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Calculation of chiral-symmetry breaking and pion properties as a Goldstone boson

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A procedure for picking out the solution of the covariant Bethe-Salpeter equation corresponding to that Goldstone boson in a spontaneous-chiral-symmetry-breaking theory is described under very general assumptions. Physical quantities for chiral-symmetry breaking and the pion are calculated with this procedure taking into account gluon ladder exchange and a covariant chiral-symmetric generalization of the confinement potential. Satisfactory results are obtained with reasonable values of the parameters.

I. INTRODUCTION

As is well known, phenomenological nonrelativistic models based on extrapolation of an asymptotically free one-gluon potential at short distances and a linear confinement potential at large distances fit excellently the heavy-quarkonioum spectra (see, e.g., Refs. 1 and 2). The potential in the range 0.1–1.0 fm, where theoretical predictions are not available, is essentially fixed by experimental data.[3] With some relativistic corrections included in the calculation, the same potential models can even give satisfactory results for spectra of mesons composed of light quarks except for the lightest 0^- octet mesons (see, e.g., Ref. 2). These kinds of calculation, when applied to a pion considered as a bound state of quarks with a constant constituent mass, usually give too large a value for m_π. It has been realized that the failure of this approach to the lightest 0^- octet mesons is due to the peculiar feature of these mesons being simultaneously relativistic bound states and Goldstone bosons. The approaches which fail to incorporate this point cannot give a satisfactory description of these mesons.

There have been a large number of works investigating chiral-symmetry breaking in QCD-like theories with the Schwinger-Dyson equation (see, e.g., Refs. 4–6). In Refs. 5 and 6, a scheme was first developed in which the Bethe-Salpeter wave function of the pion was obtained from a solution of the Schwinger-Dyson equation for a dynamical quark mass. This scheme properly incorporates the Goldstone-boson nature of the pion. However, the authors of these works made some approximations which are good only for the nonrelativistic system. In particular the authors of Ref. 5 used an instantaneous Coulomb interaction and neglected the contributions of transverse gluons. In Ref. 7, the effect of the transverse gluon was considered with the retardation effect replaced by an effective transverse-gluon mass. Since the pion is a relativistic bound state, it is worthwhile to reanalyze the problem with a relativistic formulation. In this article we shall use the Schwinger-Dyson equation for a dynamical quark mass and the Bethe-Salpeter equation for the pion in their Lorentz-invariant form. The kernels of these two equations are taken to be the same. In a previous note,[8] we have shown under very general conditions how to pick out the solution of the Bethe-Salpeter equation corresponding to the Goldstone boson in a spontaneous chiral-symmetry-breaking theory. Our scheme is essentially a generalization of the procedure used in Refs. 5 and 6 to the full relativistic theory. We shall use this scheme to the case of gluon ladder exchange with a running coupling constant, as well as to the case where a covariant generalization of the confinement potential is contained in the kernel. A confinement potential of vector type was considered in Refs. 6 and 7. The kernel corresponding to the confinement interaction used by us is a chiral-symmetric combination which contains a superposition of the vector and scalar confinement potential in the v/c expansion. Physical quantities for chiral-symmetry breaking and the pion, including quark dynamical mass, quark condensate $\langle\bar\psi\psi\rangle$, pion decay constant f_π, and pion charge radius r_π, are calculated with these solutions. We would like to investigate the possibility of fitting the experimental data for chiral-symmetry breaking and the pion properties from realistic assumptions. Therefore, we use Lorentz-covariant equations and try to restrict the values of parameters within the region allowed by other experimental data or theoretical considerations.

In Sec. II we shall explain our method and write down basic equations. Formulas used for calculating physical quantities and numerical results will be presented in Sec. III. These results will be discussed in Sec. IV.

II. BASIC ASSUMPTIONS AND METHODS

The same interaction kernel appearing in the Wick-rotated Schwinger-Dyson equation for quark self-energy and Bethe-Salpeter equation for quark-antiquark pair is assumed to be of the form

$$\Gamma_i\otimes\Gamma_i U_i, \quad i=S,P,V,A,T,L \ , \tag{1}$$

where U_i are Lorentz-invariant functions of momenta involved, q is the momentum transfer of the quark, Γ_i are 16 Hermitian Dirac matrices for $i\neq L$, and

$$\Gamma_L=\gamma\cdot q/\sqrt{q^2} \ . \tag{2}$$

The Schwinger-Dyson equation takes the form

$$\Sigma(p)=-(Z-1)\hat{p}+i\delta m-\int\frac{d^4q}{(2\pi)^4}\Gamma_i S'_F(p-q)\Gamma_i U_i\ , \tag{3}$$

where $\Sigma(p)=S_F'^{-1}(p)+\gamma\cdot p$. The same kernel should also be used in the integral equation for the axial vertex function $\Gamma^5_\mu(p_1,p_2)$, which reads

$$\Gamma^5_\mu(p_1,p_2)=Zi\gamma_5\gamma_\mu+\int\frac{d^4q}{(2\pi)^4}\Gamma_i S'_F(p_1-q)\Gamma^5_\mu(p_1-q,p_2-q)\times S'_F(p_2-q)\Gamma_i U_i\ . \tag{4}$$

We shall put the quark current mass $m=0$ throughout this article. The Ward-Takahashi (WT) identity for Γ^5_μ in a theory with conserved local axial-vector current is

$$(p_1-p_2)_\mu\Gamma^{5i}_\mu(p_1,p_2)=i[S_F'^{-1}(p_1)\gamma_5+\gamma_5 S_F'^{-1}(p_2)]\frac{\tau^i}{2}\ . \tag{5}$$

This identity can be satisfied by the solutions of the equation (3) and corresponding equation (4) if $U_S=U_P=U_T=0$. But we shall also consider the less stringent case

$$U_S+U_P=0,\quad U_T=0\ , \tag{6}$$

which corresponds to the nonlocal chiral-symmetric interaction

$$\int\int d^4x\,d^4y[\bar\psi(x)\psi(x)\bar\psi(y)\psi(y)-\bar\psi(x)\gamma_5\psi(x)\bar\psi(y)\gamma_5\psi(y)]\tilde U_S(x-y)\ .$$

The γ_5 component of the identity (5) is satisfied under condition (6). As we shall see, this condition is sufficient for realization of the Goldstone theorem.

Write the self-energy of the quark as

$$\Sigma(p)\equiv iA(p^2)-\gamma\cdot pB(p^2)\ . \tag{7}$$

Then the Schwinger-Dyson equation (3) for $\Sigma(p)$ can be decomposed into the two equations

$$A(p^2)=\int\frac{d^4p'}{(2\pi)^4}(4U_V+4U_A+U_S+U_P+12U_T+U_L)\frac{A(p'^2)}{A^2(p'^2)+p'^2[1+B(p'^2)]^2}\ , \tag{8}$$

$$\gamma\cdot pB(p^2)=\int\frac{d^4p'}{(2\pi)^4}\left[(2U_V-2U_A-U_S+U_P)\gamma\cdot p'+\left[2\frac{q\cdot p'}{q^2}\gamma\cdot q-\gamma\cdot p'\right]U_L\right]\frac{1+B(p'^2)}{A^2(p'^2)+p'^2[1+B(p'^2)]^2}\ . \tag{9}$$

Equation (9) needs one subtraction at some point p_0 if $Z\neq 1$.

From the symmetry principle, the Bethe-Salpeter amplitude of the pion can be written as

$$\chi_P(p)=\gamma_5F_1+i\gamma_\mu\gamma_5[P_\mu F_2+p_\mu(p\cdot P)F_3]+\frac{i}{2}\gamma_5\sigma_{\mu\nu}(P_\mu p_\nu-p_\mu P_\nu)F_4\ , \tag{10}$$

where P and p are total and relative momentum, respectively. $F_i=F_i(p^2,(p\cdot P)^2)$ are even functions of p. The Bethe-Salpeter equation can be decomposed into four equations, the first of which is

$$\left\{\left[p^2-\frac{P^2}{4}\right]\left[1+B\left[p-\frac{P}{2}\right]\right]\left[1+B\left[p+\frac{P}{2}\right]\right]+A\left[p-\frac{P}{2}\right]A\left[p+\frac{P}{2}\right]\right\}F_1(P,p)$$
$$-P^2F_2(P,p)-(P\cdot p)^2F_3(P,p)-[(P\cdot p)^2-p^2P^2]F_4(P,p)$$
$$=\int\frac{d^4p'}{(2\pi)^4}(4U_V+4U_A-U_S-U_P-12U_T+U_L)F_1(P,p')\ . \tag{11}$$

Assume F_i has the form

$$F_i(p^2,(p\cdot P)^2)=f_i(p^2)+(p\cdot P)^2\tilde f_i(p^2,(p\cdot P)^2)\ , \tag{12}$$

where $\tilde f_i$ are finite at $(p\cdot P)^2=0$ and $f_i(p^2)$ are independent of $(p\cdot P)$. Since

$$\int d^4p'U_i(p'\cdot P)^2\tilde f_i(p'^2,(p'\cdot P)^2)$$

must be proportional to P^2 or $(p\cdot P)^2$, we can obtain the following closed system of equations for $f_i(p^2)$ of a bound state of zero mass ($P^2=0$):

$$\{A^2(p^2)+[1+B(p^2)]^2p^2\}f_1(p^2)=\int\frac{d^4p'}{(2\pi)^4}(4U_V+4U_A-U_S-U_P-12U_T+U_L)f_1(p'^2)\ , \tag{13}$$

$$A(p^2)[1+B(p^2)]f_1(p^2)+\{A^2(p^2)-[1+B(p^2)]^2p^2\}f_2(p^2)-2A(p^2)[1+B(p^2)]p^2f_4(p^2)$$
$$=-\int\frac{d^4p'}{(2\pi)^4}(U_S-U_P+2U_V-2U_A)\left[f_2(p'^2)+\frac{1}{3p'}[p^2p'^2-(p\cdot p')^2]f_3(p'^2)\right]$$
$$+\int\frac{d^4p'}{(2\pi)^4}2U_L\left\{-\frac{1}{3}\left[\frac{(p\cdot q)^2}{p^2q^2}+\frac{1}{2}\right]f_2(p'^2)+\left[\left[\frac{(p\cdot p')^2}{p^2p'^2}-\frac{1}{6}\right]p'^2+\frac{1}{3}(q\cdot p')\left[\frac{(p\cdot q)^2}{p^2q^2}-1\right]\right]f_3(p'^2)\right\}, \tag{14}$$

$$2\{A(p^2)B'(p^2)-[1+B(p^2)]A'(p^2)\}f_1(p^2)+2[1+B(p^2)]^2f_2(p^2)+\{A^2(p^2)+[1+B(p^2)]^2p^2\}f_3(p^2)$$
$$+2A(p^2)[1+B(p^2)]f_4(p^2)$$
$$=-\int\frac{d^4p'}{(2\pi)^4}(U_S-U_P+2U_V-2U_A)\frac{1}{3p^4}[4(p\cdot p')^2-p^2p'^2]f_3(p'^2)$$
$$+\int\frac{d^4p'}{(2\pi)^4}2U_L\frac{1}{p^2}\left\{\frac{1}{3}\left[4\frac{(p\cdot q)^2}{p^2q^2}-1\right]f_2(p'^2)\right.$$
$$\left.+\left[\frac{1}{q^2}(q\cdot p)(q\cdot p')-\frac{1}{3}(q\cdot p')\left[4\frac{(p\cdot q)^2}{p^2q^2}-1\right]-\frac{1}{6}\left[4\frac{(p\cdot p')^2}{p^2p'^2}-1\right]p'^2\right]f_3(p'^2)\right\}, \tag{15}$$

$$[1+B(p^2)]^2f_1(p^2)+2A(p^2)[1+B(p^2)]f_2(p^2)+\{A^2(p^2)-[1+B(p^2)]^2p^2\}f_4(p^2)$$
$$=-\frac{1}{p^2}\int\frac{d^4p'}{(2\pi)^4}(U_S+U_P-4U_T)(p\cdot p')f_4(p'^2)$$
$$+\frac{1}{p^2}\int\frac{d^4p'}{(2\pi)^4}U_L\left[(p\cdot p')+\frac{2}{3}\left[\frac{(p\cdot q)^2}{p^2q^2}-1\right]p^2-\frac{2}{q^2}(p\cdot q)(p'\cdot q)\right]f_4(p'^2). \tag{16}$$

If the condition (6) is satisfied, we find by comparing Eq. (13) with (8) that the former has a solution

$$f_1(p^2)=N\frac{A(p^2)}{A^2(p^2)+p^2[1+B(p^2)]^2} \tag{17}$$

when the latter has a nontrivial chiral-symmetry-breaking solution. Here N is the normalization constant. The relation (17) has been obtained from the axial-vector WT identity (5) and the relation

$$S_F(p_1)\Gamma^5_\mu(p_1,p_2)S_F(p_2)=\frac{f_\pi}{2}\frac{P_\mu}{P^2}\chi(p_1,p_2)+\text{terms regular at } P^2=0 \tag{18}$$

between the axial-vector vertex function Γ^5_μ and the Bethe-Salpeter amplitude of the pion χ with the normalization constant N fixed to[9]

$$N=f_\pi^{-1}\;. \tag{19}$$

Our solution indeed satisfied this relation.

The Bethe-Salpeter wave functions f_2, f_3, and f_4 can then be solved from (14)–(16) in terms of $f_1(p^2)$. In principle, $\tilde{f}_i(p^2,(p\cdot P)^2)$ are determined by $f_i(p^2)$. This is the realization of the Nambu-Goldstone theorem in our formalism. The solution obtained in this way is the correct solution for the pion as a Goldstone boson. An important point here is that we should use dynamical quark self-energy obtained from Schwinger-Dyson equation with the same kernel instead of constant constituent quark mass in Bethe-Salpeter equation. When $P^2=0,\tilde{f}_i(p^2,(p\cdot P)^2)$ do not contribute to the normalization constant N and the pion decay constant f_π.

For a kernel independent of P, the normalization constant N can be determined either by the condition[10]

$$\int\frac{d^4p}{(2\pi)^4}\left\{\text{Tr}\left[\bar{\chi}_P\frac{\partial}{\partial P_\mu}S_F'^{-1}\left[p+\frac{P}{2}\right]\chi_PS_F'^{-1}\left[p-\frac{P}{2}\right]\right]+\text{Tr}\left[\chi_PS_F'^{-1}\left[p+\frac{P}{2}\right]\chi_P\frac{\partial}{\partial P_\mu}S_F'^{-1}\left[p-\frac{P}{2}\right]\right]\right\}=-2P_\mu \tag{20}$$

or by the condition $F_\pi(0)=1$, where $F_\pi(q^2)$ is the pion electromagnetic form factor. From (20), we obtain at $P^2=0$ the formula (A1) in the Appendix. The charge radius of the pion r_π depends on $\tilde{f}_1(p^2)\equiv\tilde{f}_1(p^2,0)$ as well. This function can be obtained by the equation

$$\{A^2(p^2)+p^2[1+B(p^2)]^2\}\tilde{f}_1(p^2)-(A'^2(p^2)-A(p^2)A''(p^2)+p^2\{B'^2(p^2)-[1+B(p^2)]B''(p^2)\})f_1(p^2)$$
$$-2\{A(p^2)B'(p^2)-A'(p^2)[1+B(p^2)]\}f_2(p^2)$$
$$-(2\{A(p^2)B'(p^2)-A'(p^2)[1+B(p^2)]\}p^2+A(p^2)[1+B(p^2)])f_3(p^2)-[1+B(p^2)]^2f_4(p^2)$$
$$=\frac{1}{3p^2}\int\frac{d^4p'}{(2\pi)^4}(4U_V+4U_A-U_S-U_P-12U_T+U_L)\left[4\frac{(p\cdot p')^2}{p^2p'^2}-1\right]p'^2\tilde{f}_1(p'^2) \quad (21)$$

derived from Eq. (11).

III. CALCULATION OF PHYSICAL QUANTITIES

In the following we shall assume $U_A=U_T=0$, and consider the following cases.

(a) Gluon ladder approximation in the Landau gauge:

$$U_S=U_P=0\ ,$$
$$U_V=-U_L=\frac{4}{3}g^2_{s\text{eff}}\frac{1}{q^2}\ ,$$

where $q=p'-p$ is the momentum transfer of the quark, and

$$g^2_{s\text{eff}}=\begin{cases}\dfrac{16\pi^2}{9}\dfrac{1}{\ln\dfrac{\text{Max}(p^2,p'^2)}{\Lambda^2}} & \text{for } g^2_{s\text{eff}}\le 4\pi\alpha_0,\\ 4\pi\alpha_0 & \text{otherwise}\ .\end{cases} \quad (22)$$

In this case, $B(p^2)=0$. It is reasonable to interpret Λ in (22) as the QCD scale parameter in momentum subtraction scheme calculated with quark-gluon vertex. Therefore it should lie in the region of 200–500 MeV.

(b) Gluon ladder in the Landau gauge plus confinement.

Experimental data of the fine structure of heavy-quarkonian spectra favor the scalar confinement potential.[2,11] Therefore we shall allow for the possibility that the confinement interaction for light quarks also contains a scalar component. However, pure scalar interaction violates chiral symmetry explicitly. Therefore we assume the confinement interaction is a superposition of a vector term and the chiral-symmetrical combination $U_S=-U_P$. The contribution of the U_P term vanishes in the nonrelativistic limit but is important for the pion. The simple choice of q^{-4} kernel for confinement interaction leads to infrared divergence of the integral equations. We shall assume

$$U_V(q)=-U_L(q)=\xi\frac{8\pi\kappa}{(q^2+\mu^2)^2}\ ,$$
$$U_S(q)=-U_P(q)=-(1-\xi)\frac{8\pi\kappa}{(q^2+\mu^2)^2}\ . \quad (23)$$

Here μ is an infrared cutoff which may represent the screening effect of light-quark pairs in the physical vacuum and ξ is a constant. The nonrelativistic potential between a pair of quarks is equal to U_V-U_S at $q_0=0$ in the limit $|\mathbf{q}|\ll$ constituent quark mass. This quantity is expected to be flavor independent. This implies that κ should be equal to the string tension determined from heavy-quarkonium spectra. Therefore κ lies in the region of 0.15–0.20 GeV^2. In order to avoid distorting too much the linear potential in the energy range of heavy quarkonium, we assume $\mu<100$ MeV.

(c) We have considered also the case in which the confinement interaction (23) is replaced by the form

$$\xi^{-1}U_V(q)=-\xi^{-1}U_L(q)$$
$$=-(1-\xi)^{-1}U_S(q)=(1-\xi)^{-1}U_P(q)$$
$$=8\pi\left[\frac{\kappa}{(q^2+\mu^2)^2}\frac{M^2}{M^2+q^2}-C\delta^4(q)\right]\ , \quad (24)$$

where

$$C=\int d^4q\frac{\kappa}{(q^2+\mu^2)^2}\frac{M^2}{M^2+q^2}\ . \quad (25)$$

The $\delta^4(q)$ term is introduced to ensure that the Fourier transform $\tilde{U}_S(x)$ of $U_S(q)$ vanishes at $x=0$. The contribution of this term automatically removes the infrared divergence in integral equations even in the limit $\mu\to 0$. It is insensitive to μ for sufficiently small values of μ. The corresponding procedure in three dimensions has been discussed in Ref. 6. However, the physical meaning for these two cases is not exactly the same. To understand the physical meaning of (24), let us calculate its Fourier transform. In the region $M^2x^2\ll 1$, we find

$$\tilde{U}_S(x)\simeq(1-\xi)\frac{\kappa}{\pi}\ln\left[\frac{M\sqrt{x^2}}{2e^{(1/2-\gamma)}}\right]\ . \quad (26)$$

The result coincides with the dimensionally regularized confinement kernel used in Ref. 12, if M is identified to the parameter $\tilde{\rho}^{-1}$ introduced in that article. It was shown in Ref. 12 that for sufficiently small values of M, (26) is reduced to the linear potential with a negative constant term

$$\kappa(r-2e^{(1/2-\gamma)}M^{-1})\ .$$

in the nonrelativistic limit. This form of linear potential has been used for heavy-quarkonium spectra. The introduction of M is needed not only for ultraviolet convergence of the integral (25), but also for the existence of a local potential in the nonrelativistic limit. This can be seen from the discussion in Ref. 12. From the requirement of heavy-quarkonium spectra, M is of the order of several hundreds of MeV.

The dynamical mass function of the quark can be

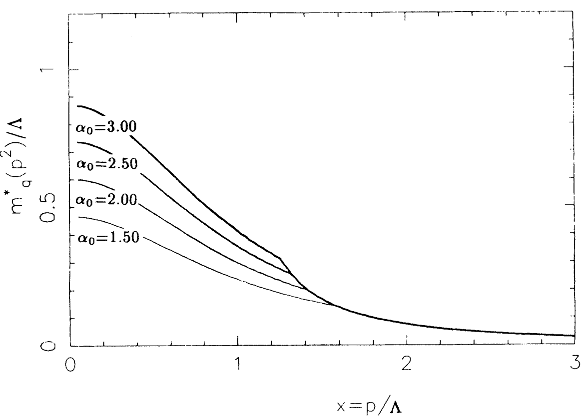

FIG. 1. Dynamical mass function $m_q^*(p^2)$ for case (a) with parameters $\alpha_0=1.50$, 2.00, 2.50, and 3.00.

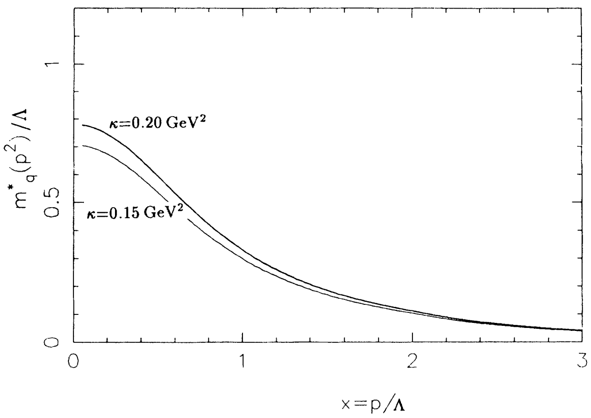

FIG. 2. Same as Fig. 1 for case (b) with parameters $\alpha_0=1.00$, $\Lambda=450$ MeV, $\mu=100$ MeV, and $\kappa=0.15$ GeV^2 and 0.20 GeV^2.

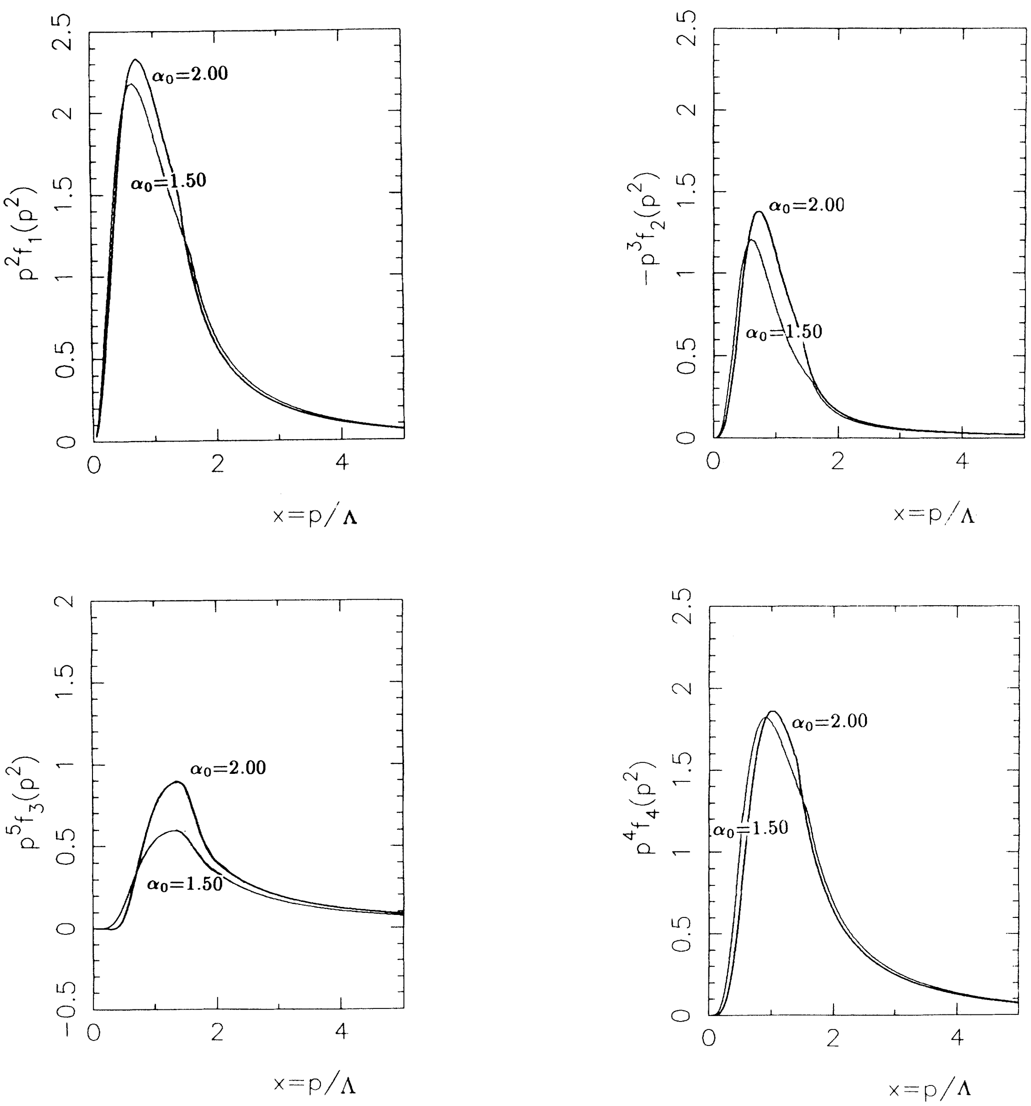

FIG. 3. Bethe-Salpeter wave functions for case (a) with parameters $\alpha_0=1.50$ and 2.00.

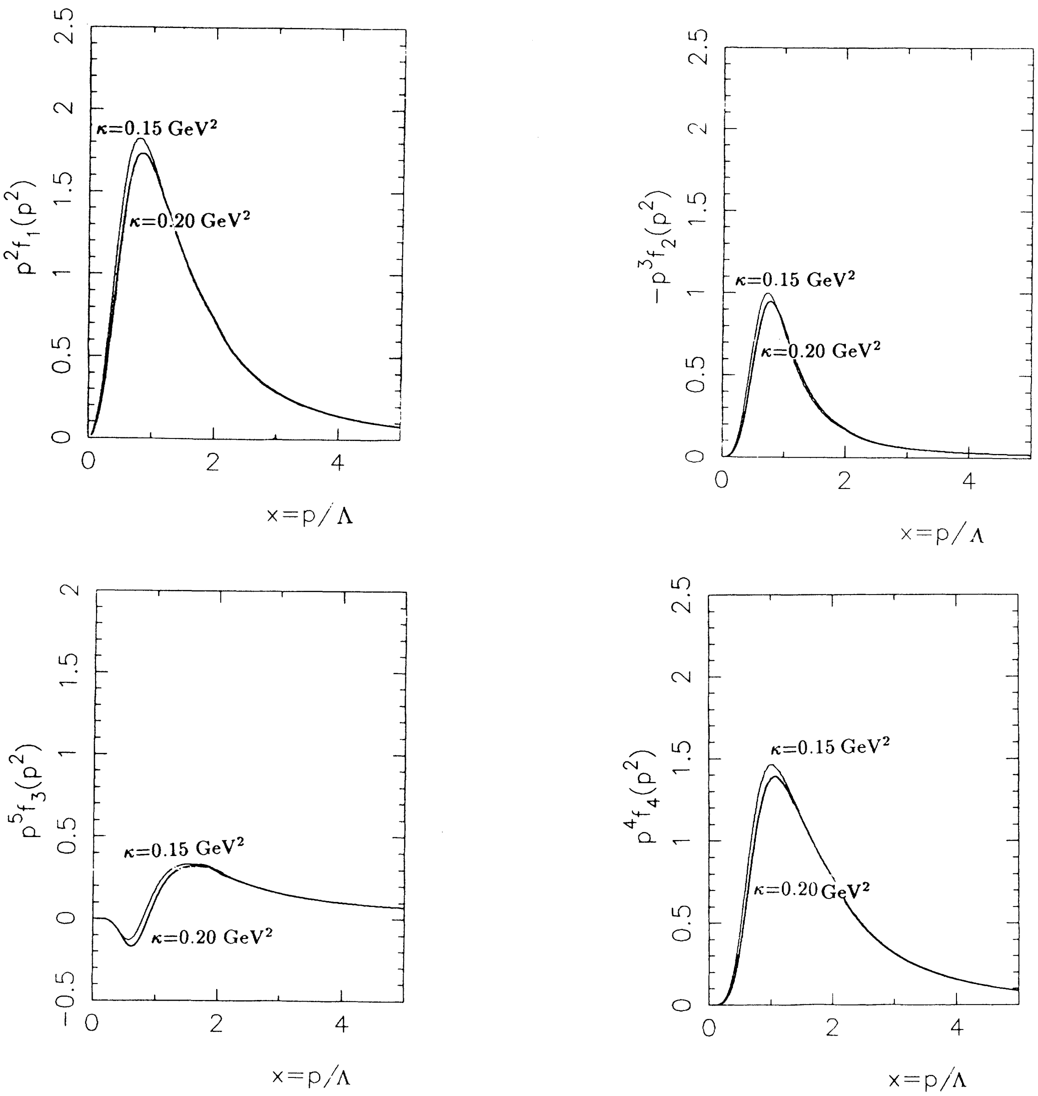

FIG. 4. Same as Fig. 3 for case (b) with parameters $\alpha_0=1.00$, $\Lambda=450$ MeV, $\mu=100$ MeV, and $\kappa=0.15$ GeV^2 and 0.20 GeV^2.

defined as

$$m_q^*(p^2)=\frac{A(p^2)}{1+B(p^2)} . \tag{27}$$

In Ref. 3, the vacuum expectation value of the renormalized composited operator $\bar{\psi}\psi$ is related to the asymptotic behavior of $m_q^*(p^2)$ by the following formula derived from the renormalization group and operator-product expansion:

$$\langle \bar{\psi}\psi \rangle_\mu = -m_q^*(p^2)\left[\frac{1}{3}\frac{g_s^2(p^2)}{p^2}\left[\frac{\ln p^2/\Lambda^2}{\ln \mu^2/\Lambda^2}\right]^{4/9}\right]^{-1} , \tag{28}$$

TABLE I. The quantities of chiral-symmetry breaking and pion properties for case (a).

	$m_q^*(0)$	$\langle \bar{\psi}\psi \rangle$	f_π	r_π
$\alpha_0=1.50$, $\Lambda=600$ MeV	280 MeV	$-(250\text{ MeV})^3$	91.8 MeV	0.55 fm
$\alpha_0=2.00$, $\Lambda=500$ MeV	300 MeV	$-(216\text{ MeV})^3$	89.4 MeV	0.54 fm

TABLE II. Same as Table I for case (b), with $\alpha_0=1.00$, $\Lambda=450$ MeV, and $\xi=1.0$.

	$m_q^*(0)$	$\langle\bar{\psi}\psi\rangle$	f_π	r_π
$\kappa=0.15$ GeV^2 $\mu=100$ MeV	317 MeV	$-(210\text{ MeV})^3$	85.3 MeV	0.60 fm
$\kappa=0.15$ GeV^2 $\mu=70$ MeV	357 MeV	$-(202\text{ MeV})^3$	74.0 MeV	0.59 fm
$\kappa=0.20$ GeV^2 $\mu=100$ MeV	350 MeV	$-(217\text{ MeV})^3$	86.1 MeV	0.57 fm

where μ is the renormalization point of the composite operator $\bar{\psi}\psi$. The asymptotic behavior of the solution of Eq. (8) in all cases considered by us is consistent with (28). Therefore we shall use (28) for the calculation of $\langle\bar{\psi}\psi\rangle$ for $\mu=1.0$ GeV.

The pion decay constant is obtained from

$$f_\pi=-12\int\frac{d^4p}{(2\pi)^4}[f_2(p^2)+\tfrac{1}{4}p^2f_3(p^2)] \ . \tag{29}$$

After tedious calculations, the charge radius of the pion r_π can also be expressed in terms of $f_i(p^2)$ and $\tilde{f}_1(p^2)$ when P^2 is taken to be zero. The result obtained after partial integrations is the formula (A2) in the Appendix. In the calculation of r_π, we have neglected the radiative corrections of the photon vertex. Correspondingly, we change the normalization of the Bethe-Salpeter wave functions in the calculation of the integral in (A2) so that the form factor $F_\pi(q^2)$ obtained in the same approximation is equal to 1 at $q^2=0$. The original normalization condition (A1) corresponds to the diagram containing the radiative correction of the photon vertex.

The numerical solutions of Schwinger-Dyson and Bethe-Salpeter equations for cases (a) and (b) obtained by our method are shown in Figs. 1–4. The results for physical quantities $m_q^*(0)$, $\langle\bar{\psi}\psi\rangle$, f_π, and r_π calculated from these solutions are summarized in Tables I–III. We have checked that the normalization constant N obtained from (A1) agrees with the relation (20), $N=f_\pi^{-1}$, within a few percent.

IV. DISCUSSION

From the numerical results, we can obtain the following conclusions.

TABLE III. Dynamical quark mass and quark condensate for case (b) with $\alpha_0=1.00$, $\Lambda=450$ MeV, and $\xi=0.00$ (pure *S-P* confinement).

	$m_q^*(0)$	$\langle\bar{\psi}\psi\rangle$
$\kappa=0.20$ GeV^2 $\mu=100$ MeV	38 MeV	$-(171\text{ MeV})^3$

(1) Comparing numerical results for the set of parameters $\Lambda=600$ MeV and $\alpha_0=1.50$ in the case (a) with that obtained for the same set of parameters in Ref. 5 considering only instantaneous Coulomb interaction, we see that, although they are qualitatively similar, the corrections due to transverse gluons can be as large as 50%. The results are in agreement with theoretical expectation.

(2) For case (b), pure scalar-pseudoscalar (*S-P*) confinement gives too small a value for the dynamical quark mass. The best fit is obtained for pure vector interaction. This is to be contrasted with the heavy-quark case where the scalar confinement is favored. Satisfactory agreement with experimental data can be obtained for $\Lambda=400-500$ MeV, $\mu=70-100$ MeV, $\kappa=0.15-0.20$ GeV^2, and $\alpha_0\gtrsim 1.0$, except that f_π is about 40% smaller than the experimental value 130 MeV. These values of parameters are quite reasonable. This is an improvement to the results obtained in Ref. 7 where a Λ value of about 2.0 GeV was used to fit the dynamical quark mass. Our results are not sensitive to the values of α_0.

(3) On the other hand, if we use the subtracted form

TABLE IV. The quantities, $m_q^*(0)$, $\langle\bar{\psi}\psi\rangle$, and f_π for case (c), with $\alpha_0=1.0$, $\Lambda=300$ MeV, and $\xi=0.00$ (pure *S-P* confinement).

	$m_q^*(0)$	$\langle\bar{\psi}\psi\rangle$	f_π
$\kappa=0.15$ GeV^2 $M=400$ MeV	341 MeV	$(-171\text{ MeV})^3$	133 MeV
$\kappa=0.15$ GeV^2 $M=500$ MeV	387 MeV	$(-184\text{ MeV})^3$	138 MeV
$\kappa=0.18$ GeV^2 $M=400$ MeV	356 MeV	$(-179\text{ MeV})^3$	149 MeV
$\kappa=0.18$ GeV^2 $M=500$ MeV	399 MeV	$(-188\text{ MeV})^3$	155 MeV

(25) for the confinement potential, a good fit to $m_q^*(0)$, $\langle \bar{\psi}\psi \rangle$, and f_π can be obtained for pure S-P confinement with reasonable values of Λ and κ. (See Table IV.) The results are not sensitive to the value of μ for $\mu \leq 50$ MeV. However, in this case solutions of the integral equations undergo rapid change in the intermediate region. Our numerical solutions are not of good quality sufficient for a reliable calculation of r_π, because the expression (A2) for r_π contains derivatives of these solutions. We hope to study this case with more accurate numerical methods in the future. This problem is important for understanding the nature of confinement interaction.

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APPENDIX

The normalization condition obtained from (20) is

$$N^{-2}=-\frac{3}{8\pi^2}\int dy\, y[C_1(y)f_1^2(y)+C_2(y)f_1(y)f_2(y)f_2+C_3(y)f_1(y)f_3(y)+C_4(y)f_1(y)f_4(y)]\ , \tag{A1}$$

where

$$C_1(y)=[1+B(y)]^2+y[A'^2(y)+yB'^2(y)]$$
$$-2\{A(y)A'(y)+y[1+B(y)]B'(y)\}$$
$$-y\{A(y)A''(y)+[1+B(y)]B''(y)\}\ ,$$
$$C_2(y)=4A(y)[1+B(y)]$$
$$+2y\{A(y)B'(y)-[1+B(y)]A'(y)\}\ ,$$
$$C_3(y)=yA(y)[1+B(y)]$$
$$+2y^2\{A(y)B'(y)-[1+B(y)]A'(y)\}\ ,$$
$$C_4(y)=-3y[1+B(y)]^2\ ,$$

and $y=p^2$.

If the radiative correction of the photon vertex is neglected the charge radius of the pion at $P^2=0$ is

$$\langle \bar{r}_\pi^2 \rangle=-\frac{3}{8\pi^2}\int dy\, y[r_{11}(y)+r_{12}(y)+r_{13}(y)+r_{14}(y)$$
$$+r_{22}(y)+r_{23}(y)+r_{24}(y)$$
$$+r_{33}(y)+r_{34}(y)$$
$$+r_{44}(y)$$
$$+r_{51}(y)+r_{52}(y)+r_{53}(y)+r_{54}(y)]\ , \tag{A2}$$

where

$$r_{11}(y)=\tfrac{1}{2}y\{3[1+B(y)]+yB'(y)\}f_1'^2(y)\ ,$$
$$r_{12}(y)=6yA(y)f_1'(y)f_2'(y)\ ,$$
$$r_{13}(y)=y^2\{-A'(y)f_3(y)+A(y)f_3'(y)\}f_1'(y)\ ,$$
$$r_{14}(y)=-y(\{3[1+B(y)]+yB'(y)\}f_4(y)$$
$$+5y[1+B(y)]f_4'(y))f_1'(y)\ ,$$
$$r_{22}(y)=0\ ,$$
$$r_{23}(y)=6y^2[1+B(y)]f_2'(y)f_2(y)\ ,$$
$$r_{24}(y)=-6yA(y)f_2(y)f_4'(y)\ ,$$
$$r_{33}(y)=-y^2\{[1+B(y)]+yB'(y)\}f_3^2(y)\ ,$$
$$r_{34}(y)=-y\{[15A(y)+4yA'(y)]f_3(y)$$
$$+6yA(y)f_3'(y)\}f_4(y)\ ,$$
$$r_{44}(y)=-2y\{3[1+B(y)]+yB'(y)\}f_4^2(y)\ ,$$
$$r_{51}(y)=-y^2[1+B(y)]\tilde{f}_1(y)f_1'(y)\ ,$$
$$r_{52}(y)=0\ ,$$
$$r_{53}(y)=2y^2A(y)\tilde{f}_1(y)f_3(y)\ ,$$
$$r_{54}(y)=2y^2[1+B(y)]\tilde{f}_1(y)f_4(y)\ ,$$

and $y=p^2$.

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