{-1}r_0^2r)(r-\eta')+r_0^4(2r^3-3H^{-1}r^2+6H^{-1}r_0^2)]\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]-6\sqrt{\pi/2}r_0^7\operatorname{erf}\left[\frac{r-\eta'}{\sqrt{2}r_0}\right]+(\eta'\to-\eta' \text{ and } H^{-1}\to-H^{-1})\right] , \quad (3.28)$$

and

$$F_{LM}^h=-\frac{HA}{24r_0^7r^3}\left[[r^4(r-\eta')^5-r^3(r^2-2r_0^2-H^{-1}r)(r-\eta')^4-r_0^2r^2(7r^2-3r_0^2-2H^{-1}r)(r-\eta')^3+r_0^2r(6r^4-7r_0^2r^2+3r_0^4-6H^{-1}r^3+3H^{-1}r_0^2r)(r-\eta')^2+3r_0^4(2r^4-r_0^2r^2+r_0^4-2H^{-1}r^3+H^{-1}r_0^2r)(r-\eta')-r_0^4(3r^5-r_0^2r^3-3H^{-1}r^4+3H^{-1}r_0^2r^2-3H^{-1}r_0^4)]\times\exp\left[-\frac{(r-\eta')^2}{2r_0^2}\right]-3\sqrt{\pi/2}r_0^9\operatorname{erf}\left[\frac{r-\eta'}{\sqrt{2}r_0}\right]+(\eta'\to-\eta' \text{ and } H^{-1}\to-H^{-1})\right] . \quad (3.29)$$

First, we show, in Figs. 1(a) and 1(b), the time evolution of the extrinsic curvature K_{ij} for various scales of inhomogeneities ($r_0=(2H)^{-1}$, $(4H)^{-1}$). It can be seen from the figures that K_{ij} vanishes in one expansion time H^{-1}. Here one expansion time is measured by the cosmic time $t\equiv\int^{\eta}\Omega(\eta)d\eta=-H^{-1}\ln(-H\eta)$. We can also easily see this behavior from the basic equations as follows. From Eq. (3.4) and the definition of t, $K_{ij}=\exp(-Ht)k_{ij}$. Since k_{ij} satisfies the ordinary wave equation in Minkowski space-time [Eq. (3.9)], a localized wave packet of k_{ij} propagates to spatial infinity without divergence. Hence K_{ij} vanishes exponentially within $t\sim H^{-1}$. We can understand this homogenization mechanism in a de Sitter space-time as follows. Rewriting the basis equations (3.6) and (3.7), we find

$$\frac{\partial h_{ij}}{\partial\eta}=-2K_{ij} , \quad (3.30)$$

$$\frac{\partial K_{ij}}{\partial\eta}=-\frac{1}{2}\Delta h_{ij}+\frac{2K_{ij}}{\eta} . \quad (3.31)$$

This is just the first-order form of an ordinary wave equation except for the last term in Eq. (3.31). This last term describes the effect of background expansion of the Universe and behaves as a viscosity because $\eta<0$. This viscosity term diverges when the Universe evolves into future infinity, corresponding to $t=\infty$ or $\eta=0$. That is why K_{ij} vanishes so soon. The quantities described by K_{ij} and its derivatives, such as the Newman-Penrose variables Ψ_0 and Ψ_4,[24] have the same behavior. This is the homogenization mechanism in de Sitter background in the linear regime.

As for the metric perturbations, we show the time evolutions of h_{rr} in Figs. 2(a) and 2(b). Although h_{rr} seems to disappear within one expansion time H^{-1} by spherical damping from Fig. 2, we can show analytically that it survives at the future infinity. However, the values of h_{ij} themselves have no physical meaning because of the freedom of general coordinate transformations. Instead, we can examine the behavior of the three-space Riemann curvature ${}^{(3)}R_{ijkl}$. Any three-dimensional Riemann curvature is always described by the Ricci and scalar curva-

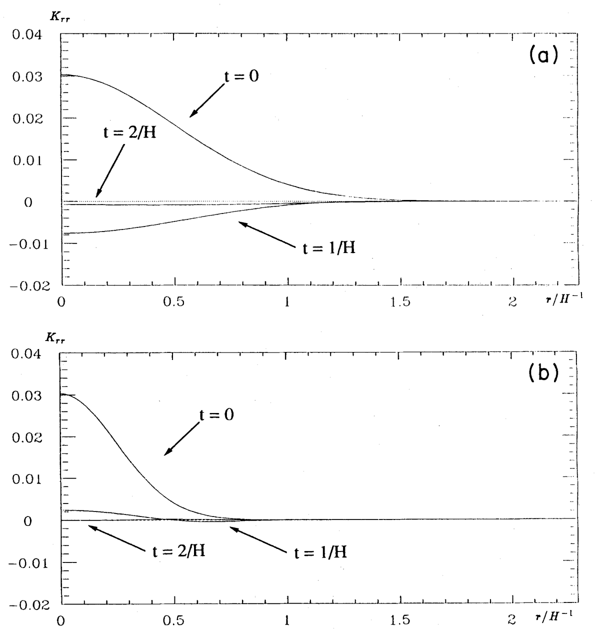

FIG. 1. Time evolution of the r-r component of the extrinsic curvature K_{rr} on $\theta=\pi/2$, $\varphi=0$. (a) depicts the case with the scale of the inhomogeneity $r_0=0.5H^{-1}$, and (b) is that with $r_0=0.25H^{-1}$. The amplitude A is taken to be $0.1r_0^5$ for later comparison.

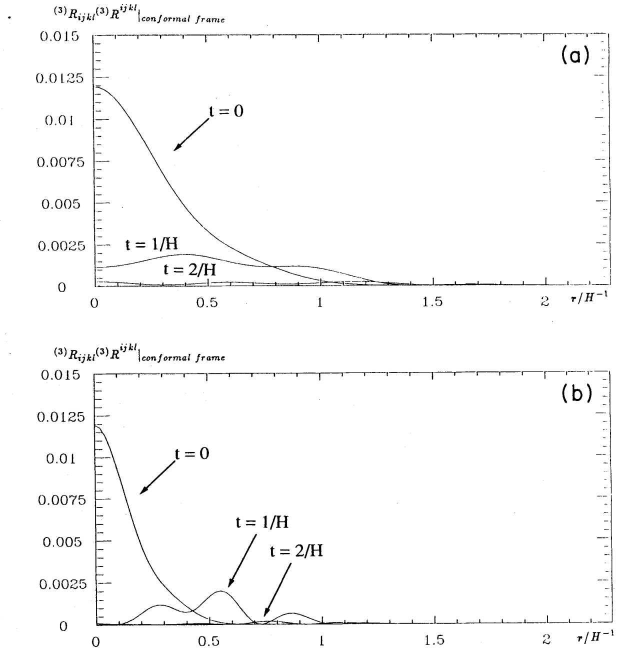

FIG. 3. Time evolution of the three-dimensional Riemann invariant on $\theta=\pi/2$, $\varphi=0$ in the conformally transformed space. (a) depicts the case with the scale of the inhomogeneity $r_0=0.5H^{-1}$, and (b) is the case of $r_0=0.25H^{-1}$. The amplitude A is equal to $0.1r_0^5$.

tures. Through the equations of motion

$$^{(3)}R_{ij}=\frac{\partial K_{ij}}{\partial\eta}-\frac{2}{\eta}K_{ij}\ , \tag{3.32}$$

$^{(3)}R_{ij}$ asymptotically approaches $-Hk_{ij}(\eta=0)$, which is finite. Hence $^{(3)}R_{ijkl}$ does not vanish at future infinity,

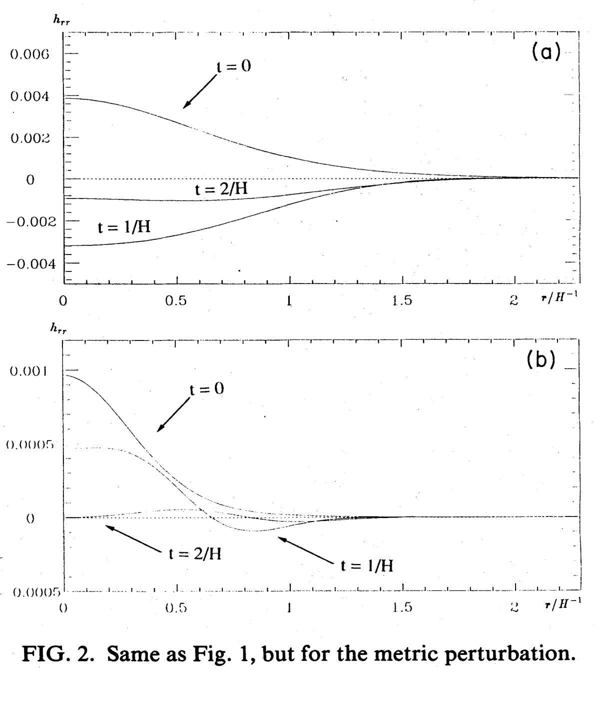

FIG. 2. Same as Fig. 1, but for the metric perturbation.

and then the metric perturbations of the conformally transformed three-space never disappear.

However, we should remember, as discussed by Boucher and Gibbons, that the metric perturbations $\{\Pi^h_{LM}\}$ and the three-curvature $^{(3)}R_{ijkl}$ surviving at future infinity are not a counterexample of the cosmic no-hair conjecture,[25] because for any timelike observer the cosmological event horizon exists in de Sitter background, and by use of our expanding universe frame, its circumference shrinks infinitesimally in proportion to $\Omega^{-1}=e^{-Ht}$. Hence the wavelength of the metric perturbations becomes longer than the scale of the event horizon, and consequently the observer finds null metric perturbations. We can also show

$$^{(3)}R^{ij}{}_{kl}|_{\text{physical space}}=\Omega^{-2}\,{}^{(3)}R^{ij}{}_{kl}|_{\text{conformal frame}}\ . \tag{3.33}$$

In Figs. 3(a) and 3(b), we depict the time evolution of the three-dimensional Riemann invariant $^{(3)}R_{ijkl}{}^{(3)}R^{ijkl}$ for the conformally transformed frame. Though the perturbation survives, its amplitude becomes very small within a few expansion times due to spherical damping. Thus, from the above equation, we can show that in physical space the three-dimensional Riemann invariant almost vanishes within one expansion time.

IV. CONCLUSIONS AND REMARKS

We have presented a formalism to treat a three-dimensional universe without any symmetry in the context of numerical relativity. The factorization of the metric into a scalar portion and a new conformal metric has been proposed. One advantage is that the boundary conditions become simple. In particular, for a universe with an asymptotically flat three-space, we can impose the same boundary conditions as in the case of gravitational collapse. We have also found general solutions of the linearized Einstein equations in a de Sitter background and discussed a mechanism of homogenization of inhomogeneous space-time.

We would also like to remark briefly about the nonlinear effects, which will be the main purpose of our numerical study. The homogenization mechanism in the linear regime can be understood from Eq. (3.31). So, to know whether or not an inhomogeneous universe is homogenized, we may also look at the same equation without the linear approximation, which is

$$\frac{\partial K_{ij}}{\partial \eta} = {}^{(3)}R_{ij} + KK_{ij} - 2K_i^{\ m}K_{mj} + \frac{2}{\eta}K_{ij} \ . \tag{4.1}$$

The last term is from the background expansion, which has the same homogenization effect as in the linear case. For the linearized pure gravitational waves, this "viscosity" term overcomes the growth rate of the metric perturbations, always resulting in homogeneous de Sitter space-time. However, if nonlinear effects dominate, the situation becomes different. In the asymptotically flat space-time, the gravitational waves can grow and collapse to black holes through this nonlinear interaction.[15] Furthermore, our case also has the last term in Eq. (4.1), which may suppress the deviation from the homogeneous background de Sitter space-time. So we have two competing processes; one is a nonlinear effect of gravity, by which the compact system becomes more preferable, and the other is the above homogenization effect. Because the role of each term is very clear in our description of the system, we expect to find some criterion for the cosmic no-hair conjecture, which may be another advantage of our formalism.

This formalism can be easily extended into the more general case with matter fluid, including an inflaton scalar field. We will also discuss this case elsewhere. In a future paper we will describe how to construct initial data in the present approach and how to find the apparent horizon.

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APPENDIX A

The tensor harmonics defined by Zerilli are

$$a_{LM} = \begin{bmatrix} Y_{LM} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \tag{A1}$$

$$b_{LM} = \begin{bmatrix} 0 & \partial Y_{LM}/\partial\theta & \partial Y_{LM}/\partial\varphi \\ * & 0 & 0 \\ * & 0 & 0 \end{bmatrix} , \tag{A2}$$

$$c_{LM} = \begin{bmatrix} 0 & (1/\sin\theta)\partial Y_{LM}/\partial\varphi & -\sin\theta\partial Y_{LM}/\partial\theta \\ * & 0 & 0 \\ * & 0 & 0 \end{bmatrix} , \tag{A3}$$

$$d_{LM} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -X_{LM}/\sin\theta & \sin\theta W_{LM} \\ 0 & * & \sin\theta X_{LM} \end{bmatrix} , \tag{A4}$$

$$g_{LM} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sin^2\theta \end{bmatrix} Y_{LM} , \tag{A5}$$

$$f_{LM} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & W_{LM} & X_{LM} \\ 0 & * & -\sin^2\theta W_{LM} \end{bmatrix} , \tag{A6}$$

$$X_{LM} = 2\frac{\partial}{\partial\varphi}\left[\frac{\partial}{\partial\theta} - \cos\theta\right] Y_{LM} , \tag{A7}$$

and

$$W_{LM} = \left[\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] Y_{LM} \ . \tag{A8}$$

The asterisk denotes components derived from symmetry of the tensors. These tensors are orthogonal to each other with respect to the inner product

$$(T,S) \equiv \int \gamma^{(B)ij}\gamma^{(B)kl} T_{ik}^{*} S_{jl}\, d\Omega \ . \tag{A9}$$

Therefore, because of the above orthogonality, we can consider each L and M mode independently in the case of the linearized theory as used in Sec. III.

APPENDIX B

The relations between the conformal transformed quantities and the original true quantities are as follows.

The lapse function and the shift vector are

$$\alpha = \Omega\tilde{\alpha} , \tag{B1}$$

$$\beta^l = \tilde{\beta}^l \ . \tag{B2}$$

The hypersurface unit normal vector is

$$n^\mu = \Omega^{-1}\tilde{n}^\mu \ . \tag{B3}$$

The intrinsic metric and the extrinsic curvature are

$$\gamma_{lm} = \Omega^2\tilde{\gamma}_{lm} , \tag{B4}$$

$$K_{lm} = \Omega\tilde{K}_{lm} - \chi\tilde{\gamma}_{lm} , \tag{B5}$$

$$K = \Omega^{-1}\tilde{K} - 3\Omega^{-2}\chi \ . \tag{B6}$$

APPENDIX C

We present another method to obtain the solution of the linearized Einstein equations in a de Sitter background in a closed form. We abbreviate a tilde in denoting conformally transformed geometrical quantities. From Eq. (3.9), the monotonic wave

$$k_{ij}=\kappa_{ij}\exp[i(\mathbf{k}\cdot\mathbf{x}-\omega\eta)] \tag{C1}$$

is the solution. In order to guarantee the transverse-traceless nature of the solution, we use the polar coordinate in $\mathbf{k}$ space as $\mathbf{k}=(k,\theta_k,\varphi_k)$. Then, in the coordinate system (x',y',z'), in which the z' direction coincides with $\mathbf{k}$, the solution k'_{ij} is expressed as

$$(k'_{ij})=\begin{bmatrix}\kappa_{(+)} & \kappa_{(\times)} & 0\\ \kappa_{(\times)} & -\kappa_{(+)} & 0\\ 0 & 0 & 0\end{bmatrix}\exp[i(kr\cos\Theta-\omega\eta)] , \tag{C2}$$

where $\kappa_{(+)}$ and $\kappa_{(\times)}$ are arbitrary constants and

$$\cos\Theta=\cos\theta\cos\theta_k+\sin\theta\sin\theta_k\cos(\varphi-\varphi_k) . \tag{C3}$$

Using the rotation matrix

$$\mathcal{R}^j{}_i=\begin{bmatrix}\cos\varphi_k & \sin\varphi_k & 0\\ -\cos\theta_k\sin\varphi_k & \cos\theta_k\cos\varphi_k & \sin\theta_k\\ \sin\theta_k\sin\varphi_k & -\sin\theta_k\sin\varphi_k & \cos\theta_k\end{bmatrix} , \tag{C4}$$

k_{ij} becomes

$$k_{ij}=(\mathcal{R}^{-1})^l_i k'_{lm}(\mathcal{R})^m{}_j , \tag{C5}$$

and simple calculation yields

$$k_{ij}=(\kappa_{ij}^{(+)}+\kappa_{ij}^{(\times)})\exp[i(kr\cos\Theta-\omega\eta)] , \tag{C6}$$

where

$$\kappa_{ij}^{(+)}=\kappa_{(+)}\begin{bmatrix}\cos^2\varphi_k-\cos^2\theta_k\sin^2\varphi_k & (1+\cos^2\theta_k)\sin\varphi_k\cos\varphi_k & \sin\theta_k\cos\theta_k\sin\varphi_k\\ * & \sin^2\varphi_k-\cos^2\theta_k\cos^2\varphi_k & -\sin\theta_k\cos\theta_k\cos\varphi_k\\ * & * & -\sin^2\theta_k\end{bmatrix} , \tag{C7}$$

and

$$\kappa_{ij}^{(\times)}=\kappa_{(\times)}\begin{bmatrix}-\sin2\varphi_k\cos\theta_k & \cos\varphi_k\cos\theta_k & \cos\varphi_k\sin\theta_k\\ * & \sin2\varphi_k\cos\theta_k & \sin\varphi_k\sin\theta_k\\ * & * & 0\end{bmatrix} . \tag{C8}$$

Then the general solution is expressed as

$$k_{ij}=\int\{f^{(+)}(\mathbf{k})\kappa_{ij}^{(+)}\exp[ik(r\cos\Theta-\eta)+i\varphi_{\mathbf{k}}^{(+)}]+f^{(\times)}(\mathbf{k})\kappa_{ij}^{(\times)}\exp[ik(r\cos\Theta-\eta)+i\varphi_{\mathbf{k}}^{(\times)}]\}d^3\mathbf{k} , \tag{C9}$$

where $f^{(+)}(\mathbf{k})$ and $f^{(\times)}(\mathbf{k})$ are amplitudes, and $\varphi_{\mathbf{k}}^{(+)}$ and $\varphi_{\mathbf{k}}^{(\times)}$ phases of each $\mathbf{k}$ mode. The metric perturbations are again obtained for the above solution through Eq. (3.18).

To see the asymptotic behavior of h_{ij}, we use the relation

$$\exp(ikr\cos\Theta)=\sum_{l=0}^{\infty}(2l+1)i^l j_l(kr)P_l(\cos\Theta) , \tag{C10}$$

and

$$P_l(\cos\Theta)=P_l(\cos\theta)P_l(\cos\theta_k)+2\sum_{m=-1}^{l}\frac{(l-m)!}{(l+m)!}P_l^m(\cos\theta)\cos m(\varphi-\varphi_{k'}) , \tag{C11}$$

where j_l is the spherical Bessel function. Since $j_l(kr)\propto\cos[kr-(l+1)\pi/2]/kr$ for $r\to\infty$, $k_{ij}\propto 1/r$ for $r\to\infty$ in general. To make the perturbation localized, a possible choice of the amplitude $f(\mathbf{k})$ is the Gaussian $\propto\exp(-\mathbf{k}^2/2k_0^2)$ with random phase φ_k, for example.

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