Absence of exotic hadrons in flux-tube quark models

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Flux-tube quark models based upon strong-coupling QCD are studied in the limit of small coupling between different flux-tube topologies. In this limit, it is shown that no bound exotic states exist, in contrast with predictions made with bag and other mean-field and two-body potential quark models. Results are presented for the $2q$-$2\bar{q}$, $4q$-$\bar{q}$, and $6q$ sectors, and the various quark models are compared. The extent to which "exotic" states are observed experimentally can be regarded to some degree as a test of the relevance of strong-coupling-based flux-tube models.

I. INTRODUCTION

It is believed that color fields are confined, and hence only colorless-singlet states can exist as isolated states in nature. The qq and $3q$ color-singlet states are generally associated with the observed mesons and baryons. These hadrons may have additional virtual $q\bar{q}$ pairs (sea quarks); however, for the present discussion we ignore them and consider only valence quarks. It is of course possible to construct color-singlet states from more than three quarks. The most trivial of these contain two or more hadrons, either free or loosely bound as in a nucleus. In these states the color fields are confined within two or more disconnected regions of space. In a nucleus it is possible that these regions overlap to some extent, but they are believed to be distinct in the dominant configurations. In contrast, one can construct so-called multiquark hadrons (MQH's) which have more than three valence quarks in a single volume within which the color fields are confined. Such states have not been experimentally verified, and therefore are often called exotic.

MQH's have been studied within the framework of the bag model by placing more than three quarks in a single bag.[1-4] They have also been studied with other quark potential models, including alternative mean-field-type models[5,6] and models in which the quarks interact only through two-body potentials.[7-11] In the mean-field studies it is generally found that an H dibaryon consisting of $2u$, $2d$, and $2s$ quarks has a mass less than two isolated Λ's and hence is stable against strong decay. Experimental programs to search for such a particle are under way.[12] Nonstrange dibaryons, in particular one having $J^{\pi}=3^{+}$ and isospin zero, have also been predicted to exist as narrow resonances in nucleon-nucleon scattering. No exotic narrow resonances have yet been confirmed.[13]

Quark models based upon two-body potentials have also been employed to study systems of four,[7] five,[11] and six[8-10] quarks. These models necessarily encounter difficulties with van der Walls interactions, but may nevertheless be useful for qualitative studies. Variational calculations generally find that the confining interaction is not attractive in the multiquark sectors (the kinetic energy required to cluster two hadrons is greater than the potential energy gained). However, the hyperfine interaction is strong enough in some spin-isospin channels to produce exotic configurations which are either strong resonances or absolutely stable against decay into normal mesons and baryons.

These predictions of the H and other exotic hadrons must, to some extent, be influenced by the assumptions of the various models, in particular the mean-field-type approximation inherent in models used in Refs. 1–5. In the present work we study MQH's having $2q$-$2\bar{q}$ $4q$-$\bar{q}$, and $6q$ within the flux-tube model inspired by the strong-coupling limit of the QCD lattice Hamiltonian.[14-19] The "exotic" multiquark states have not previously been calculated within the flux-tube model, although they have been discussed in related harmonic-oscillator string models.[20]

This model has been phenomenologically quite successful, and does not make use of a mean-field-type interaction. It predicts rather interesting flux-tube structures for MQH's as discussed in Sec. II. Variational and Green's-function Monte Carlo (GFMC) calculations of the energies of the various MQH's are reported in Sec. III. Our main conclusion, discused in Sec. IV, is that in this quark model there are no stable or narrow resonant MQH states.

II. MQH'S IN THE FLUX-TUBE MODEL

In the flux-tube quark model it is assumed that the color-electric flux is confined to narrow, stringlike tubes joining the quarks in accordance with Gauss' law. A flux tube starts from every quark i and ends on an antiquark $\bar{i}$. Three flux tubes i,j,k can end or start from an antisymmetric ϵ_{ijk} junction called the Y junction. The resulting flux-tube patterns for the familiar $q\bar{q}$ and $3q$ states, as well as the exotic MQH having four to six particles, are shown in Fig. 1. As is apparent in the figure, a state with $N+1$ particles is generated by replacing a quark or an anti-

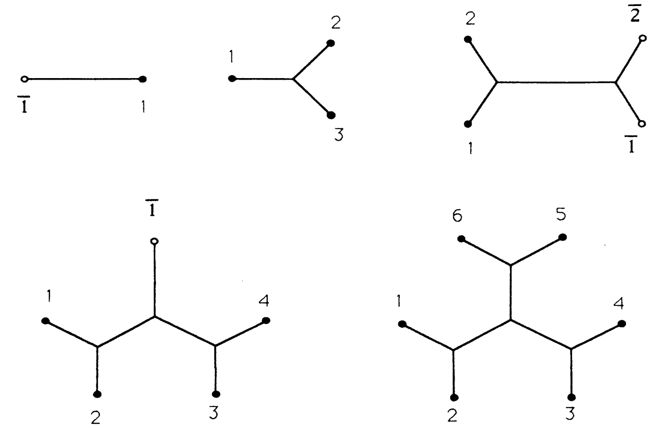

FIG. 1. Flux-tube configurations for confined states of two to six quarks.

quark in an N-particle state by a Y junction and two antiquarks or quarks.

The quarks are treated as semirelativistic spin-$\frac{1}{2}$ Pauli particles. In contrast, bag[1-4] and other mean-field[5,6] models treat quarks more accurately as relativistic Dirac spinors. We solve for the eigenstates of a Hamiltonian H_Q which operates only on the quark degrees of freedom. It contains three main terms:

$$H_Q = H_0 + H_F + H_I \ , \tag{1}$$

where H_0 is the relativistic kinetic energy of the quarks;

$$H_0 = \sum_{i=1,\ldots,N} (m_i^2 + p_i^2)^{1/2} \ . \tag{2}$$

H_f, the energy of the flux tubes, is obtained by minimizing the total length of the tubes by varying the position of the Y junctions. Let $L(\mathbf{r}_i)$ be the minimum length for a given configuration $\{\mathbf{r}_i, i=1,\ldots,N\}$ of the N quarks; then

$$H_f = \sqrt{\sigma} L(\mathbf{r}_i) - N\delta M \ . \tag{3}$$

Here $\sqrt{\sigma}$ is the string tension of the tubes and δM is a constant term added to the Hamiltonian which is proportional to the number of quarks N. This term represents the constants $M_{q\bar{q}}$ and M_{qqq} used in Ref. 15 for the $N=2$ and 3 states. Since these constants appear to be proportional to N it is natural to associate them with the free ends of the tubes. We note that a similar term proportional to N appears in pairwise additive potentials of the form

$$V_{ij} = \sum_{i,j} v(r_{ij}) \lambda_i \cdot \lambda_j \ , \tag{4}$$

whenever the potential $v(r)$ contains a constant. The term proportional to N appears as a consequence of the fact that an operator $\sum_i \lambda_i$ acting on an overall color-singlet state Φ gives zero, and hence

$$\sum_{i<j} \lambda_i \cdot \lambda_j \Phi = -\frac{1}{2} \left[\sum_i \lambda_i^2 \right] \Phi = -\frac{8}{3} N \Phi \ . \tag{5}$$

H_I represents the short-range one-gluon interaction between the quarks. It consists of a Coulomb, spin-spin, tensor, and spin-orbit terms. Our primary interest here is in low-energy S-wave hadrons in which the tensor and spin-orbit interactions are not very important. Hence, for the sake of simplicity, we use the approximation

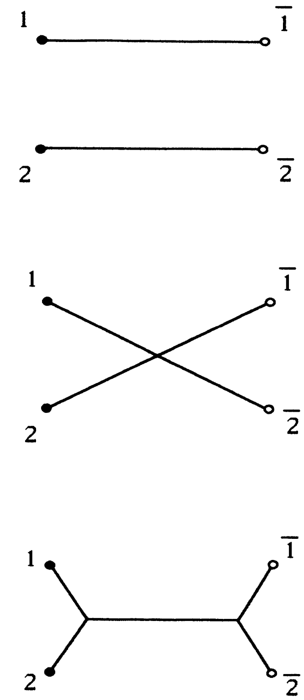

FIG. 2. Possible flux-tube configurations for a $2q$-$2\bar{q}$ system.

$$H_I = \alpha_c \sum_{i<j} \sum_{\alpha=1,\ldots,8} F_i^\alpha F_j^\alpha \left[\frac{1}{r_{ij}} - \frac{2\pi}{3\bar{m}^2} s(r_{ij}) \sigma_i \cdot \sigma_j \right] \ . \tag{6}$$

Here F_i^α is $\frac{1}{2}\lambda_i^\alpha$ for quarks and $-\frac{1}{2}(\lambda_i^\alpha)^*$ for antiquarks, and α_c is the QCD fine-structure constant. The usual δ function in the spin-spin interaction is broadened by assuming $\exp(-\frac{1}{2}\Lambda^2 q^2)$ vertex form factors, so that

$$s(r) = \left[\frac{2}{\sqrt{\pi}\Lambda} \right]^3 \exp\left[-\frac{r^2}{4\Lambda^2} \right] \ . \tag{7}$$

The interaction parameters used are obtained by fitting the masses of light mesons and baryons:[15]

$$\begin{aligned} &\tfrac{4}{3}\alpha_c = 0.5 \ , \\ &\Lambda = 0.13 \text{ fm} \ , \\ &m = 0.313 \text{ GeV} \ , \\ &\bar{m} = 0.36 \text{ GeV} \ , \\ &\sqrt{\sigma} = 1 \text{ GeV/fm} \ , \\ &\delta M = 0.41 \text{ GeV} \ . \end{aligned} \tag{8}$$

The interpretation of this model is trivial for $q\bar{q}$ and $3q$ states, however many questions have been raised about it for $N \geq 4$ states. There are three different flux-tube arrangements for the $2q$-$2\bar{q}$ state as shown in Fig. 2. In the lattice QCD terminology[14] these states are given by

$$|\mathrm{I}\rangle = \Psi_i^\dagger(\mathbf{r}_{q1})U_{im}(\mathbf{r}_{q1},\mathbf{r}_{\bar{q}1})\Psi_m(\mathbf{r}_{\bar{q}1})\Psi_{i'}^\dagger(\mathbf{r}_{q2})U_{i'm'}(\mathbf{r}_{q2},\mathbf{r}_{\bar{q}2})\Psi_{m'}(\mathbf{r}_{\bar{q}2})|0\rangle\ , \tag{9}$$

$$|\mathrm{II}\rangle = \Psi_i^\dagger(\mathbf{r}_{q1})U_{im}(\mathbf{r}_{q1},\mathbf{r}_{\bar{q}2})\Psi_m(\mathbf{r}_{\bar{q}2})\Psi_{i'}^\dagger(\mathbf{r}_{q2})U_{i'm'}(\mathbf{r}_{q2},\mathbf{r}_{\bar{q}1})\Psi_{m'}(\mathbf{r}_{\bar{q}1})|0\rangle\ , \tag{10}$$

$$|\mathrm{III}\rangle = \Psi_i^\dagger(\mathbf{r}_{q1})U_{im}(\mathbf{r}_{q1},\mathbf{r}_3)\Psi_j^\dagger(\mathbf{r}_{q2})U_{jn}(\mathbf{r}_{q2},\mathbf{r}_3)\epsilon_{mnk}U_{k'k}(\mathbf{r}_4,\mathbf{r}_3)\epsilon_{k'm'n'}U_{m'i'}(\mathbf{r}_4,\mathbf{r}_{\bar{q}1})\Psi_{i'}(\mathbf{r}_{\bar{q}1})U_{n'j'}(\mathbf{r}_4,\mathbf{r}_{\bar{q}2})\Psi_{j'}(\mathbf{r}_{\bar{q}2})|0\rangle\ , \tag{11}$$

where $\Psi_i^\dagger$ and Ψ_m are quark operators and $U_{im}(\mathbf{r},\mathbf{r}')$ denote products of link operators,

$$U_{im}(\mathbf{r},\mathbf{r}') = U_{ij}(l_1)U_{jk}(l_2)\cdots U_{lm}(l_n)\ , \tag{12}$$

the links $l_1, l_2, \ldots, l_n$ join $\mathbf{r}$ and $\mathbf{r}'$, and a sum over the repeated color indices $i, j, \ldots, m$ is implied. Even when the quark positions are the same the states $|\mathrm{I}\rangle$, $|\mathrm{II}\rangle$, and $|\mathrm{III}\rangle$ are orthogonal to each other due to differences in the link operators. They have different flux topologies, and hence are not coupled by the Hamiltonian H_Q. The underlying assumption of H_Q is that the flux adiabatically follows the motion of the quarks without changing its topology.

The full QCD Hamiltonian has other terms in addition to H_Q. Of these, the one most studied is H_B, the term responsible for breaking and rejoining the flux tubes. It couples states with different numbers of particles, and has been used to study the decay of mesons into two mesons,[21] the mesons-baryon couplings,[22,23] and the $\Delta \to N + \pi$ width.[24] It gives second-order corrections to the hadron energies[24] which can partly be absorbed into the values of the parameter $\sqrt{\sigma}$ and δM. As illustrated in Fig. 3, H_B can couple states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ in fourth order. It cannot change the number of Y junctions, and hence does not couple $|\mathrm{III}\rangle$ to either $|\mathrm{I}\rangle$ or $|\mathrm{II}\rangle$.

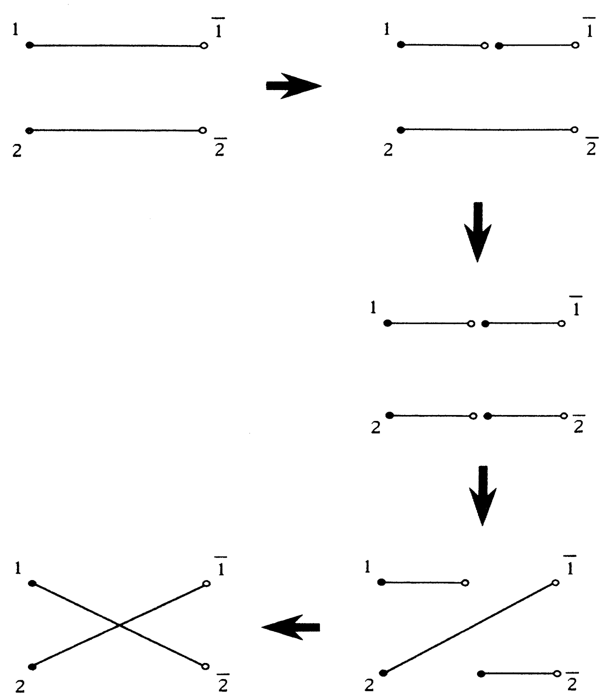

FIG. 3. Coupling of states $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ in Fig. 2 by H_B.

The term H_P containing $\mathrm{Tr}U(1)U(2)U(3)U(4)$ over closed plaquettes can create or annihilate Y junctions as illustrated in Fig. 4. This term can change flux topologies and couple the states $|\mathrm{I}\rangle$, $|\mathrm{II}\rangle$, and $|\mathrm{III}\rangle$. Even though it has been mentioned several times in the literature,[16,17] its effect on hadron spectroscopy has not been studied. In the limit that the quark mass is large one can take the point of view that these terms can change the flux topology faster than the motion of the quarks. In the limit one may be able to use the topology that gives the lowest H_F for a given configuration of quarks, as has been assumed in some models.[25] In the present work we assume that for light quarks it is more reasonable to start with H_Q and treat effects of H_B and H_P perturbatively.

Any pair of quarks in a $3q$ hadron may be exchanged without influencing the flux-tube topology. This exchange gives a factor of -1 from the antisymmetric Y junction, and for this reason the quarks in a $3q$ hadron have symmetric bosonlike wave functions in the combined coordinate, spin, and flavor space. Similarly, the pairs of quarks and antiquarks in the $2q$-$2\bar{q}$ MQH (Fig. 1) have to be symmetric excluding color. For $N \geq 5$, however, the symmetry requirements are more novel. Consider the $4q$-$\bar{q}$ state, and let quarks 1 and 2 be connected to one Y junction, and quarks 3 and 4 to the other as shown in Fig. 1. Obviously pairs 1,2 and 3,4 can be exchanged and must therefore be symmetric under the pair interchange. Exchanging either 1 or 2 with 3 or 4, however, changes the topology, and the resulting state cannot interfere with the original. Hence exchanges of this type can be neglected in this model. On the other hand, we can consider the simultaneous exchange of pair 1,2 with pair 3,4. This exchange acquires a factor -1 from the Y junction to which the $\bar{q}$ is connected, and hence has to be antisym-

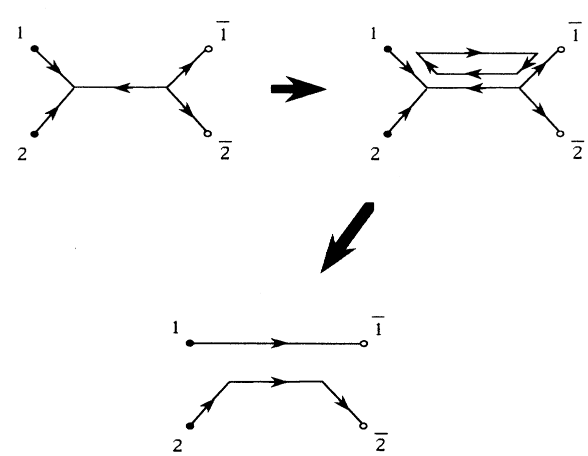

FIG. 4. Coupling of different flux-tube topologies by H_Q.

metric in the space which excludes color. The symmetry requirements in the six-quark states are now obvious. Excluding color, quarks connected to the same Y junction must be symmetric, while pairs of diquarks connected to the same Y junction must be antisymmetric and exchange between quarks connected to different Y junctions is to be neglected.

The flux-tube topologies specify how the colors of the quarks and antiquarks are coupled to form the color-singlet state. Hence the color factors $F_i^\alpha F_j^\alpha$ of the one-gluon-exchange interaction are uniquely determined by the flux-tube topology. It can be verified that the value of this factor is

$$\sum_\alpha F_i^\alpha f_j^\alpha = -\tfrac{4}{3}(\tfrac{1}{2})^{n_y(ij)} , \tag{13}$$

where $N_y(ij)$ is the number of Y junctions encountered in going from i to j.

III. CALCULATIONS

We have studied the MQH states in the flux-tube model with both variational Monte Carlo[14,26] (VMC) and GFMC[27–29] methods. In VMC calculations the Monte Carlo method is used to evaluate and minimize $\langle H \rangle$ with respect to changes in the variational parameters of the trial function Ψ_T. The VMC method is described in Ref. 14; here we only present the specific forms chosen for the trial wave functions and discuss the evaluation of the nontrivial terms in the Hamiltonian.

Our choice of variational (trial) wave functions for the "exotic" hadrons (states in which all quarks are confined into one region by flux tubes) was guided by previous studies of the mesons and baryons. Initially, we consider only the spatial part of the wave functions, and calculate the lowest state of the spin-independent Hamiltonian, which contains the kinetic, flux-tube, and color Coulomb energies. We are assuming here that the symmetry requirements described in the preceding section can be satisfied by an appropriate choice of spin-flavor functions, and give specific examples of the complete wave function later in this section. For the $2q$-$2\bar{q}$ system, the spatial part of the wave function Ψ_t is taken to be

$$\Psi_T = f(r_{12}) f(r_{34}) F(R_{12}, R_{34}) , \tag{14}$$

where particles 1 and 2 are quarks and 3 and 4 are antiquarks. The f_{ij} are functions of the distance between particles i and j, and have the same functional form used previously in the meson and baryon studies:

$$f(r) = r^\delta \exp\{-w(r)(\gamma_1 r + \gamma_2 r^2) - [1 - w(r)]\gamma_{1.5} r^{1.5}\} . \tag{15}$$

The factors γ and the constant δ are variational parameters, and $w(r)$ is a Woods-Saxon function whose strength and range are additional variation parameters. This form interpolates between a Coulomb-like solution near the origin to the behavior appropriate to a linear potential at large separations. The function F describes correlations between the center of mass of the quarks (1 and 2) and the antiquarks (3 and 4), and it is chosen as

$$F(R) = \exp(-\gamma_c R^{1.5}) . \tag{16}$$

The constant γ_c is an additional variational parameter.

The spatial part of the trial function for the other systems is similar. For example, that of the six-quark wave function is given by

$$\Psi_T = f(r_{12}) f(r_{34}) f(r_{56}) f(R_{12}, R_{34}) F(R_{12}, R_{56}) \times F(R_{34}, R_{56})[1 - \beta V_3(R_{12}, R_{34}, R_{56})] . \tag{17}$$

The quarks 12, 34, and 56 are paired in this wave function, and we use the same form for the two-body correlations f and the pair center-of-mass correlations F as in the $2q$-$2\bar{q}$ case. The last term in this expression is a small three-body correlation between the centers of mass of the pairs; its functional form is the same as the three-body correlations used previously in studies of the baryon wave function:

$$V_3(r_1, r_2, r_3) = \sqrt{\sigma}\left[L(\mathbf{r}_i) - \tfrac{1}{2}\sum_{i,j} r_{ij}\right] . \tag{18}$$

The function L has been defined previously; here it is simply the minimum length of a string connecting points r_1, r_2, and r_3, and it is well approximated by half the sum of two-body distances $\sum_{i<j} r_{ij}$. The $(1 - \beta V_3)$ correlation is an attempt to incorporate the most important nonpairwise components of the six-body potential into the trial wave function. These variational wave functions give accurate energies for the ground states of the spin-independent Hamiltonian, typically within nearly one standard error ($\sim$20 MeV) of the exact GFMC result.

The $4q$-$\bar{q}$ wave function is obtained by replacing the coordinates of quarks 5 and 6 in the six-quark wave function by the antiquark coordinate R_5:

$$\Psi_T = f(r_{12}) f(r_{34}) F(R_{12}, R_{34}) f(R_{12}, R_5) \times F(R_{34}, R_5)[1 - \beta V_3(R_{12}, R_{34}, R_5)] , \tag{19}$$

Once the hyperfine interaction is included in the Hamiltonian, it is also necessary to specify the spin-flavor parts of the wave function. The wave functions of S-wave hadrons may be written as $\Psi_T X$, where X are the spin-flavor wave functions given below. The simplest case is the $2q$-$2\bar{q}$ system, for which a spin-0 wave function may be taken to be

$$X_4(S=0) = \chi_{12}^0 \chi_{34}^0 (ud - du)_{12} (\bar{u}\bar{d} - \bar{d}\bar{u})_{34} , \tag{20}$$

where χ_{ij}^S is the state in which spins i and j are coupled to spin S. In this wave function the two quarks (1 and 2) and the two antiquarks (3 and 4) are coupled to spin zero; these are the pairs which are strongly correlated by the confining interaction. Since the hyperfine interaction is strongest between pairs 1,2 and 3,4 [Eq. (14)], and is attractive for singlet states, this state has the most attractive hyperfine interaction. The product $\Psi_T X_4(S=0)$ thus represents the trial wave function for the ground state of the exotic $2q$-$2\bar{q}$ system.

It is also instructive to consider an excited spin-2 state of $2q$-$2\bar{q}$. In this case the spin-flavor wave function is taken as

$$X_4(S=2)=\left[\prod_i \uparrow_i\right](uu)_{12}(\bar{d}\,\bar{d})_{34}\ , \tag{21}$$

where $\uparrow_i$ denotes a spin-up state of quark i. Clearly this state has a repulsive hyperfine interaction. However, if the spatial extent of this four-particle state is large, it is possible that the strength of this repulsion is weaker than in two isolated spin-1 mesons, and that consequently it may exist as a narrow resonance.

The most often-studied MQH is the H dibaryon; which is a total spin-0 state of $2u$, $2d$, and $2s$ quarks. In the flux-tube model, the spin-flavor part of the wave function is given by

$$X_6(H)=(\chi^0_{12}\chi^0_{34}\chi^0_{56})\times\sum_P(-1)^P[(ud-du)_{12}(ds-sd)_{34}(us-su)_{56}]\ . \tag{22}$$

The sum over flavor states runs over the permutations P of the three pairs and ensures an overall antisymmetric spin-flavor wave function when two pairs are interchanged. Recalling that another factor of -1 comes from the color factors at the central Y junction, we obtain an overall plus sign when two pairs of quarks are interchanged. As required by the color symmetry, the combination of spin and flavor terms within a pair are symmetric. Note that this wave function couples quarks attached to the same Y junction to spin 0, and hence has the most attractive hyperfine interaction in the six-quark system. For the sake of simplicity we neglect the mass difference between u, d, and s quarks. In this case the H is the ground state of the $6q$ exotic state.

Another six-quark state has recently been proposed as a candidate for a narrow exotic resonance. If one assumes that a confined $6q$ state is spatially lager than $3q$ $S=\frac{3}{2}$ Δ baryon, the $6q$ $S=3$ state may have a less repulsive hyperfine interaction than the two isolated Δ's, and hence may exist as a narrow dibaryon resonance. The flux-tube spin-flavor wave function for such a state is

$$X_6(S=3)=\prod_i(\uparrow_i)\sum_P(-1)^P(uu)_{12}(dd)_{34}(ud+du)_{56}\ . \tag{23}$$

All spins are aligned in this state; therefore, the flavor pairs must be symmetric also. We note that this state has zero isospin.

States consisting of $4q$ and a $\bar{q}$ may be easily constructed by replacing the pair 56 in a $6q$ state with a single antiquark. Note, however, that to retain the highest spatial symmetry and spin-0 diquarks, we must still include a strange quark in one pair (similar to the H dibaryon above). However, this is not necessary in states having spin-1 diquarks. In this case the wave function can be written as, for example,

$$X_5(S=\tfrac{5}{2})=\prod_i(\uparrow_i)[(uu)_{12}(dd)_{34}\bar{u}_5-(dd)_{12}(uu)_{34}\bar{u}_5]\ . \tag{24}$$

Given these forms for the trial wave functions, we must be able to evaluate the expectation values of the Hamiltonian. Evaluation of the two-body terms in the interaction is straightforward, and the many-body confining interaction H_F is relatively simple to calculate numerically. For example, given the positions of the six quarks, and assuming a position for the center Y junction (see Fig. 1), the formulas given in Ref. 14 can be used to solve for the positions of the remaining Y junctions and consequently the total length of the flux tubes. The remaining task of minimizing this total length by varying the position of the central Y junction is easily accomplished through the simplex method, for example, which typically requires of the order of 10 iterations to provide a very accurate potential.

We also need to evaluate the expectation value of the semirelativistic kinetic energy operator. This is not trivial due to the nonlocal nature of the operator (it involves all powers of the momentum operator); and therefore we evaluate the free-particle propagator

$$\langle\Psi_T|\exp[-\sqrt{(p_i^2+m_i^2)}\epsilon]|\Psi_T\rangle \tag{25}$$

for a small parameter ϵ and extrapolate the result to linear terms in ϵ. This method has been checked in two-body calculations where it is trivial to calculate the Fourier transform of the coordinate-space wave function and evaluate the kinetic energy directly in momentum space. The free-particle propagator is given in Ref. 29:

$$G(R)=\frac{4\pi}{(2\pi)^3}\frac{\beta\alpha^2}{R^3(1+\beta^2)}K_2(\alpha(1+\beta^2)^{1/2})\ , \tag{26}$$

with

$$\beta=\frac{\hbar c\epsilon}{R}\ ,\qquad \alpha=mcR/\hbar\ . \tag{27}$$

The propagation distance is R, and K_2 is a Bessel function of order 2. This same propagator is employed when calculating the ground-state energies of the MQH with the GFMC method.

The GFMC method projects the true ground-state wave function ψ_0 through

$$\Psi_0=\lim_{\tau\to\infty}\exp(-H\tau)\Psi_T\ . \tag{28}$$

For these calculations we employ the short-time approximation

$$\exp(-H\tau)=\prod_{i=1,N}\exp(-H\Delta\tau)\approx\prod_{i=1,N}\exp(-V\Delta\tau/2)\times\exp(-T\Delta\tau)\exp(-V\Delta\tau/2)\ . \tag{29}$$

The solution of the Schrödinger-like equation is obtained through an analogy with the diffusion equation in imaginary time; the kinetic energy term is responsible for the diffusion, while the effects of the potential energy are incorporated into a branching algorithm. This method is exact up to finite-time-step errors due to the Trotter breakup of the Hamiltonian; we employ time steps of approximately 0.03 GeV^{-1} in order to minimize this error.

For the cases where the hyperfine interaction is included, we consider only wave functions in which the spin-flavor part of the wave function is that given above, and solve for the spatial wave function which gives the lowest energy.

IV. RESULTS AND CONCLUSIONS

We first consider result for the dominant spin-independent Hamiltonian summarized in Table I. The statistical error in the VMC calculations are ≤ 10 MeV, whereas the symmetric and statistical error in the GFMC calculations are ~ 20 MeV. The GFMC energies are lower than the VMC by only ~ 20 MeV; hence, the two calculations give nearly identical results. In the q-$\bar{q}$ system the GFMC result has been checked by diagonalizing in a basis of Gaussian functions.

The most striking feature of these results is that each additional particle added to the system raises the energy by a roughly constant amount. The additive constant is large (~ 540 MeV), and hence the exotic MQH states are four to five hundred MeV higher than two hadron states. This is not a trivial result since for some configurations of the particles the MQH confining potential is less repulsive than for two isolated baryons. Overall, however, in the flux-tube model it costs more kinetic and potential energy to form the multiquark systems (Table I).

Of course, the effects of the hyperfine interaction must be included in any reasonable calculation. We consider first the case of the six-quark systems, both the $S=0$ H dibaryon and the proposed $S=3$ exotic. The results for these systems are given in Table II. These are obtained with the GFMC method as follows. In the $S=3$ state the $\sigma_i \cdot \sigma_j$ in H are replaced by unity, and an effective spin-independent interaction is obtained. In the $S=0$ state the $\sigma_i \cdot \sigma_j$ are replaced by -3 when the quarks belong to the same Y junction (and therefore are coupled to spin 0), and zero otherwise. Thus the $\sigma_i \cdot \sigma_j$ interaction between quarks belonging to different Y junctions is treated in perturbation theory. In first order its contribution is zero, and its higher-order contributions are expected to be negligibly small.

The strength of the hyperfine interaction has been adjusted to roughly reproduce the experimental N-Δ splitting of ~ 290 MeV; with these parameters we obtain a splitting of 0.27 GeV. Other effects, including coupling to pions (string breaking) may also contribute ~ 0.1 GeV to this splitting.[24] This model reproduces the experimental π-ρ splitting of 630 MeV; however, the pion has a rather small energy of 55 MeV. The results are slightly different than presented in Ref. 15, almost entirely due to the fact that in the previous study the constant term δM was adjusted independently in the meson and baryon sector.

We have computed the H dibaryon in the SU(3) limit in which all u, d, and s quarks have the same mass. The hyperfine interaction does provide a relative attraction for the H dibaryon when compared to two baryons. Two isolated spin-$\frac{1}{2}$ baryons gain approximately 350 MeV from the spin-dependent term, while the H dibaryon gains ~ 500 MeV. This shift is close to what one would expect from first-order perturbation theory. The $3q$ baryons have one $S=0$ pair each; thus, their total energy shift is roughly equal to the N-Δ splitting. The H dibaryon has three such pairs, and consequently gains an energy of nearly 1.5 times the N-Δ splitting. However, this additional attraction is not nearly strong enough to overcome the much higher energy of the six-quark state in the confining potential; indeed the full calculation reveals that the energy difference between the H and two baryon states is 300 MeV in the SU(3) limit.

To be more realistic we must consider the effect of the strange-quark mass being larger that that of u and d quarks. The energy of the H in this case can be estimated as follows. The mass difference between the Σ and the nucleon (~ 250 MeV) essentially represents the difference in the energies of $S=0$ us or ds diquarks and the $S=0$ ud diquark. Hence a better estimate of the energy of H is ~ 2.8 GeV obtained by adding 500 MeV to the energy in the SU(3)-symmetric limit. This estimate is much larger than $2M_\Lambda = 2.23$ GeV.

We next consider the proposed $S=3$ dibaryon. In this case the two isolated baryons (Δs) are pushed up approximately 200 MeV by the hyperfine interaction. This shift is somewhat smaller than that of two nucleons due to nonperturbative effects in the nucleon channel.[15] The energy shift in the six-quark state (~ 190 MeV) is only slightly smaller. Consequently, the six-quark state again remains higher in energy than two baryons, by approximately 450 MeV. It was noted in Ref. 5 that this state is bound in all previously studied quark models. It is not bound in the flux-tube model because of the relatively small spatial extent of the six-quark system. Mean-field models typically produce a six-quark state of significantly larger spatial extent than baryons, and hence a smaller hyperfine repulsion in the spin-3 dibaryon. This effect is

TABLE I. Ground-state energies in GeV, for the spin-independent part of H_Q. Green's-function Monte Carlo results for the ground states of the spin-independent part of H_Q. Statistical and systematic errors are approximately 20 MeV the total energy and the "Coulomb" potential term, and twice that for the kinetic energy (H_0) and H_F.

State	Energy	H_0	H_f	V_{Coul}
$q\bar{q}$	0.60	1.17	-0.30	-0.27
$3q$	1.17	1.81	-0.29	-0.35
$2q$-$2\bar{q}$	1.72	2.49	-0.32	-0.47
$4q$-$\bar{q}$	2.29	3.30	-0.42	-0.59
$6q$	2.79	3.80	-0.34	-0.68

TABLE II. Energies of six-quark states. Green's-function Monte Carlo results for the two hadron and MQH six-quark states with the full interaction, assuming SU(3) symmetry.

State	Energy (GeV)
$2\times N$	2.00
$6q$ ($S=0$)	2.30
$2\times\Delta$	2.54
$6q$ ($S=3$)	2.99

very small in the present model, whose results are dominanted by the spin-independent interaction.

Results in the $2q$-$2\bar{q}$ and $4q$-$\bar{q}$ systems are similar. The spin-0 four-quark state is very high in energy compared to two pions, as expected, since the pions feels an extraordinarily strong hyperfine interaction. We would also predict no strong spin-2 exotic resonance with the flux-tube model; the hyperfine repulsion in the $2q$-$2\bar{q}$ state is only slightly weaker than that in two ρ mesons. The lowest-energy five-quark state with spin $\frac{1}{2}$ has an energy of 1.95 GeV in the SU(3) limit of this model. Since this state has an $S=0$ *us* diquark, a more realistic estimate of its energy is 2.2 GeV which is much larger than M_N+M_K or $M_\Lambda+M_\pi$. The charge 3 $4q$-$\bar{q}$ state made up of four u and one $\bar{d}$ quarks has also been discussed in the literature.[30] If it exists as a narrow resonance it may be possible to study it by double-charge-exchange (π^+,π^-) reactions on the proton. In the flux-tube model this state has a rather high energy of $\sim$2.3 GeV.

It must be emphasized that we do not expect these MQH states to exist as sharp resonances at high energy. We have explored an extreme limit of the theory in order to better understand the possible range of quark models. In this limit there is no coupling between the MQH and multihadron states. Physically the H_P (neglected in this work) provides this coupling, and thus all MQH states will have a width. Unfortunately, since H_p is largely unknown, we cannot provide any reliable estimate of its width. This term in the Hamiltonian is an important topic for future study.

In summary, we have explored the consequences of flux-tube models based upon strong-coupling QCD in multiquark hadron spectroscopy. We find that in the limit of weak coupling between different flux-tube configurations there will be no bound multiquark states. Our results stand in sharp contrast to mean-field models which explain traditional meson and baryon spectroscopy with a similar degree of accuracy. They also differ considerably from potential-based quark models, in which the hyperfine interaction provides enough attraction to produce sharp low-energy resonances in certain spin-isospin channels. Consequently, the presence or absence of these exotic states in the experimental spectrum may be an important guide in our understanding of QCD.

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