Neutrino mass matrices in horizontal models

Krishnanath Bandyopadhyay* and Debajyoti Choudhury
Theory Group, Physical Research Laboratory, Ahmedabad 380 009, India

Utpal Sarkar†
Department of Physics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
(Received 14 May 1990)

We have studied the Majorana neutrino mass matrices in the context of the standard electroweak model including different horizontal symmetries: namely, $U(1)_H$, $SU(2)_H$, $SU(3)_H^V$, and $SU(3)_H^{VL}$. We have shown that these different horizontal symmetries will imply a different structure for these mass matrices.

I. INTRODUCTION

Although the standard electroweak model[1] correctly describes weak-interaction phenomenology at presently available energies, an important and profound question of what determines the mass spectra of leptons and quarks still remains unanswered. To obtain relations between the masses of fermions some additional gauge interactions among the generations were considered. In this context, horizontal interactions[2–6] among the fermion families have been introduced to restrict the independent parameters of the fermion mass matrix in electroweak theories and to obtain some insight on the problem of fermion family repetition. Several horizontal symmetries have been used by different authors to understand the generation problem of fermions including their masses, mixing angles, and *CP* violation. In the present work, we will assume that this symmetry is local because of the privileged role local gauge symmetries are playing in our present theories. To suppress the flavor-changing neutral currents the horizontal symmetry has to be broken at a scale $\gtrsim 10^6$ GeV. Most of the work on local horizontal symmetry has been to understand the hierarchy of quark masses, their mixing, and *CP* violation in the quark sector. Although there are extensions that attempt to obtain particular scenarios in the leptonic sector, in general the leptonic sectors of these models have not been widely studied. In this paper we focus on possible mass matrices for the Majorana neutrinos in the standard electroweak models including different horizontal symmetries G_H, where $G_H = U(1)_H$, $SU(2)_H$, $SU(3)_H^V$, and $SU(3)_H^{VL}$.

A striking and puzzling phenomenon is that neutrinos appear to be nearly massless compared to all other fermions. To understand this smallness of the neutrino mass it is assumed that the left-handed neutrinos combine with some exotic particles due to some additional interactions and as a result we get light Majorana neutrinos with a mass $\sim m_q^2/M$, where m_q are the quark masses and M is the symmetry breaking scale of the additional interaction. There are models with left-right symmetry or with horizontal symmetries in which this situation can naturally arise.[7]

It is interesting to note that the Majorana masses of the neutrinos $\sim m_q^2/M$ are related to the quark mass matrices. In particular, if one assumes that there is no additional symmetry at energies slightly higher than the horizontal-symmetry-breaking scale and that no additional exotic particle lighter than the next symmetry-breaking scale exists, then the structure of the quark mass matrix can give us the structure of that for the neutrinos. One can write the effective Lagrangian for the Majorana neutrino masses as $(f/M)(\bar{\psi}_\nu^c \psi_\nu)\phi\phi$, which is invariant under both the standard electroweak theory and horizontal symmetry. f/M is an unknown parameter in the problem; M is the scale of new physics; f is a combination of Yukawa couplings; ϕ is the Higgs scalar, which gives the quark mass.

The plan of this paper is as follows. In Sec. II we study the structure of the neutrino mass matrices and their mixing in various phenomenologically interesting minimal horizontal-symmetric models. The word minimal implies that only those Higgs bosons, which are necessary to give the particular structure of the quark mass matrices, acquire vacuum expectation values (VEV's). Section III contains the summary of our work and discussion of the results. It is worthwhile to mention that for simplicity and convenience, we suppress various indices (left handed, horizontal, etc.) in all models.

II. NEUTRINO MASSES IN VARIOUS HORIZONTAL-SYMMETRIC MODELS

In this section, we discuss the various horizontal-symmetric models, the representations of fermions and Higgs fields, and the structure of the VEV's of the Higgs fields. The mass matrices for the quarks and neutrinos are also presented here.

A. $SU(2)_L \times U(1)_Y \times U(1)_H$ model

The minimal horizontal gauge group $U(1)_H$ has been used[2] to distinguish between two fermionic generations. $U(1)_H$ is expected to provide the desired nontrivial horizontal symmetry, and it is associated with the Y_H quantum number. This quantum number is not a new one and can be expressed as $Y_H = \mp\frac{1}{3}[Y + 2(B-L)]$ for two generations of fermions. The (plus) and (minus) signs correspond to the first and second generations, respectively. B, L, and Y are the baryon number, lepton number, and Weinberg-Salam hypercharge, respectively. The model can be generalized to 2^n $(n=1,2,\ldots)$ generations. Presently, we consider the two- and four-generation cases only. The fermion classification within the two-generation scheme is shown in Table I. To construct the neutrino mass matrix we consider two Higgs fields $h_1(2,1,\frac{1}{3})$ and $h_2(2,1,-\frac{1}{3})$, which are used to give quark masses. The Higgs-fermion Yukawa interactions for two-generation neutrinos can be written as

$$\mathcal{L} = \frac{f}{M}(\bar{\psi}^c_{1L}\psi_{2L} + \bar{\psi}^c_{2L}\psi_{lL})h_1 h_2 + \text{H.c.} \tag{1}$$

The neutrino mass matrix can be written as

$$M_\nu = F\begin{bmatrix} 0 & \chi_1\chi_2 \\ \chi_1\chi_2 & 0 \end{bmatrix}, \tag{2}$$

where χ_i's $(i=1,2)$ are the complex VEV's of the Higgs fields h_i's and $F = f/M$. On account of the two eigenvalues being degenerate one would expect only a single Dirac neutrino in the model. However, in the absence of a detailed study of the Higgs sector one cannot be sure of a conserved lepton number and, hence, this degeneracy could be lifted by higher-order corrections giving rise to a pseudo Dirac particle.[8] If we extend this model

$$[\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_H \times \mathrm{U}(1)_H]$$

to incorporate four generations of leptons then the neutrino mass matrix will have the form

$$M_\nu = \begin{bmatrix} 0 & m_1 & 0 & m_2 \\ m_1 & 0 & m_3 & 0 \\ 0 & m_3 & 0 & m_4 \\ m_2 & 0 & m_4 & 0 \end{bmatrix}, \tag{3}$$

where m_i's are the neutrino mass terms. From this mass matrix it is evident that the four Majorana neutrinos, after combination, giving rise to two pseudo-Dirac neutrinos. There is a small mixing between the neutrinos but we cannot speculate much due to the lack of experimental data on the fourth generation.

TABLE I. Fermion classification in the two-generation scheme.

	T_3	Y	Y_H
ν^e_L, ν^μ_L	$+\frac{1}{2}$	-1	$+1,-1$
e_L, μ_L	$-\frac{1}{2}$	-1	$+1,-1$
e_R, μ_R	0	-2	$+\frac{4}{3}, -\frac{4}{3}$
u_L, c_L	$+\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}, +\frac{1}{3}$
d_L, s_L	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}, +\frac{1}{3}$
d_R, s_R	0	$-\frac{2}{3}$	0,0
u_R, c_R	0	$\frac{4}{3}$	$-\frac{2}{3}, +\frac{2}{3}$

B. $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(2)_H$ model

$\mathrm{SU}(2)_H$ horizontal symmetry has been introduced by several authors[3,9] to achieve *CP* violation and calculate the weak mixing angles in the quark sector. We consider an

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(2)_H$$

model for three generations of fermions. In addition to their usual $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ representation all the fermions transform as triplets under $\mathrm{SU}(2)_H$. We use two Higgs fields $\rho(2,1,5)$ and $\xi(2,1,3)$ in the model for generating the fermion masses. It is to be noted that with these two Higgs fields Wilczek-Zee-type[9] quark mass matrices can be obtained. The fermion and Higgs fields are assigned in this model as follows:

$$\psi_L(2,-1,3) \equiv \begin{bmatrix} \nu e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{bmatrix}_L, \quad Q_L(2,\tfrac{1}{3},3) \equiv \begin{bmatrix} u & c & t \\ d & s & b \end{bmatrix}_L,$$

$$U_R(1,\tfrac{4}{3},3) \equiv (u,c,t)_R, \quad d_R(1,-\tfrac{2}{3},3) \equiv (d,s,b)_R,$$

$$\langle\rho\rangle = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ v_1 & v_2 & v_3 & v_4 & v_5 \end{bmatrix}, \quad \langle\xi\rangle = \begin{bmatrix} 0 & 0 & 0 \\ a_1 & a_2 & a_3 \end{bmatrix}.$$

where all v's and a's are complex in general. The Higgs-boson–quark interaction Lagrangian can be written as

$$\mathcal{L} = \Gamma_1 \bar{Q}_L U_R \tilde{\rho} + \Gamma_2 \bar{Q}_L U_R \tilde{\xi} + \Gamma_3 \bar{Q}_L d_R \rho + \Gamma_4 \bar{Q}_L d_R \xi + \text{H.c.} \tag{4}$$

where Γ's are the coupling constants and $\tilde{\rho} = i\tau_2\rho^*$ and $\tilde{\xi} = i\tau_2\xi^*$, respectively. Now, the mass matrix for the up-quark sector can be written as

$$M^u_q = \begin{bmatrix} \sqrt{\tfrac{2}{15}}v_3\Gamma_1 & -\sqrt{\tfrac{2}{5}}v_2\Gamma_1 + \sqrt{\tfrac{2}{3}}a_3\Gamma_2 & \sqrt{\tfrac{2}{5}}v_4\Gamma_1 - \sqrt{\tfrac{2}{3}}a_1\Gamma_2 \\ -\sqrt{\tfrac{2}{5}}v_2\Gamma_1 - \sqrt{\tfrac{2}{3}}a_3\Gamma_2 & \sqrt{\tfrac{4}{5}}v_1\Gamma_1 + \sqrt{\tfrac{2}{15}}v_3\Gamma_1 & -\sqrt{\tfrac{4}{5}}v_5\Gamma_1 + \sqrt{\tfrac{2}{3}}a_2\Gamma_2 \\ \sqrt{\tfrac{2}{5}}v_4\Gamma_1 + \sqrt{\tfrac{2}{3}}a_1\Gamma_2 & -\sqrt{\tfrac{4}{5}}v_5\Gamma_1 - \sqrt{\tfrac{2}{3}}a_2\Gamma_2 & \sqrt{\tfrac{4}{5}}v_1\Gamma_1 - \sqrt{\tfrac{2}{15}}v_3\Gamma_1 \end{bmatrix}. \tag{5}$$

The down-quark mass matrix can similarly be written. The following possible conditions, which are consistent with the minimization of the Higgs potential, lead to the desired quark mass matrix (in the absence of higher-order corrections):

$$\begin{gathered} v_1 \neq 0, \quad v_2 = 0, \quad v_3 = 0, \quad v_4 = 0, \quad v_5 = 0, \\ a_1 = 0, \quad a_2 \neq 0, \quad a_3 = 0. \end{gathered} \tag{6}$$

Using these conditions, the quark mass matrix [Eq. (5)] reduces to

$$M_q^u=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{\tfrac{4}{4}}v_1\Gamma_1 & \sqrt{\tfrac{2}{3}}a_2\Gamma_2 \\ 0 & -\sqrt{\tfrac{2}{3}}a_2\Gamma_2 & \sqrt{\tfrac{4}{5}}v_1\Gamma_1 \end{bmatrix} . \tag{7}$$

The effective Lagrangian for the Majorana neutrino mass term is

$$\mathcal{L}=\frac{1}{M}(F_1\bar{\psi}_L^c\psi_L\rho\rho+F_2\bar{\psi}_L^c\psi_L\xi\xi+F_3\bar{\psi}_L^c\psi_L\rho\xi)+\text{H.c.} , \tag{8}$$

where F's are the coupling constants. With the conditions written in Eq. (6), we can get a Majorana neutrino mass matrix which is given by

$$M_\nu=\frac{1}{M}\begin{bmatrix} \begin{array}{l} F_2a_2^2\left[\frac{1}{3}+\frac{4}{6\sqrt{5}}\right] \\ +F_1v_1^2\left[\frac{8}{\sqrt{420}}-\frac{2}{\sqrt{15}}\right] \end{array} & 0 & 0 \\ 0 & \begin{array}{l} F_2a_2^2\left[\frac{4}{6\sqrt{5}}-\frac{2}{3}\right] \\ +F_1v_1^2\left[\frac{4}{\sqrt{15}}+\frac{8}{\sqrt{420}}\right] \end{array} & -2\sqrt{\tfrac{2}{15}}a_2v_1F_3 \\ 0 & -2\sqrt{\tfrac{2}{15}}a_2v_1F_3 & \begin{array}{l} F_2a_2^2\left[\frac{2}{3}-\frac{4}{6\sqrt{5}}\right] \\ -F_1v_1^2\left[\frac{8}{\sqrt{420}}+\frac{4}{\sqrt{15}}\right] \end{array} \end{bmatrix} . \tag{9}$$

If we consider radiative corrections to the VEV's (represented by $a_3\neq 0$), then both the mass matrices M_q^u and M_ν will be modified such that the (1,2) elements in each will be nonzero. The up-quark and neutrino mass matrices will then have the forms

$$M_q^u=\begin{bmatrix} 0 & (\tfrac{2}{3})^{1/2}a_3\Gamma_2 & 0 \\ -(\tfrac{2}{3})^{1/2}a_3\Gamma_2 & (\tfrac{4}{5})^{1/2}v_1\Gamma_1 & (\tfrac{2}{3})^{1/2}a_2\Gamma_2 \\ 0 & -(\tfrac{2}{3})^{1/2}a_2\Gamma_2 & (\tfrac{4}{5})^{1/2}v_1\Gamma_1 \end{bmatrix} \tag{10}$$

and

$$M_\nu=\frac{1}{M}\begin{bmatrix} \begin{array}{l} F_2\left[a_2^2\left[\frac{1}{3}+\frac{2}{3\sqrt{5}}\right]\right. \\ \left.\quad +a_3^2\left[\frac{2}{3}+\frac{2}{3\sqrt{5}}\right]\right] \\ +F_1v_1^2\left[\frac{8}{\sqrt{420}}-\frac{2}{\sqrt{15}}\right] \end{array} & \sqrt{\tfrac{2}{15}}a_3v_1F_3 & \frac{2}{\sqrt{5}}a_2a_3F_2 \\ \sqrt{\tfrac{2}{15}}a_3v_1F_3 & \begin{array}{l} F_2\left[a_2^2\left[\frac{2}{3\sqrt{5}}-\frac{2}{3}\right]\right. \\ \left.\quad +a_3^2\left[\frac{2}{\sqrt{5}}+\frac{2}{3\sqrt{5}}-\frac{4}{3}\right]\right] \\ +F_1v_1^2\left[\frac{8}{\sqrt{420}}+\frac{4}{\sqrt{15}}\right] \end{array} & -2\sqrt{\tfrac{2}{15}}a_2v_1F_3 \\ \frac{2}{\sqrt{5}}a_2a_3F_2 & -2\sqrt{\tfrac{2}{15}}a_2v_1F_3 & \begin{array}{l} F_2\left[a_2^2\left[\frac{2}{3}-\frac{2}{3\sqrt{5}}\right]\right. \\ \left.\quad +a_3^2\left[\frac{2}{\sqrt{5}}-\frac{2}{3\sqrt{5}}+\frac{4}{3}\right]\right] \\ -F_1v_1^2\left[\frac{8}{\sqrt{420}}+\frac{4}{\sqrt{15}}\right] \end{array} \end{bmatrix} . \tag{11}$$

It is evident from Eq. (9) that there is a pseudo-Dirac neutrino and one Majorana neutrino ν_e and they follow the mass relationship such that $2m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau}$. There is no mixing among the neutrinos. However, the radiative correction leads to the mass matrix [Eq. (11)] which still yields a pseudo-Dirac neutrino and a Majorana neutrino with a small mixing.

C. $SU(2)_L \times U(1)_Y \times SU(3)_H^V$ model

Another natural horizontal symmetry is $SU(3)_H$ since three generations of fermions have been observed so far. This symmetry has been proposed by several authors[4] to study fermion masses, CP violation and flavor-changing neutral currents. Both the left- and the right-handed fermions transform as triplets and their charge conjugates as antitriplets under $SU(3)_H$. A right-handed neutrino must be introduced to avoid the triangle anomaly. We refer to this symmetry as vectorial $SU(3)_H^V$ symmetry.

It is obviously difficult to generate a small Dirac neutrino naturally in this model. Also the minimal Higgs scalars in the model necessary to give masses to the quarks and charged leptons, namely, $T(2,1,8)$ and $T(2,1,1)$, do not lead to Majorana masses for either the left- or right-handed neutrinos. Hence, we shall desist from any further discussion of this symmetry.

D. $SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ model

Another version of $SU(3)_H$ symmetry, referred to as $SU(3)_H^{VL}$ has been considered by many authors.[5] Under this group all the left-handed fermions and antifermions transform as triplets while the right-handed fields transform as antitriplets. Anomaly cancellation requires the introduction of a mirror set of fermions, which one assumes decouple from the usual particles at a high energy. While two $SU(3)_H^{VL}$ Higgs six-plets ϕ_1, ϕ_2 can give masses to the down quarks and the charged leptons, a $\overline{\mathbf{6}}$ is needed for the up-type masses. It is easy to see that to one loop the latter does not contribute to the neutrino masses.

The effective interaction Lagrangian for the neutrinos can be written as

$$\mathcal{L} = \frac{1}{M}(\mu_1 \bar{\psi}_L^c \psi_L \phi_1 \phi_1 + \mu_2 \bar{\psi}_L^c \psi_L \phi_2 \phi_2 + \mu_3 \bar{\psi}_L^c \psi_L \phi_1 \phi_2) + \text{H.c.} , \tag{12}$$

where μ's are the effective couplings. The VEV's for ϕ_1, ϕ_2 are

$$\langle \phi_1 \rangle = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ K_1 & K_2 & K_3 & K_4 & K_5 & K_6 \end{bmatrix} ,$$

$$\langle \phi_2 \rangle = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 \end{bmatrix} ,$$

where all the entries are complex in general. Now the Yukawa interaction for the down quarks and the charged leptons reads

$$\mathcal{L} = \Gamma_1 \bar{Q}_L d_R \phi_1 + \Gamma_2 \bar{Q}_L d_R \phi_2 + \Gamma_3 \bar{\psi}_L e_R \phi_1 + \Gamma_4 \bar{\psi}_L e_R \phi_2 + \text{H.c.} , \tag{13}$$

leading to

$$M_q^d = \frac{1}{\sqrt{6}} \begin{bmatrix} \Gamma_1 K_1 + \Gamma_2 \zeta_1 & -\frac{1}{\sqrt{2}}(\Gamma_1 K_2 + \Gamma_2 \zeta_2) & \frac{1}{3\sqrt{2}}(\Gamma_1 K_4 + \Gamma_2 \zeta_4) \\ -\frac{1}{\sqrt{2}}(\Gamma_1 K_2 + \Gamma_2 \zeta_2) & \Gamma_1 K_3 + \Gamma_2 \zeta_3 & -\frac{1}{3\sqrt{2}}(\Gamma_1 K_5 + \Gamma_2 \zeta_5) \\ \frac{1}{3\sqrt{2}}(\Gamma_1 K_4 + \Gamma_2 \zeta_4) & -\frac{1}{3\sqrt{2}}(\Gamma_1 K_3 + \Gamma_2 \zeta_5) & -\frac{5}{3}(\Gamma_1 K_6 + \Gamma_2 \zeta_6) \end{bmatrix} , \tag{14}$$

and similarly for M_e. It is to be noted that one can obtain realistic Fritzsch-type[10] mass matrices by imposing

$$K_1 = 0 \quad K_2 \neq 0 \quad K_3 = 0 \quad K_4 = 0 , \quad K_5 \neq 0 , \quad K_6 \neq 0 ,$$

$$\zeta_i = 0 , \quad \zeta_2 \neq 0 , \quad \zeta_3 = 0 , \quad \zeta_4 = 0 , \quad \zeta_5 \neq 0 , \quad \zeta_6 \neq 0 ,$$

conditions consistent with potential minimization. Then M_q^d and M_e reduce to

$$M_q^d = \frac{1}{\sqrt{6}} \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}}(\Gamma_1 K_2 + \Gamma_2 \zeta_2) & 0 \\ -\frac{1}{\sqrt{2}}(\Gamma_1 K_2 + \Gamma_2 \zeta_2) & 0 & -\frac{1}{3\sqrt{2}}(\Gamma_1 K_5 + \Gamma_2 \zeta_5) \\ 0 & -\frac{1}{3\sqrt{2}}(\Gamma_1 K_5 + \Gamma_2 \zeta_5) & -\frac{5}{3}(\Gamma_1 K_6 + \Gamma_2 \zeta_6) \end{bmatrix} , \tag{15}$$

$$M_e=\frac{1}{\sqrt{6}}\begin{bmatrix} 0 & -\frac{1}{\sqrt{2}}(\Gamma_3 K_2+\Gamma_4\zeta_2) & 0 \\ -\frac{1}{\sqrt{2}}(\Gamma_3 K_2+\Gamma_4\zeta_2) & 0 & -\frac{1}{3\sqrt{2}}(\Gamma_3 K_5+\Gamma_4\zeta_5) \\ 0 & -\frac{1}{3\sqrt{2}}(\Gamma_\zeta K_5+\Gamma_4\zeta_5) & -\frac{5}{3}(\Gamma_3 K_6+\Gamma_4\zeta_6) \end{bmatrix}, \tag{16}$$

and for the neutrino mass matrix one obtains

$$M_\nu=\frac{1}{\sqrt{6}}\frac{\mu}{M}\begin{pmatrix} -\frac{1}{\sqrt{3}}(K_5^2+\zeta_5^2+k_5\zeta_5) & -\frac{1}{\sqrt{6}}(2k_2k_6+2\zeta_2\zeta_6+K_6\zeta_2+K_2\zeta_6) & \frac{-1}{6\sqrt{3}}(2k_2k_5+2\zeta_2\zeta_5+K_5\zeta_2+\zeta_5K_2) \\ -\frac{1}{\sqrt{6}}(2K_2K_6+2\zeta_2\zeta_6+K_6\zeta_2+K_2\zeta_6) & 0 & 0 \\ \frac{-1}{6\sqrt{3}}(2K_2K_5+2\zeta_2\zeta_5+K_5\zeta_2+\zeta_5K_2) & 0 & \frac{5}{3}(K_2^2+\zeta_2^2+K_2\zeta_2) \end{pmatrix}, \tag{17}$$

where we have assumed that μ_i's$=\mu$. We thus have three eigenvalues which are quite different. In fact putting in the values of the quark masses one obtains $m_{\nu_e}:m_{\nu_\mu}:m_{\nu_\tau}\sim 300:10:1$, and, for the neutrino mixing matrix N, we have $N_{\nu_e\text{-}\nu_\mu}\sim 0.08, N_{\nu_e\text{-}\nu_\tau}\sim 0.04$ and $N_{\nu_\mu\text{-}\nu_\tau}\sim 0.24$.

III. SUMMARY AND DISCUSSIONS

We shall now summarize our results for the neutrino masses and mixings obtained from dimension-five operators in $SU(2)_L\times U(1)_Y\times G_H$ models where $G_H=U(1)_H$, $SU(2)_H$, $SU(3)_H^V$, and $SU(3)_H^{VL}$. The Majorana mass term, consistent with gauge symmetry, is of the form $\psi_L\psi_L\langle\phi^0\rangle^2$ which, being a nonrenormalizable term, can only arise as a one-loop correction induced by an exchange of massive particles. In the present work we have constructed the neutrino mass matrices with the minimal set of scalar VEV's required to generate quark mass matrices of the desired type. It thus is of the form $(f/M)\langle\phi^0\rangle^2$, where f is a dimensionless model-dependent constant and M represents the mass suppression which we take to be a singlet of the low-energy group.

We have shown that with a $U(1)_H$ symmetry for two generations one gets two degenerate Majorana neutrinos. Though there is a conserved lepton number to this order, in the absence of a full analysis involving the Higgs fields, an exact conservation is not obvious, and it would be safer to call the resultant a pseudo-Dirac rather than a Dirac neutrino. In the four-generation case the situation is similar and we get two such particles with a small amount of mixing, the extent of which cannot be commented upon in the absence of any data on a possible fourth generation.

In the case of $SU(2)_H$ symmetry we have considered the scenario wherein a Higgs triplet and a five-plet are used to generate Wilczek-Zee-type mass matrices for the quarks. Here the second- and third-generation neutrinos are nearly degenerate, but there exists no conserved lepton number. To this order they do not mix with ν_e but the radiative corrections to the Higgs VEV's that induce masses for the first-generation fermions would also generate a small mixing.

In the context of $SU(3)_H^V$ symmetry we have already pointed out that it is difficult to generate naturally small Dirac masses for the neutrinos. Furthermore, the minimal set of Higgs VEV's do not give rise to a Majorana mass term, thus precluding a seesaw-type mechanism.[7,11] Hence this particular symmetry has not been considered in the present work.

For the $SU(3)_H^{VL}$ case we used two Higgs six-plets to generate Fritzsch-type mass matrices for the down quarks and the charged leptons. Amusingly one gets a reverse hierarchy for the neutrino masses. The mixing matrix, not too different from unity, also has the interesting feature that ν_μ-ν_τ mixing is more than ν_e-ν_μ, which in turn exceeds the ν_e-ν_τ mixing.

Thus we see that the main difference between these models lie in the wide range of deviation from the Dirac character and consequently in the ratio R of the neutrinoless double-β-decay rate and the mass. In the $U(1)_H$ case ν_e forms part of a pseudo-Dirac particle and hence R is expected to be small, while in $SU(2)_H$ and $SU(3)_H^{VL}$ ν_e is a Majorana particle and R would be larger. The last two can be distinguished only by a study of the second- and third-generation neutrino masses. While in SU(2) one has a pseudo-Dirac particle about twice as heavy as ν_e, SU(3) admits only the very remarkable reverse hierarchy.

*Department of Physics, Visva-Bharati University, Santiniketan 731 235, West Bengal, India.

†On leave of absence from Theory Group, Physical Research Laboratory, Ahmedabad 380 009, India.

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