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Why current algebra and PCAC are applicable for charmed-meson and -baryon weak decays

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By accounting for rapidly varying pole terms, the current-algebra–PCAC (partial conservation of axial-vector current) procedure properly measures large decay momentum corrections in charmed $D^+ \to \bar{K}^0\pi^+$, $D^+ \to \bar{K}^0K^+$, and $\Lambda_c^+ \to p\bar{K}^0$ weak decays.

It is usually assumed that the current-algebra–PCAC (partially conserved axial-vector current) procedure cannot be applied to charmed-meson and -baryon weak decays because the momentum of the decaying pion is then in the relativistic region beyond the realm of soft-pion techniques. However, even for $K \to \pi\pi$ decays the decaying momentum is $p \sim 200$ MeV, which is large scaled to the pion mass of 140 MeV. Before studying charmed decays we review $K \to \pi\pi$ but allow for a large momentum variation away from the soft-pion limit.

More specifically it is known that current algebra and PCAC can be applied to $K \to 2\pi$ provided one accounts for the rapidly varying tadpole terms[1,2] $M_P + \bar{M}$ for a background amplitude $\bar{M}$ which is smoothly varying with pion momenta. This formally leads to the amplitude structure

$$M = M_{CC} + M_P - M_P(0) , \tag{1}$$

where M_{CC} is the soft-pion current commutator, M_P is the on-mass-shell pole amplitude, and $M_P(0)$ is the pole amplitude when one pion becomes soft ($p_\pi \to 0$) and the other pion still conserves momentum in $p_K = p_\pi + p'_\pi$. In effect, the rapidly varying $M_P - M_P(0)$ part in (1) allows one to extend the soft-pion current-algebra procedure away from low energies.

Then the three $K \to \pi\pi$ decays have the current-algebra–PCAC amplitude structures derived from (1) with $[Q_5, H_w] = -[Q, H_w]$ and $f_\pi \approx 93$ MeV:

$$\langle \pi^0\pi^0|H_w|K^0\rangle = (i/f_\pi)\langle \pi^0|H_w|K^0\rangle(1 - m_\pi^2/m_K^2) , \tag{2a}$$

$$\langle \pi^+\pi^-|H_w|K^0\rangle = (-i/\sqrt{2}f_\pi)\langle \pi^+|H_w|K^+\rangle(1 - m_\pi^2/m_K^2) , \tag{2b}$$

$$\langle \pi^+\pi^0|H_w|K^+\rangle = (-i/2f_\pi)(\langle \pi^+|H_w|K^+\rangle + \sqrt{2}\langle \pi^0|H_w|K^0\rangle)(1 - m_\pi^2/m_K^2) . \tag{2c}$$

Note that the amplitudes in (2a) and (2b) are twice as large as M_{CC} alone[1,3] and yet they are also compatible with Cronin's nonlinear Lagrangian answer.[4] Note also (2c) automatically obeys the manifest $\Delta I = 3/2$ structure of $K^+ \to \pi^+\pi^0$. In effect, these last two statements are realized because of Eq. (1), due in turn to the large (not small) pion momentum in $K \to \pi\pi$ decays. Another way to interpret Eqs. (2) is that this large momentum requires *both* pions in $K_{2\pi}$ to separately become soft, adding to $M = M_{CC1} + M_{CC2}$ and one can show by direct computation[3] that the latter prescription is equivalent to (1), then giving (2).

To demonstrate that Eqs. (2) are physically meaningful, we take the observed $K_{2\pi}$ rates[5] to infer

$$|\langle \pi^0\pi^0|H_w|K^0\rangle| \approx 26 \times 10^{-8} \text{ GeV}$$

and

$$|\langle \pi^+\pi^-|H_w|K^0\rangle| \approx 28 \times 10^{-8} \text{ GeV} ,$$

which requires, from (2a) and (2b) up to an overall sign,

$$\langle \pi^0|H_w|K^0\rangle \approx -2.6 \times 10^{-8} \text{ GeV}^2 , \quad \langle \pi^+|H_w|K^+\rangle \approx 4.0 \times 10^{-8} \text{ GeV}^2 . \tag{3}$$

In fact, the former scale is close to the π^0 pole model for $K_L \to 2\gamma$ decay, which gives[1] $|\langle \pi^0|H_w|K^0\rangle| \approx 2.3 \times 10^{-8}$ GeV^2. Furthermore, using the reduced matrix elements (3), the small $\Delta I = 3/2$ current-algebra–PCAC amplitude (2c) is then predicted to be

$$|\langle \pi^+\pi^0|H_w|K^+\rangle| \approx (19.8 - 18.2) \times 10^{-8} \text{ GeV} = 1.6 \times 10^{-8} \text{ GeV} , \tag{4}$$

close to experiment[5] $(1.83 \pm 0.01) \times 10^{-8}$ GeV.

For charmed-meson decays $D \to \bar{K}\pi$ the decay momentum $p \sim 860$ MeV is extremely relativistic compared to the K and π masses, yet the current-algebra–PCAC procedure analogous to (2) for $K \to 2\pi$ decays also appears to "work." More specifically if we compute $M_{CC1} + M_{CC2}$ for the pure $I = 3/2$, $D^+ \to \bar{K}^0\pi^+$ transition (where final-state interactions are irrelevant), we find

$$\langle \pi^+\bar{K}^0|H_w|D^+\rangle = (i/\sqrt{2}f_K)\langle \pi^+|H_w|F^+\rangle + (i/\sqrt{2}f_\pi)\langle \bar{K}^0|H_w|D^0\rangle , \tag{5}$$

quite similar in structure to (2c). An approximate estimate of the Cabibbo-enhanced $\langle \bar{K}^0|H_w|D^0\rangle$

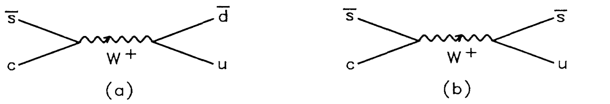

FIG. 1. W-pole quark graphs for (a) $\langle \pi^+|H_w|F^+\rangle$ and (b) $\langle K^+|H_w|F^+\rangle$.

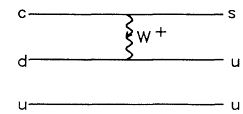

FIG. 2. W-exchange quark graph for $\langle \Sigma^+|H_w|\Lambda_c^+\rangle$.

scale in (5) relative to the Cabibbo-suppressed transition $\langle \pi^0|H_w|K^0\rangle$ in (3) is

$$\langle \bar{K}^0|H_w|D^0\rangle=\sqrt{2}(c_1/s_1)\langle \pi^0|H_w|K^0\rangle \approx -0.16\times 10^{-6}\ \text{GeV}^2\ . \tag{6}$$

Even though $\langle \pi^+|H_w|F^+\rangle$ in (5) is not the SU(4) analog of $\langle \pi^+|H_w|K^+\rangle$ in (3), we can estimate $\langle \pi^+|H_w|F^+\rangle$ from the W pole linking $F^+(c\bar{s})$ to $\pi^+(u\bar{d})$ as depicted in Fig. 1(a), generating the hadronic matrix element

$$\langle \pi^+|H_w|F^+\rangle=(G_F/\sqrt{2})c_1^2 f_\pi f_F m_D^2 \approx 0.42\times 10^{-6}\ \text{GeV}^2 \tag{7}$$

for $f_\pi\approx 93$ MeV, $f_F\approx 1.8 f_\pi$. Substituting these estimates (6) and (7) into the current-algebra–PCAC amplitude (5) then predicts, for $f_K/f_\pi\approx 1.25$,

$$|\langle \pi^+\bar{K}^0|H_w|D^+\rangle|\approx(2.56-1.22)\times 10^{-6}\ \text{GeV} =1.34\times 10^{-6}\ \text{GeV}\ , \tag{8}$$

which is quite near the observed amplitude[5] $(1.32\pm 0.10)\times 10^{-6}$ GeV. Note that the partial cancellation of the two terms in (8) is analogous to the cancellation of the two terms in (4).

The current-algebra–PCAC technique is also valid for the Cabibbo-suppressed charmed decays $D\to \bar{K}K$ when we apply the $M_{\text{CC1}}+M_{\text{CC2}}$ version, which corrects for the large decay momentum $p_K\sim 790$ MeV. More specifically the analog of (5) containing no final-state interactions is

$$\langle K^+\bar{K}^0|H_w|D^+\rangle=(i/\sqrt{2}f_K)\langle K^+|H_w|F^+\rangle\ , \tag{9}$$

where two current commutator terms cancel in (9). The reduced matrix element $\langle K^+|H_w|F^+\rangle$ is the Cabibbo-suppressed W-pole analog of $\langle \pi^+|H_w|F^+\rangle$ in (7) as depicted in Fig. 1(b), giving

$$\langle K^+|H_w|F^+\rangle=(G_F/\sqrt{2})s_1 c_1 f_K f_F m_D^2 \approx 0.12\times 10^{-6}\ \text{GeV}^2\ . \tag{10}$$

Substituting (10) into (9) predicts the current-algebra–PCAC amplitude magnitude

$$|\langle K^+\bar{K}^0|H_w|D^+\rangle|=(G_F/2)s_1 c_1 f_F m_D^2 \approx 0.75\times 10^{-6}\ \text{GeV}\ , \tag{11}$$

also near experiment[5] $(0.76\pm 0.12)\times 10^{-6}$ GeV.

To extend the above analysis to baryon decays $B\to B'\pi$ one must also distinguish parity-violating (PV) s-wave amplitudes A from parity-conserving (PC) p-wave amplitudes B, giving an approximate charmed-baryon decay rate

$$\Gamma(B\to B'\pi)\approx(p/8\pi m_B^2)[(m_B+m_{B'})^2A^2 +(m_B-m_{B'})^2B^2]\ . \tag{12}$$

For the lower-energy hyperon decays $\Lambda\to n\pi^0$, etc., the s-wave and p-wave contributions to (12) are about the same size as deduced from the data. From the current-algebra–PCAC perspective one can either treat the $p\sim 100$–200 MeV decay momenta as large relative to the pions so that (1) could be employed[1] or small relative to the final-state baryons, so the simpler structure $M=M_{\text{CC}}+M_P$ could be used. Although both methods are roughly equivalent, the former approach will prove to be more useful for heavier charmed-baryon decays. In this case, one finds from (1) that the p-wave pole terms become suppressed by the double-pole structure of $M_P-M_P(0)$ for charmed-baryon masses. Even though the decay momentum $p\sim 800$ MeV is large, the above p-wave pole suppression means that the s waves (via the current algebra) dominate the charmed-baryon decay rate (12).

In particular for the $\Lambda_c^+\to p\bar{K}^0$ decay, Eq. (1) reduces to the current-algebra s-wave amplitude magnitude

$$|A|=(1/\sqrt{2}f_K)|\langle \Sigma^+|H_w|\Lambda_c^+\rangle|\ . \tag{13}$$

In order to estimate the reduced matrix element $\langle \Sigma^+|H_w|\Lambda_c^+\rangle$ in (13), we look at the W-exchange (Wx) quark model of Riazuddin and Fayyazuddin.[6] [In fact the quark model can also be used to justify the meson reduced matrix elements (3) and (6).] The extension of the Ref. 6 value $|\langle n|H_w|\Lambda\rangle|_{Wx}\approx 35$ eV to charm gives, by analogy with Eq. (6),

$$|\langle \Sigma^+|H_w|\Lambda_c^+\rangle|_{Wx}=(c_1/s_1)|\langle n|H_w|\Lambda\rangle|_{Wx}\approx 150\ \text{eV}\ . \tag{14a}$$

Alternatively, from Fig. 2 one can write[7]

$$|\langle \Sigma^+|H_w|\Lambda_c^+\rangle|_{Wx}=(G_F/\sqrt{2})\sqrt{6}c_1^2\langle \psi_0|\delta^3(\mathbf{r})|\psi_0\rangle\ , \tag{14b}$$

where $\langle \psi_0|\delta^3(\mathbf{r})|\psi_0\rangle\approx 8\times 10^{-3}\ \text{GeV}^3$ is obtained from the Δ-N mass splitting.[8] Substituting either (14a) or (14b) into (13) then predicts $|A|\approx 0.91\times 10^{-6}$ and this in turn requires the $\Lambda_c^+\to p\bar{K}^0$ decay width to be

$$\Gamma\approx(p/8\pi m_{\Lambda_c}^2)(m_{\Lambda_c}+m_p)^2A^2\approx 5.7\times 10^{-14}\ \text{GeV}\ , \tag{15}$$

where we have neglected the $\sim 20\%$ contribution from the suppressed B octet pole term. This current-algebra–PCAC prediction (15) is also near the observed rate[5] $(5.5\pm2.1)\times10^{-14}$ GeV.

In conclusion, we point out that the current-algebra–PCAC procedure coupled with rapidly varying pole corrections (1) accounts for large decay momenta away from the soft-pion limit. Then the predicted decays for $K^+\to\pi^+\pi^0$, $D^+\to\bar{K}^0\pi^+$, $D^+\to\bar{K}^0K^+$, and $\Lambda_c^+\to p\bar{K}^0$ in (4), (8), (11), and (15), respectively, are all in good agreement with data.

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