Pole model of B-meson decays into baryon-antibaryon pairs

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We reconsider in detail the pole model for baryonic B decays, first crudely estimated by Deshpande *et al.* We make a complete calculation on the basis of the quark model by an extensive evaluation of strong- and weak-interaction vertices. We also consider charmed decays, and the previously lacking $J^P=\frac{1}{2}^-$ baryon poles that lead to large parity-violating amplitudes. The strong couplings are calculated in absolute magnitude by our quark-pair-creation model in a crossed channel. The effects of unequal quark masses are fully taken into account, correcting previous published results, and introducing a large set of new calculations for baryon couplings. We emphasize the selection rules and algebraic ratios of rates that follow from either color antisymmetry of the baryon wave functions or from the pole model. We pay particular attention to the ratio of parity-conserving to parity-violating rates, as it enters in tests of CP violation involving modes such as $p\bar{p}$. At the same time, we make a detailed discussion of the very principles of the pole model, which exhibits a series of yet overlooked problems: ambiguity in the choice of relativistic couplings, extrapolation of strong couplings to complex momentum transfer, and of weak baryon-baryon matrix elements to nonzero and large transfers. For both theoretical and experimental reasons, it is suggested that decays with a Δ are largely overestimated because of a basic weakness of the pole model for high spin. On the other hand, results with $\frac{1}{2}^+$ baryons in the final state share several features of the QCD-sum-rule calculation by Chernyak and Zhitnisky, while differing in detailed predictions that must await experimental test.

I. INTRODUCTION

With the baryon-antibaryon decays of B mesons we are in a new situation in hadronic transitions: for the first time we have a stable meson heavy enough for these decay modes to be allowed. Hence the theoretical interest of such processes that are not, even for the hardest mode $p\bar{p}$, in a region where perturbative QCD can be applied. One cannot avoid appealing to models, and the object of this paper is to examine such decays, with charmed and uncharmed final states, in a (relatively) simple pole model. As we will conclude, our work, although rather complete in its goal within a definite model, leaves unanswered questions and should be viewed as a tentative to estimate these difficult new processes.

Another interest of exclusive decays of the type $b\to u$ such as $B\to p\bar{p}$ is to have an independent proof of a sizable value of the matrix element V_{ub}, recently estimated from the semileptonic spectrum.[1]

The general level of noncharmed B baryonic weak decays is best displayed by comparison with a typical mesonic one. Using a model of Bauer, Stech, and Wirbel,[2] we get

$$B(B^0\to\pi^+\pi^-)\simeq 10^{-3}\left|\frac{V_{ub}}{V_{cb}}\right|^2 . \tag{1.1}$$

We have used the experimental measure of $B(B^0\to D^{*-}\pi^+)\sim 3\times10^{-3}$ and the theoretical ratio of the two reactions.

A comparable rate comes out for $B^0\to p\bar{p}$ if one trusts our present calculation:

$$B(B^0\to p\bar{p})\simeq 0.7\times 10^{-3}\left|\frac{V_{ub}}{V_{cb}}\right|^2 . \tag{1.2}$$

This is due to the fact that while baryonic decay suffers stronger form-factor suppression than the mesonic one (in analogy with the steeper falloff of baryonic form factors), the baryon-to-baryon weak transition matrix element is, on the other hand, favored in absolute magnitude by the large baryon wave function at center, as estimated in potential models[3,4]

$$\langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle\sim 10^{-2}\ \mathrm{GeV}^3 \tag{1.3}$$

compared to a mesonic very small factor for pion emission f_π.

$B^0\to p\bar{p}$ could be also interesting for CP violation, at least if parity-violating (PV) and parity-conserving (PC) waves sufficiently differ in magnitude.[5,6] In fact, we find the PV wave to be of the same order of magnitude as the PC one, with a ratio submitted to large uncertainties. The CP asymmetry is therefore not so large as in $B^0\to\pi\pi$, which is purely parity violating. Nevertheless, it may add complementary information, and it may be possible to open new possibilities with other baryonic decays such as $B_d^0, \bar{B}_d^0 \to \Lambda\bar{\Lambda}$, $\Lambda_c^+ \bar{\Lambda}_c^+$.[6] Polarization measurements of charmed-baryon decays through their hyperon products could provide other interesting tests of P and CP violation.[7]

But the main interest of baryonic B decays at present is to test our ability to handle hadronic physics through the quark model in a rather new and complex situation. First, we are confronted here, as in the case of hyperon nonleptonic decays,[8] or weak radiative decays,[9] or in the case of neutron electric dipole moment,[10] with higher-order processes, which pass through intermediate baryonic states. This we handle through the pole model, as first suggested by Deshpande, Soni and Trampetic.[11] Second, the B decays have the additional feature of presenting large mass differences, corresponding to the very unequal quark masses m_b, m_c, m_u. As a consequence, we have to treat adequately the separation of center-of-mass motion and the Pauli principle in the presence of unequal quark masses, a nontrivial but conceptually already resolved problem. But another consequence is the presence of large momentum transfers, which has required original treatments in the context of the pole model, as we point out below. And finally, the situation is new also because it requires, to apply the quark model, a crossing procedure. On the whole, the interest of the present study is to present a set of calculations of new hadronic quantities, involving a large number of conceptual problems.

It is already possible to test quantitatively our method by considering the case of charmed-baryon decays, where the Kobayashi-Maskawa (KM) parameter V_{cb} is already known; this fact allows us to make absolute predictions to be compared to experimental rates. For noncharmed decays, the matrix element V_{ub} is now becoming known: $V_{ub} \sim 0.05$; but the rates seem too small for present experimental accuracy. We show that we can already learn something from the existing bounds.

A previous pole-model calculation has been made by Deshpande, Soni, and Trampetic.[11] We therefore insist a bit on the comparison between our works. Our work is more extensive in that it treats also charmed-baryon decays, which are crucial to test the model. But it differs essentially in the treatment of the pole model itself.

(1) We take into account the $J^P = \frac{1}{2}^-$ intermediate states and not only the $J^P = \frac{1}{2}^+$, and this allows us to account for the parity-violating waves. This reveals to be important for modes such as $B \to p\bar{p}$, since we find the PV wave to be as large as the PC one in this case.

(2) We treat the strong vertex explicitly through the quark model instead of using guesses to relate b-hyperon couplings to nucleon couplings. We then make absolute predictions for the strong-interaction coupling constants. This is obtained by an original use of the $^3P_0$ quark-pair-creation (QPC) model,[12] which is applied in the crossed channel of the physical reaction and extrapolated to an unphysical domain of momentum transfer. Indeed, we show that to calculate the pole residue, negative square transfers are involved at the strong vertex, which require a careful discussion.

(3) We show that to calculate the pole residue, we must consider the baryon-to-baryon transition weak matrix element at large momentum transfer, and we present the results of the quark model for this matrix element at nonzero momentum transfer, which are new (Appendix D). We show that the large transfer results in a strong depression of noncharm decays.

(4) We take into account systematically the inequality of the masses of the various quarks u,c,b. We then present results which are new for the $^3P_0$ model of baryon decays and for the PV weak matrix elements, including the $b \to c$ transition with three different masses. We find that such quark mass effects are especially large for the PV wave with $b \to u$. We correct a result presented previously for the PC case.[6]

All these ingredients being put together, we observe a certain compatibility with the alternative QCD-sum-rule calculation of Chernyak and Zhitnisky[13] (CZ) for the charmed-baryon decays, in the $\frac{1}{2}^+ \bar{\frac{1}{2}}^+$ case.

We reduce the gap concerning the asymptotic m_b power-counting behavior noted by these authors between the pole model of Deshpande, Soni, and Trampetic[11] and the QCD asymptotic predictions, especially in the $\frac{1}{2}^+ \bar{\frac{3}{2}}^+$ case. Nevertheless, our numerical results happen to give, contrarily to CZ, very large branching ratio for the $\frac{3}{2}^+$ decays, which are almost excluded by the experimental results. We ascribe this *to an essential failure of the pole model for high-spin particles.* This should not spoil the relevance of our results in the $\frac{1}{2}^+ \bar{\frac{1}{2}}^+$ case, and in this latter case, our calculation may seem to have some advantage over CZ, as it is simpler and perhaps more transparent.

We do not compute in this paper all possible baryon-antibaryon decay modes of B mesons, but only a few, significant ones from the theoretical or the experimental point of view.

B mesons, composites of a b quark and a light quark, $\bar{B}_u(b\bar{u})$, $\bar{B}_d^0(b\bar{d})$, and $\bar{B}_s^0(b\bar{s})$, can give a variety of baryon-antibaryon final states. The final states will depend (1) on the weak transitions, and (2) on the strong quark pair creation needed to have a baryon-antibaryon pair. The weak transitions can be the following: (i) Kobayashi-Maskawa, Cabibbo (C) allowed, i.e., $b \to c$, and $u \to d$ or $c \to s$; (ii) KM-allowed, C-suppressed, $b \to c$, and $u \to s$ or $c \to d$; (iii) KM-suppressed, C-allowed, $b \to u$, and $u \to d$ or $c \to s$; (iv) KM suppressed, C suppressed, $b \to u$, and $u \to s$ or $c \to d$. As for the needed strong quark pair creation, we can have $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$. All these combinations are possible, unless there is no phase space allowing them, as in the case of three charmed quarks in the final state. In our pole model, the intermediate states will differ from one of these combinations to another.

Notice that among the final baryons, only a few are stable with respect to strong and electromagnetic interactions. Among the cuu, cud baryons, only Λ_c^+ with flavor wave function $c(ud-du)/\sqrt{2}$ will be stable. Among the Ξ_c's, $\Xi_c^{+(a)} = c(us-su)/\sqrt{2}$ and $\Xi_c^{0(a)} = c(ds-sd)/\sqrt{2}$ are stable, the symmetric combinations $\Xi_c^{+(s)} = c(us+su)/\sqrt{2}$ and $\Xi_c^{0(s)} = c(ds+sd)/\sqrt{2}$ decaying by strong or electromagnetic interactions into the $\Xi_c^{(a)}$ ones. Also, $\Omega(css)$ will be stable.

In this paper we only compute the rates for a few of these decays. To be definite, we only make the full calculation of the decay rates for the transitions of $B_u^-(b\bar{u})$ and $\bar{B}_d^0(b\bar{d})$ into KM-allowed, C-allowed decays without strange quarks in the final state: $B_u^- \to (cdu, \bar{u}\bar{u}\bar{u})$ or $(cdd, \bar{u}\bar{u}\bar{d})$ and $\bar{B}_d^0 \to (cdu, \bar{d}\bar{u}\bar{u})$ or $(cdd, \bar{d}\bar{u}\bar{d})$ and KM-suppressed, C-allowed decays of B_u^- and $\bar{B}_d^0$ without strange quarks in the final state: $B_u^- \to (udu, \bar{u}\bar{u}\bar{u})$ or $(udd, \bar{u}\bar{u}\bar{d})$ and $\bar{B}_d^0 \to (udu, \bar{d}\bar{u}\bar{u})$ or $(udd, \bar{d}\bar{u}\bar{d})$.

Moreover, we only consider ground-state baryon final states $B \to \Sigma_c \bar{N}$, $\Lambda_c \bar{N}$ or $\Sigma_c \bar{\Delta}, \Lambda_c \bar{\Delta}$ and $B \to N\bar{N}$ or $N\bar{\Delta}$. In our pole model, where baryon poles dominate, the lowest-lying intermediate states will be, in both cases, Σ_b, Λ_b and Λ_b^*, Σ_b^*. The study of these modes will allow us to account for *the ground-state contribution* to be inclusive decay rates $B_u^- \to \Lambda_c^+ + \cdots, \bar{B}_d^0 \to \Lambda_c^+ + \cdots$ and $B_u^- \to p\bar{p} +$ pions, $\bar{B}_d^0 \to p\bar{p} +$pions.

Although we only consider, for reasons of simplicity, a few final states, many other modes are allowed. Let us restrict to B_u and B_d. If a $s\bar{s}$ or a $c\bar{c}$ pair is created, we will have other stable strange or charmed particles in the final state, and those will not contribute to $B_u^- \to \Lambda_c^+ + \cdots; \bar{B}_d^0 \to \Lambda_c^+ + \cdots$ and $B_u^- \to p\bar{p}+$pions; $\bar{B}_d^0 \to p\bar{p}+$pions. Among the rest of the decays into two ground-state baryons, not computed in this paper, a few can be easily obtained from Clebsch-Gordan coefficients, such as, for instance, the KM-suppressed, C-allowed mode $\bar{B}_d^0 \to \Lambda\bar{\Lambda}$, interesting for tests of *CP* violation.[6] However, there are interesting modes for which our results cannot be applied easily, as they involve intermediate states with three quarks of widely different masses, not considered here. One example is the KM-allowed, C-suppressed mode $\bar{B}_d^0 \to \Lambda_0^+ \bar{\Lambda}_c^+$, also interesting in tests of *CP* violation.[6] In these modes we will have as intermediate states Ξ_{bc} baryons, composites of a light u or d and b,c quarks, three quarks with very different masses. There is no difficulty of principle in treating these cases, although the formalism would be heavier, as it needs the diagonalization of the Hamiltonian (A6) for three unequal masses. However, as we discuss in the text, and we have argued,[6] the effect of the quark mass differences on the weak and strong couplings is not very large, a factor less than one order of magnitude in rate.

Our paper is organized as follows. Section II is devoted to a rather long discussion of the various aspects of the model: to define precisely what we mean by a pole model, to discuss other competing mechanisms, and to analyze the subtleties of this nonstandard application of the quark model. In Sec. III we define the couplings, decay amplitudes in the pole model, and give the rates. In Sec. IV we give the final quark model expressions of the couplings and discuss its ingredients and symmetry properties. In Sec. V we give the numerical results and in Sec. VI we make a discussion of the different effects. Finally, in Sec. VII we conclude. We postpone all heavy calculations to the Appendixes. In Appendix A we give the hadron wave functions in the harmonic-oscillator model with unequal masses. In Appendix B we compute the weak-interaction matrix elements at zero-momentum transfer, and in Appendix C we compute the strong couplings with the QPC model. Finally, in Appendix D we compute the weak matrix elements at nonzero-momentum transfer.

II. GENERAL DISCUSSION OF THE MODEL

A. Diagrams contributing to B baryonic decays

The pole model we are considering here corresponds to the process $B \to \mathcal{B}_1 \bar{\mathcal{B}}_2$ with the general configuration described by the diagram of Fig. 1, treated in the particular way pictured in Fig. 2, with low-lying b-flavored baryon intermediate states $\mathcal{B}_b$.

The first question to answer is why one chooses the configuration of Fig. 1 rather than many others which are possible. For noncharm decays, we can draw five of them according to Fig. 3.

The first one [Fig. 3(a)] can be neglected with respect to Fig. 1 for the well-known reason that it is the annihilation into a light-quark pair. The matrix element can be factorized into

$$M \sim f_B q_\mu \langle H | j^\mu | 0 \rangle \ , \qquad (2.1)$$

where q^μ is the B momentum, and H is the hadronic final state, produced through the current j^μ, which is a *light*-quark current (u,d). Formula (2.1) then implies the divergence of this current $\partial_\mu j^\mu$ and M contains as a factor the very small *current* masses m_u, m_d; f_B is also small (100–200 MeV). In addition, one could observe that exclusive decays of the type $H = \bar{p}N^*$, for instance, will be suppressed by form factors of the current matrix element taken at the very large transfer $q^2 \sim 25$ GeV^2. It must be recalled however that similar suppressions occur in the diagram of Fig. 1, as calculated by the pole model: a denominator corresponding to the propagator of the Σ_b intermediate state at small momentum $p_1^2 = m_N^2$, and an additional weak vertex form factor taken also at large virtuality (See Secs. II B and II C). Therefore the form-factor suppression is not effective if we want to compare to Fig. 1.

It must be noticed that Paver and Riazuddin[14] found very large values for $\bar{B}_d^0 \to p\bar{p}$ through annihilation, while according to our argument, annihilation should be very small. Their result comes from the pion-pole contribution. However, it must be observed that the pion contribution by itself does not satisfy the PCAC (partial conservation of axial-vector current) requirement that $\partial_\mu j^{5\mu} \sim m_u + m_d$. To satisfy this requirement, one must take into account the $\gamma^\mu \gamma^5$ contribution, which almost cancels the pion-pole contribution.

Figures 3(b) and 3(e) are characterized by the fact that

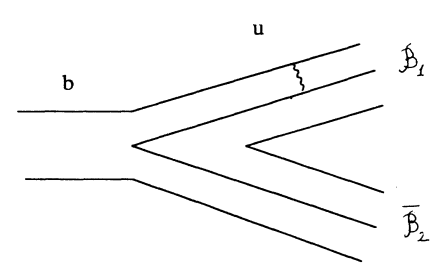

FIG. 1. B decay into baryon-antibaryon pair with weak interaction within a baryon. $\mathcal{B}_1$ and $\mathcal{B}_2$ are both made of light quarks.

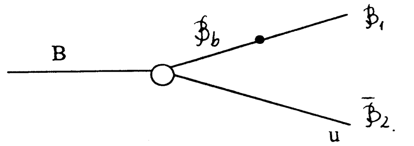

FIG. 2. Low-lying intermediate baryons in the pole model contributing to the diagram of Fig. 1.

one pion is emitted *directly* through the weak interaction. They are not relevant, of course, for $p\bar{p}$ decay. But they are relevant as a background to decay with additional pions coming from direct production of N^* resonances such as the Δ, the process that we consider. Moreover, one can see that the direct meson production is small

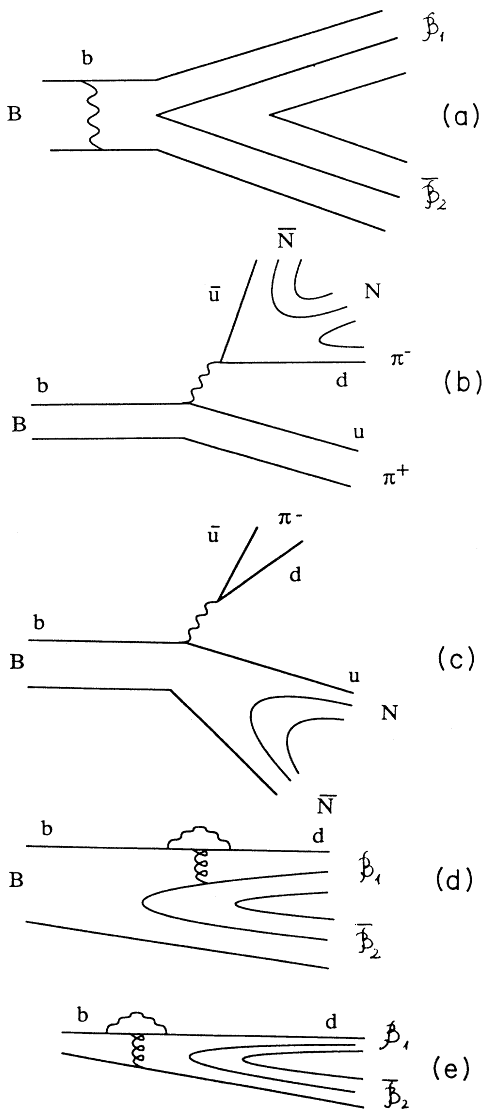

FIG. 3. Other diagrams contributing to noncharmed final states. (a) Meson annihilation into baryon-antibaryon pair. (b)–(c) Pion emission by the weak interaction. (d)–(e) Penguin-diagram contributions to B decay int a baryon-antibaryon pair.

with respect to what we calculate: indeed, we find at the end of our calculation that our rates greatly exceed the ones calculated in Ref. 15.

Figures 3(d) and 3(e), penguin diagrams, are expected to be small because the enhancement of L-R densities with respect to the usual $(V-A)\times(V-A)$ interactions is overcompensated by the smallness of the coefficient obtained for the effective operator. In the same vein, we think that the coefficients obtained through the QCD radiative corrections for the $(V-A)\times(V-A)$ operators will not change qualitatively our picture (see the similar discussion of hyperon nonleptonic decays in our book[4]).

A general remark is now in order. The dominance of baryon internal W exchange (Figs. 1 and 2) for the decay to $p\bar{p}+\pi$'s depends on the particular evaluation we have made of Fig. 1. If the CZ evaluation were valid,[13] then it would be so small that the processes of Figs. 3(b) and 3(c) (Ref. 15) could dominate.

In the case of decays to baryons with a single charmed quark, the discussion is quite analogous. We draw the diagrams Figs. 4 and 5 (see discussion by Avery[16]). Figure 4 is the configuration that we retain, with the W exchanged within one baryon. Figures 5(b)–5(d) are analogous to the diagrams considered in Ref. 15 for $p\bar{p}+\pi$'s. There is direct production of one meson by the weak interaction. If we stick to decays through direct production of baryon resonances only, we can disregard them. In addition in Figs. 5(b) and 5(d), a charmed meson is necessarily produced, and therefore it corresponds to a distinct process, since a charmed meson could not come from the secondary decay of one low-lying charmed baryon (unless through virtual propagation). As to Fig. 5(c), it can reasonably be expected to be small as the similar process Fig. 3(c). The annihilation diagram, Fig. 5(a), on the contrary, does not have the very strong suppression factor of the corresponding Fig. 3(a), because small quark masses are replaced by the charm-quark mass $m_c \sim 2$ GeV. Still there is the small factor f_B. Evaluating the process $B^0 \to \bar{\Lambda}_c p$ through the annihilation mechanism, and retaining only γ^μ and $\gamma^\mu\gamma^5 p-\Lambda_c$ transition currents (disregarding other form factors), we find

$$B(B^0 \to \bar{\Lambda}_c p) \sim 10^{-4} \tag{2.2}$$

without the baryon form factors. These form factors taken at $q^2 \simeq 25$ GeV^2 will still lower the result by several orders of magnitude, so that we can neglect it safely by

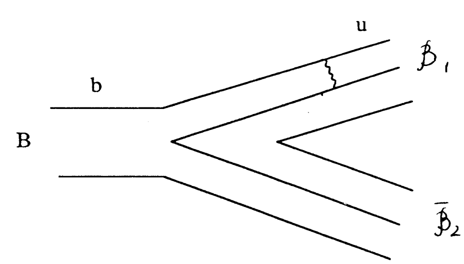

FIG. 4. B decay into a baryon-antibaryon pair with weak interaction within a baryon. $\mathcal{B}_1$ is a charmed baryon and $\mathcal{B}_2$ is made of light quarks.

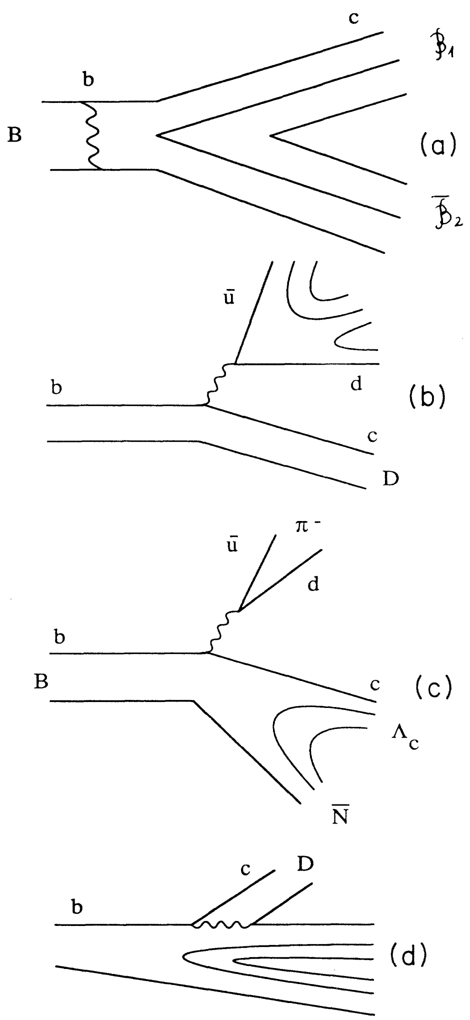

FIG. 5. Other diagrams contributing to charmed final states. (a) Meson annihilation into a charmed-baryon and an antibaryon. (c) Pion emission and charmed-baryon production. (b)–(d) Charmed-meson emission by the weak interaction.

comparison to our estimate of Fig. 4:

$$B(B^0 \rightarrow \overline{\Lambda}_c p) \sim 10^{-3} , \tag{2.3}$$

B. General discussion of the pole model

We now proceed to the discussion of the specific method we use to calculate Fig. 1, i.e., the pole model. We think that there are two distinct levels in this idea of using a "pole model." There is first a quite general idea which consists in saying that the process under consideration passes through certain hadronic intermediate states, and that it can be decomposed into two steps: production of these intermediate states in the strong process $B \rightarrow \mathcal{B}_b \overline{\mathcal{B}}_2$, where $\overline{\mathcal{B}}_2$ is the accompanying baryon, and $\mathcal{B}_b$ the intermediate baryon which then undergoes a weak transition to the baryon $\mathcal{B}_1$.

This is the general idea of the pole model. In fact, it includes the specific pole-model method that we adopt here, as well as other approaches, such as the use of second-order Hamiltonian perturbation theory, which will also intervene in the subsequent discussion.

By itself, this general idea can be contrasted with various alternative approaches. For instance, an alternative evaluation would be possible by connecting directly the quarks inside the initial and final states through free quark propagators, as pictured in Fig. 6. This approach has been followed for a variety of processes.[17] Another possible evaluation is through perturbative QCD, and involves, in addition to free-quark lines, gluon exchange (Fig. 7). And finally, there is the QCD-sum-rule approach, which cannot be pictured so easily.[13]

The advantage of our two-step approach with a selection of a few intermediate baryonic states is that it is especially fitted for quark-model calculations. The latter will yield easily the respective amplitudes for each step, which correspond to "basic processes" in the sense described in our book.[18] Such basic processes are calculable as matrix elements of a simple transition operator between hadron states, represented by their wave functions. Moreover, one can use harmonic-oscillator wave functions which lead to simple calculations.

Now, at a second level, the pole model is something much more precise than the above two-step general idea; it involves the appeal to pure pole terms, a notion that we shall define with some detail below. We will distinguish it from, for instance, the above-mentioned Hamiltonian perturbation approach.

For the sake of clarity, we first present the technical formulation of our pole model, and thereafter discuss the physical background behind the technique. The calculation consists in calculating Fig. 2 as a Feynman diagram with certain given couplings at the strong vertex $B\mathcal{B}_b\mathcal{B}_2$ and at the weak one $\mathcal{B}_b\mathcal{B}_1$, which shall be subsequently determined by the quark model. The final states $\mathcal{B}_1,\mathcal{B}_2$ that we consider will be only $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The intermediate states $\mathcal{B}_b$ will be only $\frac{1}{2}^+$ and $\frac{1}{2}^-$ baryons and moreover only $\mathcal{B}_1$ with $\frac{1}{2}^+$ will be produced at the weak vertex. This selection of states results from general principles (forbidding higher-spin intermediate states), and from quark-model selection rules at the weak vertex (to be discussed later), as well as from our choice, motivated by simplicity, to evaluate only the decay to ground state ($L=0$) baryons. The relativistic couplings are chosen to be the *minimal* ones, i.e., Yukawa for $\frac{1}{2}^+\frac{1}{2}^+0^-$ and $\frac{1}{2}^+\frac{1}{2}^-0^-$ strong vertices, the standard derivative coupling for $\frac{3}{2}^+\frac{1}{2}^+0^-$ and $\frac{3}{2}^+\frac{1}{2}^-0^-$ strong ver-

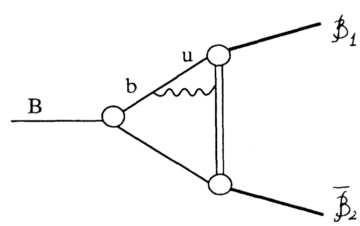

FIG. 6. Diagram contributing to B decay into a baryon-antibaryon pair through free-quark intermediate states.

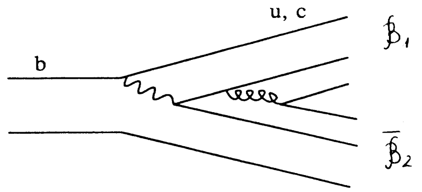

FIG. 7. The perturbative QCD picture of B decay into a baryon-antibaryon pair. The wavy line is a W, the curled one a gluon.

tices, and for the weak part, bilinear vertices of the form $\bar{\psi}\psi$ for both $\frac{1}{2}^+\frac{1}{2}^+$ and $\frac{1}{2}^+\frac{1}{2}^-$ transitions (respectively, parity-conserving and parity-violating).

The result of this choice of couplings can be displayed on the particular case $B \to p\bar{p}$. The decay amplitude writes (for definiteness, we choose $\mathcal{B}_1$ to be a baryon, and $\overline{\mathcal{B}}_2$ to be the antibaryon)

$$M = \bar{u}(\mathbf{p}_1)(A + B\gamma^5)v(\mathbf{p}_2) , \tag{2.4}$$

where A and B are Lorentz invariants corresponding, respectively, to the parity-violating (PV) and parity-conserving (PC) weak interactions. Then A and B are simply given by the products of the strong and weak coupling constants for each intermediate state α, divided by the mass difference $m_\alpha - m_{\mathcal{B}_1}$. For instance

$$B \sim \frac{ga}{m_p - m_\alpha} , \tag{2.5}$$

where g and a are the strong and PC weak couplings.

This is what one would like to call a pure "pole term," but for this expression to make sense, we have to make precise what is the function which is equal to a pole term, since, up to now, in a two-body decay, everything is fixed by the masses. In general, to introduce a function which really presents such poles, we must extrapolate the amplitude in some way. The quark model is defined in Hamiltonian formalism, and the simplest way to define such an extrapolation is to maintain the external particles $B\mathcal{B}_1\overline{\mathcal{B}}_2$ *on their mass shell,* and to relax only the energy-momentum conservation by allowing a transfer k at the weak vertex (Fig. 8) $p_B = p_1 + p_2 + k$. The amplitude is still defined by

$$M = (2\pi)^3 \langle \mathcal{B}_1\overline{\mathcal{B}}_2 | \mathcal{H}_w(0) | B \rangle . \tag{2.6}$$

We can now look at the amplitude as representing a *three-body* decay $B \to \mathcal{B}_1\overline{\mathcal{B}}_2\sigma$, where σ is a "spurion", i.e., a fictitious particle carrying momentum k^μ and the quantum numbers of $\mathcal{H}_w$ ($J^P = 0^\pm$). This *three-body decay* amplitude can be analyzed in a manner entirely similar to a two-body scattering amplitude. There will be three Mandelstam variables s,t,u:

$$\begin{aligned} s &= (p_B - p_2)^2 = (p_1 + k)^2 , \\ t &= (p_B - k)^2 = (p_1 + p_2)^2 , \\ u &= (p_B - p_1)^2 = (p_2 + k)^2 . \end{aligned} \tag{2.7}$$

At the physical point $k_\mu = 0$ (corresponding to the decay under study), $s = m_1^2$, $t = m_B^2$, $u = m_2^2$. Since we want to study the point $k_\mu = 0$, we can maintain $k^2 = 0$, while varying freely s,t,u, subject only to the condition

$$s + t + u = m_1^2 + m_2^2 + m_B^2 . \tag{2.8}$$

We meet singularities whenever s,t,u, reach the mass squared of some intermediate state: baryons for s and u, mesons for t. We note that there is in fact no u-channel baryon due to the b quantum numbers. The singularities in t are those of ordinary mesons $\pi,\rho,\ldots$ In fact, the quark configuration corresponding to the singularities in the t channel is exactly that of Fig. 3(a), i.e., annihilation of B into light quarks.

If we furthermore maintain t at its physical value $t = m_B^2$, M is then a function of the single variable s, which has singularities for $s = m_\alpha^2$, α being a b-flavored baryon. More precisely, we have with our choice of couplings for the PC amplitude of $B \to p\bar{p}$,

$$M \sim g\, a\, \bar{u}_p \gamma^5 \frac{(m_p + m_\alpha) - \not{k}}{s - m_\alpha^2} v_{\bar{p}} . \tag{2.9}$$

In fact, we have twice the number of invariants we had in the two-body decay:

$$M = \bar{u}[A + A'\not{k} + \gamma^5(B + B'\not{k})]v \tag{2.10}$$

but A' and B' will not contribute to the real decay. The four invariants are given by pure pole terms in the s variable, corresponding to the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ discrete intermediate states, i.e., a constant divided by $s - m_\alpha^2$.

Our assumption is then that even at low $s = m_1^2$, i.e., very far from the pole position (which begins at $m_{\Sigma_b}^2 \sim 25$ GeV^2), the actual amplitude is given by a few such pole terms corresponding to the lowest-lying baryonic b-flavored intermediate states.

This assumption amounts to two distinct statements.

(i) There are appreciable pole contributions from only a few discrete intermediate states.

(ii) There is no polynomial background $\mathcal{P}(s)$ as would be generated by couplings with higher-order derivatives, or t-channel-exchange couplings, or contact terms, ... For instance, we reject the s dependence which would be introduced by a $\frac{1}{2}^+$ intermediate state with derivative strong coupling f that would give the contribution

$$M \sim f\, a\, \bar{u}_p \gamma^5 \frac{m_p^2 + 2m_p m_\alpha + s - (m_p + m_\alpha)\not{k}}{2 - m_\alpha^2} v_{\bar{p}} . \tag{2.11}$$

We have little to say in defense of these assumptions. In the analogous case of nonleptonic hyperon decays, one

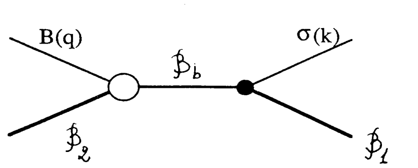

FIG. 8. The three-body decay amplitude $B \to \mathcal{B}_1\overline{\mathcal{B}}_2\sigma$, where σ a "spurion," a fictitious particle carrying momentum k.

uses Regge arguments to determine the high-energy behavior of the fictitious two-body scattering amplitude,[19] and from this one deduces the well-known fact that the PC wave (P wave) obeys an unsubtracted dispersion relation without arbitrary constant, while the PV wave (S wave) requires a subtraction; then in turn this justifies the pure pole model for P waves, while the remaining arbitrary constant in S waves is given by the commutator.

Nothing similar is available for our case. The Regge analysis was formulated for small t, and is therefore justified for the hyperon decays where $t=m_\pi^2$, but certainly not for our case where $t=m_B^2$. Moreover, in the case of hyperon decays, the selection of a small number of baryon poles could be justified by proximity arguments, while here we are very far from any of the poles.

In the absence of any definite argument, we formulate only some remarks.

(i) It is usual to assume that all the scattering is assigned to discrete intermediate states and that no additional continuum contribution is needed. This is the old duality idea. It may be that in addition the residues of the poles are rapidly decreasing above the lowest-lying b-baryon states, giving on the whole a contribution that is not too large. That this happens at least for radial excitations is suggested by the somewhat analogous case of the pion form factor, where the ρ pole residue seems to be by far the largest, and also by the decrease of electromagnetic couplings of the radial excitations of ψ and Υ.

(ii) The most serious problem we have to face is the possibility of a polynomial background. We can say that the contribution of t-channel exchanges at $t=m_B^2$ is small, since it corresponds to the B annihilation diagrams, whose magnitude has been shown to be small in Sec. II A. But we have reasons to expect that the s-channel poles have to be multiplied by factors varying rapidly with s, generating a polynomial background.

Indeed, the quark model suggests something of this sort. One could imagine the four-point amplitude to be calculated in second-order Hamiltonian perturbation theory, with the weak Hamiltonian and the strong-interaction pair-creation Hamiltonian or transition operator as interaction Hamiltonians:

$$M\sim\sum_{\mathcal{B}_b}\frac{\langle\mathcal{B}_1|\mathcal{H}_w|\mathcal{B}_b\rangle\langle\mathcal{B}_b\overline{\mathcal{B}}_2|H_{\text{strong}}|B\rangle}{E_B-E_{\mathcal{B}_b}-E_2}\,. \tag{2.12}$$

The matrix element will depend on s, which is equivalent, in the rest frame of the intermediate baryon $\mathcal{B}_b$, to a dependence on $\mathbf{k}^2$, the momentum of the two other particles at the vertex $B\to\mathcal{B}_b\overline{\mathcal{B}}_2,\mathcal{B}_b\to\mathcal{B}_1\sigma$. In fact, using, for example, harmonic-oscillator wave functions, the nonrelativistic quark-model calculation will always yield, in addition to centrifugal-barrier factors $|\mathbf{k}|^l$, an exponential factor $\exp(-\mathbf{k}^2R_1^2)$, where R_1^2 is some radius squared related to the wave functions, and also a polynomial in $\mathbf{k}^2R_1^2$ for orbital and radial excitations, all varying rapidly with s.

We will not use this method to calculate the decay amplitude at the physical point because while the weak vertex $\mathcal{B}_b-\mathcal{B}_1$ is then taken at zero transfer $\mathbf{p}_{\mathcal{B}_b}=\mathbf{p}_1$, the strong-interaction vertex must be considered at $|\mathbf{p}_2|\sim m_B/2$ in the B rest frame, therefore at very large transfer $|\mathbf{p}_2|^2\sim 6$ GeV^2, outside the scope of the quark model, especially in the channel $B\to\mathcal{B}_b\overline{\mathcal{B}}_2$ which requires two pair creations (Fig. 9). We prefer to use a pure pole model, where only the calculations of residues is required, more accessible to the quark model as explained below.

Nevertheless, from these quark-model considerations, we suspect that the contribution of each intermediate state is *a priori*, apart from the pole factor $1/(s-m_\alpha^2)$, affected by a rapidly varying numerator. Then, in order that pure pole terms dominate, higher powers in s must cancel, either by cancellations between poles, or with some contact term, . . . , or a new regime must hold when one passes to very large transfers not accessible to the quark model. The analogy with the pion form factor and the ρ dominance shows that such things can effectively happen.

In any case, it must be emphasized that the effective "minimal" couplings we introduce to define the pole terms should make sense only to define the residues of the poles, and not to describe the couplings away from these poles; they should not be expected to fit the above-mentioned variations of couplings implied by the quark model in Hamiltonian formalism.

Under these assumptions, what we have to predict is the strong and weak couplings at the corresponding pole. For that purpose, we have to make precise the kinematical situation under which these couplings must be calculated. At the pole, Feynman and Hamiltonian formalisms coincide in that (i) every particle at the vertex, either external or intermediate, is on shell, and (ii) energy-momentum is conserved. This is not always possible with real momentum.

As to the weak vertex, this corresponds to the energetically allowed, although quite fictitious, process $\mathcal{B}_b\to\mathcal{B}_1+\sigma$ or to the matrix element of the weak density at $\mathbf{k}^2=0$. The momentum transfer $|\mathbf{k}|$ in the B_b rest frame is

$$|\mathbf{k}|=\frac{m_{\mathcal{B}_b}^2-m_1^2}{2m_{\mathcal{B}_b}}\sim\frac{m_{\mathcal{B}_b}}{2}\,. \tag{2.13}$$

As explained below, $|\mathbf{k}|^2\sim 6$ GeV^2 is in principle outside the scope of the quark model, but we can dispose of it reasonably well.

For the strong vertex, the process $B\to\mathcal{B}_b+\overline{\mathcal{B}}_2$ is, of course, not allowed, since B is stable, and either of course

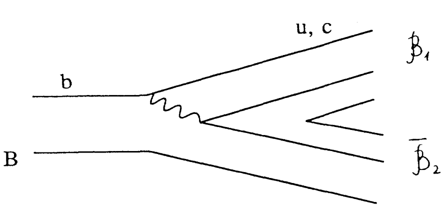

FIG. 9. The decay of a B meson into baryon-antibaryon pair according to the quark model, that involves two quark pair creations.

is $\mathcal{B}_2 \to \mathcal{B}_b \bar{B}$ which requires $b\bar{b}$ creation. The only possible real process would be $\mathcal{B}_b \to B + \mathcal{B}_2$, but this is still forbidden by phase space for the lowest-lying excitations in which we are interested. It may be just possible for the $\Sigma_b(\frac{1}{2}^-)$ with $\mathcal{B}_2$=nucleon.

Then, the couplings at the pole for the strong vertex are just like the *YKN* couplings ($Y = \Lambda, \Sigma$). They can be defined only by analytic continuation. After having defined the amplitude as a function of s by introducing the spurion momentum k_μ, we can define the residue of a pole at $s = m^2_{\mathcal{B}_b}$ independently of any interpretation of the vertex as a real process. We can also relate it to Hamiltonian matrix elements (disregarding any consideration of the mathematical problems of continuation) in the following way. The pole residues shall be the same as found in the Hamiltonian perturbation theory contribution of the $\mathcal{B}_b$ intermediate state (2.12):

$$M \sim \frac{\langle \mathcal{B}_1 | \mathcal{H}_w | \mathcal{B}_b \rangle \langle \mathcal{B}_b \bar{\mathcal{B}}_2 | H_{\text{strong}} | B \rangle}{E_1 + k^0 - E_{\mathcal{B}_b}} , \tag{2.14}$$

where the energy denominator $E_1 + k^0 - E_{\mathcal{B}_b} = E_B - E_{\mathcal{B}_b} - E_2$.

We have assumed a global energy conservation and we have assumed that the Z graphs which would also contribute to the pole $s = m^2_{\mathcal{B}_b}$ are negligible. The pole at $s = m^2_{\mathcal{B}_b}$ can be reached by choosing the spurion momentum (k_μ) in the weak matrix element so that $E_1 + k^0 = E_{\mathcal{B}_b}$, which corresponds to (2.13). On the other hand, $E_B = E_{\mathcal{B}_b} + E_2$ implies that to reach the pole, we have to choose *complex momenta* at the strong-interaction vertex.

As already indicated, the strong-interaction matrix element in (2.14) requires two quark pair creations (Fig. 9), and there is no standard calculation of this in the quark model. One can bypass this additional difficulty by considering, instead of the decay $B \to \mathcal{B}_1 \bar{\mathcal{B}}_2$, the amplitude in the channel $\mathcal{B}_2 B \to \mathcal{B}_1 \sigma$. It shall have the same invariant amplitudes, i.e., there shall be the same functions of s, t, u, with the same poles and residues. Therefore we can alternatively identify the residues we are looking for on the new Hamiltonian perturbation-theory formula:

$$M \sim \frac{\langle \mathcal{B}_1 | \mathcal{H}_w | \mathcal{B}_b \rangle \langle \mathcal{B}_b | H_{\text{strong}} | \mathcal{B}_2 B \rangle}{E_2 + E_B - E_{\mathcal{B}_b}} \tag{2.15}$$

instead of (2.14). The pole at $s = m^2_{\mathcal{B}_b}$ will be reached this time by choosing complex momenta to ensure $E_2 + E_B = E_{\mathcal{B}_b}$ in the calculation of the new matrix element

$$\langle \mathcal{B}_b | H_{\text{strong}} | \mathcal{B}_2 B \rangle \tag{2.16}$$

which has the advantage of involving a one-pair-creation process, which is well studied. In addition, we have now an extrapolation to complex momenta which is more moderate than in $B \to \mathcal{B}_b \bar{\mathcal{B}}_2$. In the rest frame of $\mathcal{B}_b$, we reach the pole by setting the $\mathcal{B}_2$ and B momenta to

$$|\mathbf{q}| = \frac{\sqrt{m^2_{\mathcal{B}_b} - (m_B + m_2)^2}\sqrt{m^2_{\mathcal{B}_b} - (m_B - m_2)^2}}{2m_{\mathcal{B}_b}} \tag{2.17}$$

As we shall see, with (2.17) we can handle (2.16) reasonably well by the quark model, with some adaptation.

To avoid confusion, it must be emphasized again that we have rejected direct use of the standard Hamiltonian perturbation theory to calculate the process $B \to \mathcal{B}_1 \bar{\mathcal{B}}_2$. We have used it several times in the discussion, but only as an auxiliary mean to discuss the pole model. In particular, in the last steps, to correlate the pole residues with Hamiltonian matrix elements which are more accessible to the quark model. We do not mean either that the standard Hamiltonian perturbation theory is incorrect: we simply cannot use it to calculate the physical process with large transfers and two quark pair creations. Moreover it may be that more intermediate states are required in Hamiltonian perturbation theory than in the pole model.

C. Discussion of the nonrelativistic quark-model calculations of the pole couplings

As we have said, the main motivation for the pole model is that we are able to calculate through standard quark-model methods the "basic processes" corresponding, respectively, to the strong and weak vertices, as matrix elements of simple transition operators. In particular, this has been rendered possible for the strong vertex by crossing the vertex, so as to reduce it to the form (2.16), which implies only one pair creation.

Yet, we have also to ask whether we are really under conditions which allow us to apply the standard quark-model calculations at the vertices. Indeed, we are using a nonrelativistic model under conditions which may seem *a priori* completely outside the scope of such an approach. And even after having answered the objection, we will have to cut off the ambiguities which are unavoidably present in applying the model to a relativistic situation.

Let us recall first that, from experience of more than twenty years, the quark model seems to be able to describe soft vertices and transitions rather than only strictly nonrelativistic situations. Soft vertices could be defined as the ones not involving momentum transfers greater than 1 GeV. By contrast, large momentum transfers would require perturbative QCD methods. It must be emphasized that such a definition applies far outside a true nonrelativistic situation. Internal velocities are always of the order $v/c \sim 1$ wherever light quarks are involved. And, if the momentum transfer is 1 GeV^2, a light hadron will also have a relativistic center-of-mass motion.

Here we deal with light-quark hadrons ($\mathcal{B}_2$ and possibly $\mathcal{B}_1$) and with hadrons composite of light and heavy quarks (B and possibly $\mathcal{B}_1$). The fact that v/c remains ~ 1 for the light quarks has striking consequences: terms of order v/c in the nonrelativistic calculation are not suppressed with respect to order $(v/c)^0$. Indeed, the PV weak and the strong transitions of $\frac{1}{2}^-$ intermediate states, which formally contain such a v/c factor (see the detailed calculations in Sec. IV), are as large as the PC contribution of $\frac{1}{2}^+$ states, whence the possibility of a large ratio PV/PC.

It is known that in spite of the large values of v/c, the nonrelativistic quark-model calculations do rather well in related processes such as hyperon decays[8] or weak radiative decays[9]. In these cases, the contribution of $\frac{1}{2}^-$ intermediate states, as calculated by the naive quark model, is crucial in solving the problems of S/P wave ratios or the large $\Sigma^+ \to p\gamma$ asymmetry. On the other hand, we would not *a priori* trust calculations at large transfer, as is illustrated by pion and nucleon form factors. From the discussion of Sec. II B, we know that the strong- and weak-interaction vertices are in very different situations with regards to momentum transfer, and we discuss them separately.

(i) *Strong vertex.* As we recall, we have to calculate $g_{B\mathcal{B}_b\mathcal{B}_2}$ by continuation of the transition matrix element $\mathcal{B}_b \to \mathcal{B}_2 B$. Considered in the rest frame $\mathbf{p}_{\mathcal{B}_b}=0, \mathbf{p}_2 = -\mathbf{q}, \mathbf{p}_B = +\mathbf{q}$, the condition $E_{\mathcal{B}_b} = E_2 + E_B$ implies

$$\mathbf{q}^2 = \mathbf{q}_0^2 \simeq (m_{\mathcal{B}_b} - m_2)^2 - m_B^2 \ . \tag{2.18}$$

We have used (2.17) and exploited the fact that m_b is very large to simplify the equation.

For low-lying excitations, $\mathbf{q}_0^2 < 0$ since $m_{\mathcal{B}_b} - m_B \sim m_u$ while $m_1 \sim 3m_u$. Using calculations of Isgur,[20] we find Λ_b at 5.64 GeV, Σ_b at 5.82 GeV, and Σ_b and Λ_b ($\frac{5}{2}^-$) at 6.25 and 6.4 GeV, respectively. These are naive-quark-model predictions. The $\frac{5}{2}^-$ masses can serve as upper bounds for $\frac{1}{2}^-$, relevant to our problem, which we locate in the band 6–6.2 GeV. From this we conclude, for $\frac{1}{2}^+$ states and the $\mathcal{B}_2$=nucleon,

$$\mathbf{q}_0^2 \simeq -1 \text{ GeV}^2 \tag{2.19}$$

and, for $\frac{1}{2}^-$,

$$\mathbf{q}_0^2 \simeq -0.5 \text{ to } 0.0 \text{ GeV}^2 \ . \tag{2.20}$$

In absolute magnitude, these momenta are in a range allowing for a quark-model approach; we think however that a specific problem arises for complex momenta around the value (2.19), as will be commented below. As to precise values, we observe that the ones corresponding to $\frac{1}{2}^-$ states are very sensitive to the state considered and to the uncertainties in the spectroscopic model. We therefore expect large uncertainties in the PV wave.

The quark-pair-creation model (Fig. 10) tells us how to calculate the M matrix for the transition $\mathcal{B}_b \to \mathcal{B}_2 B$, which in the case of a real decay, is related to the S matrix by

$$S = 1 - 2\pi i \delta(E_f - E_i)\delta(\mathbf{p}_f - \mathbf{p}_i) M \ . \tag{2.21}$$

But M is in fact defined when $E_f \neq E_i$ as well, since it depends only on the momenta. We can interpret it, in this case, as the effective Hamiltonian matrix element

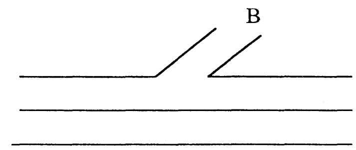

FIG. 10. The strong vertex $\mathcal{B}_b\mathcal{B}_1 B$ according to the quark-pair-creation model.

[such as (2.16)] with $\delta(\mathbf{p}_f - \mathbf{p}_i)$ factorized. Therefore, we can get the coupling at the pole we are looking for by setting $\mathbf{q}^2 = \mathbf{q}_0^2 < 0$ in the general expression $M(\mathbf{q})$, and identifying with the relativistic expression for the amplitude in terms of the coupling constant.

In fact, this identification is subjected *a priori* to ambiguities. In the quark-pair-creation model, M_{QPC} is defined with the standard nonrelativistic normalization of states with a $\delta^3(\mathbf{p}-\mathbf{p}')$. However, given that the transition is really relativistic while the calculation of M is nonrelativistic, it would be also logical to identify it with the relativistic M matrix, defined with an invariant normalization $(E/m)\delta^3(\mathbf{p}-\mathbf{p}')$, or with the noninvariant $\delta^3(\mathbf{p}-\mathbf{p}')$. In the same vein, instead of the expression (2.17) for $|\mathbf{q}_0|$, one could have used to calculate M_{QPC} any other expression having the same limit as $|\mathbf{q}_0| \to 0$.

In our papers,[4] we have understood the calculation rules for M as giving the M matrix for noninvariant normalization $\delta^3(\mathbf{p}-\mathbf{p}')$ with $\mathbf{q}$ fixed at its relativistic value (2.17), all calculations being made in the rest frame of the initial particle. For a real decay, this would give

$$\Gamma = 8\pi^2 \frac{E_2 E_B}{m_{\mathcal{B}_b}} |M(\mathbf{q}_0)|^2 \ . \tag{2.22}$$

This prescription has been tested over a very large set of data, with an overall agreement obtained[21] with a value for the light-quark pair-creation constant $\gamma \simeq 3$. However, when we apply it to the present problem, this prescription would raise serious difficulties. One would write, for a $\frac{1}{2}^+\frac{1}{2}^+0^-$ vertex, the equation

$$(2\pi)^3 \langle \mathcal{B}_2 B | H_{\text{strong}} | \mathcal{B}_b \rangle = \frac{1}{(2\pi)^{3/2}} \frac{1}{(2E_B)^{1/2}} \left[\frac{m_2}{E_2} \right]^{1/2} g_{\mathcal{B}_b\mathcal{B}_2 B} \bar{u}_{\mathcal{B}_b} \gamma^5 u_2 = -M_{\text{QPC}} \ . \tag{2.23}$$

The values (2.19) or (2.20) allow us to set safely $E_B \simeq m_B$. On the contrary, $E_{\mathcal{B}_2} \simeq m_{\mathcal{B}_b} - m_B$ is very variable according to the preceding remarks, and then the factors $(E_2)^{1/2}$ or $E_2 + m_2$ in the left-hand side (LHS) of (2.23) will lead to physically improbable variations of the coupling constant expressed in terms of M_{QPC}, whose variation is much smoother. In fact one readily sees that our identification contradicts ideas about analyticity around $\mathbf{q}^2 \simeq 0$. Nonrelativistic approximations of amplitudes such as $\mathcal{B}_2 B \to \mathcal{B}_1 \sigma$ will have analytical properties (smooth function with poles), which, in the relativistic treatment, are presented by the invariant amplitudes (with normalization and spinor factors dropped). The normalization and spinor factors generate in addition the so-called "kinematical" singularities, which precisely bother us in the present problem. To avoid this contradiction, we simply set everywhere $E_2 \equiv m_2$.

Suppose now that we use the harmonic-oscillator wave functions to calculate $M_{\rm QPC}$. From (2.23) or similar equations, we see that the coupling constant as calculated from $M_{\rm QPC}$ will present a simple form: $|\mathbf{q}_0|^l$ centrifugal barrier factors cancel between each side, and the coupling constant equals a polynomial in $\mathbf{q}_0^2$, times a Gaussian factor $\exp(-\mathbf{q}_0^2 A^2)$ where A^2 depends only on radii and quark masses, times hadron mass factors, and in particular the factor $(2m_B)^{1/2}$ which will always spoil the assumption of flavor symmetry for the coupling constants.

In our calculations we have taken fully into account the effect of unequal quark masses, but we have found that the result is qualitatively similar for $M(\mathbf{q}_0)$ as if one had set $m_b = m_u$.

We find then for the couplings $\frac{1}{2}^+\frac{1}{2}^+0^-$ expressions of the form (cf. Sec. IV)

$$g_{\Sigma_b NB} \propto \sqrt{2m_B}\, 2m_p \tilde{B} \exp(-\mathbf{q}_0^2 \tilde{A}^2) , \tag{2.24}$$

where $\tilde{B}$ and $\tilde{A}$ are combinations of the wave function radii of the usual light-quark hadrons. The factor leads to rather large values of $g^2/4\pi$ as compared with the corresponding $g_{\Sigma NK}$. For the analogous $g_{\Lambda NK}$ (which is better known than $g_{\Sigma KN}$) we find, in the QPC model,

$$\frac{g^2_{\Lambda NK}}{4\pi} \sim 20 \tag{2.25}$$

which is very encouraging since we have no fitted parameter. For the sake of completing the discussion, let us recall that the corresponding result for $g^2_{NN\pi}/4\pi$ is predicted too small in the QPC model due to the factor $(2m_\pi)^{1/2}$. However, we cannot believe such a crude model in this case due to the exceptional smallness of the pion mass related to its quasi-Goldstone character.

In the case of the $\frac{1}{2}^+\frac{1}{2}^-0^-$ S-wave couplings, there is in the expression of $M_{\rm QPC}$ (cf. Sec. IV) a rapidly varying polynomial $(1-\lambda \mathbf{q}_0^2)$ with $\lambda > 0$ and large magnitude (~ 2). For $\mathbf{q}_0^2 \lesssim 0$, the sign is safely determined, but the absolute magnitude can be very much enhanced according to the precise mass difference $\mathcal{B}_b - B$. This is one serious source of uncertainty concerning the magnitude of PV waves.

In the PC case ($\frac{1}{2}^+$ intermediate state), we encounter a much deeper problem. Although the *absolute* value of $\mathbf{q}_0$ seems to allow for the use of the quark model, it can be seen that something very unphysical happens at $\mathbf{q}_0^2 \simeq -1$ GeV^2 when we use the quark model. This is most clearly seen by using Coulomb wave functions fitting the *same* radii which we have determined by the standard phenomenology. Then, calculating the QPC matrix element, one finds a strong maximum near $\mathbf{q}^2 \simeq -1$ GeV^2. This maximum is seen to be related to the pole of the Coulomb wave function at negative $\mathbf{q}^2$. That such an enhancement must not be considered as physical is clearly seen by considering the corresponding form factor, which gets a pole for the same reason. When one considers instead a confining potential, there is not a maximum, but a very steep increase of the matrix element. *The softer the confining potential* is, *the steeper the increase* near $\mathbf{q}^2 \simeq -1$ GeV^2. We must therefore consider that this increase is unphysical, because it corresponds to an unphysical enhancement in the form factor. In fact, we already observe this increase in the harmonic-oscillator case. We cut off this unphysical effect by relying on a smoothness assumption: we assume that the matrix element remains a linear function of $\mathbf{q}^2$, with coefficients fixed around $\mathbf{q}^2 \simeq 0$, a region in which we trust the quark model.

(ii) *Weak vertex.* We have to calculate

$$(2\pi)^3 \langle \mathcal{B}_1(-\mathbf{k}_0) | \mathcal{H}_w(0) | \mathcal{B}_b(\mathbf{p}=0) \rangle . \tag{2.26}$$

The three-momentum transfer $\mathbf{k}_0^2$ is very large in contrast with the one at the strong vertex $\mathbf{q}_0^2$. According to (2.13), $\mathbf{k}_0^2 \sim 6-9$ GeV^2. This is of course due to the fact that, through the weak interaction, we have a decay with a very large change of the quark mass $b \to c$ or $b \to u$.

In principle, we cannot claim to make a direct quark-model calculation of such a hard process. We follow the line of Altomari and Wolfenstein[22] in the $B \to D, D^*$ semileptonic decays. We calculate the couplings with the quark model at $\mathbf{k} \simeq 0$ and we extrapolate at $\mathbf{k}_0^2$ by using polelike form factors. For orientation, it is nevertheless useful to discuss first the $\mathbf{k}^2$ dependence predicted by the quark model. It also appears that finally the nonrelativistic calculation would not be too bad for the $b \to c$ transition.

The quark model for the matrix element (2.26) is visualized in Fig. 12, with a wavy line for the spurion. This calculation is new, since usually one calculates only at $\mathbf{k}=0$. Also, we have made a complete calculation of the effect of unequal quark masses $m_b \neq m_c \neq m_u$, $\mathcal{B}_1$ being possibly a charmed baryon. This calculation is especially nontrivial for the complicated PV weak operator, and the effect of unequal masses is especially important in that case.

As to $\mathbf{k}^2$ dependence, one would find none for a meson-meson transition, since it is a pointlike interaction. A form factor is generated for baryon transitions by the presence of the spectator quark. In fact, the baryon problem is analogous to the one of a current density matrix element between meson states, since the baryon can be viewed as a quark-diquark composite, with a spectator quark and the diquark having a point interaction analogous to the photon coupling for a quark. Using harmonic-oscillator wave functions, we find (Appendix D) the usual exponential factor, with a smaller slope: $\exp(-\mathbf{k}^2 R^2/24)$ in the equal quark mass case, instead of $\exp(-\mathbf{k}^2 R^2/6)$ for a current interaction. In the case of the decay of an orbital excitation $\mathcal{B}_b$ ($\frac{1}{2}^-$), one would expect, just like in the case of radiative decays, a further po-

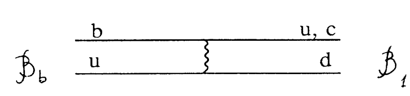

FIG. 11. The weak vertex $\mathcal{B}_b\mathcal{B}_1$ where $\mathcal{B}_1$ can be a charmed or a light baryon.

lynomial dependence on $\mathbf{k}^2$. It is found indeed, although it disappears in the equal-mass limit.

The effect of unequal quark masses is especially important on the exponential factor, which becomes

$$\exp - \frac{R^2}{24}\left[\frac{3m}{2m+m_{q_1}}\right]^2 \mathbf{k}^2 \qquad (2.27)$$

where $q_1 = c$ or u. For a c quark, it depresses considerably the slope, just as in the case of a $b \to c$ semileptonic decay[22], while for $q = u$, one still obtains $\exp(-\mathbf{k}^2 R^2/24)$ (the dissymmetry between the final and the initial state is due to the choice of frame: the $\mathcal{B}_b$ rest frame). Therefore, with $\mathbf{k}^2 \sim 6$ GeV^2, we would find a large suppression (a factor ~ 0.07) for the $\mathcal{B}_b - N$ transition and a moderate one (~ 0.7) for $\mathcal{B}_b - \Lambda_c$.

Let us now pass to the polelike form factors which appear more reasonable, at least for the $\mathcal{B}_b - N$ transition. We write

$$F(\mathbf{k}^2) = \frac{1-(m_{\mathcal{B}_b} - m_1)^2/m_{\tilde{B}}^2}{1-\mathbf{k}^2/m_{\tilde{B}}^2} . \qquad (2.28)$$

$\tilde{B}$ is the mediating hadron with quantum numbers of B for flavor and $0^{\pm}$ for J^P (as $\mathcal{H}_w$ is a scalar or pseudoscalar operator). The form factor is normalized at $k=0$. We make the quark-model calculation at $\mathbf{k}^2 = (m_{\mathcal{B}_b} - m_1)^2$, and we extrapolate at the spurion mass^2, $\mathbf{k}^2 = 0$. Therefore, one ends with a reduction factor

$$F(0) = 1 - \frac{(m_{\mathcal{B}_b} - m_1)^2}{m_{\tilde{B}}^2} . \qquad (2.29)$$

From the above analogy with a current density matrix element, and with the emitting quark substituted by a diquark (emitting a spurion), we expect that $\tilde{B}$ is a diquark-antidiquark system (Fig. 12). To be simple, we evaluate the masses very crudely: $m_{\mathcal{B}_b} = m_b + 2m$, $m_1 = m_{q_1} + 2m$, $m_{\tilde{B}} = m_b + m_{q_1} + 2m$, whence

$$F(0) = 1 - \frac{(m_b - m_{q_1})^2}{(m_b + m_{q_1} + 2m)^2} = \begin{cases} 0.8 & \text{for } q_1 = c \ , \\ 0.4 & \text{for } q_1 = u \ . \end{cases} \qquad (2.30)$$

We find therefore the same trend as for the quark-model form factor: although $\mathbf{k}^2$ is nearly equal for the two cases, the decay to u-quark baryons is more suppressed than the decay to charmed baryons, although the effect is much more moderate. We assume for simplicity that the same form factor applies to PC and PV matrix elements, although we could expect a more complicated behavior in the latter case.

To sum up, we have found that, at the strong vertex, it was possible to apply the standard $^3P_0$ quark-pair-creation approach under reasonable conditions. On the other hand, at the weak vertex, the large momentum transfer would *a priori* prevent a quark-model approach. Nevertheless, we can rely on the analogy with semileptonic $b \to u,c$ weak decays of mesons to make reasonable assumptions. This is to be contrasted with what the situation would have been if one had tried to apply directly the method of second order perturbation theory (2.12): the weak vertex would have been calculated at $\mathbf{k}=0$, a certain improvement indeed, but the strong vertex would have required two pair creations and large transfers a rather uncontrollable situation at present, whence the advantage of the pole model, which would not occur in hyperon nonleptonic decays, where any transfer is small, and where the analogue of (2.12) presents definite advantages.

As a last remark, we note that, at zero-momentum transfer, the conservation of J implies that the spins of $\mathcal{B}_b$ and $\mathcal{B}_1$ should be equal. Therefore one finds the result, used in our papers on nonleptonic[8] and weak radiative decays,[9] and on the neutron electric dipole moment,[10] that with $\mathcal{B}_1$ of $J^P = \frac{1}{2}^+$ ($\frac{3}{2}^+$ are forbidden[23]), we have the contributions of intermediate states $\mathcal{B}_b$ of $J^P = \frac{1}{2}^+$ or $\frac{1}{2}^-$ for PC and PV waves, respectively. This is no longer true if $\mathbf{k} \neq 0$, and one should include other intermediate states, for example, $\frac{1}{2}^+$ for the PV wave and $\frac{1}{2}^-$ for the PC wave. For simplicity, we have not tried to evaluate such contributions.

For the evaluation of the quark-model matrix elements at $\mathbf{k}=0$, we have to know the wave functions. We still use the harmonic-oscillator wave functions, but we are conscious that here the only argument is one of simplicity. In contrast with the strong vertex, the weak interaction is not a peripheral one, it implies essentially short distances, specially in the $\frac{1}{2}^+ - \frac{1}{2}^+$ transition, and this requires an accurate knowledge of the wave function. The harmonic oscillator is in gross error for the crucial quantity determining the $\frac{1}{2}^+ - \frac{1}{2}^+$ matrix element, it gives, for purely light-quark masses,[24]

$$\langle \psi_0 | \delta(\mathbf{r}_1 - \mathbf{r}_2) | \psi_0 \rangle \sim 4 \times 10^{-3} \text{ GeV}^3 \qquad (2.31)$$

while a realistic estimate would be rather $0.8-1 \times 10^{-2}$ GeV^3. The estimate 10^{-2} GeV^3 is confirmed by a recent calculation of Gignoux and Silvestre-Brac.[3] On the other hand, we have no simple way to estimate the effect of unequal quark masses or the $\frac{1}{2}^-$ orbital excitation transitions except by using harmonic-oscillator wave functions. A possible attitude is then to rely on the harmonic oscillator to estimate ratios of PV/PC or unequal/equal quark masses matrix elements, but to calibrate the absolute magnitudes by the estimate for the case of purely light-

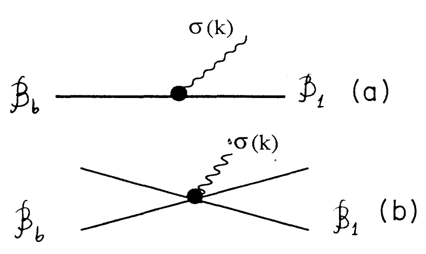

FIG. 12. The weak vertex $\mathcal{B}_b\mathcal{B}_1$ at nonzero-momentum transfer k, carried by the spurion σ.

quark baryons:

$$\langle \psi_0 | \delta(\mathbf{r}_1 - \mathbf{r}_2) | \psi_0 \rangle \sim 10^{-2}\ \mathrm{GeV}^3 . \tag{2.32}$$

D. QCD asymptotic behavior and the pole model

Quite apart from the QCD-sum-rule calculation by CZ,[13] we think that their discussion,[25] based on QCD asymptotics of a previous pole model calculation[11] is very interesting, and we can answer some of their objections or comments.

If one counts powers of m_b for $m_b \to \infty$ as predicted by QCD, one finds, according to CZ

$$\Gamma(B \to \mathcal{B}_1 \overline{\mathcal{B}}_2) = O(1/m_b^4) , \tag{2.33}$$

$$\Gamma(B \to N\overline{\Delta})/\Gamma(B \to N\overline{N}) = O(1) . \tag{2.34}$$

None of these relations is satisfied by the pole model of Deshpande, Soni, and Trampetic[11] who would find, as pointed out by CZ

$$\Gamma(B \to N\overline{\Delta}) = O(m_b^3) , \tag{2.35}$$

$$\Gamma(B \to N\overline{N}) = O(1/m_b) \tag{2.36}$$

and neither by our results, which are however closer to the ones of CZ. We find, *without* the weak form factor $F(0)$ [formula (2.30)],

$$\Gamma(B \to N\overline{\Delta}) = O(m_b^2) , \tag{2.37}$$

$$\Gamma(B \to N\overline{N}) = O(1) \tag{2.38}$$

and including this factor we find

$$\Gamma(B \to N\overline{\Delta}) = O(1) , \tag{2.39}$$

$$\Gamma(B \to N\overline{N}) = O(1/m_b^2) \tag{2.40}$$

[the intermediate results (2.37) and (2.38) are given only to clarify the discussion].

Having these types of behavior in mind, we can proceed to the discussion. First, we must emphasize that the full predictions of QCD asymptotic behavior are not expected to prevail in principle except for very large transfers or equivalently for very large m_b. Even in the weaker form advocated by CZ, with nonasymptotic wave functions, they have been controversial.[26] And finally, in the case under concern, CZ themselves recognize that *the asymptotic contribution is anomalously small.*

These objections must be balanced with the fact that the power counting works in the case of form factors at rather small Q^2; then it could well be indicative for the mass behavior at moderate m_b. At least, it is in this sense that CZ are invoking QCD asymptotic behavior in their report[25] and it is within this context that we discuss the power behavior of our own model. The use of asymptotic behavior will be the following in short: if the falloff of the pole model is slower than predicted by QCD, we take it as an indication that it probably overestimates the rate at large m_b, although we cannot say at what precise value of m_b it must fail. In addition, we suspect that the bigger the discrepancy in power behavior, the more we expect the rate to be overestimated. These conclusions, although rather naive, seem confirmed by the failure of the predictions for the modes with a Δ.

Our pole prediction, *without* the form factor for the weak transition, could be compared to a calculation of mesonic decays (e.g., $B \to \pi\pi$) through a B^*-pole-dominated form factor (Fig. 13)

$$M \sim f_\pi \frac{f_{B^*} g_{B^* B\pi}}{m_B^2} (p_B + p_{\pi_2}) \cdot p_{\pi_1} , \tag{2.41}$$

The power behavior of $g_{B^*B\pi}$ is found with help of the QPC model, or, more safely, through PCAC and the quark-model estimate for the axial-vector $B^* - B$ current coupling: $g_{B^*B\pi} = O(m_b)$. The behavior of f_{B^*} is determined by the wave function at origin of the B^*, which depends only on the reduced mass $f_{B^*} = O(m_b^{1/2})$. Finally

$$\Gamma(B \to \pi\pi) = O(m_b^2) , \tag{2.42}$$

This disagrees with the CZ asymptotic prediction $\Gamma(B \to \pi\pi) \sim 1$, but less than (2.37) and (2.38) do with the similar CZ prediction (2.34). This difference is understandable from the discussion below on the further suppression of baryonic decays.

For baryons, somewhat similar to the electromagnetic nucleon form factors, CZ expect a steeper falloff, which is attributed to the necessity to create an additional $q\bar{q}$ pair out of the vacuum. This additional effect also explains the difference between charmed and noncharmed decays according to CZ, because the momentum of the created quarks is constrained to be different according to the final baryon wave functions: it is hard in a proton, and much softer for a Λ_c. In $p\bar{p}$, this would explain the additional factor $1/m_b^4$ for baryons.

It is remarkable that our weak form factor includes qualitatively the same effect. As we have observed in the preceding paragraphs, in our method the strong-interaction vertex is soft, and it is the weak vertex which is hard. Nevertheless, the effect is essentially the same: the weak form factor implies a further depression of baryonic decay (relative to the meson decay) due to the spectator quark. In the weak transition $B^* - \pi$ such a weak form factor is indeed not present (due to factorization or, equivalently, to the point character of the interaction). Moreover, this effect differentiates between a Λ_c and a N at the weak vertex.

As one can easily show, the marked difference in the form factors as calculated in the quark model is essentially due to the fact that in a system with a very heavy quark, this one carries almost all the momentum. Therefore the spectator is soft, while if all quarks are light the momentum is shared between all quarks, and the specta-

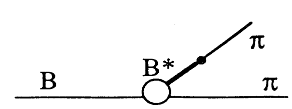

FIG. 13. Pole model for B-meson decay into a pair of light mesons. The intermediate state B^* is a 1^- meson.

tor is harder. We note that the spectator quark is precisely the same as the one coming from the created pair in the CZ scheme.

This weak form factor introduces an additional $1/m_b^2$ factor in the power counting. But since the strong-interaction coupling has an additional power $m_b^{1/2}$, in our quark-model calculation for $B \to p\bar{p}$ we find in the rate only one power less than Deshpande, Soni, and Trampetic [compare (2.36) and (2.40)]. The case of $N\bar{\Delta}$ will be discussed below.

The remaining differences with QCD asymptotics are that (i) our effect is apparently much weaker, especially when we substitute, as it seems logical, the Gaussian form factors by pole-dominated ones, and (ii) the additional power introduced by our weak form factor is only $1/m_b^2$ in rate, which is not sufficient to get (2.33).

Concerning the ratio $N\bar{\Delta}/N\bar{N}$, we also observe an improvement over Deshpande *et al:* we get $O(m_b^2)$ instead of $O(m_b^4)$. This is due to our more homogeneous treatment of strong coupling constants by the quark model instead of flavor-symmetry arguments. Indeed, while they assume

$$g_{B_b B\Delta}/g_{B_b BN} \sim 1 \qquad (2.43)$$

(equality to couplings for light quarks), we find, through the ${}^3P_0$ model,

$$g_{B_b B\Delta}/g_{B_b BN} = O(1/m_b) \ . \qquad (2.44)$$

It is in order here to support our prediction (2.44) to note that in the case of a coupling constant such as $g_{\mathcal{B}_b \mathcal{B}'_b \pi}$, our prediction through the ${}^3P_0$ model is in agreement with the power counting deduced from PCAC and the axial-vector current coupling $\mathcal{B}_b - \mathcal{B}'_b$.

We note that an important contribution to the relative enhancement of $B^0 \to p\bar{p}$ is due to the introduction of the PV wave. We are then in agreement with the qualitative arguments of CZ.[13,25]

Nevertheless, we do not get, as far as m_b dependence is concerned, $\Gamma(B \to N\bar{\Delta})/\Gamma(B \to N\bar{N}) \sim 1$ as seems implied by QCD.[13,25] We can trace the difference back to the behavior of the Rarita-Schwinger spinors as compared to ordinary Dirac spinors. Indeed, on passing from the vertex $\mathcal{B}_b \to B\mathcal{B}_2$ to the reaction $B \to \mathcal{B}_1\bar{\mathcal{B}}_2$, one observes that, leaving aside factors with identical m_b power behavior, spinor factors $\bar{u}u$ or $\bar{u}\gamma^5 u$ with soft momenta are substituted by $\bar{u}v$ or $\bar{u}\gamma^5 v$ with hard momenta in the case $\mathcal{B}_2 = \frac{1}{2}^+$, while in the case $\mathcal{B}_2 = \frac{3}{2}^+$, we have $\bar{u}u_0$ or $\bar{u}\gamma^5 u_0$ (0 component of the Rarita-Schwinger spinor u_μ) substituted by $\bar{u}v_0$ or $\bar{u}\gamma^5 v_0$ with hard momenta. While we have

$$\bar{u}v \sim \bar{u}\gamma^5 v = O(m_b) \qquad (2.45)$$

we get

$$\bar{u}v_0 \sim \bar{u}\gamma^5 v_0 = O(m_b^2) \qquad (2.46)$$

due to the factor $e_0 = O(m_b)$. We do not know at present whether this difference has any physical significance. It can be seen to be general in pole models, necessarily *at odds with QCD power counting when a high spin is present.*

Take, for instance, the $\pi - \omega$ electromagnetic transition form factors, as compared to the pion form factor $F_\pi(\mathbf{q}^2)$. QCD predicts, according to CZ, $F_{\pi\omega}(q^2) \sim 1/q^4$, while a ρ pole model of course gives $F_{\pi\omega}(q^2) \sim 1/q^2$, just as for $F_\pi(q^2)$. Indeed, while the ω spin implies a ω-π coupling with *more* derivatives than for the π-π coupling, the power counting finds a *smaller algebraic* power of q^2 for the matrix element involving the spin-one particle (the power decreases with helicity change). We do not know how to cure this defect of pole models. From the above argument, we can simply expect that for very large m_b, the $N\bar{\Delta}$ rate will be overestimated by the pole model as compared to the $N\bar{N}$ rate, but we do not know how large m_b must be.

Similar considerations would explain the different power-counting rules of mesons (2.42) as compared to $B \to N\bar{N}$ without the weak form factor (2.38). It can be ascribed to the fact that in $B \to \pi\pi$ the B^* requires derivatives, not present in the spin-$\frac{1}{2}$ spinor couplings: the scalar product in (2.41) grows more rapidly with m_b than the spinor factor $\bar{u}v$ when the soft momenta of the quark-model calculation of residues are substituted by the large momenta ($\sim m_b$) of the overall reaction.

Then, an important comment is in order: the remaining discrepancies between our calculation and the QCD asymptotic behavior must be attributed to *principles inherent to the pole model itself,* and not to the *quark-model calculation of residues* which seems always in agreement with more general approaches. For instance, f_{B^*} is in agreement with asymptotic counting, and our estimate of strong coupling constants, when a π is involved, is in agreement with the PCAC estimate (with the rather safe power counting of the axial-vector current of the quark model, also adopted by the QCD-sum-rule people).

In summary, we have done the best that we can do to estimate the residues of the pole model, and the question is whether experiment will confirm or invalidate the use of the pole model at the physical value of the bottom-quark mass.

III. COUPLINGS AND DECAY RATES IN THE POLE MODEL

Once we have discussed the basis for our pole model, we will now write the following.

(i) The expressions of the partial decay rates of the different processes $B \to \mathcal{B}_1\bar{\mathcal{B}}_2$ in terms of the couplings $B\mathcal{B}_1\mathcal{B}_2$. We restrict ourselves to ground-state $\frac{1}{2}^+, \frac{3}{2}^+$ baryons. $\mathcal{B}_1, \mathcal{B}_2$ can also be charmed baryons or N, Δ.

(ii) The expressions of the effective couplings $B\mathcal{B}_1\mathcal{B}_2$ (parity conserving and parity violating) in terms of strong ($B\mathcal{B}_b\mathcal{B}_2$ and $B\mathcal{B}_b^*\mathcal{B}_2$) and weak $\mathcal{B}_b\mathcal{B}_1$ (parity-conserving) and $\mathcal{B}_b^*\mathcal{B}_1$ (parity-violating) couplings. $\mathcal{B}_b$ and $\mathcal{B}_b^*$ denote, respectively, a $\frac{1}{2}^+, \frac{1}{2}^-$, b-composed baryon.

(iii) Finally, we will obtain the quark-model expressions for the total effective couplings in terms of the strong and weak couplings calculated by the quark model in the Appendixes.

A. Effective couplings and decay rates

CP invariance constrains the total $B\mathcal{B}_1\mathcal{B}_2$ couplings to be (neglecting possible derivative couplings)

$$iA(\bar{\psi}_{\mathcal{B}_2}\psi_{\mathcal{B}_1}\phi_B-\bar{\psi}_{\mathcal{B}_1}\psi_{\mathcal{B}_2}\phi_B^+)\ ,\qquad iB(\bar{\psi}_{\mathcal{B}_2}\gamma^5\psi_{\mathcal{B}_1}\phi_B+\bar{\psi}_{\mathcal{B}_1}\gamma^5\psi_{\mathcal{B}_2}\phi_B^+) \tag{3.1}$$

for $\mathcal{B}_1,\mathcal{B}_2$ $\frac{1}{2}^+$ baryons and

$$\frac{C}{M_B}(\bar{\psi}_{\mathcal{B}_{2\mu}}\gamma^5\psi_{\mathcal{B}_1}\partial_\mu\phi_B^+-\bar{\psi}_{\mathcal{B}_1}\gamma^5\psi_{\mathcal{B}_{2\mu}}\partial_\mu\phi_B)\ ,\qquad \frac{D}{M_B}(\bar{\psi}_{\mathcal{B}_{2\mu}}\psi_{\mathcal{B}_1}\partial_\mu\phi_B^+ +\bar{\psi}_{\mathcal{B}_1}\bar{\psi}_{\mathcal{B}_{2\mu}}\partial_\mu\phi_B) \tag{3.2}$$

for $\mathcal{B}_1$ $\frac{1}{2}^+$ and $\mathcal{B}_2$ $\frac{3}{2}^+$ baryons. ψ_μ denotes the corresponding Rarita-Schwinger field and the factor M_B^{-1} is introduced to ensure that C,D are dimensionless. A,C are parity-violating and B,D, parity-conserving couplings. Notice that we have changed the notation relatively to Ref. 6. We do not consider two $\frac{3}{2}^+$ baryons in the final state due to the selection rule of our model that forbids the process[23] $B\to\frac{1}{2}^+\frac{3}{2}^+$ due to the vanishing of the matrix elements $\langle\mathcal{B}_b$ or $\mathcal{B}_b^*|\mathcal{H}_w|\frac{3}{2}^+\rangle$, as discussed in Sec. II.

After some algebra, we find the following decay rates in the B center of mass:

$$\Gamma(B\to\mathcal{B}_1\bar{\mathcal{B}}_2)=\frac{k}{4\pi}\left\{|A|^2\frac{(M_B+m_1+m_2)^2k^2}{(E_1+m_1)(E_2+m_2)M_B^2}+|B|^2\frac{[(E_1+m_1)(E_2+m_2)+k^2]^2}{(E_1+m_1)(E_2+m_2)M_B^2}\right\} \tag{3.3}$$

for $\mathcal{B}_1,\mathcal{B}_2$ $\frac{1}{2}^+$ baryons with masses and energies m_1,m_2,E_1,E_2.

For the case of $\mathcal{B}_1$ $\frac{1}{2}^+$ and $\mathcal{B}_2$ $\frac{3}{2}^+$ baryons we find, instead,

$$\Gamma(B\to\mathcal{B}_1\bar{\mathcal{B}}_2)=\frac{k^3}{6\pi}\frac{1}{m_2^2}\left\{|C|^2\frac{[(E_1+m_1)(E_2+m_2)+k^2]^2}{(E_1+m_1)(E_2+m_2)M_B^2}+|D|^2\frac{(M_B+m_1+m_2)^2k^2}{(E_1+m_1)(E_2+m_2)M_B^2}\right\}\ , \tag{3.4}$$

where m_2,E_2 are the mass and energy of the spin $\frac{3}{2}$ baryon. Both in (3.3) or (3.4), B_1 is a $\frac{1}{2}^+$ baryon, charmed (Λ_c^+,Σ_c) or a nucleon.

Notice that in (3.3), (3.4) the powers of k correspond to the expected behavior for the allowed partial waves:

$B\to\frac{1}{2}^+\frac{1}{2}^+$, $l=0$ for parity conserving ,

$l=1$ for parity violating ,

$B\to\frac{1}{2}^+\frac{3}{2}^+$, $l=2$ for parity conserving ,

$l=1$ for parity violating .

B. Effective couplings in terms of weak and strong couplings of intermediate states

In our pole model, the intermediate states are $\frac{1}{2}^+(\mathcal{B}_b)$ and $\frac{1}{2}^-(\mathcal{B}_b^*)$ baryons containing a single b quark. We now write down the couplings involving these intermediate states.

The weak couplings are

$$a_{\mathcal{B}_b\mathcal{B}_1}(\bar{\psi}_{\mathcal{B}_b}\psi_{\mathcal{B}_1}+\bar{\psi}_{\mathcal{B}_1}\psi_{\mathcal{B}_b})\ ,\qquad ib_{\mathcal{B}_b^*\mathcal{B}_1}(\bar{\psi}_{\mathcal{B}_b^*}\psi_{\mathcal{B}_1}-\bar{\psi}_{\mathcal{B}_1}\psi_{\mathcal{B}_b^*})\ , \tag{3.5}$$

respectively, for parity-conserving and parity-violtating amplitudes, and the strong couplings will be

And the strong couplings will be

$$ig_{B\mathcal{B}_2\mathcal{B}_b}(\bar{\psi}_{\mathcal{B}_b}\gamma^5\psi_{\mathcal{B}_2}\phi_B+\bar{\psi}_{\mathcal{B}_2}\gamma^5\psi_{\mathcal{B}_b}\phi_B^+)\ ,\qquad g_{B\mathcal{B}_2\mathcal{B}_b^*}(\bar{\psi}_{\mathcal{B}_b^*}\psi_{\mathcal{B}_2}\phi_B+\bar{\psi}_{\mathcal{B}_2}\psi_{\mathcal{B}_b^*}\phi_B^+)\ , \tag{3.6}$$

if $\mathcal{B}_2$ is a $\frac{1}{2}^+$ baryon, and

$$\frac{h_{B\mathcal{B}_2\mathcal{B}_b}}{M_B}(\bar{\psi}_{\mathcal{B}_b}\psi_{\mathcal{B}_{2\mu}}\partial_\mu\phi_B+\bar{\psi}_{\mathcal{B}_{2\mu}}\psi_{\mathcal{B}_b}\partial_\mu\phi_B^+)\ ,\qquad i\frac{h_{B\mathcal{B}_2\mathcal{B}_b^*}}{M_B}(\bar{\psi}_{\mathcal{B}_b^*}\gamma^5\psi_{\mathcal{B}_{2\mu}}\partial_\mu\phi_B+\bar{\psi}_{\mathcal{B}_{2\mu}}\gamma^5\psi_{\mathcal{B}_b^*}\partial_\mu\phi_B^+)\ , \tag{3.7}$$

if $\mathcal{B}_2$ is a $\frac{3}{2}^+$ baryon.

From (3.5)–(3.7) we will obtain the effective total couplings (3.1), (3.2) by the corresponding Feynman diagrams. Remember that the s-channel meson intermediate states give a negligible contribution, as discussed in Sec. II. We obtain

$$A_{B\mathcal{B}_1\mathcal{B}_2}=-\sum_{\mathcal{B}_b^*}\frac{g_{B\mathcal{B}_2\mathcal{B}_b^*}b_{\mathcal{B}_b^*\mathcal{B}_1}}{m_1-m_{\mathcal{B}_b^*}}\ ,\qquad B_{B\mathcal{B}_1\mathcal{B}_2}=\sum_{\mathcal{B}_b}\frac{g_{B\mathcal{B}_2\mathcal{B}_b}a_{\mathcal{B}_b\mathcal{B}_1}}{m_1-m_{\mathcal{B}_b}}\ , \tag{3.8}$$

if $\mathcal{B}_2$ is a $\frac{1}{2}^+$ baryon, and

$$C_{B\mathcal{B}_1\mathcal{B}_2}=-\sum_{\mathcal{B}_b^*}\frac{h_{B\mathcal{B}_2\mathcal{B}_b^*}b_{\mathcal{B}_b^*\mathcal{B}_1}}{m_1-m_{\mathcal{B}_b^*}}\ ,\qquad D_{B\mathcal{B}_1\mathcal{B}_2}=\sum_{\mathcal{B}_b}\frac{h_{B\mathcal{B}_2\mathcal{B}_b}a_{\mathcal{B}_b\mathcal{B}_1}}{m_1-m_{\mathcal{B}_b}}\ , \tag{3.9}$$

if $\mathcal{B}_2$ is a $\frac{3}{2}^+$ baryon.

IV. QUARK-MODEL EXPRESSIONS OF THE COUPLINGS

We need now to identify the Lorentz-invariant weak (3.5), and strong couplings (3.6), (3.7) with their corresponding expressions in the nonrelativistic quark model, computed, respectively, in Appendixes B and C. Once these expressions are obtained, we must sum over the intermediate states following (3.8), (3.9). For example, for a transition such as $B^- \to n\bar{p}$ we must sum over the intermediate b-flavored baryons $\Sigma_b^0, \Lambda_b \frac{1}{2}^+$ for the PC amplitude, and Σ_b^{0*}, $\Lambda_b^* \frac{1}{2}^-$ for the PV amplitude. For $B^- \to \Lambda_c^+ \bar{\Delta}^{++}$, for example, we must consider Σ_b^+ and Σ_b^{+*}

We will first give the final results for the transitions computed in this paper, and we will at the end comment on the factors entering in the expressions.

A. $B \to N\bar{N}$ transitions

We find for parity-conserving couplings, from (3.8), (B13), (B14), (C8), (C10),

$$B_{BNN} = i\frac{G}{\sqrt{2}} V_{ub}\gamma \frac{1}{m_N - m_{\mathcal{B}_b}} \sqrt{M_B}\frac{4}{\sqrt{3}}\pi^{1/4} m_N I_0 R'^{3/2} c_b (\alpha_b \beta_b A_b)^{3/2} B_b \exp(-C_b R^2 k^2) \times \begin{cases} -1, & \bar{B}^0 \to p\bar{p} \ , \\ -2, & B^- \to n\bar{p} \ , \\ 1, & \bar{B}^0 \to n\bar{n} \ , \end{cases} \tag{4.1}$$

and for the parity-violating ones, from (3.8), (B34), (B35), (C15), (C18)

$$A_{BNN} = i\frac{G}{\sqrt{2}} V_{ub}\gamma \frac{1}{m_N - m_{\mathcal{B}_b^*}} \sqrt{M_B}\, 2\sqrt{3}\,\pi^{1/4} \frac{I_0}{mR}\frac{R'^{3/2}}{R} c_b'' \alpha_b^{3/2}$$

$$\times (\beta_b A_b)^{5/2}[3 - B_b D_b (k^2 R^2)] \exp(-C_b R^2 k^2) \times \begin{cases} 1, & \bar{B}^0 \to p\bar{p} \ , \\ 1, & B^- \to n\bar{p}, \\ 0, & \bar{B}^0 \to n\bar{n} \ . \end{cases} \tag{4.2}$$

Notice that we have assumed degeneracy of the intermediate states, allowing factorization of the energy denominators. The factor I_0 comes from the wave function at a zero interquark distance (B11). The factor $M_B^{1/2}$ comes from the relativistic expressions of the strong couplings in the Lorentz-invariant couplings, that involve a factor $1/M_B^{1/2}$, that must be identified to the quark-model results.[12] The mass-dependent factors $c_b, c_b'', \alpha_b, \beta_b, A_b, B_b, C_b, D_b$ are given in Appendixes B and C and k^2 is the momentum squared entering in the quark-model strong couplings. We will discuss its actual value in the next section.

Equations (4.1) and (4.2) satisfy separately the isospin relations

$$M(\bar{B}^0 \to p\bar{p}) - M(\bar{B}^0 \to n\bar{n}) = M(B^- \to n\bar{p}) \tag{4.3}$$

that follow from the $\Delta I = \frac{1}{2}$ rule, a consequence of the color antisymmetry of the baryon wave functions and the fact that we consider only weak interactions between quarks within a baryon.[23]

B. $B \to N\bar{\Delta}$ transitions

We find, from (3.9), (B13), (B14), (C9), (C10) for the parity-conserving couplings:

$$D_{B\Delta N} = i\frac{G}{\sqrt{2}} V_{bu}\gamma \frac{1}{m_N - m_{\mathcal{B}_b}} m_\Delta \sqrt{M_B}\pi^{1/4} I_0 R'^{3/2} c_b (\alpha_b \beta_b A_b)^{3/2} B_b \exp(-C_b R^2 k^2) \times \begin{cases} -\sqrt{3}, & B^- \to p\bar{\Delta}^{++}, \\ -1, & B^- \to n\bar{p}, \\ 1, & \bar{B}^0 \to p\bar{\Delta}^+, \\ 1, & \bar{B}^0 \to n\bar{\Delta}^0 \ , \end{cases} \tag{4.4}$$

and from (3.9), (B34), (B35), (C16), (C19),

$$C_{B\Delta N} = 0 \ , \tag{4.5}$$

i.e., the $B \to N\bar{\Delta}$ transitions turn out to be purely *parity conserving*. These couplings satisfy the $\Delta I = \frac{1}{2}$ isospin relations

$$M(B^- \to p\bar{\Delta}^{++}) : M(B^- \to n\bar{\Delta}^+) : M(\bar{B}^0 \to p\bar{\Delta}^+) : M(\bar{B}^0 \to n\bar{\Delta}^0) = -\sqrt{3} : -1 : 1 : 1 \ . \tag{4.6}$$

C. $B \to \mathcal{B}_c \bar{N}$ transitions

$\mathcal{B}_c$ is now a ground-state $\frac{1}{2}^+$ charmed baryon. We obtain, from (3.8), (B44), (B45), (C8), (C10), for the parity-conserving couplings,

$$B_{BN\mathcal{B}_c}=i\frac{G}{\sqrt{2}}V_{cb}\gamma\frac{1}{m_{\mathcal{B}_c}-m_{\mathcal{B}_b}}\sqrt{M_B}m_N\frac{2\sqrt{2}}{3}\pi^{1/4}I_0R'^{3/2}c_{bc}(\alpha_b\beta_b A_b)^{3/2}B_b\exp(-C_bR^2k^2)\times\begin{cases}-2\sqrt{6}, & B^-\to\Sigma_c^0\bar{p}\ ,\\ -\sqrt{3}, & \bar{B}^0\to\Sigma_c^+\bar{p}\ ,\\ -1, & \bar{B}^0\to\Sigma_c^+\bar{p}\ ,\\ \sqrt{6}, & \bar{B}^0\to\Sigma_c^0\bar{n}\ ,\end{cases} \tag{4.7}$$

and, from (3.8), (B56), (B57), (C15), (C18) for the parity-violating ones,

$$A_{BN\mathcal{B}_c}=i\frac{G}{\sqrt{2}}V_{cb}\gamma\frac{1}{m_{\mathcal{B}_c}-m_{\mathcal{B}_b^*}}\sqrt{M_B}\left[-\frac{2}{3\sqrt{6}}\right]\pi^{1/4}\frac{I_0}{mR}\frac{R'^{3/2}}{R}\alpha_b^{3/2}(\beta_b A_b)^{5/2}[3-B_bD_b(R^2k^2)]\exp(-C_bR^2k^2)$$
$$\times\left[c''_{bc}\begin{bmatrix}0\\9\\3\sqrt{3}\\9\sqrt{2}\end{bmatrix}+d''_{bc}\begin{bmatrix}2\sqrt{2}\\1\\-\sqrt{3}\\-\sqrt{2}\end{bmatrix}\right],\quad\text{for}\quad\begin{cases}B^-\to\Sigma_c^0\bar{p},\\ \bar{B}^0\to\Sigma_c^+\bar{p},\\ \bar{B}^0\to\Lambda_c^+\bar{p},\\ \bar{B}^0\to\Sigma_c^0\bar{n},\end{cases} \tag{4.8}$$

where the mass-dependent factors c_{bc}, c''_{bc}, d''_{bc} are given in Appendix B. These amplitudes satisfy the $\Delta I=1$ ($b\to c, u\to d$) isospin relations

$$M(\bar{B}^0\to\Sigma_c^0\bar{n})+M(B^-\to\Sigma_c^0\bar{p})=\sqrt{2}\,M(\bar{B}^0\to\Sigma_c^+\bar{p})\ . \tag{4.9}$$

D. $B\to\mathcal{B}_c\bar{\Delta}$ transitions

Finally, we obtain, from (3.9), (B44), (B45), (C9), (C10), the parity-conserving couplings,

$$D_{B\Delta\mathcal{B}_c}=i\frac{G}{\sqrt{2}}V_{cb}\gamma\frac{1}{m_{\mathcal{B}_c}-m_{\mathcal{B}_b}}\sqrt{M_B}m_\Delta\frac{4}{\sqrt{6}}\pi^{1/4}I_0R'^{3/2}c_{bc}(\alpha_b\beta_b A_b)^{3/2}$$
$$\times B_b\exp(-C_bR^2k^2)\times\begin{cases}1, & \bar{B}^0\to\Lambda_c^+\bar{\Delta}^=\ ,\\ -\sqrt{3}, & B^-\to\Lambda_c^+\bar{\Delta}^{++}\ ,\\ \sqrt{3}, & \bar{B}^0\to B^-\to\Sigma_c^+\bar{\Delta}^{++}\ ,\\ -3, & B^-\to\Sigma_c^+\bar{\Delta}^{++}\ ,\\ \sqrt{6}, & \bar{B}^0\to\Sigma_c^0\bar{\Delta}^0\ ,\\ -\sqrt{6}, & B^-\to\Sigma_c^0\bar{\Delta}^+\ ,\end{cases} \tag{4.10}$$

and, from (3.9), (B56), (B57), (C16), (C19), for the parity-violtating couplings,

$$C_{B\Delta\mathcal{B}_c}=i\frac{G}{\sqrt{2}}V_{cb}\gamma\frac{1}{m_{\mathcal{B}_c}-m_{\mathcal{B}_b^*}}\sqrt{M_B}\frac{8\sqrt{2}}{3}\pi^{1/4}m_\Delta^2\frac{I_0}{mR}RR'^{3/2}\alpha_b^{3/2}(\beta_b A_b)^{5/2}B_bD_b\exp(-C_bR^2k^2)$$
$$\times d''_{bc}\times\begin{cases}-\sqrt{3}, & \bar{B}^0\to\Lambda_c^+\bar{\Delta}^+\ ,\\ 3, & B^-\to\Lambda_c^+\bar{\Delta}^{++}\ ,\\ 1, & \bar{B}^0\to\Sigma_c^+\bar{\Delta}^+\ ,\\ -\sqrt{3}, & B^-\to\Sigma_c^+\bar{\Delta}^{++}\ ,\\ \sqrt{2}, & \bar{B}^0\to\Sigma_c^0\bar{\Delta}^0\ ,\\ -\sqrt{2}, & B^-\to\Sigma_c^0\bar{\Delta}^+\ .\end{cases} \tag{4.11}$$

Now the PV amplitudes do not vanish, being proportional, through d''_{bc} to m_c-m. Equations (4.10) and (4.11) satisfy the $\Delta I=1$ isospin relations

$$\begin{aligned}&M(\bar{B}^0\to\Lambda_c^+\bar{\Delta}^+)/M(B^-\to\Lambda_c^+\bar{\Delta}^{++})=-1/\sqrt{3}\ ,\\ &M(\bar{B}^0\to\Sigma_c^+\bar{\Delta}^+):M(B^-\to\Sigma_c^+\bar{\Delta}^{++}):M(\bar{B}^0\to\Sigma_c^0\bar{\Delta}^0):M(B^-\to\Sigma_c^0\bar{\Delta}^+)=1:-\sqrt{3}:\sqrt{2}:-\sqrt{2}\ .\end{aligned} \tag{4.12}$$

TABLE I. Numerical results for the branching ratios and Γ^{PV}/Γ^{PC} for the different charmed- and uncharmed-baryon–antibaryon final states. We have taken the values $V_{cb}=0.046$, $V_{ub}=0.005$, and $\tau_B=10^{-12}$ sec.

Mode	Branching ratio	Γ^{PV}/Γ^{PC}	Mode	Branching ratio	Γ^{PV}/Γ^{PC}
$\bar{B}^0\to p\bar{p}$	0.74×10^{-5}	0.79	$B^-\to p\bar{\Delta}^{++}$	0.32×10^{-3}	0
$B^-\to n\bar{p}$	0.17×10^{-4}	0	$B^-\to n\bar{\Delta}^{+}$	0.11×10^{-3}	0
$\bar{B}^0\to n\bar{n}$	0.74×10^{-5}	0.79	$\bar{B}^0\to p\bar{\Delta}^{+}$	0.11×10^{-3}	0
$B^-\to\Sigma_c^0\bar{p}$	0.15×10^{-1}	0.5×10^{-4}	$\bar{B}^0\to n\bar{\Delta}^{0}$	0.11×10^{-3}	0
$\bar{B}^0\to\Sigma_c^+\bar{p}$	0.29×10^{-2}	0.52	$\bar{B}^0\to\Lambda_c^+\bar{\Delta}^{+}$	0.12×10^{-1}	0.63×10^{-1}
$\bar{B}^0\to\Sigma_c^0\bar{n}$	0.58×10^{-2}	0.54	$B^-\to\Lambda_c^+\bar{\Delta}^{++}$	0.36×10^{-1}	0.63×10^{-1}
$\bar{B}^0\to\Lambda_c^+\bar{p}$	0.11×10^{-2}	0.59	$\bar{B}^0\to\Sigma_c^+\bar{\Delta}^{+}$	0.29×10^{-1}	0.65×10^{-2}
			$B^-\to\Sigma_c^+\bar{\Delta}^{++}$	0.86×10^{-1}	0.65×10^{-2}
			$\bar{B}^0\to\Sigma_c^0\bar{\Delta}^{0}$	0.57×10^{-1}	0.65×10^{-2}
			$B^-\to\Sigma_c^0\bar{\Delta}^{+}$	0.57×10^{-1}	0.65×10^{-2}

Let us now give the numerical results.

V. NUMERICAL RESULTS

We give first the final results for the different modes, the branching ratios and the ratio between the parity-violating and the parity-conserving rates (Table I). These results take into account all of the effects: (i) quark mass differences ($m_b=4.950$ GeV, $m_c=2.0$ GeV, and $m=0.33$ GeV for the light quarks); (ii) strong-coupling form factors $F_s(k^2)$ with linear extrapolation, keeping only the order k^2R^2 in the expansion of the exponential; (iii) weak form factors $F_w(k^2)$. The rest of the mass parameters are $M_{\Lambda_c}=2.284$ GeV, $M_{\Sigma_c}=2.455$ GeV, $M_B=5.278$ GeV, and $M_{\mathcal{B}_b}=5.640$ GeV, $M_{\mathcal{B}_b^*}=6.040$ GeV, where $\mathcal{B}_b$ and $\mathcal{B}_b^*$ are the intermediate b baryons, of, respectively, $J^P=\frac{1}{2}^+,\frac{1}{2}^-$. Moreover we take $R^2=6$ GeV^{-2} and $R'^2=10$ GeV^{-2} for the light baryon and meson square radii, and $I_0=\langle\psi_0|\delta(\mathbf{r}_1-\mathbf{r}_2)|\psi_0\rangle=10^{-2}$ GeV^3 for the light baryon wave function at zero interquark distance squared. We have discussed all these parameters in the preceding sections. Moreover, we take $V_{cb}=0.046$ and $V_{ub}=0.005$ and $\tau_B=10^{-12}$ sec.

In these results we have many effects involved (quark mass differences, form factors, etc.). It is necessary to make a discussion of how these different effects enter in the final results. To this aim, we have selected a few transitions, significant of the different situations. For our discussion we select the branching ratios and the ratio $r=\Gamma^{PV}/\Gamma^{PC}$ between the parity-violating and the parity-conserving rates for a few transitions such as $\bar{B}^0\to p\bar{p}$, $B^-\to p\bar{\Delta}^{++}$, $\bar{B}^0\to\Lambda_c^+\bar{p}$, and $B^-\to\Lambda_c^+\bar{\Delta}^{++}$. We select also some weak and strong couplings: the weak couplings $a(\Sigma_b^+p)$, $a(\Sigma_b^+\Sigma_c^+)$ (parity conserving) and

TABLE II. A few significant branching ratios, ratios $r=\Gamma^{PV}/\Gamma^{PC}$, weak and strong couplings to show the sensitivity of the results to the different effects: (1) includes all the effects (quark mass differences between m_b, m_c, m, weak and strong form factors) with linear approximation for the strong form factor; in (2) we show all the effects plus the full exponential of the strong form factor; in (3) we set the strong form factor $F_s(k^2)=F_s(0)$; in (4) we keep the strong form factor with the linear extrapolation but we set the weak form factor $F_w(k^2)=1$; in (5) we set both $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$; in (6) we take the limit $m_c\to m=0.33$ GeV together with $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$; finally in (7) we take the limits $m_b,m_c\to m=0.33$ GeV together with $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$B(\bar{B}^0\to p\bar{p})$	0.7×10^{-5}	0.1×10^{-4}	0.2×10^{-5}	0.4×10^{-4}	0.1×10^{-4}	0.1×10^{-4}	0.3×10^{-4}
$B(B^-\to p\bar{\Delta}^{++})$	0.3×10^{-3}	0.2×10^{-2}	0.4×10^{-4}	0.2×10^{-2}	0.3×10^{-3}	0.3×10^{-3}	0.5×10^{-3}
$B(\bar{B}^0\to\Lambda_c^+\bar{p})$	10^{-3}	0.2×10^{-2}	0.2×10^{-3}	0.1×10^{-2}	0.3×10^{-3}	0.2×10^{-3}	0.5×10^{-3}
$B(B^-\to\Lambda_c^+\bar{\Delta}^{++})$	0.4×10^{-1}	0.17	0.4×10^{-2}	0.5×10^{-1}	0.6×10^{-2}	0.4×10^{-2}	0.8×10^{-2}
$r(\bar{B}^0\to p\bar{p})$	0.79	0.48	0.76	0.78	0.77	0.77	1.15
$r(B^-\to p\bar{\Delta}^{++})$	0	0	0	0	0	0	0
$r(\bar{B}^0\to\Lambda_c^+\bar{p})$	0.59	0.36	0.58	0.58	0.58	0.56	0.86
$r(B^-\to\Lambda_c^+\bar{\Delta}^{++})$	0.6×10^{-1}	0.2×10^{-1}	10^{-1}	0.6×10^{-1}	10^{-1}	0	0
$a(\Sigma_b^+p)$	0.1×10^{-8}	0.1×10^{-8}	0.1×10^{-8}	0.3×10^{-8}	0.3×10^{-8}	0.3×10^{-8}	0.2×10^{-8}
$a(\Sigma_b^+\Sigma_c^+)$	0.2×10^{-7}	0.2×10^{-7}	0.2×10^{-7}	0.3×10^{-7}	0.3×10^{-7}	0.2×10^{-7}	0.2×10^{-7}
$b(\Sigma_b^{+*}(\psi'\chi')p)$	-0.4×10^{-9}	-0.4×10^{-9}	-0.4×10^{-9}	-0.1×10^{-8}	-0.1×10^{-8}	-0.1×10^{-8}	-0.1×10^{-8}
$b(\Sigma_b^{+*}(\psi'\chi')\Sigma_c^+)$	0.6×10^{-8}	0.6×10^{-8}	0.6×10^{-8}	0.8×10^{-8}	0.8×10^{-8}	0.7×10^{-8}	10^{-8}
$g(\Sigma_b^+p\bar{B}^0)$	-0.1×10^2	-0.2×10^2	-0.6×10	-0.1×10^2	-0.6×10	-0.6×10	-0.1×10^2
$h(\Sigma_b^+\Delta^{++}B^-)$	-0.7×10^2	-0.2×10^3	-0.2×10^2	-0.7×10^2	-0.2×10^2	-0.2×10^2	-0.5×10^2
$g(\Sigma_b^{+*}(\psi'\chi')p\bar{B}^0)$	-0.4×10	-0.4×10	-0.2×10	-0.4×10	-0.2×10	-0.2×10	-0.2×10
$h(\Sigma_b^{+*}(\psi'\chi')\Delta^{++}B^-)$	-0.3×10^3	-0.4×10^3	-0.1×10^3	-0.3×10^3	-0.1×10^3	-0.1×10^3	-0.2×10^3

$b(\Sigma_b^{+*}(\psi'\chi')p)$, $b(\Sigma_b^{+*}(\psi'\chi')\Sigma_c^+)$ (parity violating) and the strong couplings $g(\Sigma_b^+ p\bar{B}^0)$, $h(\Sigma_b^+\Delta^{++}B^-)$, $g(\Sigma_b^{+*}(\psi'\chi')p\bar{B}^0)$, $h(\Sigma_b^{+*}(\psi'\chi')\Delta^{++}B^-)$ involving N and Δ and $\mathcal{B}_b$ and $\mathcal{B}_b^*$ baryons, respectively. These couplings are dimensionless, defined in Eqs. (3.5)–(3.7). In Table II we give the numerical results for these couplings in the different interesting limits.

In column (1) we plot the results taking into account all the effects: the large mass differences between m_b, m_c, and m, the strong from factors $F_s(k^2)$ keeping only the linear terms in k^2R^2, and the weak form factors $F_w(k^2)$. In column (2) we plot what we would obtain in the same conditions but if we had kept the full exponential form factor of the QPC model. Of course, then the strong couplings would increase enormously. In column (3) we take into account the quark mass differences and the weak form factor $F_w(k^2)$, but we set $F_s(k^2)=F_s(0)$. The strong couplings decrease. In column (4) we take into account the quark mass differences and the strong form factor $F_s(k^2)$ (with the linear terms), but we set $F_w(k^2)=1$ for all transitions. The weak couplings increase. In column (5) we take into account all the quark mass differences and we set both $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$. In column (6) we keep $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$ and moreover we set $m_c \rightarrow m=0.33$ GeV to see the effect of the charmed-quark mass; in particular we see that the parity-violating waves involving a charmed baryon and a Δ vanish in this limit. Finally, keeping $F_s(k^2)=F_s(0)$ and $F_w(k^2)=1$, we make $m_b, m_c \rightarrow m=0.33$ GeV to see the effect of the large quark masses relatively to the light-equal-mass case.

We will comment on these results in the following section.

VI. DISCUSSION OF THE RESULTS

As already emphasized, there are really two independent ingredients in the calculation of $B \rightarrow \mathcal{B}_1\bar{\mathcal{B}}_2$. One is the pole model; the other is the quark model prediction for the pole residues. Although experiment shall test only their combination, the latter have to be discussed first by themselves, because the coupling constants of strong and weak vertices have their own physical significance. Indeed, they could be measured separately in other processes (although we do not have presently these processes at our disposal), and the predictions of the quark model for them could be valid even if the pole model was ruled out. In addition, it is the pole-model idea which includes the most doubtful assumptions in the present context, although it has been successful elsewhere.

A. Predictions of coupling constants and branching ratios

The predictions for "coupling constants" (generic denomination for the strong- and weak-interaction vertices) depend on a series of parameters which have been discussed in our book.[4] These include quark masses, the light-quark meson and baryon radii R^2 and R'^2, the squared baryon wave function at $\mathbf{r}_1=\mathbf{r}_2, \langle\psi_0|\delta(\mathbf{r}_1-\mathbf{r}_2)|\psi_b\rangle=|\psi(0)|^2$, and the quark-pair-creation constant γ. Quark masses and wave functions can be determined in principle from spectroscopy alone. R^2 and R'^2 are safely fixed by the quark masses and the orbital splitting, rather independently of the potential. $|\psi(0)|^2$ depends on more refined and more disputable knowledge of the potential, but it exceeds certainly the harmonic-oscillator estimate. For the strong vertex, γ has been chosen to fit decay widths once radii have been determined.

We warn the reader that quite different values are encountered in the literature. We do not want to discuss here the various reasons leading to such discrepancies. But one must be aware that they would lead to significantly different predictions, especially for the weak matrix element. The best we can say is that our choice of parameters results in an overall satisfactory picture of radiative decay widths, Okubo-Zweig-Iizuka-allowed strong decay widths, and baryon nonleptonic decays.[4] This gives us confidence in the extension of such predictions to heavy quarks. However, this extension itself requires new assumptions which have been discussed in detail in Sec. II, and which lack experimental support. Some are motivated only by calculational simplicity and could be tested by numerical computations. These include the use of the harmonic oscillator to treat the unequal quark mass effects. For strong vertices, the rough success of the analogous calculation of OZI-allowed charmonium decays gives encouragement. Others are really new physical assumptions and would deserve independent confirmation. These include the weak "nonleptonic form factor," and the extrapolation of the strong vertices form factors to negative $\mathbf{q}^2$. The latter is supported only weakly by our estimate of $g_{KN\Lambda}$ (2.25), since there $\mathbf{q}^2$ is small and therefore the form factor effect is moderate.

Let us then just quote the trend and magnitude of the new effects we have considered.

First, we observe that, by mere definition of the relativistic coupling constants, our treatment leads to a factor $(2M_B)^{1/2}$ in their quark-model expression [note that in the $\frac{3}{2}^+$ case, this is true with our own definition of the coupling, that includes a factor M_B, in analogy with M_π, to make the coupling dimensionless; otherwise, one gets a factor $(2M_B)^{-1/2}$]. This results systematically in larger couplings for heavy flavors, and rules out from the beginning any prescription of algebraic symmetry with usual hadron couplings.

Now, we have two main specific effects in our quark model: (i) the effect of unequal quark masses on the expression of the matrix elements in terms of internal wave functions, and on the internal functions themselves (modifications of the radii); (ii) the effect of the large transfers both at strong ($\mathbf{q}^2<0$) and weak ($\mathbf{k}^2>0$) vertices. These we label, respectively, as "unequal-quark-mass effects," and "form-factor effects." We emphasize them by comparing them to a "systematic" calculation, where the quark masses would be all equal to m, the light-quark mass, and where the transfer is negligible.

The numerical results that justify the following discussion are given in Table II. Let us first describe this table, and then comment on it. The comparison of columns (1) (strong form factors with linear extrapolation) and (2) (strong form factors with the full exponential) shows stronger departures for the Δ (a factor ~ 2) than for the N couplings (a factor close to 1) since the value of $|\mathbf{k}^2 R^2|$ is bigger for the former. The comparison of columns (3) and (1) shows the effect of the strong-coupling extrapolation from $\mathbf{k}^2=0$ to $\mathbf{k}^2<0$; this effect *increases* the N couplings by a factor ~ 2, and the Δ ones by a factor ~ 3. The comparison of columns (4) and (1) shows the effect of the weak form factor; its effect is to *decrease* the weak couplings: a factor ~ 0.5 for the $\mathcal{B}_b - N$ transitions, and a factor closer to 1 for the $\mathcal{B}_b - \mathcal{B}_c$ ones. The comparison of columns (5) and (1) shows the combined effects of the strong and weak form factors. Their overall effect is to *decrease* the $N\bar{N}$ rates by a factor ~ 0.7, the $N\bar{\Delta}$ ones by a smaller factor. On the contrary, the $\mathcal{B}_c\bar{N}$ rates *increase* by a factor ~ 3 and the $\mathcal{B}_c\bar{\Delta}$ increase by a factor ~ 7. This different behavior results from the fact that the weak $\mathcal{B}_b - N$ transitions have a larger weak form-factor suppression than the $\mathcal{B}_b - \mathcal{B}_c$ ones. The comparison of columns (6) and (5) allows us to appreciate the effect of the $m_c - m$ mass difference (neglecting the weak and strong form-factor extrapolations). The parity-violating $\mathcal{B}_c\bar{\Delta}$ waves go to zero in the limit $m_c \to m$, but the other results are rather stable: the rates $\mathcal{B}_c\bar{N}$ and $\mathcal{B}_c\bar{\Delta}$ decrease by a factor ~ 1.5 to 2 in this limit. Finally the comparison of columns (7) and (6) allows us to see the effect of the large m_b mass in the weak and strong couplings: all couplings remain about the same magnitude within a factor 2. For the ground state, the weak couplings increase relatively to the $m_b \to m$ limit, since the wave function at small distances is larger for a heavier quark; but this pattern is reversed for the excited states. On the contrary, the strong couplings decrease relatively to the $m_b \to m$ limit. Note that the $\Gamma^{\mathrm{PV}}/\Gamma^{\mathrm{PC}}$ ratio for the N transitions is stable in all the limits considered, except somewhat for the situations of columns (2) and (7).

Let us first discuss the noncharmed vertices. Unequal-quark-mass effects result in an important depression (factor ~ 2) of $\frac{1}{2}^+\frac{1}{2}^+$ strong vertices, and a weaker depression of $\frac{1}{2}^-\frac{1}{2}^+$ ones. As to the weak vertex, they enhance moderately the $\frac{1}{2}^+\frac{1}{2}^+$ ones, and they depress the $\frac{1}{2}^-\frac{1}{2}^+$ ones. The strong vertices $\frac{1}{2}^+\frac{3}{2}^+$ are markedly depressed. Form-factor effects are simpler: strong couplings are enhanced by a factor $\sim$2–3, and weak couplings are divided by a factor ~ 2.5. Combining the two effects, one sees finally that strong vertices $\frac{1}{2}^+\frac{1}{2}^+$ are roughly equal to their symmetric, while $\frac{1}{2}^-\frac{1}{2}^+$, $\frac{1}{2}^+\frac{3}{2}^+$, and $\frac{1}{2}^-\frac{3}{2}^+$ are somewhat enhanced. Weak vertices, on the other hand, are suppressed, through the combined effects, by a factor ~ 2 and 4, respectively, for $\frac{1}{2}^+$ (PC) and $\frac{1}{2}^-$ (PV). Charmed decays differ only by the weak vertex. At the weak vertex, the effects of unequal quark masses and form factors are much smaller than in the noncharmed case, and the values are close to the symmetric case.

For the noncharmed decays, the final result of the two effects on the branching ratios is roughly the same as the one of unequal quark masses, since the effect of form factors nearly cancel between the weak and the strong vertex. The parity-conserving wave in $\frac{1}{2}^+\frac{1}{2}^+$ and $\frac{1}{2}^+\frac{3}{2}^+$ which is pure PC is a bit suppressed (1/3 in width), while the $\frac{1}{2}^+\frac{1}{2}^+$ PV wave is more markedly suppressed (1/5 in width) with respect to the symmetric value. Since charmed decays are only slightly affected at the weak vertex, the effects in that case are controlled by the strong vertex. According to the discussion made above, the resulting effect is a small enhancement for $\frac{1}{2}^+\frac{1}{2}^+$ decays, and a strong enhancement for $\frac{1}{2}^+\frac{3}{2}^+$. On the whole, although each factor may seem appreciable, the general picture obtained is not too different from what is given by the "symmetric" values. Indeed, we must expect very large uncertainties of all origins in such a calculation, and factors of $\sim$3–5, for instance, must therefore not be considered as very significant.

The result of our predictions for the branching ratios can be appreciated only in comparison with parallel calculations, or other models, or experiment. This we do separately in the next paragraphs. Let us only emphasize two main typical features of our predictions. First, the ratio $\Gamma(B^- \to p\bar{\Delta}^{++})/\Gamma(\bar{B}^0 \to p\bar{p})$ is very large. This is due to large algebraic factors at the strong vertex, which are also present, for instance, in the ratio $g_{K\Delta\Sigma}/g_{KN\Sigma}$, and this is not affected by the above effects of unequal quark masses and form factors. One ends with a ratio ~ 50. A similar ratio is found for $\Gamma(B^- \to \Lambda_c^+\bar{\Delta}^{++})/\Gamma(\bar{B}^0 \to \Lambda_c^+\bar{p})$, since it depends on the same strong vertices. Second, the ratio $\Gamma^{\mathrm{PV}}/\Gamma^{\mathrm{PC}}$ in $p\bar{p}$ is close to one. This does not result from algebraic factors, because, in the nonrelativistic limit, the ratio would be suppressed, in width, by $(v/c)^4$. It reflects the fact that, in the quark model, whenever light quarks are present, there are large internal velocities. On the other hand, the ratio $\Gamma^{\mathrm{PV}}/\Gamma^{\mathrm{PC}}$ is zero for the Δ decays in the symmetric $m_c = m$ limit, and it remains small after introducing quark mass differences. This could be a striking feature to be tested, especially since it can be expected to survive the difficulties of the pole model for the Δ, which we have underlined.

Let us now compare our calculation with previous theoretical work. We will limit ourselves to the discussion of the pole model of Deshpande, Soni, and Trampetic and the QCD-sum-rule calculation of Chernyak and Zhitnitsky.[13,25] For completeness, let us quote other theoretical works. Li and Wu[27] have studied flavor-symmetry predictions for charmed baryon-antibaryon decays. Motivated by the old ARGUS data[28] on the modes $p\bar{p}\pi^+$ and $p\bar{p}\pi^+\pi^-$, Gronau and other Rosner develop a statistical model of multipion production in B decays.[29]

B. Comparison with Deshpande, Soni, and Trampetic

The work of Deshpande, Soni, and Trampetic[11] only aimed at giving an order-of-magnitude estimate of baryon-antibaryon decay rates in a crude approach to the pole model. The comparison with this work seems, however, necessary, because their results have been discussed in the report by Chernyak and Zhitnitsky at the Munich Conference,[25] and their work could be considered as representing the pole model. We must emphasize that while their final results are numerically close to ours, this is due to a numerical accident.

We note first that their phase space contains factors which we are not able to understand (of course, we disregard factors due to a different definition of the $\frac{3}{2}^+$ strong coupling constant). But it happens that such factors seem harmless in the limit $M_B \gg$ light hadron masses.

Of deeper physical significance is the difference in the calculation of the weak transitions. Their treatment of the unequal quark masses, borrowed from Ref. 30, does not take into account the full Pauli antisymmetry of the wave function. They symmetrize independently the space wave function, thus leading to spurious overlaps between permuted terms, which should disappear in a correct calculation because they should be associated with orthogonal spin-isospin wave functions (if the quark b is first chosen in the position 3, then by permutation it comes to position 2, and the associated flavor wave function is automatically orthogonal to the previous one). Another mistake we noticed is that, after having taken into account the mass of the b quark, they borrow the radius R_ρ (R in our notation), which should be the unbroken one, from a B-meson calculation,[31] which already takes into account the effect of the b-quark mass. Moreover, even with equal masses, there is no direct relation between the meson and baryon radii, since their definition is different, and since the effective potential is different. Instead, one should simply take R_ρ to be the same as in the nucleon case. Quite accidentally, the combination of all that results in something not too far from the harmonic-oscillator result in the symmetric case.

As to the strong couplings, Deshpande, Soni, and Trampetic write certain *equalities* between heavy quark and nonstrange coupling constants, which have simply no reason to hold [their equation (12)]; indeed, at least some SU(6) algebraic coefficients should be present, identical to those which relate the usual Σ couplings to the nonstrange couplings. In addition, our analysis finds several other factors which differenciate Σ_b couplings from usual Σ couplings; one is the kinematical factor $(2M_B)^{1/2}$, the others are the unequal-quark-masses and form-factor effects (see Sec. VI A). All these factors, some of which are large, but act in opposite directions, result quite accidentally in a rough equality of the couplings.

All in all, we see that our calculation is at odds with the one of Deshpande, Soni, and Trampetic, although there is a rough similarity of the final numerical results. The differences, which touch the analytical expression, may be illustrated by the different asymptotic powers of m_b found in the two calculations (see Sec. II D).

C. Comparison with the work of Chernyak and Zhitnitsky

The comparison with the work of CZ[13,25] has quite a different meaning. It is a comparison with a really different method not relying on the pole model, but on a direct evaluation of the process as a whole. Of course, no comparison is then possible for the separate strong and weak couplings constants, which are not calculated by CZ. We take it as an assumption, based on our confidence in the quark-model estimate of the latter, that the differences in the prediction of the branching ratios by the two approaches are to be attributed mainly to the difference of the two general methods of pole model and QCD sum rules, rather than to some incorrect treatment of the coupling constants.

The main observation is that, on the one hand, there is a very strong discordance for $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ decays, which we find much larger, while, on the other hand, the $\frac{1}{2}^+\overline{\frac{1}{2}}^+$ predictions grossly agree; yet we find them larger than CZ, the discrepancy in the latter case does not exceed the large uncertainties expected from both models. One notes the encouraging fact that for $\frac{1}{2}^+\overline{\frac{1}{2}}^+$ decays, the best agreement is for charmed decays, where the quark model is expected to be safer, and maybe QCD sum rules too. Another encouraging fact is that quite different methods lead to a rather similar Γ^{PV}/Γ^{PC} ratio, close to one, in the $\overline{B}^0 \to p\overline{p}$ mode. A careful comparison shows that the agreement extends to the sign of the amplitude ratio.

The direction of the discrepancies seems to parallel the one observed in Sec. III D for the asymptotic power of m_b: the $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ pole-model prediction exceeds the QCD prediction by four powers of m_b in width, while the difference is two powers only for $\frac{1}{2}^+\overline{\frac{1}{2}}^+$. We think that from the very general point of view of the quark model, which considers the N and Δ as entirely similar particles, this result is unsatisfactory, because it means that, in contrast with QCD asymptotics, N and Δ are behaving differently. We are tempted to put the blame on the pole model; at least with ususal couplings, it tends to overestimate amplitudes for high-spin particles. We are now also tempted to explain in the same way the very large numerical discrepancy with CZ in the $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ case. Although we know that the CZ contribution does not follow the QCD asymptotic behavior, we may suspect that they give a similar behavior in m_b to $\frac{1}{2}^+\overline{\frac{1}{2}}^+$ and $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ branching ratios, in contrast with the pole model.

D. Comparison with experiment

It must be stressed that the evaluation of the overall process in our model shall suffer from many large uncertainties, inherent to almost all stages of the calculation, which cannot be quantified *a priori*. Therefore, disagreement with experiment should not be interpreted too hastily, even with large discrepancies. Yet, the data seem to confirm our doubts about decays involving a Δ.

Although there is yet no direct measurement of any of the predicted processes, we can still learn something from two sources: (i) the inclusive branching ratio $B(B \to \Lambda_c^+ X) \sim 7\%$, well known;[32] (ii) the upper bounds on noncharmed decays $B \to p\overline{p}+$pions.[33]

The prediction for charmed decays with $\overline{\Delta}^{++}$ and a Λ_c^+ clearly exceeds the experimental inclusive value $B(B \to \Lambda_c^+ X) \sim 7\%$. Summing $\Lambda_c^+\overline{\Delta}^{++}$ and $\Sigma_c\overline{\Delta}^{++}$ leads to more than 50% (of course, these percentages are a way of expressing the partial computed rates, normalized by τ_b^{-1}). Since the inclusive rate must be a generous upper bound for $\overline{\Delta}^{++}$ decays, we find a very large discrepancy with experiment, exceeding the allowed uncertainties.

Let us recall that some years ago, the ARGUS Collaboration claimed to have seen[28] the modes $p\overline{p}\pi^+$ and $p\overline{p}\pi^+\pi^-$. Now the CLEO Collaboration[33] gives only an upper bounds for these processes:

$$B(p\bar{p}\pi^+) < 1.4\times 10^{-4}\ ,$$
$$B(p\bar{p}\pi^+\pi^-) < 2.9\times 10^{-4}\ . \tag{6.1}$$

These bounds do not favor our prediction for $B^+ \to \bar{p}\Delta^{++} \sim 3\times 10^{-4}$. It is true that V_{ub} is not yet safely known, and that, once more, there are large uncertainties within out hadronic model. Therefore, the difficulty is not conclusive by itself, but points to the same direction as previous observations: the predictions for decays involving Δ are anomalously large. As to the $\frac{1}{2}^+\overline{\frac{1}{2}}^+$ final states, there is nothing to say, except that they are compatible with the present bounds $B(B\to\Lambda_c^+X)\sim 7\%$ and $B(B\to p\bar{p})<0.4\times 10^{-4}$ (Ref. 34). We must wait for more precise data.

VII. CONCLUSION

Having carefully discussed the pole model as regards many of its aspects, ranging from general principles to detailed quark-model calculations of the relevant hadronic quantities, we have obtained a large set of predictions which converge with the alternative QCD-sum-rule approach[13,25] towards certain similar conclusions, including a very rough order of magnitude, for the $\frac{1}{2}^+\overline{\frac{1}{2}}^+$ final states. This is not true for the $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ branching ratios, which are much larger in our case. Since we have found a rationale concluding to the failure of the $\frac{1}{2}^+\overline{\frac{3}{2}}^+$ predictions, namely, that the pole model is badly behaved for high-spin particles, we think that this rough convergence is encouraging for both approaches which have each their own merit.

On the other hand, our calculation cannot be considered as a firm prediction, since it relies on many problematic assumptions: a certain type of coupling, selection of a few intermediate states, crossing of the strong vertex, extrapolation to complex or large momentum transfer. Neither is the QCD-sum-rule approach to be considered as more trustable, in view of the complexity of the treatment and of the many assumptions which it also includes. In fact, there is not a detailed critical discussion available in the CZ papers. Therefore, only experience will tell us whether the assumptions on both sides are correct, and which succeeds the best. It seems to us that if it is correct, the pole approach would have the advantage of a relative simplicity. It should also be suggested that even if the pole model would reveal quantitatively unsatisfactory, some results could be of more general validity: e.g., the ratio of PV to PC waves or the algebraic relations that correspond to the $\Delta I=\frac{1}{2}$ rule.

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APPENDIX A: WAVE FUNCTION OF HEAVY-FLAVOR HADRONS

1. Baryons

Let us consider the harmonic-oscillator Hamiltonian

$$H=\sum_i \frac{\mathbf{p}_i^2}{2m_i}+K\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2 \tag{A1}$$

and the relative coordinates

$$\mathbf{R}=\frac{\sum_i m_i\mathbf{r}_i}{\sum_i m_i},\quad \boldsymbol{\lambda}=\left[\frac{2}{3}\right]^{1/2}\left[\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_1+m_2}-\mathbf{r}_3\right],$$
$$\boldsymbol{\rho}=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) \tag{A2}$$

or

$$\mathbf{r}_{1,2}=\mathbf{R}+\left[\frac{3}{2}\right]^{1/2}\frac{m_3}{\sum_i m_i}\boldsymbol{\lambda}\pm\sqrt{2}\frac{m_2}{m_1+m_2}\boldsymbol{\rho}\ ,$$
$$\mathbf{r}_3=\mathbf{R}-\left[\frac{3}{2}\right]^{1/2}\frac{m_1+m_2}{\sum_i m_i}\boldsymbol{\lambda} \tag{A3}$$

and from $d\psi=\sum_i(\partial\psi/\partial r_{i\alpha})dr_{i\alpha}$ $(\alpha=x,y,z)$ we can read relative momenta:

$$\mathbf{p}_R=\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3\ ,$$
$$\mathbf{p}_\rho=\sqrt{2}\frac{m_2}{m_1+m_2}\mathbf{p}_1-\sqrt{2}\frac{m_1}{m_1+m_2}\mathbf{p}_2\ , \tag{A4}$$
$$\mathbf{p}_\lambda=\left[\frac{3}{2}\right]^{1/2}\frac{m_3}{\sum_i m_i}(\mathbf{p}_1+\mathbf{p}_2)-\left[\frac{3}{2}\right]^{1/2}\frac{m_1+m_2}{\sum_i m_i}\mathbf{p}_3\ ,$$

or

$$\mathbf{p}_{1;2}=\frac{m_{1,2}}{\sum_i m_i}\mathbf{p}_R+\left[\frac{2}{3}\right]^{1/2}\frac{m_{1,2}}{m_1+m_2}\mathbf{p}_\lambda\pm\frac{1}{\sqrt{2}}\mathbf{p}_\rho$$
$$\mathbf{p}_3=\frac{m_3}{\sum_i m_i}\mathbf{p}_R-\left[\frac{2}{3}\right]^{1/2}\mathbf{p}_\lambda\ . \tag{A5}$$

In terms of these variables we can write therefore

$$H=\frac{\mathbf{p}_R^2}{2\sum_i m_i}+\frac{1}{3}\left[\frac{1}{m_1+m_2}+\frac{1}{m_3}\right]\mathbf{p}_\lambda^2+\frac{1}{4}\left[\frac{1}{m_1}+\frac{1}{m_2}\right]\mathbf{p}_\rho^2$$
$$+K\left[3\lambda^2+2\left[1+\frac{m_1^2+m_2^2}{(m_1+m_2)^2}\right]\rho^2+2\sqrt{3}\left[\frac{m_2-m_1}{m_1+m_2}\right]\boldsymbol{\rho}\cdot\boldsymbol{\lambda}\right]\ . \tag{A6}$$

Note that, in the case $m_1 \neq m_2 \neq m_3$, the Hamiltonian is not diagonal in the base $\boldsymbol{\lambda},\boldsymbol{\rho}$. However, we will limit ourselves here to the case $m_1 = m_2 = m \neq m_3$. We obtain, in this case,

$$H = \frac{\mathbf{p}_R^2}{2M} + \frac{\mathbf{p}_\lambda^2}{2m_\lambda} + \frac{\mathbf{p}_\rho^2}{2m_\rho} + 3K(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2) \tag{A7}$$

with

$$M = 2m + m_3, \quad m_\lambda = \frac{3mm_3}{2m + m_3}, \quad m_\rho = m \ . \tag{A8}$$

We will consider transitions involving only the ground-state and $L=1$ baryons, whose wave functions will be given by

$$\begin{aligned}
\psi_0(\boldsymbol{\rho},\boldsymbol{\lambda}) &= N_0 \exp\left[-\left[\frac{\lambda^2}{2R_\lambda^2} + \frac{\rho^2}{2R_\rho^2}\right]\right] , \\
\psi_1'(\boldsymbol{\rho},\boldsymbol{\lambda}) &= N_1 \mathcal{Y}_1^m \left[\frac{\boldsymbol{\rho}}{R_\rho}\right] \exp\left[-\left[\frac{\lambda^2}{2R_\lambda^2} + \frac{\rho^2}{2R_\rho^2}\right]\right] , \\
\psi_1''(\boldsymbol{\rho},\boldsymbol{\lambda}) &= N_1 \mathcal{Y}_1^m \left[\frac{\boldsymbol{\lambda}}{R_\lambda}\right] \exp\left[-\left[\frac{\lambda^2}{2R_\lambda^2} + \frac{\rho^2}{2R_\rho^2}\right]\right] .
\end{aligned} \tag{A9}$$

The Schrödinger equation gives

$$R_\lambda^4 = \frac{2m + m_3}{3m_3} R^4, \quad R_\rho^4 = R^4 , \tag{A10}$$

where R is the radius in the equal-mass limit

$$6mK = R^{-4} . \tag{A11}$$

We normalize the wave functions according to

$$\int \prod_i d\mathbf{r}_i \delta\left[\tfrac{1}{3}\sum_i \mathbf{r}_i\right] |\psi(\{\mathbf{r}_i\})|^2 = 1 . \tag{A12}$$

Performing the change of variables, we find the Jacobian

$$\left|\frac{\partial(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)}{\partial(\mathbf{R},\lambda,\rho)}\right| = 3\sqrt{3} , \tag{A13}$$

i.e., the same value as for equal masses. We obtain the normalizations

$$N_0 = \frac{1}{(3\sqrt{3}R_\rho^3 R_\lambda^3 \pi^3)^{1/2}}, \quad N_1 = \left[\frac{8\pi}{3}\right]^{1/2} N_0 \tag{A14}$$

with R_λ, R_ρ given by (A10).

We will need also the internal wave functions in momentum space, $\tilde{\psi}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$, normalized according to

$$\int \prod_i d\mathbf{p}_i \delta\left[\sum_i \mathbf{p}_i\right] |\tilde{\psi}(\{\mathbf{p}_i\})|^2 = 1 . \tag{A15}$$

The expressions are

$$\begin{aligned}
\tilde{\psi}_0(\mathbf{p}_\lambda,\mathbf{p}_\rho) &= \tilde{N}_0 \exp\left[-\left[\frac{\mathbf{p}_\lambda^2 R_\lambda^2}{2} + \frac{\mathbf{p}_\rho^2 R_\rho^2}{2}\right]\right] , \\
\tilde{\psi}_1'(\mathbf{p}_\lambda,\mathbf{p}_\rho) &= \tilde{N}_0 \mathcal{Y}_1^m(\mathbf{p}_\rho R_\rho) \exp\left[-\left[\frac{\mathbf{p}_\lambda^2 R_\lambda^2}{2} + \frac{\mathbf{p}_\rho^2 R_\rho^2}{2}\right]\right] , \\
\tilde{\psi}_1'(\mathbf{p}_\lambda,\mathbf{p}_\rho) &= \tilde{N}_0 \mathcal{Y}_1^m(\mathbf{p}_\rho R_\rho) \exp\left[-\left[\frac{\mathbf{p}_\lambda^2 R_\lambda^2}{2} + \frac{\mathbf{p}_\rho^2 R_\rho^2}{2}\right]\right] ,
\end{aligned} \tag{A16}$$

with

$$\tilde{N}_0 = \left[\frac{3\sqrt{3}R_\rho^3 R_\lambda^3}{\pi^3}\right]^{1/2}, \quad \tilde{N}_1 = \left[\frac{8\pi}{3}\right]^{1/2} \tilde{N}_0 . \tag{A17}$$

Notice that all the dependence on the large mass m_3 is contained in R_λ.

To construct now the total wave functions for baryons with one heavy quark, containing space, spin, flavor, and color degrees of freedom, we have to take into account the Pauli principle. In the spatial wave functions written above we have singled out the quark labeled 3 as the heavy quark. Let us call these spatial wave functions $\psi_{0(123)}$, $\psi'_{1(123)}$, and $\psi''_{1(123)}$, where the ordering 123 singles out the last quark 3 as the heavy quark. Let us now construct the total wave functions. We will be interested in the lowest-lying heavy baryons, namely, those made out of u, d and a heavy quark Q that could be c or b. We are then restricted to consider Λ or Σ type baryons and their $L=1$ excitations.

To construct the wave functions for unequal masses it will be convenient first to write down the equal-mass limit wave function, in a particular way. Consider the octet and decuplet ground states in the well-known notation

$$\begin{aligned}
\Psi(\mathbf{56},S=\tfrac{1}{2},8) &= \psi_s \frac{1}{\sqrt{2}}(\phi'\chi' + \phi''\chi'') , \\
\Psi(\mathbf{56},S=\tfrac{3}{2},10) &= \psi_s \phi^s \chi^s ,
\end{aligned} \tag{A18}$$

where the ϕ and χ are the flavor and spin wave functions given in Chapter 1 of our book.[4]

Using the relations

$$\begin{aligned}
\chi''_{312} &= \frac{\sqrt{3}}{2}\chi'_{123} - \tfrac{1}{2}\chi''_{123} , \\
\chi''_{231} &= -\frac{\sqrt{3}}{2}\chi'_{123} - \tfrac{1}{2}\chi''_{123} , \\
\chi'_{312} &= -\frac{\sqrt{3}}{2}\chi''_{123} - \tfrac{1}{2}\chi'_{123} , \\
\chi'_{231} &= \frac{\sqrt{3}}{2}\chi''_{123} - \tfrac{1}{2}\chi'_{123}
\end{aligned} \tag{A19}$$

we can rewrite the flavor-spin parts of the wave functions (A18) in the form

$$\frac{1}{\sqrt{2}}(\phi'_\Sigma\chi'+\phi''_\Sigma\chi'')=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Sigma_{123}\chi''_{123}\ ,$$
$$\frac{1}{\sqrt{2}}(\phi'_\Lambda\chi'+\phi''_\Lambda\chi'')=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Lambda_{123}\chi'_{123}\ , \qquad (A20)$$
$$\phi^s_\Sigma\chi^s=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Sigma_{123}\chi^s\ ,$$

where the sum extends over circular permutations of (123) and ϕ^Σ_{123}, ϕ^Λ_{123} are the flavor wave functions symmetric or antisymmetric under the exchange $1\leftrightarrow2$:

$$\phi^\Sigma_{123}=uuQ,\frac{1}{\sqrt{2}}(du+ud)Q,ddQ\ ,$$
$$\phi^\Lambda_{123}=\frac{1}{\sqrt{2}}(du-ud)Q \qquad (A21)$$

that single out the quark 3 as the heavy quark.

From the forms (A20) and the spatial wave function $\psi_{0(123)}$ (A9) or (A16), that treats the quark 3 as heavy, we have the total wave functions that obey the Pauli principle:

$$\Psi(56,S=\tfrac{1}{2},8,\Sigma)=\frac{1}{\sqrt{3}}\sum_{P(123)}\psi_{0(123)}\phi^\Sigma_{123}\chi''_{123}\ ,$$
$$\Psi(56,S=\tfrac{3}{2},8,\Sigma)=\frac{1}{\sqrt{3}}\sum_{P(123)}\psi_{0(123)}\phi^\Sigma_{123}\chi^s_{123}\ . \qquad (A22)$$
$$\Psi(56,S=\tfrac{3}{2},8,\Sigma)=\frac{1}{\sqrt{3}}\sum_{P(123)}\psi_{0(123)}\phi^\Sigma_{123}\chi^s_{123}\ .$$

For χ^s we do not need to specify the ordering as it is totally symmetric. Note the difference in sign between the first two wave functions in (A20) and (A22). We will adopt this last more natural convention in all our calculations.

Let us now write down the wave functions for the $(\mathbf{70},L=1)$ states. Let us consider first the equal-mass wave functions and transform them in a convenient way to read the wave functions in the case of a heavy quark.

The total wave functions are

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},8)=\tfrac{1}{2}\{([\psi''_1\chi']^M_J+[\psi'_1\chi'']^M_J)\phi' +([\psi'_1\chi']^M_J-[\psi''_1\chi'']^M_J)\phi''\}\ ,$$
$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{3}{2},8)=\frac{1}{\sqrt{2}}([\psi'_1\chi^s]^M_J\phi'+[\psi''_1\chi^s]^M_J\phi'')\ , \qquad (A23)$$
$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},10)=\frac{1}{\sqrt{2}}([\psi'_1\chi']^M_J+[\psi''_1\chi'']^M_J)\phi^s\ ,$$
$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},1)=\frac{1}{\sqrt{2}}([\psi''_1\chi']^M_J-[\psi'_1\chi'']^M_J)\phi^a\ ,$$

where the mixed symmetric wave functions $\psi'_1,\psi''_1,\ldots$ are, respectively, antisymmetric and symmetric relatively to the exchange $1\leftrightarrow2$.

Using the same method as before we can write down the *equal-mass* wave functions in the form

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},8,\Sigma)=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Sigma_{123}\frac{1}{\sqrt{2}}[\psi'_{1(123)}\chi'_{123}-\psi''_{1(123)}\chi''_{123}]^M_J\ ,$$

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},8,\Lambda)=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Lambda_{123}\frac{1}{\sqrt{2}}[\psi''_{1(123)}\chi'_{123}+\psi'_{1(123)}\chi''_{123}]^M_J\ ,$$

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{3}{2},8,\Sigma)=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Sigma_{123}[\psi''_{1(123)}\chi^s_{123}]^M_J\ ,$$

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{3}{2},8,\Lambda)=-\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Lambda_{123}[\psi'_{1(123)}\chi^s_{123}]^M_J\ , \qquad (A24)$$

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},10,\Sigma)=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Sigma_{123}\frac{1}{\sqrt{2}}[\psi'_{1(123)}\chi'_{123}+\psi''_{1(123)}\chi''_{123}]^M_J\ ,$$

$$\Psi^M_J(\mathbf{70},L=1,S=\tfrac{1}{2},1,\Lambda)=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi^\Lambda_{123}\frac{1}{\sqrt{2}}[\psi''_{1(123)}\chi'_{123}-\psi'_{1(123)}\chi''_{123}]^M_J\ .$$

Of course, SU(3) is not a good symmetry when we are dealing with hadrons composites of u,d quarks and a heavy quark Q, and the states with the Σ (or Λ) quantum numbers will be substantially mixed. The eigenfunctions corresponding to the variables ρ and λ correspond to different eigenvalues. The correct wave functions in the unequal-mass case will then be

$$\begin{aligned}
\Psi_J^M(70,L=1,S=\tfrac{1}{2},\Sigma_a)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Sigma}[\psi'_{1(123)}\chi'_{123}]_J^M ,\\
\Psi_J^M(70,L=1,S=\tfrac{1}{2},\Sigma_b)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Sigma}[\psi''_{1(123)}\chi''_{123}]_J^M ,\\
\Psi_J^M(70,L=1,S=\tfrac{1}{2},\Lambda_a)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Lambda}[\psi''_{1(123)}\chi'_{123}]_J^M ,\\
\Psi_J^M(70,L=1,S=\tfrac{1}{2},\Lambda_b)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Lambda}[\psi'_{1(123)}\chi''_{123}]_J^M ,\\
\Psi_J^M(70,L=1,S=\tfrac{3}{2},\Sigma)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Sigma}[\psi''_{1(123)}\chi^s_{123}]_J^M ,\\
\Psi_J^M(70,L=1,S=\tfrac{3}{2},\Lambda)&=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Lambda}[\psi'_{1(123)}\chi^s_{123}]_J^M ,
\end{aligned} \tag{A25}$$

where a,b denote the two $S=\frac{1}{2}$ linearly independent states. As in the case of the ground state, we will adopt the conventions (A25) for the $L=1$ wave functions when dealing with a heavy quark.

2. Mesons

We will need in some calculations the harmonic-oscillator wave functions of mesons made out of quarks of unequal masses.

Let us consider the Hamiltonian

$$H=\sum_i \frac{\mathbf{p}_i^2}{2m_i}+C(\mathbf{r}_1-\mathbf{r}_2)^2 \tag{A26}$$

that, making the change of variables

$$\mathbf{R}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_1+m_2},\quad \rho=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) ,$$
$$\mathbf{p}_R=\mathbf{p}_1+\mathbf{p}_2,\quad \mathbf{p}_\rho=\sqrt{2}\frac{m_2\mathbf{p}_1-m_1\mathbf{p}_2}{m_1+m_2} , \tag{A27}$$

gives the expression

$$H=\frac{\mathbf{p}_R^2}{2(m_1+m_2)}+\frac{\mathbf{p}_\rho^2}{4m_1m_2/(m_1+m_2)}+2C\rho^2 \tag{A28}$$

Let us compare with the equal-mass limit. Assuming flavor independence of the potential we have, for $m_1=m_2=m$, the ground-state wave function

$$\tilde{\psi}_0(\mathbf{p}_\rho)=\left[\frac{R'^2}{\pi}\right]^{3/4}\exp\left[-\frac{\mathbf{p}_\rho^2R'^2}{4}\right] ,$$
$$\psi_0(\rho)=\left[\frac{1}{\pi R'^2}\right]^{3/4}\exp\left[-\frac{\rho^2}{R'^2}\right] \tag{A29}$$

with

$$4mC=R'^{-4} . \tag{A30}$$

For $m_1\neq m_2$ we will have exactly the same form, but with $R'^2\to\tilde{R}^2$, with

$$\frac{8m_1m_2}{m_1+m_2}C=\tilde{R}^{-4} . \tag{A31}$$

These equations will allow us to compare the equal mass radius R' to the case of unequal masses $\tilde{R}$. We will use the flavor wave functions for B mesons:

$$\phi(\bar{B}_d^0)=b\bar{d},\quad \phi(B_u^-)=-b\bar{u},\quad \phi(\bar{B}_s^0)=b\bar{s} . \tag{A32}$$

APPENDIX B: WEAK MATRIX ELEMENTS

Let us now consider the matrix elements of H_w between baryons, of the type $\langle N|H_w^{\rm PC}|\Lambda_b\rangle$, $\langle N|H_w^{\rm PV}|\Lambda_b^*\rangle$, $\langle\Lambda_c|H_w^{\rm PC}|\Lambda_b\rangle$, $\langle\Lambda_c|H_w^{\rm PV}|\Lambda_b^*\rangle$, etc., where Λ_b,Λ_b^* are, respectively, $\frac{1}{2}^+$ or $\frac{1}{2}^-$ states.

To compute these matrix elements we will have to consider the wave functions for baryons containing a different heavy quark in the initial and final state. To have the general case, let us consider the Hamiltonian

$$H_w=\frac{G}{\sqrt{2}}V_{cb}[\bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha]\times[\bar{d}_\beta\gamma_\mu(1-\gamma_5)u_\beta]+\text{H.c.} , \tag{B1}$$

where α,β are color indices and for $b\to u$ transitions we must replace c, $V_{cb}\to u, V_{ub}$. We will neglect for the moment all other operators that appear when taking into account QCD short-distance radiative corrections.

In the lowest order in a v/c expansion we have, in first-quantization formalism and in momentum space (the color overlap gives just 1),

$$H_w^{\rm PC}=\frac{G}{\sqrt{2}}V_{cb}\sum_{i\neq j}\tau_i^{(-)}v_j^{(+)}(1-\sigma_i\cdot\sigma_j)+\text{H.c.} , \tag{B2}$$

$$H_w^{\rm PV}=\frac{G}{\sqrt{2}}V_{cb}\sum_{i\neq j}\tau_i^{(-)}v_j^{(+)}\left\{-(\sigma_i-\sigma_j)\left[\left[\frac{\mathbf{p}_i+\mathbf{p}_i'}{2m}\right]-\frac{\mathbf{p}_j}{2m_b}-\frac{\mathbf{p}_j'}{2m_c}\right]\right.$$
$$\left.+i\left[\left[\frac{\mathbf{p}_i-\mathbf{p}_i'}{2m}\right]-\left[\frac{\mathbf{p}_j}{2m_b}-\frac{\mathbf{p}_j'}{2m_c}\right]\right](\sigma_i\times\sigma_j)\right\}+\text{H.c.}\ , \tag{B3}$$

where $\tau^{(-)}u=d, v^{(+)}b=c$ (or $v^{(+)}b=u$ for $b\to u$ transitions).

To compute the matrix elements we will follow now the same steps as in our paper on nonleptonic hyperon decays.[8] We will first compute the $\Lambda_b,\Sigma_b\to N$ transitions, as they are simpler, and later the $\Lambda_b,\Sigma_b\to\Lambda_c,\Sigma_c$ ones.

1. $\Lambda_b,\Sigma_b\to N$ transitions

Since, up to the V_{ub} Kobayashi-Maskawa matrix element and mass differences, these transitions are identical to Λ_s, $\Sigma_s\to N$, we will obtain, in the equal-mass limit, the same ratios than for strange baryons.[8]

a. Parity conserving matrix elements

We follow the same steps as in Ref. 8. From the total symmetry of the wave functions (A18), (A22), (A25) we obtain, calling $\mathcal{B}_b=\Lambda_b,\Sigma_b$,

$$(2\pi)^3\langle N|H_w^{\rm PC}|\mathcal{B}_b\rangle=6\frac{G}{\sqrt{2}}V_{ub}\langle N|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tau_1^{(-)}v_2^{(+)}|\mathcal{B}_b\rangle\ , \tag{B4}$$

where the overlap is taken in spin, flavor, and space. Moreover, since $\tau_1^{(-)}\mathbf{v}_2^{(+)}\phi_{123}(\mathcal{B}_b)=\tau_1^{(-)}\mathbf{v}_2^{(+)}\phi_{231}(\mathcal{B}_b)=0$, as in $\phi_{ijk}(\mathcal{B}_b)$ the heavy quark is in position k:

$$(2\pi)^3\langle N|H_w^{\rm PC}|\mathcal{B}_b\rangle=6\frac{G}{\sqrt{2}}V_{ub}\left[-\frac{1}{\sqrt{6}}\right]\langle(\phi_{123}'\chi_{123}'+\phi_{123}''\chi_{123}'')\tilde{\psi}_0|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tau_1^{(-)}v_2^{(+)}|\tilde{\psi}_{0(312)}^{(b)}\phi_{312}^{\mathcal{B}_b}\chi_{312}\rangle\ , \tag{B5}$$

where $\tilde{\psi}_{0(312)}^{(b)}$ denotes the ground-state spatial wave function when 2 is the heavy quark and $\tilde{\psi}_0$ the spatial wave function with all quarks degenerate. We have $\chi_{312}=\chi_{312}'$ for a Λ_b, and $\chi_{312}=\chi_{312}''$ or χ_{312}^s for Σ_b. Since $(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)|\chi_{123}''\rangle=0,(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)|\chi_{123}'\rangle=4|\chi_{123}'\rangle$ we are only left with one term:

$$(2\pi)^3\langle N|H_w^{\rm PC}|\mathcal{B}_b\rangle=6\frac{G}{\sqrt{2}}V_{ub}\left[-\frac{4}{\sqrt{6}}\right]\langle\chi_{123}'|\chi_{312}\rangle\langle\phi_{123}'|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle I_b\ , \tag{B6}$$

where I_b is the space matrix element

$$I_b=\frac{1}{(2\pi)^3}\int\tilde{\psi}_0^+(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')\tilde{\psi}_{0(312)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\delta(\mathbf{p}_3-\mathbf{p}_3')\prod_i d\mathbf{p}_i d\mathbf{p}_i'\delta(\sum_j\mathbf{p}_j)\delta(\sum_j\mathbf{p}_j')$$
$$=\langle\psi_0|\delta(\mathbf{r}_1-\mathbf{r}_2)|\psi_{b(123)}\rangle\ , \tag{B7}$$

where the independent integration on $\mathbf{p}_1,\mathbf{p}_2$ and on $\mathbf{p}_1',\mathbf{p}_2'$ corresponds to the flatness in momentum space [$\delta(\mathbf{r}_1-\mathbf{r}_2)$ in configuration space] of the interaction. We formulate the calculation in $\mathbf{p}$ space as it will be then easier to compute the matrix elements of $H_w^{\rm PV}$ (B3). The spin matrix elements are, from (A19),

$$\langle\chi_{123}'|\chi_{312}'\rangle=-\tfrac{1}{2},\quad\langle\chi_{123}'|\chi_{312}''\rangle=\frac{\sqrt{3}}{2}\ ,$$
$$\langle\chi_{123}'|\chi_{312}^s\rangle=0\ . \tag{B8}$$

The flavor matrix elements will be, from (B21),

$$\sqrt{2}\langle\phi_{123}'^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Lambda_b}\rangle=\sqrt{2}\langle\phi_{123}'^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^0}\rangle$$
$$=\langle\phi_{123}'^{p}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^+}\rangle$$
$$\equiv-\frac{1}{\sqrt{2}}\ . \tag{B9}$$

To compute the spatial integral we must take into account the ordering $\tilde{\psi}_{0(312)}$, that differs from the one in (A16). A straightforward calculation gives

$$I_b=\frac{1}{(2\pi)^{3/2}R^3}\left[\frac{8\alpha_b}{1+7\alpha_b^2}\right]^{3/2},$$
$$\alpha_b\equiv\left[\frac{2m+m_b}{3m_b}\right]^{1/4}. \tag{B10}$$

We recognize the matrix element in the equal-mass limit

$$I_0=\langle\psi_0|\delta(\mathbf{r}_1-\mathbf{r}_2)|\psi_{b(312)}\rangle=\frac{1}{(2\pi)^{3/2}R^3} \tag{B11}$$

and the factor

$$c_b=\left[\frac{8\alpha_b}{1+7\alpha_b^2}\right]^{3/2} \tag{B12}$$

contains all the dependence on the heavy-quark mass, $c_b\to 1$ as $m_b\to m$.

We obtain finally the matrix elements

$$(2\pi)^3\langle N|H_w^{\rm PC}|\mathcal{B}_b\rangle=6\frac{G}{\sqrt{2}}V_{ub}Fc_bI_0\ , \tag{B13}$$

where F changes for the different transitions (note the different sign for $\Sigma^+\to p$ than in Ref. 8 due to the opposite phase convention[4]):

$$F(\Sigma_b^+\to p)=\sqrt{2}F(\Sigma_b^0\to n)=-\sqrt{6}F(\Lambda_b\to n)=6\ . \tag{B14}$$

I_0 is the spatial matrix element in the equal-mass limit, and all the dependence in the unequal masses is given by the m_b-dependent factor in (B12).

It is instructive to point out that, for instance, the ratio $\Lambda_b\to n$ to $\Lambda\to n$ will be given by

$$\frac{\langle n|H_w^{\rm PC}|\Lambda_b\rangle}{\langle n|H_w^{\rm PC}|\Lambda\rangle}=\frac{V_{ub}}{\sin\theta_C}c_b \tag{B15}$$

that contains the KM mixing parameters and a mass-dependent factor, computable in the harmonic-oscillator case. In this way we can relate the matrix elements $\mathcal{B}_b\to N$ to the measured ones in nonleptonic strange hyperon decays.

b. Parity-violating matrix elements

Let us now compute the matrix elements of $H_w^{\rm PV}$ between $\frac{1}{2}^-$ and $\frac{1}{2}^+$ states. Calling any wave function of the type (A25),

$$\Psi_{1/2}^M(70,L=1)=\frac{1}{\sqrt{3}}\sum_{P(123)}\phi_{123}^{\Sigma}[\psi_{1(123)}\chi_{123}]_{1/2}^M\ . \tag{B16}$$

where ϕ can be ϕ^Σ or ϕ^Λ, ψ can be ψ_1', or ψ_1'' and χ can be χ', χ'', or χ^s.

As we will see in Appendix C, the only nonvanishing strong couplings concern the states of the ψ_1'' type; the states of the ψ_1' type give zero strong couplings in the spectator quark hypothesis of the QPC $^3P_0$ model of strong decays. Therefore, although the weak couplings involving excitations of ψ_1' and ψ_1'' types are both nonvanishing, we only give the results for the latter.

From the total symmetry of the wave functions we obtain, calling $\mathcal{B}_b^*$ a $\frac{1}{2}^-$ b baryon:

$$(2\pi)^3\langle N|H_w^{\rm PV}|\mathcal{B}_b^*\rangle=6\frac{G}{\sqrt{2}}V_{ub}\frac{1}{\sqrt{6}}\langle(\phi_{123}'\chi_{123}'+\phi_{123}''\chi_{123}'')\psi_0|O_{\rm SS}(1,2)\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}[\psi_{1(312)}\chi_{312}]_{1/2}^M\rangle\ , \tag{B17}$$

where $O_{\rm SS}$ is the spin-space part of (B3) for $m_c=m$.

Now, $O_{\rm SS}$ is antisymmetric in spin and antisymmetric in space, in the indices (1,2). On the other hand, χ can be χ', χ'', or χ^s. Using (A19) we obtain

$$\begin{aligned}
&\langle(\phi_{123}'\chi_{123}'+\phi_{123}''\chi_{123}'')\psi_0|O_{\rm SS}(1,2)\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}[\psi_{1(312)}\chi_{312}']_{1/2}^M\rangle\\
&\qquad=-\frac{\sqrt{3}}{2}\langle\phi_{123}'|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\chi_{123}'\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}'']_{1/2}^M\rangle\\
&\qquad\quad-\tfrac{1}{2}\langle\phi_{123}''|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\chi_{123}''\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}']_{1/2}^M\rangle\ ,\\
&\langle(\phi_{123}'\chi_{123}'+\phi_{123}''\chi_{123}'')\psi_0|O_{\rm SS}(1,2)\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}[\psi_{1(312)}\chi_{312}'']_{1/2}^M\rangle\\
&\qquad=\frac{\sqrt{3}}{2}\langle\phi_{123}''|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\chi_{123}''\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}']_{1/2}^M\rangle\\
&\qquad\quad-\tfrac{1}{2}\langle\phi_{123}'|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\chi_{123}'\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}'']_{1/2}^M\rangle,\\
&\langle(\phi_{123}'\chi_{123}'+\phi_{123}''\chi_{123}'')\psi_0|O_{\rm SS}(1,2)\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}[\psi_{1(312)}\chi_{312}^s]_{1/2}^M\rangle\\
&\qquad=\langle\phi_{123}'|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\chi_{123}'\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}^s]_{1/2}^M\rangle\ .
\end{aligned} \tag{B18}$$

We are left therefore with the spin-space matrix elements

$$\langle\chi_{123}^f\psi_0|O_{\rm SS}(1,2)|[\psi_{1(312)}\chi_{123}^i]_{1/2}^M\rangle\ , \tag{B19}$$

where $\chi^f=\chi'$ or χ'', $\chi^i=\chi'',\chi^s$ or χ', and $\psi_1=\psi_1'$ or ψ_1''. It will be convenient now to consider the spatial matrix elements.

We have two spatial combinations in (B3) (for $m_c=m$):

$$\mathcal{Y}_1^m\left[\left[\frac{\mathbf{p}_1\pm\mathbf{p}_1'}{2m}\right]-\left[\frac{\mathbf{p}_2}{2m_b}\pm\frac{\mathbf{p}_2'}{2m}\right]\right]\ . \tag{B20}$$

The spatial integral to consider, analogous to (B7) will be now

$$J_b=\frac{1}{(2\pi)^3}\left[\frac{4\pi}{3}\right]^{1/2}\int \tilde{\psi}_0^\dagger(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')\mathcal{Y}_1^{m\,*}\left[\left[\frac{\mathbf{p}_1\pm\mathbf{p}_1'}{2m}\right]-\left[\frac{\mathbf{p}_2}{2m_b}\pm\frac{\mathbf{p}_2'}{2m}\right]\right]\tilde{\psi}^m_{1(312)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$$
$$\times\delta(\mathbf{p}_3-\mathbf{p}_3')\prod_i d\mathbf{p}_i d\mathbf{p}_i'\delta\left[\sum_j \mathbf{p}_j\right]\delta\left[\sum_j \mathbf{p}_j'\right], \quad \text{(B21)}$$

where $\psi_1=\psi_1'$ or ψ_1''. We obtain, for the case of interest, ψ_1'', independently of the $\pm$ sign in (B20),

$$J_b''=I_0\left[-\frac{\sqrt{3}}{2mR}\right]\cdot c_b'' ,$$
$$c_b''=\left[\frac{8\alpha_b}{1+7\alpha_b^2}\right]^{5/2}\frac{1}{12}\left[5+7\frac{m}{m_b}\right], \quad \text{(B22)}$$

where I_0 is the equal-mass matrix element (B11) and the factor c_b'' contains all the dependence on the heavy mass m_b and is equal to 1 in the limit $m_b\to m$.

Taking into account that, in the equal-mass limit, we have

$$\psi_{312}''=-\tfrac{1}{2}\psi_{123}''+\frac{\sqrt{3}}{2}\psi_{123}' \quad \text{(B23)}$$

and we can check that we obtain the same result as in Ref. 8, since the component ψ_{123}'' would not contribute due to the antisymmetry of the spatial operator under the exchange $1\leftrightarrow 2$. We obtain, as expected, in the equal-mass limit,

$$J_b''=\frac{\sqrt{3}}{2mR}I_0=\frac{\sqrt{6}}{2mR}J^{(\text{Ref. }8)} \quad \text{(B24)}$$

consistent with what we found, a factor $\sqrt{3}/2$ coming from (B23), and a factor $\sqrt{2}$ from a different definition of the spatial integrals [(B21) here and (29) in Ref. 8].

We have computed the spatial matrix elements and we are now left with the spin-space matrix elements (B19) where $O_{SS}(i,j)$ is given by

$$O_{SS}(i,j)=\left\{-(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\left[\left[\frac{\mathbf{p}_i+\mathbf{p}_i'}{2m}\right]-\frac{\mathbf{p}_j}{2m_b}-\frac{\mathbf{p}_j'}{2m}\right]+i\left[\left[\frac{\mathbf{p}_i-\mathbf{p}_i'}{2m}\right]-\left[\frac{\mathbf{p}_j}{2m_b}-\frac{\mathbf{p}_j'}{2m}\right]\right]\cdot(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\right\}. \quad \text{(B25)}$$

Because of the total symmetry of the nucleon spatial wave function $\tilde{\psi}_0(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')$, the term $(\mathbf{p}_1'-\mathbf{p}_2')/2m$ that appears in both terms of (B25) will not contribute and we are left with an effective operator for each matrix element (B19):

$$\langle \chi_{123}^M\psi_0|\boldsymbol{\Sigma}\cdot\mathbf{O}|[\psi_{1(312)}\chi_{123}]^M_{1/2}\rangle , \quad \text{(B26)}$$

where $\boldsymbol{\Sigma}$ is a spin operator

$$\boldsymbol{\Sigma}=-(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)+i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \quad \text{(B27)}$$

and $\mathbf{O}$ is an orbital part,

$$\mathbf{O}=\frac{\mathbf{p}_1}{2m}-\frac{\mathbf{p}_2}{2m_b}, \quad \text{(B28)}$$

that enters in the estimation of the integral J (B21). In (B26) χ_{123} on the left can be χ_{123}' or χ_{123}'' and on the right χ_{123}'', χ_{123}^s, or χ_{123}'.

Applying the Wigner-Eckart theorem, we obtain, after some angular momentum algebra,

$$\langle \chi_{123}^M\psi_0|\boldsymbol{\Sigma}\cdot\mathbf{O}|[\psi_{1(312)}\chi_{123}^{(S)}]^M_{1/2}\rangle$$
$$=\frac{(-1)^{S+1/2}}{\sqrt{2}}\langle\chi_{123}\|\Sigma\|\chi_{123}^{(S)}\rangle\langle\psi_0|O^{(m)*}|\psi_{1(312)}^{(m)}\rangle \quad \text{(B29)}$$

where S denotes the total quark spin: $\frac{1}{2}$ for χ',χ'', $\frac{3}{2}$ for χ^s. The spatial matrix element is just the integral J_b (B21) computed above,

$$\langle\psi_0|O^{(m)*}|\psi_{1(312)}^{(m)}\rangle\equiv J_b \quad \text{(B30)}$$

and the spin reduced matrix elements are given by

$$\langle\chi_{123}''\|\Sigma\|\chi_{123}'\rangle=0 ,$$
$$\langle\chi_{123}'\|\Sigma\|\chi^S\rangle=\sqrt{2}\langle\chi_{123}'\|\Sigma\|\chi_{123}''\rangle=8 \quad \text{(B31)}$$

and we obtain finally, for the matrix elements (B19),

$$\langle\chi_{123}'\psi_0|O_{SS}(1,2)|[\psi_{1(312)}\chi_{123}'']^M_{1/2}\rangle=-4J_b ,$$
$$\langle\chi_{123}''\psi_0|O_{SS}(1,2)|[\psi_{1(312)}\chi_{123}']^M_{1/2}\rangle=0 , \quad \text{(B32)}$$
$$\langle\chi_{123}'\psi_0|O_{SS}(1,2)|[\psi_{1(312)}\chi_{123}^s]^M_{1/2}\rangle=4\sqrt{2}J_b ,$$

where J_b is J_b'' (B22) if ψ_1 is ψ_1''. Owing to the different phase convention for χ',χ'' here[4] than in Ref. 8, we find the opposite sign for the last matrix element in (B22).

We have to compute a few flavor matrix elements from Eqs. (B18) [using the wave functions of Ref. 4 and (A21)]:

$$\begin{aligned}
\langle \phi_{123}'^{p}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^+}\rangle &= -\frac{1}{\sqrt{2}} \ ,\\
\langle \phi_{123}''^{p}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^+}\rangle &= -\frac{1}{\sqrt{6}} \ ,\\
\langle \phi_{123}'^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^0}\rangle &= -\frac{1}{2} \ ,\\
\langle \phi_{123}''^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^0}\rangle &= \frac{1}{2\sqrt{3}} \ ,\\
\langle \phi_{123}'^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Lambda_b}\rangle &= -\frac{1}{2} \ ,\\
\langle \phi_{123}''^{n}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Lambda_b}\rangle &= \frac{1}{2\sqrt{3}} \ .
\end{aligned} \tag{B33}$$

Grouping all the terms in (B18) together with (B32) and (B33) we obtain, for the different states (A25) with ψ_1'':

$$(2\pi)^3\langle N|H_w^{\rm PV}|\mathcal{B}_b^*(\psi_1''\chi)\rangle = 6\frac{G}{\sqrt{2}}V_{ub}Fc_b''\frac{I_0}{mR} \ , \tag{B34}$$

where F changes according to the state. The spin-wave function χ can be χ', χ'', χ^s, and $\mathcal{B}_b^*$ can be Σ_b^+, Σ_b^0, Λ_b. Since ψ_1'' is already made explicit we have to specify χ and the flavor state. Since we have ψ_1'', then χ must be χ'' or χ^s for Σ_b, and χ' for Λ_b. We obtain

$$\begin{aligned}
F(\Sigma_b^+(\psi_1''\chi'')\to p) &= -3 \ ,\\
F(\Sigma_b^0(\psi_1''\chi'')\to n) &= -\frac{3}{\sqrt{2}} \ ,\\
F(\Sigma_b^+(\psi_1''\chi^s)\to p) &= -6\sqrt{2} \ ,\\
F(\Sigma_b^0(\psi_1''\chi^s)\to n) &= -6 \ ,\\
F(\Lambda_b(\psi_1''\chi')\to n) &= -\frac{3\sqrt{3}}{\sqrt{2}} \ .
\end{aligned} \tag{B35}$$

It is instructive to compute the ratios

$$\frac{\langle n|H_w^{\rm PV}|\Lambda_b(\psi''\chi')\rangle}{\langle n|H_w^{\rm PC}|\Lambda_b\rangle} = 3\frac{c_b''}{c_b}\left[\frac{1}{mR}\right] , \tag{B36}$$

$$\frac{\langle n|H_w^{\rm PV}|\Lambda_b(\psi''\chi')\rangle}{\langle n|H_w^{\rm PC}|\Lambda\rangle} = 3\frac{V_{ub}}{\sin\theta_C}c_b''\left[\frac{1}{mR}\right] . \tag{B37}$$

We can thus know these matrix elements in terms of the measured ones in strange hyperon decays and factors dependent on the masses, calculable in the harmonic-oscillator model.

2. $\Sigma_b,\Lambda_b\to\Sigma_c,\Lambda_c$ transitions

a. Parity-conserving matrix elements

From (B2), we follow the same steps as in Appendix B 1 a:

$$(2\pi)^3\langle \mathcal{B}_c|H_w^{\rm PC}|\mathcal{B}_b\rangle = 6\frac{G}{\sqrt{2}}V_{cb}\langle \mathcal{B}_c|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tau_1^{(-)}v_2^{(+)}|\mathcal{B}_b\rangle \ , \tag{B38}$$

where $\mathcal{B}_b=\Sigma_b,\Lambda_b$ and $\mathcal{B}_c=\Sigma_c,\Lambda_c$, and $v_2^{(+)}b=c$.

From the wave functions (A22) we write

$$\langle \mathcal{B}_c|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tau_1^{(-)}v_2^{(+)}|\mathcal{B}_b\rangle = \tfrac{1}{3}\langle \phi_{312}^{\mathcal{B}_c}\chi_{312}|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\chi_{312}\rangle I_{cb} \ , \tag{B39}$$

where χ can be χ' or χ'', χ^s according to Λ_b or Σ_b. Since $(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)|\chi^s\rangle=0$, the $\Sigma\ \frac{3}{2}^+$ has zero matrix elements. I_{bc} in (B39) is the spatial integral

$$I_{cb} = \frac{1}{(2\pi)^3}\int \tilde{\psi}_{0(312)}^{(c)\dagger}(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')\tilde{\psi}_{0(312)}^{(b)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\delta(\mathbf{p}_3-\mathbf{p}_3')\prod_i d\mathbf{p}_i d\mathbf{p}_i'\delta\left[\sum_j \mathbf{p}_j\right]\delta\left[\sum_j \mathbf{p}_j'\right] . \tag{B40}$$

From (A16) we obtain

$$\begin{aligned}
I_{cb} &= I_0 c_{cb} \ ,\\
c_{cb} &= \left[\frac{8\alpha_b\alpha_c}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^{3/2}
\end{aligned} \tag{B41}$$

with I_0 given by (B11) and α in (B10). $c_{bc}\to c_b$ when $m_c\to m$, and $c_{bc}=1$ when $m_b=m_c=m$.

We need also the spin matrix elements, from (A19),

$$\begin{aligned}
&\langle \chi_{312}''|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)|\chi_{312}''\rangle\\
&\quad = -\sqrt{3}\langle \chi_{312}'|(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)|\chi_{312}''\rangle = 3
\end{aligned} \tag{B42}$$

and the flavor matrix elements

$$\begin{aligned}
\langle \phi_{312}^{\Sigma_c^+}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^+}\rangle &= \langle \phi_{312}^{\Sigma_c^0}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^0}\rangle\\
&= \langle \phi_{312}^{\Sigma_c^0}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Lambda_b}\rangle\\
&= -\langle \phi_{312}^{\Lambda_c^+}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\Sigma_b^+}\rangle = \frac{1}{\sqrt{2}} \ .
\end{aligned} \tag{B43}$$

From these expressions we find finally

$$(2\pi)^3\langle \mathcal{B}_c|H_w^{\rm PC}|\mathcal{B}_b\rangle = 6\frac{G}{\sqrt{2}}V_{cb}Fc_{cb}I_0 \tag{B44}$$

with F for each particular transition:

$$F(\Sigma_b^+\to\Sigma_c^+)=F(\Sigma_b^0\to\Sigma_c^0)=\sqrt{3}F(\Sigma_b^+\to\Lambda_c^+)=-\sqrt{3}F(\Lambda_b\to\Sigma_c^0)=3\sqrt{2}\ . \tag{B45}$$

We have the interesting ratio

$$\frac{\langle\Lambda_c^+|H_w^{\rm PC}|\Sigma_b^+\rangle}{\langle n|H_w^{\rm PC}|\Lambda\rangle}=-\frac{V_{cb}}{\sin\theta_C}c_{cb} \tag{B46}$$

with c_{cb} containing all dependence on the heavy quark masses, (B41).

b. Parity-violating matrix elements

From (B2) and the wave functions (A22) and (A25):

$$(2\pi)^3\langle\mathcal{B}_c|H_w^{\rm PV}|\mathcal{B}_b^*\rangle=6\frac{G}{\sqrt{2}}V_{cb}\tfrac{1}{3}\langle\phi_{312}^{\mathcal{B}_c}|\tau_1^{(-)}v_2^{(+)}|\phi_{312}^{\mathcal{B}_b}\rangle\langle\psi_{0(312)}^{(c)}\chi_{312}^M|\mathbf{\Sigma}_+\cdot\mathbf{O}_b+\mathbf{\Sigma}_-\cdot\mathbf{O}_c|[\psi_{1(312)}^{(b)}\chi_{312}]_{1/2}^M\rangle\ , \tag{B47}$$

where

$$\mathbf{\Sigma}_\pm=-(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\pm i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \tag{B48}$$

and

$$\mathbf{O}_b=\frac{\mathbf{p}_1}{2m}-\frac{\mathbf{p}_2}{2m_b},\quad \mathbf{O}_c=\frac{\mathbf{p}_1'}{2m}-\frac{\mathbf{p}_2'}{2m_c}\ . \tag{B49}$$

$\mathbf{\Sigma}_+$ is just the operator $\mathbf{\Sigma}$ (B27) and $\mathbf{O}_b$ is (B28), but now we have to consider the matrix elements of $\mathbf{O}_c$ and $\mathbf{\Sigma}_-$ since we have unequal masses in this final-state charmed baryon.

We apply, as in (B29), the Wigner-Eckart theorem to the spin-space matrix element in (B47):

$$\langle\psi_{0(312)}^{(c)}\chi_{312}^M|\mathbf{\Sigma}_+\cdot\mathbf{O}_b+\mathbf{\Sigma}_-\cdot\mathbf{O}_c|[\psi_{1(312)}^{(b)}\chi_{312}]_{1/2}^M\rangle=\frac{(-1)^{S+1/2}}{\sqrt{2}}[\langle\chi_{312}\|\mathbf{\Sigma}_+\|\chi_{312}^{(S)}\rangle\langle\psi_{0(312)}^{(c)}|O_b^{(m)*}|\psi_{1(312)}^{(b)m}\rangle+\langle\chi_{312}\|\mathbf{\Sigma}_-\|\chi_{312}^{(S)}\rangle\langle\psi_{0(312)}^{(c)}|O_c^{(m)*}|\psi_{1(312)}^{(b)m}\rangle] \tag{B50}$$

where S denotes the total quark spin in the initial baryon. The spatial matrix elements are given by

$$\begin{Bmatrix}J_{cb}\\K_{cb}\end{Bmatrix}=\left\langle\psi_{0(312)}^{(c)}\left|\begin{Bmatrix}O_b^{(m)*}\\O_c^{(m)*}\end{Bmatrix}\right|\psi_{1(312)}^{(b)m}\right\rangle=\frac{1}{(2\pi)^3}\int\prod_i d\mathbf{p}_i d\mathbf{p}_i'\delta\left[\sum_j\mathbf{p}_j\right]\delta\left[\sum_j\mathbf{p}_j'\right]\delta(\mathbf{p}_3-\mathbf{p}_3')\tilde{\psi}_{0(312)}^{(c)\dagger}(\mathbf{p}_3',\mathbf{p}_1',\mathbf{p}_2')\begin{Bmatrix}O_b^{(m)*}(\mathbf{p}_1,\mathbf{p}_2)\\O_c^{(m)*}(\mathbf{p}_1',\mathbf{p}_2')\end{Bmatrix}\tilde{\psi}_{1(312)}^{(b)m}(\mathbf{p}_3,\mathbf{p}_1,\mathbf{p}_2)\ , \tag{B51}$$

where $O^{(m)}$ are the spherical components of the operator O. Note that $J_{bc}\to J_b$ (B21) and $K_{bc}\to 0$ when $m_c=m$. From the wave functions (A16) we obtain, if ψ_1 is ψ_1'',

$$(J_{cb}'',K_{cb}'')=I_0\frac{\sqrt{3}}{2mR}(c_{cb}'',d_{cb}'')\ , \tag{B52}$$

where

$$c_{bc}''=\left[\frac{8\alpha_b\alpha_c}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^{3/2}\left\{\left[\frac{m+m_b}{2m_b}\right]\left[\frac{4\alpha_b}{1+3\alpha_b^2}\right]-\frac{1}{3}\frac{\alpha_b(1+3\alpha_c^2)}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\left[1-\frac{4}{1+3\alpha_b^2}\left[\frac{m+m_b}{2m_b}\right]\right]\right\}$$
$$d_{bc}''=\left[\frac{8\alpha_b\alpha_c}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^{3/2}(-\tfrac{1}{3})\frac{\alpha_b(1+3\alpha_c^2)}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\left[1-\frac{4}{1+3\alpha_c^2}\left[\frac{m+m_c}{2m_c}\right]\right] \tag{B53}$$

It is clear that $c_{bc}''\to 1$, $d_{bc}''\to 0$ when $m_b,m_c\to m$, and after some algebra one can check that $c_{bc}''\to c_b''$ (B22), and $d_{bc}''\to 0$ when $m_c\to m$.

The flavor matrix elements that we need in (B47) are given by (B43). We still need the reduced matrix elements of $\mathbf{\Sigma}_+,\mathbf{\Sigma}_-$. Those of $\mathbf{\Sigma}_+=\mathbf{\Sigma}$ are given by (B31), and those of $\mathbf{\Sigma}_-$ are given by

$$\sqrt{2}\langle\chi_{123}''\|\mathbf{\Sigma}_-\|\chi_{123}'\rangle=\sqrt{2}\langle\chi_{123}'\|\mathbf{\Sigma}_+\|\chi_{123}''\rangle=\langle\chi_{123}'\|\mathbf{\Sigma}_+\|\chi^s\rangle=8\ ,$$
$$\langle\chi_{123}'\|\mathbf{\Sigma}_-\|\chi^s\rangle=\langle\chi_{123}'\|\mathbf{\Sigma}_-\|\chi_{123}''\rangle=\langle\chi_{123}''\|\mathbf{\Sigma}_+\|\chi_{123}'\rangle=0\ . \tag{B54}$$

Now, in terms of the ordering $\langle \chi_{312} \| \Sigma_{\pm} \| \chi_{312}^{(S)} \rangle$ in (B50) we have, from (A19),

$$\begin{aligned}
&\langle \chi''_{312} \| \Sigma_+ \| \chi'_{312} \rangle = \langle \chi'_{312} \| \Sigma_- \| \chi''_{312} \rangle = -3\sqrt{2} , \\
&\langle \chi'_{312} \| \Sigma_+ \| \chi''_{312} \rangle = \langle \chi''_{312} \| \Sigma_- \| \chi'_{312} \rangle = \sqrt{2} , \\
&\langle \chi''_{312} \| \Sigma_+ \| \chi''_{312} \rangle = \langle \chi''_{312} \| \Sigma_- \| \chi''_{312} \rangle = -\sqrt{6} , \\
&\langle \chi'_{312} \| \Sigma_+ \| \chi'_{312} \rangle = \langle \chi'_{312} \| \Sigma_- \| \chi'_{312} \rangle = \sqrt{6} , \\
&\langle \chi''_{312} \| \Sigma_+ \| \chi^s \rangle = -\sqrt{3} \langle \chi'_{312} \| \Sigma_+ \| \chi^s \rangle = 4\sqrt{3} , \\
&\langle \chi''_{312} \| \Sigma_- \| \chi^s \rangle = \langle \chi'_{312} \| \Sigma_- \| \chi^S \rangle = 0 .
\end{aligned} \tag{B55}$$

We can now regroup all these terms to give the final result

$$(2\pi)^3 \langle \mathcal{B}_c | H_w^{\rm PV} | \mathcal{B}_b^* (\psi''_1 \chi) \rangle = 6 \frac{G}{\sqrt{2}} V_{cb} \frac{I_0}{mR} (F c''_{bc} + G d''_{bc}) , \tag{B56}$$

where F and G change according to the transition considered and the constants c''_{bc} and d''_{bc} contain all the information on the heavy-quark masses, given by (B53). F,G are given by

$$\begin{aligned}
&(F;G)(\Lambda_b(\psi''_1\chi') \to \Sigma_c^0) = (3\sqrt{3}/\sqrt{2}; -\sqrt{3}/\sqrt{2}) , \\
&(F;G)(\Sigma_b^+(\psi''_1\chi'') \to \Lambda_c^+) = (\sqrt{3}/\sqrt{2}; -3\sqrt{3}/\sqrt{2}) , \\
&(F;G)(\Sigma_b^+(\psi''_1\chi'') \to \Sigma_c^+) = (3/\sqrt{2}; 3/\sqrt{2}) , \\
&(F;G)(\Sigma_b^0(\psi''_1\chi'') \to \Sigma_c^0) = (3/\sqrt{2}; 3/\sqrt{2}) , \\
&(F;G)(\Sigma_b^+(\psi''_1\chi^s) \to \Lambda_c^+) = (2\sqrt{3}; 0) , \\
&(F;G)(\Sigma_b^+(\psi''_1\chi^s) \to \Sigma_c^+) = (6; 0) , \\
&(F;G)(\Sigma_b^0(\psi''_1\chi^s) \to \Sigma_c^0) = (6; 0) .
\end{aligned} \tag{B57}$$

APPENDIX C: STRONG MATRIX ELEMENTS

We will now compute strong couplings of the form $\mathcal{B}_b \bar{B} N$, $\mathcal{B}_b \bar{B} \Delta$, and $\mathcal{B}_b^* \bar{B} N$, $\mathcal{B}_b^* \bar{B} \Delta$ where $\mathcal{B}_b$ ($\mathcal{B}_b^*$) is a b-flavored baryon of $J^{PC} = \frac{1}{2}^+$ ($\frac{1}{2}^-$) and $\bar{B} = b\bar{d}$ or $-b\bar{u}$. We will use the quark-pair-creation model, extensively exposed in Ref. 12.

1. Ground-state baryons

We will first compute the couplings $\mathcal{B}_b \bar{B} N$, $\mathcal{B}_b \bar{B} \Delta$. After color factors and all contractions are taken into account, one ends with an expression (see Fig. 10)

$$\langle \bar{B} N | T | \mathcal{B}_b \rangle = 3\gamma \sum_m \langle 1,1; m, -m | 0,0 \rangle \frac{1}{\sqrt{3}} , \tag{C1}$$

$$\langle \Phi_{\bar{B}}(35) \Phi_N(124) | \Phi_{\mathcal{B}_b}(123) \Phi_{\rm vac}^{-m}(45) \rangle \mathcal{I}_m(N\bar{B}; \mathcal{B}_b) ,$$

where the Φ's are the spin-flavor wave functions; in particular, for the created pair out of the vacuum,

$$\Phi_{\rm vac}^{-m} = \chi_1^{-m} \phi_0, \quad \phi_0 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}) \tag{C2}$$

and $\mathcal{I}_m$ is the spatial integral

$$\begin{aligned}
\mathcal{I}_m(N\bar{B}; \mathcal{B}_b) = \int \prod_i d\mathbf{p}_i \tilde{\psi}_B^\dagger(\mathbf{p}_3, \mathbf{p}_4) \tilde{\psi}_N^\dagger(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_5) \mathcal{Y}_1^m(\mathbf{p}_4 - \mathbf{p}_5) \tilde{\psi}_{\mathcal{B}_b}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \\
\times \delta(\mathbf{p}_4 + \mathbf{p}_5) \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_5 - \mathbf{k}_N) \delta(\mathbf{p}_3 + \mathbf{p}_4 - \mathbf{k}_B) \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{k}_{\mathcal{B}_b}) .
\end{aligned} \tag{C3}$$

Note the important point that we are using, for $\mathcal{B}_b$, the wave functions (A22) for which the third quark is the heavy quark.

Using the harmonic-oscillator wave functions (A16) and (A29) for baryons and mesons, we find, after some algebra, in the $\mathcal{B}_b$ rest frame,

$$\begin{aligned}
\mathcal{I}_m(N\bar{B}; \mathcal{B}_b) = -\delta(\mathbf{k}_B + \mathbf{k}_N) \mathcal{Y}_1^m(\mathbf{k}_B) \frac{R'^{3/2}}{\pi^{3/4}} \\
\times (\alpha_b \beta_b A_b)^{3/2} B_b \exp(-C_b R^2 k_B^2) ,
\end{aligned} \tag{C4}$$

where R^2 and R'^2 are the baryon and meson squared radii, α_b is defined by (B10), and

$$\begin{aligned}
&A_b = \frac{6}{2\rho^2 \beta_b^2 + 3(1+\alpha_b^2)}, \quad B_b = 2 \frac{\rho^2 \mu_b + \beta_b^2 (1 + 3\alpha_b^2)}{2\rho^2 \beta_b^4 + 3\beta_b^2 (1 + \alpha_b^2)} , \\
&C_b = \frac{\rho^2 (4\beta_b^4 - 3)^2 + 3\alpha_b^2 (3\rho^2 + 8\beta_b^6)}{24\beta_b^6 [2\rho^2 \beta_b^2 + 3(1+\alpha_b^2)]} , \\
&\rho^2 = \frac{R'^2}{R^2}, \quad \beta_b = \left[\frac{m + m_b}{2m_b} \right]^{1/4}, \quad \mu_b = \frac{m}{m_b} .
\end{aligned} \tag{C5}$$

In the equal-mass limit we find

$$\begin{aligned}
&A_b \to \frac{3R^2}{R'^2 + 3R^2}, \quad B_b \to \frac{R'^2 + 4R^2}{R'^2 + 3R^2} , \\
&C_b \to \frac{5R'^2 + 12R^2}{24(R'^2 + 3R^2)}, \quad \alpha_b, \beta_b, \mu_b \to 1
\end{aligned} \tag{C6}$$

so that we recover the equal-mass integral given in Refs. 4 and 12 (for example, formula 2.210 of our book[4]).

We need now to compute the matrix elements from (B1):

$$H=\langle \Phi_{N\uparrow}(124)\Phi_B(35)|\Phi_{\mathcal{B}_b\uparrow}(123)\Phi^0_{\rm vac}(45)\rangle\ . \qquad (C7)$$

We obtain (remember $\bar B^0=b\bar d, B^-=-b\bar u$)

$$H(\Sigma_b^+\to\bar B^0 p)=\sqrt2 H(\Sigma_b^0\to B^- p)=\sqrt2 H(\Sigma_b^0\to\bar B^0 n)=-\tfrac{1}{18}\ ,$$
$$H(\Lambda_b\to B^- p)=-H(\Lambda_b\to\bar B^0 n)=\frac16\left[\frac32\right]^{1/2}\ . \qquad (C8)$$

We need also the $\mathcal{B}_b\to\Delta\bar B$ transitions

$$H(\Sigma_b^+\to B^-\Delta^{++})=-\sqrt3 H(\Sigma_b^+\to\bar B^0\Delta^+)=-\left[\frac32\right]^{1/2}H(\Sigma_b^0\to\bar B^0\Delta^0)=\left[\frac32\right]^{1/2}H(\Sigma_b^0\to B^-\Delta^+)=-\frac{\sqrt6}{9}\ ,$$
$$H(\Lambda_b\to\bar B\Delta)=0 \qquad (C9)$$

To sum up, we have, for the matrix elements (C1),

$$\langle \bar B N\uparrow|T|\mathcal{B}_b\uparrow\rangle=-\gamma H\mathcal{I}_0 \qquad (C10)$$

with $\mathcal{I}_0$ given by (C4) for $m=0$ and H by (C8), (C9).

2. Excited baryons

Let us now compute the couplings $\mathcal{B}_b^*\bar B N$, $\mathcal{B}_b^*\bar B\Delta$ where $\mathcal{B}_b^*$ is a b-flavored baryon $\frac12^-$ excited state.

The equivalent of formula (C1) will now become [the wave functions of $\mathcal{B}_b^*$ are given by (A25)]

$$\langle \bar B N|T|\mathcal{B}_b\rangle=3\gamma\sum_{mm'}\langle 1,1;m,-m|0,0\rangle\frac{1}{\sqrt3}\langle 1,S;m;M-m'|\tfrac12 M\rangle$$
$$\times\langle \Phi_{\bar B}(35)\Phi_N(124)|\phi_{\mathcal{B}_b}(123)\chi^{M-m'}_{S(123)}\Phi^{-m}_{\rm vac}(45)\rangle\mathcal{K}_{mm'}\ , \qquad (C11)$$

where the flavor wave functions of b baryons $\phi_{\mathcal{B}_b}$ are given by (A21), S is the total quark spin, and M is the J_z of $\mathcal{B}_b^*$.

The spatial integrals can be either $\mathcal{K}'_{mm'}$ or $\mathcal{K}''_{mm'}$ according to the spatial wave function ψ_1' or ψ_1'' (B16). We find

$$\mathcal{K}'_{mm'}=0 \qquad (C12)$$

because of the antisymmetry (symmetry) of $\psi_1'(\psi_0)$ in the exchange $1\leftrightarrow2$. We find, for the nonvanishing integral,

$$\mathcal{K}''_{mm'}=\delta(\mathbf{k}_B+\mathbf{k}_N)\frac1R\frac{R'^{3/2}}{\pi^{5/4}}\beta_b^{3/2}(\alpha_b A_b)^{5/2}[B_bD_b(k_B^2R^2)\delta_{m0}\delta_{m'0}+(-1)^{m'+1}\delta_{m-m'}]\exp(-C_bR^2k_B^2) \qquad (C13)$$

where $\alpha_b,\beta_b,A_b,B_b,C_b$ are given above, and

$$D_b=\frac{\rho^2+2\beta_b^2}{4\beta_b^2}\to\frac{R'^2+2R^2}{4R^2} \qquad (C14)$$

when $m_b\to m$.

After some algebra, we find, for the $\mathcal{B}_b^*\bar B N$ couplings,

$$\langle \bar B N|T|\Sigma_b(\psi''\chi'')\rangle=-\frac{1}{6\sqrt6}\gamma H'(2\mathcal{K}''_{+1-1}-\mathcal{K}''_{00})\ ,$$
$$\langle \bar B N|T|\Sigma_b(\psi''\chi^s)\rangle=-\frac{1}{3\sqrt3}\gamma H'(2\mathcal{K}''_{+1-1}-\mathcal{K}''_{00})\ , \qquad (C15)$$
$$\langle \bar B N|T|\Lambda_b(\psi''\chi')\rangle=\frac{1}{2\sqrt6}\gamma H'(2\mathcal{K}''_{+1-1}-\mathcal{K}''_{00})$$

and, for the $\mathcal{B}_b^*\bar B\Delta$ couplings,

$$\langle \bar B\Delta|T|\Sigma_b(\psi''\chi'')\rangle=\frac{2}{3\sqrt6}\gamma H'(\mathcal{K}''_{+1-1}+\mathcal{K}''_{00})\ ,$$
$$\langle \bar B\Delta|T|\Sigma_b(\psi''\chi^s)\rangle=-\frac{1}{6\sqrt3}\gamma H'(\mathcal{K}''_{+1-1}+\mathcal{K}''_{00})\ , \qquad (C16)$$
$$\langle \bar B\Delta|T|\Lambda_b(\psi''\chi')\rangle=0\ ,$$

where H' is a flavor matrix element

$$H'=\langle \phi_N(124)\phi_{\bar B}(35)|\phi_{\mathcal{B}_b}(123)\phi_{\rm vac}(45)\rangle\ , \qquad (C17)$$

and similarly for the $\mathcal{B}_b^*\bar B\Delta$ couplings.

We obtain, for the N transitions,

$$H'(\Sigma_b^+\to\bar B^0 p)=\sqrt2 H'(\Sigma_b^0\to B^- p)=\sqrt2 H'(\Sigma_b^0\to\bar B^0 n)=\frac{\sqrt2}{3}\ ,$$
$$H'(\Lambda_b\to B^- p)=-H'(\Lambda_b\to\bar B^0 n)=\frac{1}{\sqrt3} \qquad (C18)$$

and, for the Δ ones,

$$H'(\Sigma_b^0\to\bar B^{\,0}\Delta^0)=-H'(\Sigma_b^0\to B^-\Delta^+)$$
$$=-\left[\frac{2}{3}\right]^{1/2}H'(\Sigma_b^+\to B^-\Delta^{++})$$
$$=\sqrt{2}H'(\Sigma_b^+\to\bar B^{\,0}\Delta^+)=\frac{\sqrt{2}}{3}\ . \tag{C19}$$

It is instructive to express $2\mathcal{K}''_{+1-1}-\mathcal{K}''_{00}$ and $\mathcal{K}''_{+1-1}+\mathcal{K}''_{00}$ that enter in both types of transitions. We have

$$2\mathcal{K}''_{+1-1}-\mathcal{K}''_{00}=\delta(\mathbf{k}_B+\mathbf{k}_N)\frac{1}{R}\frac{R'^{3/2}}{\pi^{5/4}}\beta_b^{3/2}(\alpha_b A_b)^{5/2}$$
$$\times[3-B_bD_b(\mathbf{k}_B^2R^2)]\exp(-C_bR^2\mathbf{k}_B^2)\ , \tag{C20}$$

$$\mathcal{K}''_{+1-1}+\mathcal{K}''_{00}=\delta(\mathbf{k}_B+\mathbf{k}_N)\frac{2}{R}\frac{R'^{3/2}}{\pi^{5/4}}\beta_b^{3/2}(\alpha_b A_b)^{5/2}$$
$$\times B_bD_b(\mathbf{k}_B^2R^2)\exp(-C_bR^2\mathbf{k}_B^2)\ . \tag{C21}$$

We observe the typical isotropic behavior in (C20) (the factor 3 indicates that all polarizations contribute the same amount) corresponding to the S-wave process $\mathcal{B}_b^*\to\bar BN$. Also, we see the typical D-wave behavior $\sim\mathbf{k}_B^2$ in (C21), as it corresponds to $\mathcal{B}_b^*\to\bar B\Delta$.

APPENDIX D: WEAK MATRIX ELEMENTS AT NONZERO-MOMENTUM TRANSFER

In Appendix B we have computed the weak matrix elements assuming conservation of the spatial three-momentum. For example, in estimating weak matrix elements such as $\langle\mathcal{B}_c|H_w^{\rm PC}|\mathcal{B}_b\rangle$ we have assumed that both $\mathcal{B}_c(\Lambda_b,\Sigma_b)$ and $\mathcal{B}_c(\Lambda_c^+,\Sigma_c)$ are at rest. However, as we have seen in the discussion of our pole model in Sec. II, we need some extrapolation to go from these matrix elements where the momentum transfer is null, to the true residues at the pole, where the momentum transfer is not vanishing. Let us consider the general case of a b baryon in the initial state and charmed baryon in the final state, but assuming a nonzero-momentum transfer, i.e., a spurion, as in Fig. 12, that has the function of carrying the three-momentum $\mathbf{k}$.

This calculation will amount to compute the spatial integrals (B40), (B51) for nonzero-momentum transfer, i.e.,

$$I_{cb}(\mathbf{k})=\frac{1}{(2\pi)^3}\int\tilde\psi^{(c)\dagger}_{0(312)}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{p}'_3)\tilde\psi^{(b)}_{0(312)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\delta(\mathbf{p}_3-\mathbf{p}'_3)\prod_i d\mathbf{p}_i d\mathbf{p}'_i\delta\Big(\sum_j\mathbf{p}_j\Big)\delta\Big(\sum_j\mathbf{p}'_j-\mathbf{k}\Big) \tag{D1}$$

and

$$\begin{Bmatrix}J_{cb}(\mathbf{k})\\K_{cb}(\mathbf{k})\end{Bmatrix}=\frac{1}{(2\pi)^3}\int\prod_i d\mathbf{p}_i d\mathbf{p}'_i\delta\Big(\sum_j\mathbf{p}_j\Big)\delta\Big(\sum_j\mathbf{p}'_j-\mathbf{k}\Big)\delta(\mathbf{p}_3-\mathbf{p}'_3)\tilde\psi^{(c)\dagger}_{0(312)}(\mathbf{p}'_3,\mathbf{p}'_1,\mathbf{p}'_2)\begin{Bmatrix}O_b^{(m)*}(\mathbf{p}_1,\mathbf{p}_2)\\O_c^{(m)*}(\mathbf{p}'_1,\mathbf{p}'_2)\end{Bmatrix}\tilde\psi^{(b)m}_{1(312)}(\mathbf{p}_3,\mathbf{p}_1,\mathbf{p}_2)\ , \tag{D2}$$

where $O_b^{(m)},O_c^{(m)}$ are spherical components of the operators (B49).

We find, after some algebra

$$I_{bc}(\mathbf{k})=I_{bc}(0)\exp\left[-\left[\frac{3m}{2m+m_c}\right]^2\left[\frac{8\alpha_b^2\alpha_c^2}{\alpha_b^2+\alpha_c^2+6\alpha_c^2\alpha_c^2}\right]\frac{R^2\mathbf{k}^2}{24}\right]\ , \tag{D3}$$

where $I_{bc}(0)$ is equal to the expression (B41) and α_b,α_c are defined in (B10). We obtain then, in the equal-mass limit,

$$I_0(\mathbf{k})=I_0(0)\exp\left[-\frac{R^2\mathbf{k}^2}{24}\right]\ , \tag{D4}$$

where $I_0(0)$ is given by (B11).

Let us now look at the matrix elements (D2). We obtain

$$J_{bc}^{mm'}(\mathbf{k})=[J_{bc}(0)\delta_{mm'}+L_{bc}R^2\mathbf{k}^{(m)}\mathbf{k}^{(m')*}]$$
$$\times\exp\left[-\left[\frac{3m}{2m+m_c}\right]^2\left[\frac{8\alpha_b^2\alpha_c^2}{\alpha_b^2+\alpha_c^2+6\alpha_c^2\alpha_c^2}\right]\frac{R^2\mathbf{k}^2}{24}\right]\ ,$$

$$K_{bc}^{mm'}(\mathbf{k})=[K_{bc}(0)\delta_{mm'}+M_{bc}R^2\mathbf{k}^{(m)}\mathbf{k}^{(m')*}]$$
$$\times\exp\left[-\left[\frac{3m}{2m+m_c}\right]^2\left[\frac{8\alpha_b^2\alpha_c^2}{\alpha_b^2+\alpha_c^2+6\alpha_c^2\alpha_c^2}\right]\frac{R^2\mathbf{k}^2}{24}\right]\ , \tag{D5}$$

where $J_{bc}(0)$, $K_{bc}(0)$ are given by (B52), (B53), for the wave functions of interest ψ''_1, $\mathbf{k}^{(m)}$ are the spherical components of $\mathbf{k}$ and

$$L''_{bc}=I_0\frac{\sqrt{3}}{2mR}e''_{bc},\quad M''_{bc}=I_0\frac{\sqrt{3}}{2mR}f''_{bc}\ , \tag{D6}$$

where

$$e''_{bc}=-\left[\frac{8\alpha_b\alpha_c}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^{3/2}\frac{\alpha_b}{18\sqrt{2}}\left[\frac{8\alpha_c^2}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^2\left[\frac{3m}{2m+m_c}\right]^2\left[\frac{1+3\alpha_b^2}{4}\right]\left[1-\frac{4}{1+3\alpha_b^2}\frac{m+m_b}{2mb}\right],$$

$$\begin{aligned}f''_{bc}=-&\left[\frac{8\alpha_b\alpha_c}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]^{3/2}\frac{\alpha_b}{3\sqrt{2}}\left[\frac{3m}{2m+m_c}\right]\left[\frac{8\alpha_c^2}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]\\ &\times\left[\frac{1}{6}\left[1-\frac{4}{1+3\alpha_c^2}\frac{m+m_c}{2mc}\right]\left[\frac{3m}{2m+m_c}\right]\left[\frac{2\alpha_c^2(1+3\alpha_b^2)}{\alpha_b^2+\alpha_c^2+6\alpha_b^2\alpha_c^2}\right]\right.\\ &\left.-\left[\frac{m+m_c}{2mc}\right]\left[1-\frac{4\alpha_c^2}{1+3\alpha_c^2}\frac{3m}{2m+m_c}\right]-\frac{m_c-m}{2mc}\right].\end{aligned} \tag{D7}$$

We will emphasize two points in these formulas. The factor $[3m/(2m+m_c)]^2$ in the exponential in (D3), (D5), shows a difference between light baryons and charmed baryons in the final state: the former will be significantly more suppressed than the latter.

On the other hand, the coefficients L_{bc} vanish when $m_b=m$, and the coefficients M_{bc} vanish when $m_c=m$. We obtain therefore, in the equal-mass limit,

$$J''_{mm'}(\mathbf{k})=I_0\left[\frac{\sqrt{3}}{2mR}\right]\exp\left[-\frac{R^2\mathbf{k}^2}{24}\right],\quad K''_{mm'}=0\ . \tag{D8}$$

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