

## Exotic composite vector boson

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An exotic composite vector boson  $V$  is introduced in two dynamical models of composite quarks, leptons,  $W$ , and  $Z$ . One is based on four-Fermi interactions, in which composite vector bosons are regarded as fermion-antifermion bound states and the other is based on the confining  $SU(2)_L$  gauge model, in which they are given by scalar-antiscalar bound states. Both approaches describe the same effective interactions for the sector of composite quarks, leptons,  $W$ ,  $Z$ ,  $\gamma$ , and  $V$ .

### I. INTRODUCTION

It has been widely accepted that the present-day quark and lepton physics will be replaced by new physics which describes compositeness of quarks and leptons<sup>1-4</sup> and/or respects the additional symmetry, supersymmetry. The new physics should explain the dynamical origin of the weak energy scale of  $G_F^{-1/2} \simeq 300$  GeV, which is the parameter in the standard model, i.e., the vacuum expectation value of the Higgs scalar. The naturalness argument on the Higgs-scalar sector has led to the implication that the scale of the new physics will not be much beyond the TeV energy region.<sup>5</sup> This energy scale is identified with the compositeness scale of  $\sim 1$  TeV if quarks and leptons (as well as the weak bosons,  $W$  and  $Z$ ) are composites of further fundamental particles, the subquarks (or preons).

Once compositeness is assumed, then various exotic and excited states<sup>6</sup> naturally appear in the spectrum of composite particles, which depends on the subquark species involved. The minimal set is composed of three kinds of fermionic subquarks,  $w$ ,  $c(c_l)$ , and  $h$ , for quarks ( $q$ ) and leptons ( $l$ ) made of three fermions as  $q = whc$  and  $l = whc_l$ ,<sup>2</sup> or of two kinds of subquarks,  $w$  and  $c(c_l)$ , for quarks and leptons made as  $q = wc$  and  $l = wc_l$  in which, needless to say, either  $w$  or  $c(c_l)$  is a scalar.<sup>3</sup> The assumed subquarks  $w$ ,  $c(c_l)$ , and  $h$  are, respectively, the common carriers of the weak isospin, the three colors (one leptonic color), and the generation number of quarks and leptons. The mass spectrum for exotic composite particles, which is in principle calculable,<sup>7</sup> depends on the details of the underlying dynamics. However, roughly speaking, one can expect that the exotics generally possess the mass of the compositeness scale. In particular, we expect that the ground states of the neutral exotics can have comparatively light masses as light as  $G_F^{-1/2}$  ( $\simeq 300$  GeV). Among others, an exotic composite vector boson  $V$  will receive much attention in the next decade

since  $V$  may manifest itself in  $e^+e^-$  colliders such as the present KEK TRISTAN, SLAC Linear Collider (SLC) and CERN LEP and the future Japan Linear Collider (JLC), CERN Linear Collider (CLIC), and so on.

The possible underlying dynamics for the exotic composition boson  $V$  is also responsible for the compositeness of  $W$  and  $Z$ , whose properties are now well understood.<sup>8,9</sup> The experimental constraint comes from the validity of the standard mass relation  $m_W = \cos\theta m_Z$  ( $\theta$ : the mixing angle), which is confirmed by the consistency between  $m_{W,Z}$  and  $\sin^2\theta$  observed in low-energy neutrino experiments.<sup>10</sup> Any composite model for  $W$  and  $Z$  must ensure the mass relation. It is argued that, in the kinetic mixing scheme of the photon with vector bosons,<sup>11</sup> which is appropriate for the compositeness of  $Z$ , the standard mass relation is satisfied if the mixing parameter  $\lambda$  is fixed to be  $e/g_W$  for  $e(g_W)$  being the electric [weak  $SU(2)$ ] charge. Such a fine-tuning of  $\lambda$  is dynamically incorporated<sup>12</sup> if  $W$  and  $Z$  are generated as bound states of the *spinor*  $w$  by the four-Fermi interactions of the Nambu–Jona-Lasinio–Bjorken (NJLB) type<sup>13</sup> or as composites of the *scalar*  $w$  by the subcolor confining interactions<sup>14</sup> of  $SU(2)_L$  (Ref. 15) supplemented by complementarity.<sup>16</sup>

In this article, the properties of the exotic vector boson are studied in these dynamical models: the nonlinear-interaction model of the NJLB type in the three fermion model,<sup>17</sup> where  $V$  is assumed to be a fermion-antifermion bound state, and the non-Abelian confining model based on  $SU(2)_L$  for the fermion-boson model with the scalar  $w$  and spinor  $c(c_l)$ ,<sup>18</sup> where  $V$  is a boson-antiboson bound state (of a new scalar subquark,  $\xi$ ). As a candidate of  $V$ , it is reasonable to consider the following neutral vector bosons as fermion-antifermion bound states:<sup>19</sup> (i) color-singlet gluon  $G_\mu^0$  made of  $c$  as  $\sum_{A=1}^3 \bar{c}_A \gamma_\mu c_A$ , where  $A$  ( $=1,2,3$ ) denotes the three colors; (ii) leptonic gluon  $G_\mu^l$  made of  $c_l$  as  $\bar{c}_l \gamma_\mu c_l$ ; (iii) isosinglet weak boson  $W_\mu^0$  made of  $w$  as  $\sum_{i=1}^2 w_i \gamma_\mu w_i$ , where  $i$  ( $=1,2$ ) denotes the weak isospin; (iv) heavy photon  $A_\mu^*$  made of  $w$ ,  $c$ , and  $c_l$  as

$\sum_s \bar{s} \gamma_\mu Q_s s$ , where  $s$  runs over all subquark species with the electric charge  $Q_s$  (i.e.,  $Q_{c,c_l}$  for  $c$  and  $c_l$  etc.). In the  $SU(2)_L$  confining model, such an intuitive description of  $V$  cannot be available because composite vector bosons are restricted to be bound states of scalars but  $c$  and  $c_l$  are all spinors. The corresponding  $V$  that calls for an additional scalar subquark is constructed so as to yield the same effective couplings to quarks and leptons. It is realized by requiring that  $V_\mu$  couples to the baryon number ( $B$ ) for  $G_\mu^0$ , the lepton number ( $L$ ) for  $G_\mu^l$ ,  $3B+L$  for  $W_\mu^0$  and the hypercharge for  $A_\mu^*$ . In Sec. II, the low-energy effective Lagrangian is derived for the composite  $V$  boson. The physical vector fields are constructed in the Sec. III. Section IV is devoted to a summary.

## II. DYNAMICS AND EFFECTIVE INTERACTIONS

### A. Model of the Nambu–Jona-Lasinio–Bjorken type

In this model, quarks and leptons are composites of the spinor subquarks  $w$ ,  $h$ , and  $c(c_l)$  and described as  $q = whc$  and  $l = whc_l$  which can be defined more precisely as  $q$  (or  $l$ ) =  $P(w, h, c(c_l))$  with the projection,  $P(\psi_1, \psi_2, \psi_3)$ , of the direct product of the three spinors  $\psi_1, \psi_2$ , and  $\psi_3$  into a spin- $\frac{1}{2}$  state. For definiteness, we adopt the form

$$\begin{aligned} P(w, h, c) &= \gamma_\mu w (\bar{h}^c \gamma^\mu c), \\ P(w, h, c_l) &= \gamma_\mu w (\bar{h}^c \gamma^\mu c_l), \end{aligned} \quad (2.1)$$

which has the advantage that the composite fermions have the same chiral properties as its constituent  $w$ . The composite  $W$  and  $Z$  bosons are simply given by  $w_L \gamma_\mu \tau^3 w_L$ . These composite particles are generated by nonlinear interactions of the NJLB type.<sup>13</sup> It is, however, expected that this nonlinear interaction model of the NJLB type is considered as a somewhat phenomenologically useful low-energy model of a more fundamental theory such as the one based on subcolor gauge interactions. If this is dynamically relevant, then our subquarks carry the additional subcolor multiplicity  $N_{sc}$ .

The local flavor-color symmetry is taken to be  $SU(3)_c$  for QCD and  $U(1)_{em}$  for QED. Let  $\mathcal{L}_0$  be the QED+QCD Lagrangian for the fermionic subquarks  $w$ ,  $h$ ,  $c$ , and  $c_l$ , the photon  $A_{0\mu}$ , and the gluon  $G_{0\mu}^{(a)}$  ( $a=1-8$ ) with the electromagnetic and strong coupling constants,  $e_0$  and  $g_{s0}$ , respectively. The suffix 0 indicates that the quantities are yet to be renormalized. The fundamental Lagrangian, which includes the interactions to form the

composite quarks  $q$ , leptons  $l$ , weak bosons  $W_\mu^{\pm,3}$  and the exotic boson  $V_\mu$ , is given by

$$\begin{aligned} \mathcal{L}_{\text{fund}} &= \mathcal{L}_0 + \sum_q F_q \bar{P}(w, h, c) P(w, h, c) \\ &\quad + \sum_l F_l \bar{P}(w, h, c_l) P(w, h, c_l) \\ &\quad + F_W (\bar{w}_L \gamma^\mu \tau^i w_L)^2 + F_V (\bar{J}_\mu^V)^2, \end{aligned} \quad (2.2)$$

where  $F_q, F_l, F_W$ , and  $F_V$  are coupling constants, and the current of  $\bar{J}_\mu^V$  is defined as

$$\bar{J}_\mu^V = \begin{cases} \bar{c} \gamma_\mu c & \text{for } V_\mu = G_\mu^0 \text{ (color-singlet gluon),} \\ \bar{c}_l \gamma_\mu c_l & \text{for } V_\mu = G_\mu^l \text{ (leptonic gluon),} \\ \bar{w} \gamma_\mu w & \text{for } V_\mu = W_\mu^0 \text{ (isosinglet weak boson),} \\ \sum_s \bar{s} \gamma_\mu Q_s s & \text{for } V_\mu = A_\mu^* \text{ (heavy photon).} \end{cases} \quad (2.3)$$

The system with the Lagrangian  $\mathcal{L}_{\text{fund}}$  is transformed into the one with

$$\begin{aligned} \mathcal{L}_{\text{aux}} &= \mathcal{L}_0 + \sum_q \bar{P}(w, h, c) \bar{q} + \text{H.c.} - \sum_q \frac{1}{F_q} \bar{q} q \\ &\quad + \sum_l \bar{P}(w, h, c_l) \bar{l} + \text{H.c.} - \sum_l \frac{1}{F_l} \bar{l} l \\ &\quad + \bar{w}_L \gamma^\mu \tau^i w_L \bar{W}_\mu^i - \frac{1}{4F_W} (\bar{W}_\mu^i)^2 \\ &\quad + \bar{J}_\mu^V \bar{V}_\mu - \frac{1}{4F_V} (\bar{V}_\mu)^2, \end{aligned} \quad (2.4)$$

by introducing the auxiliary fields,  $\bar{q}$ ,  $\bar{l}$ ,  $\bar{W}_\mu^i$ , and  $\bar{V}_\mu$ , which miss their kinetic terms. Their equations of motion yield  $\bar{q} = F_q P(w, h, c)$ ,  $\bar{l} = F_l P(w, h, c_l)$ , and  $\bar{W}_\mu^i = 2F_W \bar{w}_L \gamma^\mu \tau^i w_L$  that match our naive expectation. For the exotic boson,  $\bar{V}_\mu^i = 2F_V \bar{J}_\mu^V$  is satisfied and realizes our compositeness of the exotic boson introduced in the Introduction since  $\bar{J}_\mu^V$  is given by Eq. (2.3).

The quantum loop effects arising from  $\mathcal{L}_{\text{aux}}$  itself give rise to the kinetic and interaction terms of the auxiliary fields and convert them into genuine composite fields. Let us evaluate the dominant contributions from  $\mathcal{L}_{\text{aux}}$ . For the *singlet gluon* ( $V_\mu = G_\mu^0$ ), these contributions are summarized in  $\mathcal{L}_{\text{div}}$  as

$$\begin{aligned} \mathcal{L}_{\text{div}} &= \sum_q \bar{q} (iJ_q \not{D} - K_q m_w) \bar{q} + \sum_l \bar{l} (iJ_l \not{D} - K_l m_w) \bar{l} - e_0^2 I_\gamma (A_{0\mu\nu})^2 - \left[ \frac{1}{2} - \frac{33}{4N_{sc}} \right] g_{s0}^2 I_c (G_{0\mu\nu}^{(a)})^2 \\ &\quad - I_w [( \bar{W}_{\mu\nu}^i - e_0 \epsilon^{ij3} A_{0[\mu} \bar{W}_{\nu]}^j )^2 + e_0 \bar{W}_{\mu\nu}^3 A_{0\mu\nu} - 6m_w^2 (\bar{W}_\mu^i)^2] - 3I_c [(\bar{V}_{\mu\nu})^2 + 2e_0 Q_c \bar{V}_{\mu\nu} A^{0\mu\nu}], \end{aligned} \quad (2.5)$$

where  $I_w, I_c, I_\gamma, J_q, J_l, K_q$ , and  $K_l$  are the divergent integrals which are precisely defined in Ref. 20 and

$$D_\mu \tilde{q} = \left[ \partial_\mu - ie_0 Q_q A_{0\mu} - ig_{s0} G_{0\mu}^{(a)} \frac{\lambda^a}{2} - i\tilde{W}_\mu^i \tau^i \gamma_L - i\tilde{V}_\mu \right] \tilde{q}, \quad (2.6a)$$

$$D_\mu \tilde{l} = (\partial_\mu - ie_0 Q_l A_{0\mu} - i\tilde{W}_\mu^i \tau^i \gamma_L) \tilde{l}, \quad (2.6b)$$

$$A_{0\mu\nu} = \partial_\mu A_{0\nu} - \partial_\nu A_{0\mu}, \quad (2.6c)$$

$$G_{0\mu\nu}^{(a)} = \partial_\mu G_{0\nu}^{(a)} - \partial_\nu G_{0\mu}^{(a)} + g_{s0} f^{abc} G_{0\mu}^{(b)} G_{0\nu}^{(c)}, \quad (2.6d)$$

$$\tilde{W}_{\mu\nu}^i = \partial_\mu \tilde{W}_\nu^i - \partial_\nu \tilde{W}_\mu^i + 2\epsilon^{ijk} \tilde{W}_\mu^j \tilde{W}_\nu^k, \quad (2.6e)$$

$$\tilde{V}_{\mu\nu} = \partial_\mu \tilde{V}_\nu - \partial_\nu \tilde{V}_\mu. \quad (2.6f)$$

The operation,  $\gamma_L$ , in Eqs. (2.6a) and (2.6b) denotes the left-handed chirality projection. The singlet gluon turns out to effectively couple to quarks, but not to leptons. For the leptonic gluon ( $V_\mu = G_\mu^l$ ), one can find that the corresponding  $\mathcal{L}_{\text{div}}$  is simply given by replacing the last term in Eq. (2.5) by

$$-I_{c_l} [(\tilde{V}_{\mu\nu})^2 + 2e_0 Q_{c_l} \tilde{V}_{\mu\nu} A^{0\mu\nu}], \quad (2.7)$$

and Eqs. (2.6a) and (2.6b) by

$$D_\mu \tilde{q} = \left[ \partial_\mu - ie_0 Q_q A_{0\mu} - ig_{s0} G_{0\mu}^{(a)} \frac{\lambda^a}{2} - i\tilde{W}_\mu^i \tau^i \gamma_L \right] \tilde{q}, \quad (2.6a')$$

$$D_\mu \tilde{l} = (\partial_\mu - ie_0 Q_l A_{0\mu} - i\tilde{W}_\mu^i \tau^i \gamma_L - i\tilde{V}_\mu) \tilde{l}. \quad (2.6b')$$

The leptonic gluon, then, effectively couples to leptons instead of quarks. For the isosinglet weak boson ( $V_\mu = W_\mu^0$ ), the Lagrangian  $\mathcal{L}_{\text{div}}$  is given by using Eqs. (2.6a) and (2.6b') for  $D_\mu \tilde{q}$  and  $D_\mu \tilde{l}$  and by replacing the last term in Eq. (2.5) by

$$-2I_w [(\tilde{V}_{\mu\nu})^2 + 2e_0 \langle Q_w \rangle \tilde{V}_{\mu\nu} A^{0\mu\nu}], \quad (2.7')$$

where  $\langle Q_w \rangle = (Q_{w1} + Q_{w2})/2$ . The isosinglet weak boson effectively couples to both leptons and quarks with the same strength. For the heavy photon ( $V_\mu = A_\mu^*$ ), the coupling is the same as that of the photon and the resulting effect comes in the combination of  $e_0 A_{0\mu} + \tilde{V}_\mu$ . Thus, omitting the original  $\tilde{V}_\mu$  in Eqs. (2.5)–(2.6b) and replacing  $e_0 A_{0\mu}$  by  $e_0 A_{0\mu} + \tilde{V}_\mu$  leads to the following  $\mathcal{L}_{\text{div}}$ :

$$\begin{aligned} \mathcal{L}_{\text{div}} = & \sum_q \tilde{q} (iJ_q \not{D} - K_q m_w) \tilde{q} + \sum_l \tilde{l} (iJ_l \not{D} - K_l m_w) \tilde{l} - I_\gamma (e_0 A_{0\mu\nu} + \tilde{V}_{\mu\nu})^2 - \left[ \frac{1}{2} - \frac{33}{4N_{\text{sc}}} \right] g_{s0}^2 I_c (G_{0\mu\nu}^{(a)})^2 \\ & - I_w [(\tilde{W}_{\mu\nu}^i - \epsilon^{ij3} (e_0 A_{0[\mu} + \tilde{V}_{[\mu} \tilde{W}_{\nu]}^j])^2 + \tilde{W}_{\mu\nu}^3 (e_0 A^{0\mu\nu} + \tilde{V}^{\mu\nu})], \end{aligned} \quad (2.5')$$

where

$$D_\mu \tilde{q} = \left[ \partial_\mu - iQ_q (e_0 A_{0\mu} + \tilde{V}_\mu) - ig_{s0} G_{0\mu}^{(a)} \frac{\lambda^a}{2} - i\tilde{W}_\mu^i \tau^i \gamma_L \right] \tilde{q}, \quad (2.6a'')$$

$$D_\mu \tilde{l} = [\partial_\mu - iQ_l (e_0 A_{0\mu} + \tilde{V}_\mu) - i\tilde{W}_\mu^i \tau^i \gamma_L] \tilde{l}. \quad (2.6b'')$$

In order to cast the kinetic and interaction terms into the standard forms, we rescale the elementary photon and gluon fields as

$$\begin{aligned} A'_\mu &= \sqrt{1 + 4e_0^2 I_\gamma} A_{0\mu}, \\ G_\mu^{(a)} &= \left[ 1 + \left[ 2 - \frac{33}{N_{\text{sc}}} \right] g_{s0}^2 I_c \right]^{1/2} G_{0\mu}^{(a)}, \end{aligned} \quad (2.8a)$$

and the composite fields as

$$q = \sqrt{J_q} \tilde{q}, \quad l = \sqrt{J_l} \tilde{l}, \quad W_\mu^i = 2\sqrt{I_w} \tilde{W}_\mu^i, \quad (2.8b)$$

$$V_\mu = \begin{cases} 2\sqrt{3I_c} \tilde{V}_\mu & \text{for } V_\mu = G_\mu^0, \\ 2\sqrt{I_{c_l}} \tilde{V}_\mu & \text{for } V_\mu = G_\mu^l, \\ 2\sqrt{2I_w} \tilde{V}_\mu & \text{for } V_\mu = W_\mu^0, \\ 2\sqrt{I_\gamma} \tilde{V}_\mu & \text{for } V_\mu = A_\mu^*. \end{cases} \quad (2.8c)$$

The rescaled photon field is primed to indicate that it is mixed with other vector bosons and yet to be diagonalized. The whole effects come from  $\mathcal{L}_{\text{aux}}$  and  $\mathcal{L}_{\text{div}}$ , where the coefficients can be arranged to the following new definitions:

$$e = e_0 / \sqrt{1 + 4e_0^2 I_\gamma}, \quad (2.9a)$$

$$g_s = g_{s0} / \left[ 1 + \left[ 2 - \frac{33}{N_{\text{sc}}} \right] g_{s0}^2 I_c \right]^{1/2};$$

$$m_q = (K_q m_w + 1/F_q) / J_q, \quad (2.9b)$$

$$m_l = (K_l m_w + 1/F_l) / J_l;$$

$$M_W^2 = 3m_w^2 - 1/8F_W I_w, \quad g = 1/\sqrt{I_w}; \quad (2.9c)$$

$$M_V^2 = -1/24F_V I_c, \quad g_V = 1/2\sqrt{3I_c} \quad \text{for } V_\mu = G_\mu^0,$$

$$M_V^2 = -1/8F_V I_{c_l}, \quad g_V = 1/2\sqrt{I_{c_l}}, \quad \text{for } V_\mu = G_\mu^l, \quad (2.9d)$$

$$M_V^2 = -1/16F_V I_w, \quad g_V = 1/2\sqrt{2I_w} \quad \text{for } V_\mu = W_\mu^0,$$

$$M_V^2 = -1/8F_V I_\gamma, \quad g_V = 1/2\sqrt{I_\gamma} \quad \text{for } V_\mu = A_\mu^*.$$

The redefinitions in Eqs. (2.8a) and (2.9a) mean nothing but the renormalization of the elementary photon and gluon fields and of the coupling constants up to this order.

The effective Lagrangian  $\mathcal{L}_{\text{eff}}$ , which can be regarded as the sum of  $\mathcal{L}_{\text{div}}$  to  $\mathcal{L}_{\text{aux}}$ , is then obtained as

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \sum_q \bar{q}(i\cancel{D} - m_q)q + \sum_l \bar{l}(i\cancel{D} - m_l)l - \frac{1}{4}(A'_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu}^{(a)})^2 \\
& - \frac{1}{4}[\mathcal{W}_{\mu\nu}^i - \epsilon^{ij3}(eA'_{[\mu} + g_V\eta V_{[\mu})\mathcal{W}_{\nu]}^j)]^2 - \frac{1}{2}\lambda\mathcal{W}_{\mu\nu}^3 A'_{\mu\nu} + \frac{1}{2}M_W^2(\mathcal{W}_\mu^i)^2 \\
& - \frac{1}{4}(V_{\mu\nu})^2 - \frac{1}{2}\lambda'V_{\mu\nu}A'^{\mu\nu} - \frac{1}{2}\lambda''V_{\mu\nu}\mathcal{W}^{3\mu\nu} + \frac{1}{2}M_V^2(V_\mu)^2 + eJ_\mu^{\text{em}}A'^\mu + g_s J_{\mu a}^G G^{(a)\mu} + g_W J_{\mu i}^W \mathcal{W}^{i\mu} + g_V J_\mu^V V^\mu
\end{aligned} \quad (2.10)$$

with  $\eta=1$  (for  $V_\mu$ =heavy photon),  $=0$  (for the others), where

$$A'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu, \quad (2.11a)$$

$$G_{\mu\nu}^{(a)} = \partial_\mu G_\nu^{(a)} - \partial_\nu G_\mu^{(a)} + g_s f^{abc} G_\mu^{(b)} G_\nu^{(c)}, \quad (2.11b)$$

$$\mathcal{W}_{\mu\nu}^i = \partial_\mu \mathcal{W}_\nu^i - \partial_\nu \mathcal{W}_\mu^i + g_W \epsilon^{ijk} \mathcal{W}_\mu^j \mathcal{W}_\nu^k, \quad (2.11c)$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad (2.11d)$$

and  $J_\mu^{\text{em}}$ ,  $J_{\mu a}^G$ , and  $J_{\mu i}^W$ , respectively, stand for the electromagnetic, color-octet and weak-isotriplet currents. The current for  $V_\mu, J_\mu^V$ , is given by

$$J_\mu^V = \begin{cases} \sum_q \mathbf{q} \gamma_\mu \mathbf{q} (\equiv J_\mu^q) & \text{for } V_\mu = G_\mu^0, \\ \sum_l \bar{l} \gamma_\mu l (\equiv J_\mu^l) & \text{for } V_\mu = G_\mu^I, \\ J_\mu^q + J_\mu^l & \text{for } V_\mu = \mathcal{W}_\mu^0, \\ J_\mu^{\text{em}} & \text{for } V_\mu = A_\mu^*. \end{cases} \quad (2.12)$$

The kinetic mixing parameter  $\lambda$  for the photon ( $A'_\mu$ ) and weak boson ( $\mathcal{W}_\mu^3$ ) is calculable and fixed to be

$$\lambda = e/g_W, \quad (2.13)$$

and similarly  $\lambda'$  and  $\lambda''$  are given by

$$\begin{aligned}
\lambda' &= eQ_c/g_V, \quad \lambda''=0 & \text{for } V_\mu = G_\mu^0, \\
\lambda' &= eQ_{c_l}/g_V, \quad \lambda''=0 & \text{for } V_\mu = G_\mu^I, \\
\lambda' &= e\langle Q_w \rangle/g_V, \quad \lambda''=0 & \text{for } V_\mu = \mathcal{W}_\mu^0, \\
\lambda' &= e/g_V, \quad \lambda''=g_V/g_W & \text{for } V_\mu = A_\mu^*,
\end{aligned} \quad (2.14)$$

where  $\langle Q_w \rangle$  in Eq. (2.7'). We have suppressed the terms including the subquark fields in  $\mathcal{L}_{\text{eff}}$  in Eq. (2.10) because they are expected to be confined by some unknown mechanism.

The derived Lagrangian  $\mathcal{L}_{\text{eff}}$  turns out to be nothing but the one for the current mixing scheme of Hung and Sakurai<sup>11</sup> if the new vector field  $V_\mu$  is absent. Since we have  $\lambda=e/g_W$ , it is equivalent to the non-Higgs sector of the standard model, and phenomenologically acceptable. The terms that depend on  $V_\mu$  in  $\mathcal{L}_{\text{eff}}$  of Eq. (2.10) with Eqs. (2.12) and (2.14) exhibit the characteristic mixing and interaction patterns of the neutral exotic boson. The color-singlet gluon, leptonic gluon and isosinglet weak boson get mixed with the photon with the strength  $e/g_V \times$  (the average of the electric charges of its constituents), and, respectively, couple to the quark number ( $3B$ ), lepton number ( $L$ ), and quark plus lepton number ( $3B+L$ ) currents. The heavy photon is mixed with the  $Z$  boson as well as the photon and couples to the elec-

tromagnetic current. In this work, we have taken the photon and the gluon as elementary particles unlike in Ref. 20. The present results for the leptonic gluon coincide with those in Ref. 21, except that we miss the sum rule for the coupling constants provided by the photon compositeness condition [Eq. (22) in Ref. 21]. As for the formalism of the heavy photon, it seems essential to regard the ordinary photon as an elementary particle. Suppose that both of the photon and heavy photon are composites bound by the four-Fermi interactions which are given by the square of the electromagnetic current of subquarks. Then, the induced kinetic terms are mixed with each other to form only one complete square term of the mixed field strength; i.e., we have only one physical ‘‘photon’’ state. In general, it is difficult to construct a naive Nambu–Jona-Lasinio–type model with two independent composite states in an identical channel.

## B. Model based on the confining gauge theory of $SU(2)_L$

In this subsection, instead of the three spinor subquarks  $w, c(c_l)$ , and  $h$ , we choose two subquarks,  $w$  as a scalar and  $c(c_l)$  as a spinor, which are bound into quarks and leptons and other composite particles. Since the composite particles will be generated by a confining force based on a non-Abelian gauge theory [such as  $SU(2)_L$ ], one can argue their compositeness on the basis of the notion of complementarity,<sup>16</sup> which postulates that the physical equivalence between the Higgs (or broken) phase, where elementary particles are present, and the confining (or unbroken and hidden) phase, where composite particles are generated.

The notion relies on the observation in lattice gauge theories with scalars in the fundamental representation (such as  $w$ ),<sup>22</sup> which indicates no sharp phase boundary between the two phases and which can be paraphrased as follows: let  $g$  and  $v$  be the gauge coupling constant and the scalar vacuum expectation value and then for large  $g$  and small  $v$ , the theory falls into the confining phase while for small  $g$  and large  $v$ , the Higgs phase appears, but these two are not separated by the sharp boundary and turn out to belong to the same phase. If this is true in ordinary gauge theories with *scalars*, one expects that composites in the confining phase disguise themselves as ‘‘elementary’’ particles in the Higgs phase since both continuously map into each other (as far as the energy scale involved does not exceed the vacuum expectation value or the confinement scale) because there is no phase boundary. It can be found that the standard  $SU(2)_L \times U(1)_Y$  model based on the Higgs phase of  $SU(2)_L$  is transformed into the one based on the confining phase of  $SU(2)_L$  with the kinetic  $\gamma$ - $Z$  mixing<sup>23</sup> of the Bjorken-Hung-Sakurai type.<sup>11</sup> It is rather faithful to state that, in the confining phase,  $U(1)_Y$  is nothing but

$U(1)_{\text{em}}$  yielding  $SU(2)_L \times U(1)_{\text{em}} \rightarrow U(1)_{\text{em}}$  with  $SU(2)_L$  confined. Having this in mind, we will examine composite vector mesons made as  $s^\dagger iD_\mu s$ .

Let us turn to our discussion. Quarks and leptons are described by composites of the scalar  $w$  and the spinor  $c$  and  $c_l$  formed through the confining gauge force based on  $SU(2)_L$  (called as quantum subchromodynamics, QSCD). The composite  $W$  and  $Z$  are constructed by the scalar subquarks  $w$ . Since the exotic composite vector boson in this model is also assumed to be a bound state of a scalar subquark, a new scalar subquark denoted by  $\xi$  is introduced and is set to be subcolor  $SU(2)_L$  singlet but to carry a new subcolor  $U(1)_C$  charge. The extra subcolor  $U(1)_C$  gauge symmetry is a necessity for the existence of the exotic vector boson in the present discussion. At low energies,  $SU(2)_L$  is assumed to be confined, which is triggered by  $w^\dagger w = ww^\dagger = I$ , and low-lying composite particles are taken to have no  $U(1)_C$  charge, namely,  $U(1)_C$  is unbroken and hidden, which is triggered by  $\xi^\dagger \xi = 1$ . Our *low-lying* composite particles, i.e., quarks, leptons,  $W$ ,  $Z$  and  $V$ , are taken to be subcolor  $SU(2)_L^{\text{loc}}$  singlet and  $U(1)_C$  neutral, which are selected by the ‘‘duality’’ of the compositeness and ‘‘elementariness’’ arising from complementarity. The starting Lagrangian is the one based on  $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_C$  that controls the dynamics for  $w$ ,  $\xi$ ,  $c$ ,  $c_l$ , the photon, the gluon of QCD and the subgluon of QSCD as well as the  $U(1)_C$  gauge boson. We consider the following two cases with (1)  $U(1)_Y$ -charged (scalar)  $w_L$  and (2)  $U(1)_C$ -charged (scalar)  $w_L$ .

### 1. $U(1)_Y$ -charged $w_L$

The constituents are (1) the scalars,  $w_L$  and  $\xi$ , and the spinors,  $c_L$  and  $c_R$ :

$$w_{iL}^\alpha : (2, \tau^{(3)}; 0), \quad \xi : (1, 1; -1), \quad (2.15a)$$

$$c_{aL}^\alpha : (2, Y - Q; Q), \quad c_{iR}^\alpha : (0, Y_i - Q_i; Q_i) \quad (2.15b)$$

with  $Y = B - L$  and  $Y_i = 2Q_{\text{em}}^i = (0, -2)$  for  $\alpha = 0; = (4/3, -2/3)$  for  $\alpha = 1, 2, 3$ , where  $c^0 = c_l$  and  $c^{1,2,3} = c$ , together with (2) gauge bosons,  $\mathcal{G}_\mu$  of  $SU(2)_L$ ,  $B_\mu$  of  $U(1)_Y$  and  $C_\mu$  of  $U(1)_C$ . The charges  $Q$  and  $Q_i$  are to be specified. Composite quarks and leptons are constructed as

$$q_{iL} = \sum_a (\xi)^Q w_{iL}^\alpha c_{aL}, \quad l_{iL} = \sum_a (\xi)^Q w_{iL}^\alpha c_{iL}, \quad (2.16a)$$

$$q_{iR} = (\xi)^{Q_i} c_{iR}, \quad l_{iR} = (\xi)^{Q_i} c_{iR}, \quad (2.16b)$$

and composite vector mesons  $W_\mu$  (mainly for the  $W$  and  $Z$  bosons) and  $V_\mu$  (mainly for an exotic vector boson) are as

$$\begin{aligned} g_W (W_\mu)_i^j &= (w_L iD_\mu w_L^\dagger)_i^j \\ &= \sum_{a,b,k} w_{iL}^a \left[ (i\partial_\mu + g\mathcal{G}_\mu)_a^b w_{iL}^\dagger{}^j \right. \\ &\quad \left. - w_{aL}^\dagger{}^k g' \left[ \frac{\tau^{(3)}}{2} \right]_k^j B_\mu \right], \end{aligned} \quad (2.17a)$$

$$\frac{1}{2} g_V V_\mu = \xi iD_\mu \xi^\dagger = \xi [i\partial_\mu - \frac{1}{2}(g'B_\mu - g_C C_\mu)] \xi^\dagger. \quad (2.17b)$$

The subcolor gauge theory should generate condensates as

$$\langle w_{iL}^a w_{aL}^\dagger{}^j \rangle = \delta_i^j, \quad \langle w_{aL}^\dagger w_{iL}^b \rangle = \delta_a^b, \quad \langle \xi^\dagger \xi \rangle = 1, \quad (2.18)$$

The scalar fields with the mass dimension = 1 can be interpreted as  $\Lambda w_L$  and  $\Lambda_\xi \xi$ , where  $\Lambda$  and  $\Lambda_\xi$  are the relevant mass scales.

Equipped with the substructure of Eqs. (2.16a), (2.16b), (2.17a), and (2.17b), it is straightforward to reach the following Lagrangian for the subcolor-singlet composites from the Lagrangian of the  $[SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_C]$  gauge theory:  $\mathcal{L}_{\text{conf}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_0$  with

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2g^2} \text{Tr}(u_{\mu\nu} u^{\mu\nu}) - \frac{1}{4g_C^2} v_{\mu\nu} v^{\mu\nu} - \frac{e^2}{4g'^2} A'_{\mu\nu} A'^{\mu\nu}, \quad (2.19a)$$

$$\mathcal{L}_{\text{mass}} = \Lambda^2 \text{Tr}(g_W W_\mu)^2 + \frac{\Lambda_\xi^2}{2} (g_V V_\mu)^2, \quad (2.19b)$$

$$\begin{aligned} \mathcal{L}_0 &= \overline{\psi}_L \gamma^\mu \left[ i\partial_\mu + g_W W_\mu + g_V \frac{Q}{2} V_\mu + e Q_{\text{em}} A'_\mu \right] \psi_L \\ &\quad + \overline{\psi}_{iR} \gamma^\mu \left[ i\partial_\mu + g_V \frac{Q_i}{2} V_\mu + e Q_{\text{em}}^i A'_\mu \right] \psi_{iR}, \end{aligned} \quad (2.19c)$$

up to the radial excitations in  $w_L w_L^\dagger$  and  $\xi^\dagger \xi$ , where  $e A'_\mu = g' B_\mu$ ;  $u_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu - i[\mu_\mu, u_\nu]$  and  $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$  with

$$u_\mu = g_W W_\mu + e \frac{\tau^{(3)}}{2} A'_\mu, \quad v_\mu = g_V V_\mu + e A'_\mu. \quad (2.20)$$

The coupling constants,  $g_W$ ,  $g_V$ , and  $e$ , are expressed in terms of  $g$ ,  $g_C$ , and  $g'$  as

$$g_W = g, \quad g_V = g_C, \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g_C^2} + \frac{1}{g'^2} \quad (2.21)$$

for the canonical kinetic terms of  $W_\mu$ ,  $V_\mu$ , and  $A'_\mu$ .

To see the direct correspondence of  $V_\mu$  as  $\xi^\dagger iD_\mu \xi$  to  $V_\mu$  defined in Eqs. (2.3) and (2.8c), let us examine the kinetic mixing term. It turns out to be described by  $\mathcal{L}_{\text{mix}}$  as

$$\mathcal{L}_{\text{mix}} = -\frac{1}{2} (\lambda W_{\mu\nu}^{(3)} + \lambda_{\gamma V} V_{\mu\nu}) A'^{\mu\nu} \quad (2.22)$$

with

$$\lambda = \frac{e}{g_W}, \quad \lambda_{\gamma V} = \frac{e}{g_V}, \quad (2.23)$$

where  $\lambda_{\gamma V}$  will be related to  $\lambda'$  of Eq. (2.14). Since the mixing of  $W$  and  $V$  is missing, this Lagrangian is expected to describe the physics for the leptonic gluon with  $Q = Q_i = Q_c$  (coupling to  $L$ ), the color-singlet gluon with  $Q = Q_i = Q_c$  (coupling to  $B$ ) and the isosinglet weak boson with  $Q = Q_i = \langle Q_w \rangle (= 1/2)$  [coupling to  $(3B + L)/2$ ]. The heavy photon will be found in the model with the  $U(1)_C$ -charged  $w_L$ . It is not difficult to observe that the replacement of the coupling constant  $g_V$  as  $g_V \rightarrow Q_{c,l,c} g_V$  or  $\langle Q_w \rangle g_V$  gives the same effective Lagrangian as Eq. (2.10) with  $\lambda' = Q_{c,l,c} \lambda_{\gamma V}$  or  $\langle Q_w \rangle \lambda_{\gamma V}$  and with

$M_W = g_W \Lambda$  and  $M_V = g_V \Lambda_\xi$ . Other possibilities may include a ‘‘hypercharge boson’’ with  $Q = Y$  and  $Q_i = Y_i$ .<sup>24</sup>

The physical photon,  $A_\mu$ , is described by

$$A_\mu = A'_\mu + \lambda W_\mu^3 + \lambda_{\gamma V} V_\mu. \quad (2.24)$$

Similarly, the massive vector mesons with the diagonalized kinetic terms take the form of

$$\mathcal{W}_\mu^3 (= Z_\mu^0) = \sqrt{1 - \lambda^2} \left[ W_\mu^3 - \frac{\lambda \lambda_{\gamma V}}{1 - \lambda^2} V_\mu \right], \quad (2.25)$$

$$V_\mu = \sqrt{(1 - \lambda^2 - \lambda_{\gamma V}^2)/(1 - \lambda^2)} V_\mu, \quad \mathcal{W}_\mu^\pm = W_\mu^\pm.$$

The correspondence to the massive gauge bosons and the photon in the Higgs phase can be understood by the alternative identification of  $A_\mu$ ,  $\mathcal{W}_\mu^3$ ,  $V_\mu$ , and  $\mathcal{W}_\mu^\pm$  (with  $w_L = I$  and  $\xi = 1$ ):

$$A_\mu = \sin\theta \mathcal{G}_\mu^3 + \cos\theta b_\mu, \quad \mathcal{W}_\mu^3 = \cos\theta \mathcal{G}_\mu^3 - \sin\theta b_\mu, \quad (2.26)$$

$$V_\mu = \cos\theta_V C_\mu - \sin\theta_V B_\mu, \quad \mathcal{W}_\mu^\pm = \mathcal{G}_\mu^\pm,$$

where  $b_\mu$  ( $= \sin\theta_V C_\mu + \cos\theta_V B_\mu$ ) is associated with  $U(1)_D [\leftarrow U(1)_Y \times U(1)_C]$ ;  $g \sin\theta = g_D \cos\theta$  ( $= e$ ) with

$$g_D = g' g_C / \sqrt{g'^2 + g_C^2} = g' \cos\theta_V = g_C \sin\theta_V.$$

Then, the equivalence of the Higgs phase to the unbroken phase occurs as far as the scalar degrees of freedom are frozen.

## 2. $U(1)_C$ -charged $w_L$

The scalar, which was assigned to  $(2, \tau^3; 0)$ , now carries the  $U(1)_C$  charge instead of the  $U(1)_Y$  charge as  $w_{iL}^a$ :  $(2, 0; \tau^3)$ . The content of other particles remains intact. The substructure for the subcolor  $SU(2)_L \times U(1)_C$ -singlet particles is assumed to be, for quarks and leptons,

$$q_{iL}^A = \sum_a \xi^{(\tau^3 + Q)} w_{iL}^a c_{aL}^A, \quad l_{iL} = \sum_a \xi^{(\tau^3 + Q)} w_{iL}^a c_{aL}^0, \quad (2.27)$$

$$q_{iR}^a = \xi^{(\tau^3 + Q_i)} c_{iR}^a, \quad l_{iR} = \xi^{(\tau^3 + Q_i)} c_{iR}^0,$$

where  $\tau^3 = (1, -1)$  for  $i = (1, 2)$ , and, for vector mesons,

$$g_W (W_\mu)_i^j = (w_L i D_\mu w_L^\dagger)_i^j \\ = \sum_{a,b} w_{iL}^a \left[ (i \partial_\mu + g \mathcal{G}_\mu)_a^b w_{Lb}^\dagger - w_{aL}^\dagger g_C \frac{\tau^3}{2} C_\mu \right] \quad (2.28)$$

with the same substructure of  $V_\mu$ , Eq. (2.17b).

The Lagrangian for composite particles is found to be  $\mathcal{L}_{\text{conf}}^B = \mathcal{L}'_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}'_0$  with

$$\mathcal{L}'_{\text{kin}} = \frac{1}{2g^2} \text{Tr}(u'_{\mu\nu} u'^{\mu\nu}) - \frac{1}{4g_c^2} v_{\mu\nu} v^{\mu\nu} - \frac{e^2}{4g'^2} A'_{\mu\nu} A'^{\mu\nu}, \quad (2.29a)$$

$$\mathcal{L}'_0 = \overline{\psi}_L \gamma^\mu \left[ i \partial_\mu + g_W W_\mu + g_V \frac{\tau^3 + Q}{2} V_\mu + e Q_{\text{em}} A'_\mu \right] \psi_L \\ + \overline{\psi}_{iR} \gamma^\mu \left[ i \partial_\mu + g_V \frac{Q_i}{2} V_\mu + e Q_{\text{em}}^i A'_\mu \right] \psi_{iR} \quad (2.29b)$$

with

$$u'_\mu = g_W W_\mu + \frac{\tau^3}{2} (g_V V_\mu + e A'_\mu). \quad (2.30)$$

and the same  $v'_\mu$  in Eq. (2.20). The kinetic mixing terms contained in  $\mathcal{L}_{\text{conf}}^B$  are determined as  $\mathcal{L}'_{\text{mix}}$ :

$$\mathcal{L}'_{\text{mix}} = -\frac{1}{2} (\lambda W_{\mu\nu}^3 + \lambda_{\gamma V} V_{\mu\nu}) A'^{\mu\nu} - \frac{1}{2} \lambda'' V_{\mu\nu} W^{3\mu\nu} \quad (2.31)$$

with

$$\lambda = \frac{e}{g_W}, \quad \lambda_{\gamma V} = -\frac{e}{g_V}, \quad \lambda'' = \frac{g_V}{g_W}, \quad (2.32)$$

where the additional mixing between  $W_\mu^3$  and  $V_\mu$  emerges. As previously announced, this Lagrangian coincides with Eq. (2.10) for the heavy photon if the extra  $U(1)_C$  charge is identified with the hypercharge, i.e.,

$$Q = Y = B - L, \quad Q_i = Y_i = 2Q_{\text{em}}^i, \quad (2.33)$$

which yield  $J_\mu^Y = J_\mu^{\text{em}}$  since the extra charge becomes nothing but the electric charge given by  $(\tau^3 + Y)/2$  for  $\psi_L$  and  $Q_{\text{em}}^i$  for  $\psi_{iR}$ . The equivalence of the unbroken phase to the Higgs phase proceeds in the same way as in model (1) since  $f'_W W_\mu + g_V V_\mu$  and  $g_V V_\mu$  satisfy the same relation as Eqs. (2.17a) and (2.17b) at  $w_L = I$  and  $\xi = I$ .

Summarizing the discussions in this section, we have demonstrated that the exotic composite vector boson effectively coupled to  $3B$  (for the singlet gluon),  $L$  (for the leptonic gluon),  $3B + L$  (for the isosinglet weak boson) or  $Q_{\text{em}}$  (for the heavy photon) can be a bound state of the  $\bar{s} \gamma_\mu s$  type or the  $s^\dagger i D_\mu s$  type. The resulting effective Lagrangians (with the mass dimension  $\leq 4$ ) in both cases coincide with each other provided that the dynamics is well approximated by the four-fermi interactions for  $\bar{s} \gamma_\mu s$  and by the  $SU(2)_L$  confining interactions with the hidden local  $U(1)_C$  symmetry supplemented by complementarity for  $s^\dagger i D_\mu s$ .

## III. VECTOR-BOSON MIXING

We have seen that both models A and B lead to the same effective Lagrangian involving mixings among the photon  $A'_\mu$ , the neutral weak boson  $W_\mu^3$ , and the exotic vector boson  $V_\mu$ . The relevant part for the mixings can be read off from Eq. (2.10) in model A and from Eq. (2.22) in model B and summarized as

$$\mathcal{L}_{\text{mix}} = -\frac{1}{2} \lambda A'_{\mu\nu} W^{3\mu\nu} - \frac{1}{2} \lambda' A'_{\mu\nu} V^{\mu\nu} - \frac{1}{2} \lambda'' W_{\mu\nu}^3 V^{\mu\nu} \quad (3.1)$$

with  $\lambda$ 's given by Eqs. (2.13) and (2.14) [or equivalently Eqs. (2.23) and (2.32)]. The physical photon is then defined to be  $A_\mu = A'_\mu + \lambda W_\mu^3 + \lambda' V_\mu$  as in Eq. (2.24). Similarly, the interaction Lagrangian of quarks and leptons is specified by

$$\begin{aligned}\mathcal{L}_{\text{int}} &= eJ_\mu^{\text{em}} A'^\mu + g_W J_\mu^3 W^{3\mu} + g_V J_\mu^V V^\mu \\ &= eJ_\mu^{\text{em}} A^\mu + g_W J_\mu^Z W^{3\mu} + g_V J_\mu^X V^\mu,\end{aligned}\quad (3.2)$$

where

$$J_\mu^Z = J_\mu^3 - \lambda(e/g_W)J_\mu^{\text{em}}, \quad J_\mu^X = J_\mu^3 - \lambda'(e/g_V)J_\mu^{\text{em}}. \quad (3.3)$$

The appearance of the electromagnetic contributions in  $J_\mu^{Z,X}$  represents the phenomenon of the vector-meson dominance of the photon. It should be noted that describing the interaction terms in this way only deals with the kinetic mixings but no mass mixings and that the equality of  $\lambda(e/g_W) = (e/g_W)^2 \equiv \sin^2\theta$  yields the standard current of  $Z$ :  $J_\mu^3 - \sin^2\theta J_\mu^{\text{em}}$ .

To reach the physical vector fields with no kinetic mixings, one should make the transformation that diagonalizes the kinetic mixing terms in Eq. (3.1) without causing new mass mixing. It can be achieved by the orthogonal transformation on the  $(M_W W_\mu^3, M_V V_\mu)$  basis that preserves the mass term of  $[(M_W W_\mu^3)^2 + (M_V V_\mu)^2]/2$ . Let  $A, Z$ , and  $X$  be the diagonalized fields and  $M_Z$  and  $M_X$  be the masses of  $Z$  and  $X$ , then the diagonalization proceeds via

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} 1 & \lambda & \lambda' \\ 0 & M_W \cos\phi / M_Z & -M_V \sin\phi / M_Z \\ 0 & M_W \sin\phi / M_X & M_V \cos\phi / M_X \end{pmatrix} \begin{pmatrix} A'_\mu \\ W_\mu^3 \\ V_\mu \end{pmatrix}, \quad (3.4)$$

where  $M_{X,Z}$  and  $\phi$  are specified by  $M_{W,V}, \lambda, \lambda'$ , and  $\lambda''$ :

$$M_Z M_X = M_X = M_W M_V / \sqrt{\Delta}, \quad (3.5a)$$

$$M_Z^2 + M_X^2 = [(1 - \lambda'^2)M_W^2 + (1 - \lambda^2)M_V^2] / \sqrt{\Delta}, \quad (3.5b)$$

$$\sin 2\phi = \frac{2M_X M_Z (\lambda\lambda' - \lambda'')}{(M_X^2 - M_Z^2)\sqrt{\Delta}}, \quad (3.5c)$$

with  $\Delta = 1 - \lambda^2 - \lambda'^2 - \lambda''^2 + 2\lambda\lambda'\lambda''$ . The fields  $A_\mu, Z_\mu$ , and  $X_\mu$  can be interpreted as the physical photon,  $Z$  boson and exotic boson, respectively. One can further express the relation (3.5b) in terms of measurable masses,  $M_{W,Z,X}$ , by eliminating  $M_V$  of  $V_\mu$  by Eq. (3.5a). The relation now reads

$$\begin{aligned}[1 - (1 - \lambda^2)M_Z^2/M_W^2][1 - (1 - \lambda^2)M_X^2/M_W^2] \\ = -(\lambda'' - \lambda\lambda')^2/\Delta.\end{aligned}\quad (3.6)$$

Since  $\lambda$ 's are given by the charges,  $e, g_W$ , and  $g_V$  as in Eqs. (2.13) and (2.14), relation (3.6) imposes a constraint on the unknown parameters,  $M_X$  and  $g_V$ . The interaction Lagrangian (3.2) is transformed by Eq. (3.4) into

$$\mathcal{L}_{\text{int}} = eJ_\mu^{\text{em}} A^\mu + \mathcal{J}_\mu^Z Z^\mu + \mathcal{J}_\mu^X X^\mu, \quad (3.7)$$

where the currents  $\mathcal{J}_\mu^Z$  and  $\mathcal{J}_\mu^X$  are computed to be

$$\begin{aligned}\mathcal{J}_\mu^Z &= (g_W J_\mu^3 \cos\phi / M_W - g_V J_\mu^X \sin\phi / M_V) M_Z, \\ \mathcal{J}_\mu^X &= (g_W J_\mu^3 \sin\phi / M_W + g_V J_\mu^X \cos\phi / M_V) M_X.\end{aligned}\quad (3.8)$$

Equipped with the explicit form of the interaction La-

grangian, we can now discuss various phenomenological consequences. For low energies, where the kinetic terms and their mixings are neglected, the four Fermi neutral-current interactions take the form of

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = 4\sqrt{2}G_F \left[ (J_\mu^Z)^2 + \frac{g_V^2 M_W^2}{g_W^2 M_V^2} (J_\mu^X)^2 \right]. \quad (3.9)$$

The following comments are in order. Since  $J_\mu^X$  is the combination of  $J_\mu^V$  and  $J_\mu^{\text{em}}$ , the new contribution from the  $J^X$  piece vanishes for the neutrino-induced reactions as long as  $J_\mu^V$  does not couple to neutrinos. In our cases, it occurs in the models with the *color-singlet gluon* coupling to the baryon number because neutrinos do not carry the baryon number and with the *heavy photon* coupling to the electric charge because neutrinos are electrically neutral. In Ref. 19, we determined limits on  $M_X$  and  $g_V$  of the leptonic gluon (see also Ref. 21), color-singlet gluon and heavy photon from the presently available experimental results: (i) the weak mixing angle  $\sin\theta_W$  from the neutrino scattering; (ii) the compositeness scale  $\Lambda_c$  from the Bhabha scattering; (iii) the same as in (ii) but from  $\bar{p}p \rightarrow$  a single jet + anything; (iv)  $\sigma(\bar{p}p \rightarrow X + \text{anything})B(X \rightarrow \text{two jets})$ ; (v)  $\sigma(\bar{p}p \rightarrow X + \text{anything})B(X \rightarrow e^+e^-)$ . The results from the neutrino and Bhabha scatterings place the strongest restrictions on the leptonic gluon such as  $M_X \gtrsim 450$  GeV. Similar restrictions are expected to arise for the isosinglet weak boson, which were extensively discussed in Ref. 25. On the other hand, since the singlet gluon and heavy photon decouple from neutrinos at low energies, the weaker restrictions imposed on the singlet gluon such as  $M_X \gtrsim 170$  GeV but essentially no mass bound arises for the heavy photon. Also investigated were the effects of these exotic bosons on  $e^+e^-$  scattering in the energy regions of 50–100 GeV available at KEK, SLAC, and CERN, which were found to give sizable contributions if the color-singlet gluon or heavy photon is mediated. Further extensive analysis is under progress and will appear in a separate paper.

#### IV. CONCLUSION

We have studied properties of neutral exotic vector bosons expected in composite models of quarks and leptons, especially those of leptonic gluon (coupling to the lepton number), color-singlet gluon (coupling to the baryon number), isosinglet weak boson (coupling to the weak charge) and heavy photon (coupling to the electromagnetic charge). The gluons and photon are elementary particles of  $SU(3)_c \times U(1)_{\text{em}}$ . The underlying dynamics for composite particles are based on the following two models: (A) a nonlinear interaction model of the Nambu–Jona-Lasinio–Bjorken type and (B) a  $U(1)_{\text{em}}$  model with the confined subcolor  $SU(2)_L$  and hidden  $U(1)_C$  symmetries. Quarks, leptons and vector bosons selected as low-energy degrees of freedom are made of fermions (denoted by  $f$ ) as  $fff$  and  $(\bar{f}\bar{f})_{J=1}$  in (A) and of fermions and bosons (denoted by  $b$ ) as  $bf$  and  $(b^\dagger b)_{J=1}$  in (B). The so-selected composites in (B), which are singlets of  $SU(2)_L \times U(1)_C$ , can be found to satisfy the principle of complementarity that ensures the “duality” between  $f$

and  $bf$  (thus, the anomaly matching on chiral symmetries for  $bf$ ) and between subgluons and  $(b^\dagger b)_{J=1}$ . Other possible composites such as  $\bar{f}f$  and  $(b^\dagger b)_{J=0}$  are considered as being heavy enough.

The physically remarkable consequence is that these essentially different dynamical models give the same mixings among the photon,  $Z$  and exotic vector boson as well as the same interactions with quarks and leptons. The differences, of course, arise at higher energies of  $\sim 1$  TeV, where these models predict the different spectrum of composites. So, we expect the same phenomenology of exotic vector bosons below the energies  $\lesssim 1$  TeV, especially, at the presently available accelerators. The details

of the phenomenological analysis are left for future discussions. Our preliminary conclusion is based on Ref. 19, which deals with leptonic gluon, color-singlet gluon and heavy photon but not isosinglet weak boson.<sup>25</sup> Since color-singlet gluon and heavy photon, contrary to other particles, decouple from neutrinos at low energies ( $\ll M_W$ ), these composites are not so restricted by experiments and allowed to be relatively light. As a result, their contributions will manifest themselves through the precise measurements of the  $e^+e^-$  scattering in the energy regions of 50–100 GeV, which can distinguish the effect of the leptonic gluon from that of the heavy photon.

- <sup>1</sup>J. C. Pati and A. Salam, *Phys. Rev. D* **10**, 275 (1974); K. Matumoto, *Prog. Theor. Phys.* **52**, 1973 (1974); J. C. Pati, A. Salam, and J. Strathdee, *Phys. Lett.* **59B**, 480 (1975).
- <sup>2</sup>K. Akama and H. Terazawa, INS Report No. 257, 1976 (unpublished); H. Terazawa, Y. Chikashige and K. Akama, *Phys. Rev. D* **15**, 480 (1977).
- <sup>3</sup>M. Yasuè, *Prog. Theor. Phys.* **59**, 534 (1978); **61**, 269 (1979); Y. Tanikawa and T. Saito, *ibid.* **59**, 563 (1978); T. Ne'eman, *Phys. Lett.* **82B**, 69 (1979).
- <sup>4</sup>For reviews, see H. Terazawa, *Phys. Rev. D* **22**, 184 (1980); in *Physics at TEV Scale*, proceedings of the Meeting, Tsukuba, Japan, 1988, edited by K. Hidaka and K. Hikasa (KEK, Tsukuba, Ibaraki, 1983), p. 131; M. E. Peskin, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies*, Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (RIFP, Kyoto, 1986), p. 714; J. C. Pati, in *Superstrings, Unified Theories and Cosmology*, 1987, proceedings of the Summer Workshop, Trieste, Italy, 1987, edited by G. Furlan *et al.*, ICTP Series in Theoretical Physics, Vol. 4 (World Scientific, Singapore, 1987), p. 362.
- <sup>5</sup>M. A. Beg and A. Sirlin, *Annu. Rev. Nucl. Sci.* **24**, 379 (1974); S. Weinberg, *Phys. Rev. D* **19**, 1277 (1979); G. 't Hooft, in *Recent Developments in Gauge Theories*, proceedings of the Cargèse Summer Institute, Cargèse, France, 1979, edited by G. 't Hooft *et al.* (NATO Advanced Study Institute Series B: Physics, Vol. 59) (Plenum, New York, 1980), p. 135; L. Susskind, *Phys. Rev. D* **19**, 2691 (1979).
- <sup>6</sup>F. E. Low, *Phys. Rev. Lett.* **14**, 283 (1965); H. Terazawa, *Prog. Theor. Phys.* **37**, 204 (1967); T. Appelquist and J. D. Bjorken, *Phys. Rev. D* **4**, 3726 (1971); K. Matumoto and T. Tajima, *Prog. Theor. Phys.* **52**, 741 (1974); H. Terazawa, M. Yasuè, K. Akama, and M. Hayashi, *Phys. Lett.* **112B**, 387 (1982); M. Kuroda and D. Schildknecht, *ibid.* **121B**, 173 (1983); M. Yasuè and S. Oneda, *Phys. Rev. D* **32**, 317 (1985); **32**, 3066 (1985).
- <sup>7</sup>For an attempt of calculating quark-lepton masses including exotic fermion masses, see, for example, A. Masiero, R. Pettorino, M. Roncadelli, and G. Veneziano, *Nucl. Phys.* **B261**, 633 (1985); M. Yasuè, *Phys. Rev. D* **36**, 932 (1987); *Prog. Theor. Phys.* **78**, 1437 (1987).
- <sup>8</sup>CDF Collaboration, P. Sinervo, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions*, Stanford, California, 1989, edited by M. Riordan (World Scientific, Singapore, 1990); UA2 Collaboration, K. Eggert, *ibid.*
- <sup>9</sup>Mark II Collaboration, G. S. Abrams *et al.*, *Phys. Rev. Lett.* **63**, 2173 (1989); L3 Collaboration, B. Adeva *et al.*, *Phys. Lett. B* **231**, 509 (1989); ALEPH Collaboration, D. Decamp *et al.*, *ibid.* **231**, 519 (1989); OPAL Collaboration, M. Z. Akrawy *et al.*, *ibid.* **231**, 531 (1989); DELPHI Collaboration, P. Aarnio *et al.*, *ibid.* **231**, 539 (1989).
- <sup>10</sup>U. Amaldi, A. Bohm, L. S. Durkin, P. Langacker, A. K. Mann, W. J. Marciano, A. Sirlin, and H. H. Williams, *Phys. Rev. D* **36**, 1385 (1987); G. Costa, J. Ellis, G. L. Fogli, D. V. Nanopoulos, and F. Zwirner, *Nucl. Phys.* **B297**, 244 (1988).
- <sup>11</sup>J. D. Bjorken, in *Proceedings of the Ben Lee Memorial International Conference on Parity Nonconservation, Weak Neutral Currents and Gauge Theories*, Fermilab, Batavia, Illinois, 1977, edited by D. B. Cline and F. E. Mills (Harwood Academic, New York, 1979), p. 701; *Phys. Rev. D* **19**, 335 (1979); P. Q. Hung and J. J. Sakurai, *Nucl. Phys.* **B143**, 81 (1978).
- <sup>12</sup>K. Akama, in *Physics at TEV Scale* (Ref. 4), p. 131; M. Suzuki, *Phys. Rev. D* **37**, 210 (1988); K. Akama and T. Hattori, *ibid.* **39**, 1992 (1989); M. Yasuè, *Mod. Phys. Lett. A* **4**, 815 (1989).
- <sup>13</sup>Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); J. D. Bjorken, *Ann. Phys. (N.Y.)* **24**, 174 (1963).
- <sup>14</sup>G. 't Hooft, in *Recent Developments in Gauge Theories* (Ref. 5); R. Casalbuoni and R. Gatto, *Phys. Lett.* **93B**, 47 (1980); H. Terazawa, *Prog. Theor. Phys.* **64**, 1763 (1980); O. W. Greenberg and J. Sucher, *Phys. Lett.* **99B**, 339 (1981).
- <sup>15</sup>L. F. Abbott and E. Farhi, *Phys. Lett.* **101B**, 69 (1981); *Nucl. Phys.* **B189**, 547 (1981); T. Kugo, S. Uehara, and T. Yanagida, *Phys. Lett.* **147B**, 321 (1984); S. Uehara and T. Yanagida, *ibid.* **165B**, 94 (1985).
- <sup>16</sup>G. 't Hooft, in *Recent Developments in Gauge Theories* (Ref. 5); S. Dimopoulos, S. Raby, and L. Susskind, *Nucl. Phys.* **B173**, 208 (1980); T. Matsumoto, *Phys. Lett.* **97B**, 131 (1980); R. Casalbuoni and R. Gatto, *ibid.* **103B**, 113 (1981).
- <sup>17</sup>Terazawa, Chikashige, and Akama (Ref. 2); Terazawa (Ref. 4); I. Ito and M. Yasuè, *Phys. Rev. D* **29**, 547 (1984); M. Yasuè, *ibid.* **39**, 3458 (1989).
- <sup>18</sup>See also, M. Yasuè, *Nucl. Phys.* **B234**, 252 (1984).
- <sup>19</sup>K. Akama, T. Hattori, and M. Yasuè, *Phys. Rev. D* **42**, 789 (1990).
- <sup>20</sup>(a) Akama and Hattori (Ref. 12); (b) *Phys. Rev. D* **40**, 3688 (1989).
- <sup>21</sup>Akama and Hattori [Ref. 20(b)].
- <sup>22</sup>E. Fradkin and S. H. Shenker, *Phys. Rev. D* **19**, 3782 (1979); T. Banks and E. Rabinovici, *Nucl. Phys.* **B160**, 349 (1979).
- <sup>23</sup>Yasuè (Ref. 12). See also, V. Višnjić, *Nuovo Cimento* **101A**, 385 (1989).
- <sup>24</sup>M. Kuroda, D. Schildknecht, and K. H. Schwarzer, *Nucl. Phys.* **B261**, 432 (1985); C. Bilchak and D. Schildknecht, Report No. BI-TP 8/18, 1989 (unpublished).
- <sup>25</sup>K. Akama and T. Hattori (in preparation).