Direct CP violation in $K \to 3\pi$ decay

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In light of the fact that the presence of the Z^0 penguin diagram suppresses strongly $\epsilon'_{2\pi}/\epsilon$ in $K^0 \to 2\pi$ decay for large values of m_t, we reanalyze the direct decay-amplitude CP violation in $K^0 \to 3\pi$ decays in the Kobayashi-Maskawa model. The effects due to electroweak penguins, isospin breaking, and higher-order weak chiral Lagrangians are studied in the large-N_c approach. We find that the Li-Wolfenstein relation between $\epsilon'_{3\pi}$ and $\epsilon'_{2\pi}$ is modified dramatically: the former receives very large contributions from the higher-derivative chiral terms and sizable contributions from the isospin-breaking correction due to π^0-η-η' mixing and η, $\eta' \to 3\pi$ transitions. When the top quark becomes very heavy, effects of the electroweak penguin terms are enhanced. Unlike $\epsilon'_{2\pi}/\epsilon$, which decreases as m_t increases, the CP-violating parameter $\epsilon'_{3\pi}/\epsilon$ is of order 10^{-2} and increases with the heavy top-quark mass.

I. INTRODUCTION

CP violation in $K \to 3\pi$ decays can be tested in many different ways. For example, two of the interesting experimental quantities are

$$\eta_{+-0} = \frac{\langle \pi^+\pi^-\pi^0 | H_W^- | K_S \rangle}{\langle \pi^+\pi^-\pi^0 | H_W^+ | K_L \rangle} = \epsilon + \epsilon'_{+-0} \, ,$$
$$\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0 | H_W^- | K_S \rangle}{\langle \pi^0\pi^0\pi^0 | H_W^+ | K_L \rangle} = \epsilon + \epsilon'_{000} \, , \tag{1.1}$$

where H_W^+ (H_W^-) denotes the CP-even (-odd) component of the weak Hamiltonian, and ϵ and ϵ' are CP-violating parameters to be discussed later. Using current algebra (or the lowest-order weak chiral Lagrangians), Li and Wolfenstein[1] were able to relate CP nonconservation in $K \to 3\pi$ to those in $K \to 2\pi$. More precisely, they found

$$\epsilon'_{+-0} = \epsilon'_{000} = -2\epsilon' \, . \tag{1.2}$$

Subsequently, there has been considerable theoretical work on this subject.[2]

Recently, there have been two reasons for renewed interest in the study of decay-amplitude CP violation within the $K \to 3\pi$ system. First, modifications on ϵ'/ϵ, which is a measure of direct CP noninvariance in the $K \to 2\pi$ transition introduced by a heavy top quark, are significant. It is found that ϵ'/ϵ is strongly suppressed for large m_t through the presence of Z^0 penguin contributions.[3,4] Moreover, the standard model can behave like a superweak theory for very large m_t. It is thus important to see the effects of electroweak penguins on direct CP violation in $K \to 3\pi$ decays. Second, it was conjectured in Ref. 5 that contributions due to higher-order chiral terms in the weak chiral Lagrangian are very important since they are not subject to the $\Delta I = \frac{3}{2}$ suppression. As a consequence, values of ϵ'_{+-0} could be an order of magnitude larger than previous anticipation from the Li-Wolfenstein relation. However, this assertion was objected to in Ref. 6 based on the argument that, even in the presence of higher-order operators, the $K^0 \to \pi^+\pi^-\pi^0$ decay amplitude remains proportional to the $K^0 \to \pi^0\pi^0$ one, and hence the Li-Wolfenstein relation does not get modified in the isospin limit. As we shall see, this important issue is resolved and clarified in the present paper.

The aim of this paper is to perform a detailed analysis of $\epsilon'_{3\pi}/\epsilon$, paying special attention to the following effects: isospin breaking, the Z^0 and photon penguin diagrams, and the higher-order weak-chiral-Lagrangian terms. To set up the calculational framework and notation, we shall first give a brief overview on ϵ'/ϵ in Sec. II. The main task of the study of decay-amplitude CP violation in the $K^0 \to 3\pi$ system is presented in Sec. III. Results are summarized in Sec. IV.

Direct CP violation in $K^\pm \to 3\pi$ decays, which can manifest itself in slope asymmetry or partial rate differences, will be discussed elsewhere.

II. OVERVIEW OF ϵ'/ϵ

We will give a short review on ϵ'/ϵ in the $K \to 2\pi$ system, since many results presented in this section will be utilized when we discuss CP violation in the $K \to 3\pi$ sector.

The isospin structure of the $K \to 2\pi$ transition is

$$A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} A_0 e^{i\delta_0} + \frac{1}{\sqrt{6}} A_2 e^{i\delta_2} \, ,$$
$$A(K^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} A_0 e^{i\delta_0} - \left[\frac{2}{3}\right]^{1/2} A_2 e^{i\delta_2} \, , \tag{2.1}$$
$$A(K^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2} A_2 e^{i\delta_2} \, ,$$

where A_0 and A_2 are isospin-0 and -2 amplitudes, respectively, and δ_0 as well as δ_2 are the corresponding S-

wave $\pi\pi$ scattering phase shifts. Experimentally,[7]

$$\mathrm{Re}\,A_0=4.69\times 10^{-7}\ \mathrm{GeV}\ ,$$
$$\frac{1}{\omega}\equiv\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2}=22.2\pm 0.1\ . \tag{2.2}$$

The parameters ϵ and ϵ', which measure CP violation in the kaon mixing matrix and in the $K\to 2\pi$ amplitude, respectively, read

$$\epsilon=\frac{1}{\sqrt{2}}e^{i\theta}\left[\frac{1}{2}\epsilon_m+\xi_0\right],$$
$$\epsilon'=-\frac{\omega}{\sqrt{2}}(\xi_0-\xi_2)e^{i(\pi/2+\delta_2-\delta_0)}\ , \tag{2.3}$$

where

$$\epsilon_m=\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}},\quad \xi_{0(2)}=\frac{\mathrm{Im}\,A_{0(2)}}{\mathrm{Re}\,A_{0(2)}}\ ,$$

and $\theta=\arctan(2\Delta m/\Gamma_S)\approx\pi/4$. It is evident from Eq. (2.3) that ϵ' is suppressed by a factor of 22. The smallness of ϵ' is very special to $K\to\pi\pi$ decay. Basically, this is attributed to the fact that ϵ' must vanish in the absence of $\Delta I=\frac{3}{2}$ interactions due to the phase-convention-independent argument. Therefore, decay-amplitude CP violation in the $K\to\pi\pi$ sector is subject to $\Delta I=\frac{3}{2}$ suppression.

Sometimes it is convenient to use the original expression for ϵ':

$$\epsilon'=i\left[\frac{\mathrm{Im}\,A(K^0\to\pi^+\pi^-)}{\mathrm{Re}\,A(K^0\to\pi^+\pi^-)}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right],$$
$$-2\epsilon'=i\left[\frac{\mathrm{Im}\,A(K^0\to\pi^0\pi^0)}{\mathrm{Re}\,A(K^0\to\pi^0\pi^0)}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]. \tag{2.4}$$

This enables us to easily see the connection between ϵ' and ϵ'_{+-0} or ϵ'_{000}. Note that the presence of $\mathrm{Im}\,A_0/\mathrm{Re}\,A_0$ is necessary in order to ensure the rephasing invariance of the physical parameter ϵ'.

The $\Delta S=1$ effective Hamiltonian at low energies has the form

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S=1}=\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*\sum_i c_i(\mu)Q_i(\mu)\ . \tag{2.5}$$

To evaluate the Wilson coefficient functions $c_i(\mu)$ at the renormalization scale μ, one first computes all relevant diagrams at the mass scale M_W and then integrates out the heavy quarks and the W boson using the standard renormalization-group analysis.[8] The relevant diagrams contributing to the $\Delta S=1$ Hamiltonian are the gluon-corrected four-quark-operator diagrams, the QCD penguin diagrams with gluon exchanges, the electroweak-penguin diagrams with Z^0 and photon exchanges, and the W-mediated box diagrams involving the external b quarks. The importance of the Z^0 penguin diagram in the presence of a heavy top quark was emphasized recently.[9,10]

The relevant four-quark operators at the scale $\mu<m_c$ are

$$\begin{aligned}
Q_1&=(\bar{s}d)_{V-A}(\bar{u}u)_{V-A}\ ,\\
Q_2&=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A}\ ,\\
Q_3&=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A}\ ,\\
Q_5&=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A}\ ,\\
Q_6&=-8\sum_q(\bar{s}_Lq_R)(\bar{q}_Rd_L)\ ,\\
Q_7&=\tfrac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V+A}\ ,\\
Q_8&=-12\sum_q e_q(\bar{s}_Lq_R)(\bar{q}_Rd_L)\ ,
\end{aligned} \tag{2.6}$$

where the summation over the quark flavors is done only for the light u, d, and s quarks, and $(V\pm A)$ stand for $\gamma_\mu(1\pm\gamma_5)$. Physically, the four-quark operators Q_3, Q_5, and Q_6 are induced by the gluon-penguin diagrams, whereas Q_7 and Q_8 are electroweak-penguin operators. The numerical results for the Wilson coefficient functions $c_i(\mu)$ are given, for example, in Refs. 3 and 4.

With the shorthand notation $\langle Q_i\rangle_{0,2}=\langle(\pi\pi)_{I=0,2}|Q_i(\mu)|K^0\rangle$, it follows from Eq. (2.3) that

$$\frac{\epsilon'}{\epsilon}=\frac{G_F}{2}\frac{\omega}{|\epsilon|\mathrm{Re}\,A_0}(\mathrm{Im}\lambda_t)\times\sum_i y_i(\mu)\langle Q_i\rangle_0\left[1-\frac{1}{\omega}\frac{\mathrm{Im}\,A_2}{\mathrm{Im}\,A_0}\right], \tag{2.7}$$

where $y_i(\mu)=\mathrm{Im}c_i(\mu)/\mathrm{Im}\tau$, $\tau=-\lambda_t/\lambda_u$, and $\lambda_i=V_{id}V_{is}^*$. Since the QCD penguin diagram in general gives the dominant contribution to direct CP violation, it is convenient to recast ϵ'/ϵ in terms of $\langle Q_6\rangle_0$:

$$\frac{\epsilon'}{\epsilon}=\frac{G_F}{2}\frac{\omega}{|\epsilon|\mathrm{Re}\,A_0}(\mathrm{Im}\lambda_t)y_6\langle Q_6\rangle_0(1-\Omega_{\mathrm{tot}})\ , \tag{2.8}$$

where[3]

$$\Omega_{\mathrm{tot}}=\Omega_{\mathrm{IB}}+\Omega_{\mathrm{EWP}}+\Omega_{\mathrm{oct}}+\Omega_{27}+\Omega_P\ , \tag{2.9}$$

with

$$\begin{aligned}
\Omega_{\mathrm{EWP}}&=\frac{1}{\omega}\frac{y_7\langle Q_7\rangle_2+y_8\langle Q_8\rangle_2}{y_6\langle Q_6\rangle_0}-\frac{y_7\langle Q_7\rangle_0+y_8\langle Q_8\rangle_0}{y_6\langle Q_6\rangle_0}\ ,\\
\Omega_{\mathrm{oct}}&=-\frac{y_1\langle Q_1\rangle_0+y_2\langle Q_2\rangle_0}{y_6\langle Q_6\rangle_0}\ ,\\
\Omega_P&=-\frac{y_3\langle Q_3\rangle_0+y_5\langle Q_5\rangle_0}{y_6\langle Q_6\rangle_0}\ ,\\
\Omega_{27}&=\frac{1}{\omega}\frac{y_1\langle Q_1\rangle_2+y_2\langle Q_2\rangle_2}{y_6\langle Q_6\rangle_0}\ ,\\
\Omega_{\mathrm{IB}}&=\frac{1}{\omega}\frac{\mathrm{Im}\,A_2^{\mathrm{IB}}}{\mathrm{Im}\,A_0}\ ,
\end{aligned} \tag{2.10}$$

where Ω_{IB} is the contribution due to isospin-breaking

π-η-η' mixing.

The effect of isospin breaking is[11,12]

$$\Omega_{\rm IB}=\frac{1}{3\sqrt{2}}\,\frac{1}{\omega}\left[\frac{m_d-m_u}{m_s}\right]\chi_2\ , \tag{2.11}$$

with

$$\chi_2=(\cos\theta-\sqrt{2}\sin\theta)(\cos\theta-\sqrt{2}\frac{\rho}{1+\delta}\sin\theta)+(\sin\theta+\sqrt{2}\cos\theta)\left[\sin\theta+\sqrt{2}\frac{\rho}{1+\delta}\cos\theta\right]\times\frac{m_\eta^2-m_\pi^2}{m_{\eta'}^2-m_\pi^2}\ , \tag{2.12}$$

where $\theta\approx-20^\circ$ is the η-η' mixing angle, and the parameters ρ and δ, defined in

$$\langle\eta_8|H_W|K^0\rangle=(\sqrt{1/3})(1+\delta)\langle\pi^0|H_W|K^0\rangle\ ,$$
$$\langle\eta_0|H_W|K^0\rangle=-2(\sqrt{2/3})\rho\langle\pi^0|H_W|K^0\rangle\ , \tag{2.13}$$

measure the breakdown of nonet symmetry in the K^0-η_0 transition and of SU(3)-flavor symmetry in K^0-η_8, respectively. The experimental measurement of direct emission of $K_L\to\pi^+\pi^-\gamma$ in conjunction with the data of $K_L\to\gamma\gamma$ indicates that[13] $\rho=0.78\pm0.05$, implying that nonet symmetry for pseudoscalar mesons holds at the level of (20–25)%. The SU(3)-breaking parameter δ is estimated to be 0.17 in Ref. 14. Hence, numerically,

$$\Omega_{\rm IB}=0.23\ , \tag{2.14}$$

for $(m_d-m_u)/m_s=0.022$. The original estimate $\Omega_{\rm IB}=0.27$ given in Ref. 12 is for $\rho=1$ and $\delta=0$.

The hadronic matrix elements of Q_1,Q_2,Q_3 can be easily obtained in the vacuum-insertion approximation; they are[15] ($f_\pi=132$ MeV)

$$\langle Q_1\rangle_0=\frac{i}{\sqrt{3}}\left[\frac{2}{N}-1\right]f_\pi(m_K^2-m_\pi^2)\ ,$$
$$\langle Q_1\rangle_2=i\left[\frac{2}{3}\right]^{1/2}\left[\frac{1}{N}+1\right]f_\pi(m_K^2-m_\pi^2)\ ,$$
$$\langle Q_2\rangle_0=\frac{i}{\sqrt{3}}\left[2-\frac{1}{N}\right]f_\pi(m_K^2-m_\pi^2)\ ,$$
$$\langle Q_2\rangle_2=\langle Q_1\rangle_2\ , \tag{2.15}$$
$$\langle Q_3\rangle_0=i\frac{\sqrt{3}}{N}f_\pi(m_K^2-m_\pi^2)\ ,$$
$$\langle Q_3\rangle_2=0\ ,$$

where N is the number of colors. To evaluate the matrix elements of the QCD penguin operators Q_5,Q_6 and the electroweak penguin operators Q_7,Q_8, we note that the chiral representation of quark densities is (see Sec. 4.3 of Ref. 7 for details)

$$\bar q_{Rj}q_{Li}=-\frac{f_\pi^2}{4}vU_{ij}-2L_5v(\partial_\mu U\,\partial^\mu U^\dagger\,U)_{ij}+\cdots\ , \tag{2.16}$$

where $U=\exp(2i\phi/f_\pi)$, ${\rm Tr}(\lambda^a\lambda^b)=2\delta^{ab}$, $\phi=\phi^a\lambda^a/\sqrt{2}$, and

$$v(\mu)=\frac{m_\pi^2}{m_u(\mu)+m_d(\mu)}=\frac{m_{K^+}^2}{m_u(\mu)+m_s(\mu)}=\frac{m_{K^0}^2}{m_d(\mu)+m_s(\mu)} \tag{2.17}$$

characterizes the quark order parameter $\langle\bar qq\rangle$, which breaks chiral symmetry spontaneously. The parameter L_5 in Eq. (2.16) is the coupling constant of the higher-order chiral term[16] ${\rm Tr}[\partial^\mu U^\dagger\,\partial_\mu U(M^\dagger U+U^\dagger M)]$, which contributes to the difference of the decay constants f_K and f_π (Ref. 17):

$$\frac{f_K}{f_\pi}-1=8L_5\frac{m_K^2-m_\pi^2}{f_\pi^2}\ . \tag{2.18}$$

Let us write

$$L_5=\frac{1}{8}\,\frac{f_\pi^2}{\Lambda_\chi^2} \tag{2.19}$$

with $\Lambda_\chi\approx1$ GeV (Ref. 18) inferred from the experimental measurement of f_K/f_π.

In the large-N limit, the chiral realization of Q_6 and Q_8 can be obtained by substituting (2.16) into (2.6) (Q being the 3×3 quark charge matrix):

$$Q_6=-f_\pi^4\frac{v^2}{\Lambda_\chi^2}{\rm Tr}(\lambda_6\partial_\mu U\,\partial^\mu U^\dagger)\ ,$$
$$Q_8=-\tfrac{3}{4}f_\pi^4v^2{\rm Tr}(\lambda_6U^\dagger QU)\ , \tag{2.20}$$

where only the leading terms are kept. Equation (2.20) for the QCD penguin operator was first obtained in Ref. 19. The fact that the gluon-penguin-induced $K\to2\pi$ transition vanishes in the limit of SU(3) symmetry and in the limit of zero pion and kaon momenta, as first observed by Shifman, Vainshtein, and Zakharov,[20] is manifest in the explicit chiral representation of Q_6 [i.e., chiral suppression by $(1/\Lambda_\chi^2)$]. As we shall see in the next section, a next-order chiral expansion for both QCD and electroweak penguin operators is required for the study of $K\to3\pi$. Now it is easy to obtain the hadronic matrix elements of penguin operators from Eq. (2.20):

$$\langle Q_6\rangle_0=-i4\sqrt{3}f_\pi v^2\frac{m_K^2-m_\pi^2}{\Lambda_\chi^2},\quad \langle Q_6\rangle_2=0\ ,$$
$$\langle Q_5\rangle_0=\langle Q_6\rangle_0/N,\quad \langle Q_5\rangle_2=0\ ,$$
$$\langle Q_8\rangle_0=i2\sqrt{3}f_\pi v^2,\quad \langle Q_8\rangle_2=i\sqrt{6}f_\pi v^2\ , \tag{2.21}$$
$$\langle Q_7\rangle_0=\langle Q_8\rangle_0/N,\quad \langle Q_7\rangle_2=\langle Q_8\rangle_2/N\ .$$

As will be explained in the next section, we shall calculate $\epsilon'_{3\pi}/\epsilon$ in the limit of large N in order to have a reliable estimate on the effects of higher-order weak chiral Lagrangians. Hence we should also present the results of $\epsilon'_{2\pi}/\epsilon$ in the same $1/N$ approach. It is evident from Eqs. (2.15) and (2.21) that only the matrix elements

TABLE I. Values of Ω_i and ϵ'/ϵ for $K^0 \to 2\pi$ as a function of m_t. Coefficients y_i, which are evaluated at $\mu=1$ GeV and $\Lambda_{QCD}=100$ MeV, are taken from Ref. 3. Hadronic matrix elements are evaluated in the large-N approach.

m_t (GeV)	y_1	y_2	y_6	y_7/α	y_8/α	Ω_{oct}	Ω_{27}	Ω_{EWP}	Ω_{IB}	Ω_{tot}	ϵ'/ϵ
75	0.031	−0.036	−0.054	−0.080	0.000	0.06	−0.09	−0.12	0.27	0.11	0.81×10^{-3}
100	0.030	−0.036	−0.055	−0.065	0.013	0.06	−0.11	−0.04	0.27	0.18	0.76×10^{-3}
125	0.028	−0.036	−0.057	−0.042	0.031	0.06	−0.14	0.08	0.27	0.26	0.71×10^{-3}
150	0.027	−0.036	−0.057	−0.012	0.054	0.06	−0.16	0.22	0.27	0.39	0.59×10^{-3}
200	0.023	−0.035	−0.058	0.064	0.110	0.05	−0.21	0.57	0.27	0.68	0.31×10^{-3}
250	0.018	−0.035	−0.059	0.159	0.181	0.05	−0.29	1.00	0.27	1.03	-0.03×10^{-3}

$\langle Q_1\rangle$, $\langle Q_2\rangle$, $\langle Q_6\rangle$, and $\langle Q_8\rangle$ are relevant in the leading $1/N$ expansion. It is straightforward to show

$$\Omega_{oct}=\frac{2y_2-y_1}{12y_6}\frac{\Lambda_\chi^2}{v^2},$$

$$\Omega_{27}=-\frac{1}{6\sqrt{2}\omega}\frac{y_1+y_2}{y_6}\frac{\Lambda_\chi^2}{v^2}, \quad (2.22)$$

$$\Omega_{EWP}=-\frac{1}{2\sqrt{2}}\frac{1-\sqrt{2}\omega}{\omega}\frac{y_7+3y_8}{3y_6}\frac{\Lambda_\chi^2}{m_K^2-m_\pi^2}.$$

It should be stressed that naively the contribution of the Q_7 operator to Ω_{EWP} vanishes in the leading $1/N$ expansion. However, since the coefficient y_7 is substantially larger than y_8, its net effect could be important. This requires a calculation of $\langle Q_7\rangle$ to subleading $1/N$ corrections, which is not available at present. Hence we will follow Ref. 3 to use the vacuum-insertion relation $\langle Q_7\rangle=\langle Q_8\rangle/3$, consistent with recent lattice calculations.[21]

To have a numerical analysis of ϵ'/ϵ, we first note that

$$\mathrm{Im}\lambda_t=A^2\lambda^5\eta, \quad (2.23)$$

in the Wolfenstein parametrization,[22] where A is fixed by the measured V_{cb} to be 1.0 ± 0.1. For the CP-violating parameter η, we will take $\eta=0.25$, which is consistent with the constraints inferred from observed ϵ and B^0-$\bar{B}^0$ mixing. Equation (2.8) then leads to[23]

$$\frac{\epsilon'}{\epsilon}=-1.69\times10^{-2}y_6(1-\Omega_{tot}), \quad (2.24)$$

where use of $m_s=150$ MeV and Eqs. (2.2) and (2.21) has been made. Numerical results are displayed in Table I. It is evident that ϵ'/ϵ is strongly suppressed by the presence of Z^0 penguin diagrams for large values of m_t. The standard Kobayashi-Maskawa (KM) model of CP violation is milliweak in nature for $m_t\sim100$ GeV, but it can behave like a superweak theory for $m_t \gtrsim 200$ GeV.

III. DIRECT CP VIOLATION IN $K^0\to3\pi$

A. Li-Wolfenstein relation and its modifications

The $K\to3\pi$ decay amplitudes are conventionally parametrized in powers of the Dalitz variables:

$$A(K\to3\pi)=a+bY+cX+d\left[Y^2+\frac{X^2}{3}\right]+e\left[Y^2-\frac{X^2}{3}\right], \quad (3.1)$$

where

$$Y=(s_3-s_0)/m_\pi^2,\quad X=(s_2-s_1)/m_\pi^2,$$

$$s_i=(k-p_i)^2,\quad s_0=(s_1+s_2+s_3)/3,$$

with k and p_i being the four-momenta of the kaon and pion i (the subscript 3 is assigned to the "odd" pion). Using current algebra or the chiral Lagrangian, one can relate both CP-conserving and -violating $K\to3\pi$ amplitudes to those of $K\to2\pi$.

From Eq. (1.1) we have

$$\epsilon'_{+-0}=i\left[\frac{\mathrm{Im}A(K^0\to\pi^+\pi^-\pi^0)}{\mathrm{Re}A(K^0\to\pi^+\pi^-\pi^0)}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right],$$

$$\epsilon'_{000}=i\left[\frac{\mathrm{Im}A(K^0\to\pi^0\pi^0\pi^0)}{\mathrm{Re}A(K^0\to\pi^0\pi^0\pi^0)}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]. \quad (3.2)$$

In the chiral-Lagrangian approach,[24–26] the $K\to3\pi$ amplitudes are related to $K\to2\pi$ ones by

$$A(K^0\to\pi^+\pi^-\pi^0)=-\frac{i}{3\sqrt{6}}\frac{1}{f_\pi}\frac{m_K^2}{m_K^2-m_\pi^2}\left\{A_0-\sqrt{2}A_2+3\left[A_0+\left[\frac{5}{2\sqrt{2}}-\frac{9}{2\sqrt{2}}z\right]A_2\right]\frac{m_\pi^2}{m_K^2}Y+\frac{9}{2\sqrt{2}}(3+z)\frac{m_\pi^2}{m_K^2}A_2X\right\}, \quad (3.3)$$

$$A(K^0\to\pi^0\pi^0\pi^0)=-\frac{i}{\sqrt{6}}\frac{1}{f_\pi}\frac{m_K^2}{m_K^2-m_\pi^2}(A_0-\sqrt{2}A_2),$$

where $z=m_\pi^2/(m_K^2-m_\pi^2)$. It is worth emphasizing that the X-dependent term in $K^0\to\pi^+\pi^-\pi^0$ and the proportional factor i between $K\to 3\pi$ and $K\to 2\pi$ amplitudes are often missed in the literature. The former constitutes the leading energy dependence of $K_S\to\pi^+\pi^-\pi^0$ in the Dalitz plot, while the latter is required by PCAC (partial conservation of axial-vector current). It is evident from Eqs. (2.1) and (3.3) that the $K^0\to 3\pi$ amplitude is proportional to $K^0\to\pi^0\pi^0$ at the origin of the Dalitz plot (i.e., $X=Y=0$) for $\pi^+\pi^-\pi^0$ and at any Dalitz point for $\pi^0\pi^0\pi^0$. Therefore, when final-state interactions are neglected, we have

$$\begin{aligned}&\epsilon'_{+-0}=-2\epsilon' \quad (\text{at } X=Y=0)\ ,\\ &\epsilon'_{000}=-2\epsilon' \quad (\text{at any Dalitz point})\ ,\end{aligned} \tag{3.4}$$

which is the well-known Li-Wolfenstein relation.[1] Two comments are in order. First, we have ignored final-state isospin phase shifts in deriving Eq. (3.4). Second, the relation (3.4) holds as long as the weak nonleptonic Hamiltonian obeys the commutator relation

$$[Q_5^i,\mathcal{H}_W^{\Delta S=1}]=[Q^i,\mathcal{H}_W^{\Delta S=1}]\ , \tag{3.5}$$

where Q_5^i and Q^i are the axial-vector and vector charges, respectively. This commutator relation is respected in the KM model and in the Weinberg model of CP violation.[27]

Just as ϵ'/ϵ discussed in the previous section for the case of $K\to 2\pi$, the Li-Wolfenstein relation is expected to be modified by the effects of electroweak-penguin diagrams and the isospin-violating π-η-η' mixing. However, as pointed out in Ref. 5, there is a new feature relevant for $K\to 3\pi$: the higher-order terms in the weak chiral Lagrangian. The four-derivative chiral terms, which do not contribute to $K\to 2\pi$, are in principle important since their contributions to direct CP violation in $K\to 3\pi$ are not subject to the $\Delta I=\frac{3}{2}$ suppression. In fact, as will be shown later on, this higher-order chiral effect turns out to be the dominant correction to the Li-Wolfenstein relation.

To have a quantitative description of the aforementioned three effects, we will write

$$\begin{aligned}&A(K^0\to 3\pi)=A^{\mathrm{NL}}+A^{\mathrm{IB}}+A^{\mathrm{EWP}}+A^{\mathrm{HO}}\ ,\\ &A_{0(2)}(K^0\to 2\pi)=A_{0(2)}^{\mathrm{NL}}+A_{0(2)}^{\mathrm{IB}}+A_{0(2)}^{\mathrm{EWP}}\ ,\end{aligned} \tag{3.6}$$

where A^{NL}, A^{IB}, A^{EWP}, and A^{HO}, respectively, arise from nonleptonic weak interactions, isospin-breaking effects, electroweak penguin diagrams, and higher-derivative chiral terms. Since the nonleptonic components of $K\to 3\pi$ and $K\to 2\pi$ amplitudes are related by current algebra or chiral Lagrangian, Eq. (2.8) can be written in the form

$$\epsilon'=\epsilon'_{\mathrm{NL}}(1-\Omega_{\mathrm{EWP}}-\Omega_{\mathrm{IB}})\ , \tag{3.7}$$

with (final-state interactions being ignored)

$$\epsilon'_{\mathrm{NL}}=-i\frac{\omega}{\sqrt{2}}\frac{\mathrm{Im}A_0^{\mathrm{NL}}}{\mathrm{Re}A_0^{\mathrm{NL}}}\ . \tag{3.8}$$

Likewise, we can define the analog quantities for $K^0\to\pi^+\pi^-\pi^0$,

$$\epsilon'_{+-0}=-2\epsilon'_{\mathrm{NL}}(1-\Omega_{+-0}^{\mathrm{EWP}}-\Omega_{+-0}^{\mathrm{IB}}-\Omega_{+-0}^{\mathrm{HO}})\ , \tag{3.9}$$

with

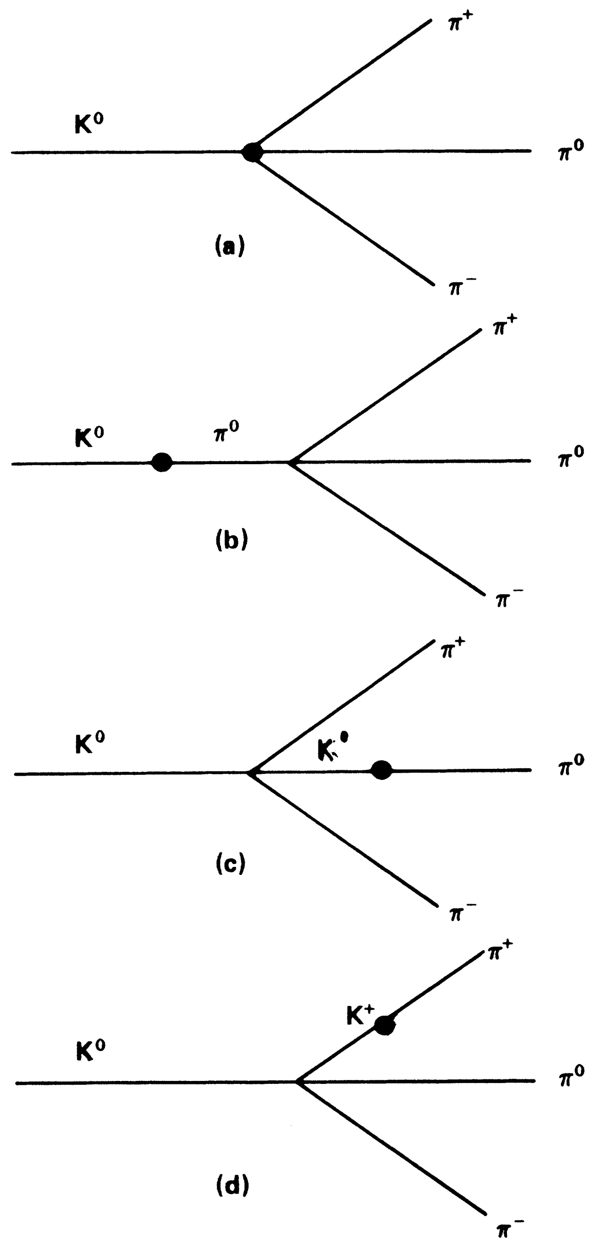

FIG. 1. Diagrams contributing to $K^0\to\pi^+\pi^-\pi^0$ decay amplitudes via the electroweak-penguin interaction denoted by a black dot.

$$\Omega_{+-0}^{\rm EWP} = -\frac{1}{\sqrt{2}\omega}\left[\frac{{\rm Im}A^{\rm EWP}(K^0\to\pi^+\pi^-\pi^0)}{{\rm Re}A(K^0\to\pi^+\pi^-\pi^0)} - \frac{{\rm Im}A_0^{\rm EWP}}{{\rm Re}A_0}\right]\left[\frac{{\rm Re}A_0^{\rm NL}}{{\rm Im}A_0^{\rm NL}}\right],$$

$$\Omega_{+-0}^{\rm IB} = -\frac{1}{\sqrt{2}\omega}\left[\frac{{\rm Im}A^{\rm IB}(K^0\to\pi^+\pi^-\pi^0)}{{\rm Re}A(K^0\to\pi^+\pi^-\pi^0)} - \frac{{\rm Im}A_0^{\rm IB}}{{\rm Re}A_0}\right]\left[\frac{{\rm Re}A_0^{\rm NL}}{{\rm Im}A_0^{\rm NL}}\right], \tag{3.10}$$

$$\Omega_{+-0}^{\rm HO} = -\frac{1}{\sqrt{2}\omega}\frac{{\rm Im}A^{\rm HO}(K^0\to\pi^+\pi^-\pi^0)}{{\rm Re}A(K^0\to\pi^+\pi^-\pi^0)}\frac{{\rm Re}A_0^{\rm NL}}{{\rm Im}A_0^{\rm NL}},$$

and similar definitions for $K^0\to\pi^0\pi^0\pi^0$. The expression for $\Omega_{+-0}^{\rm HO}$ (or $\Omega_{000}^{\rm HO}$) needs some elaboration, as we will discuss in Sec. III D.

From Eq. (3.10) it becomes clear that contributions not related by current algebra or PCAC can be potentially important owing to the lack of $\Delta I=\frac{3}{2}$ suppression. Moreover, one may argue that higher-order terms in the weak chiral Lagrangian provide the dominant corrections. This is attributed to the fact that effects of isospin breaking and electroweak penguins are subject to the suppression of the smallness of isospin violation characterized by $(m_d-m_u)/m_s$ and of the fine-structure constant α, respectively. On the other hand, the chiral effects at higher order are suppressed only by factors of m_K^2/Λ_χ^2. This conjecture is indeed confirmed in the subsequent calculations. In the following sections we will discuss each effect in detail.

In the present paper hadronic matrix elements are evaluated in the $1/N$ approach (except for the operator Q_7). This is because not only this approach is superior to other methods (see, e.g., Ref. 3) at present, but also it is the only method available today which allows us to estimate the higher-order chiral effects of four-quark weak operators. Since meson loops are suppressed in the leading $1/N$ expansion, we shall therefore neglect final-state interactions for reason of consistency.

B. Effect of electroweak penguins

From Eqs. (2.10), (2.21), and (3.10), we find that

$$\Omega_{+-0}^{\rm EWP} \cong -\frac{1}{\sqrt{2}\omega}\frac{{\rm Im}A^{\rm EWP}(K^0\to\pi^+\pi^-\pi^0)}{{\rm Re}A(K^0\to\pi^+\pi^-\pi^0)}\frac{{\rm Re}A_0^{\rm NL}}{{\rm Im}A_0^{\rm NL}} + \Omega_{2\pi}^{\rm EWP}, \tag{3.11}$$

to a good approximation. Hence, even if the electroweak penguins do not contribute, $\Omega_{+-0}^{\rm EWP}$ is nonzero. In addition to the direct $K\to3\pi$ transition induced by the electroweak-penguin operators, there are also pole diagrams as depicted in Fig. 1.

It turns out that the contributions of electroweak penguins at the center of the Dalitz plot are also chirally suppressed, just as the case for the QCD penguin. This can be seen as follows. First, it is easily seen from Eq. (2.20) that the electroweak-penguin operator Q_8 does not lead to a direct $K^0\to\pi^+\pi^-\pi^0$ transition

$$\langle\pi^+\pi^-\pi^0|Q_8|K^0\rangle_{\rm direct}=0\ . \tag{3.12}$$

Second, because[28]

$$\langle\pi^0|Q_8|K^0\rangle=0,\quad \langle\pi^+|Q_8|K^+\rangle=-3f_\pi^2v^2\ , \tag{3.13}$$

the only relevant pole diagram is Fig. 1(d) with the result

$$\langle\pi^+\pi^-\pi^0|Q_8|K^0\rangle=\frac{9}{2\sqrt{2}}\frac{v^2}{m_K^2-m_\pi^2}m_\pi^2\left[Y-\frac{1}{3}X\right]. \tag{3.14}$$

Hence there is no electroweak-penguin contribution to $K^0\to\pi^+\pi^-\pi^0$ at the origin of the Dalitz plot. For $K^0\to\pi^0\pi^0\pi^0$, the reason for chiral suppression is quite clear: The hadronic matrix elements of Q_8 and Q_6 are the same in the vacuum-insertion method except for the overall constants. In order to see the electroweak-penguin effect, we thus have to go beyond the leading-order chiral expansion. From Eqs. (2.16) and (2.6), we obtain

$$Q_8=-\frac{3}{4}f_\pi^4v^2\left[{\rm Tr}(\lambda_6U^\dagger QU)+\frac{1}{\Lambda_\chi^2}{\rm Tr}(\lambda_6U^\dagger QU\,\partial_\mu U\,\partial^\mu U^\dagger+\lambda_6\partial_\mu U\,\partial^\mu U^\dagger\,U^\dagger QU)\right]. \tag{3.15}$$

Since the same effect in $K\to2\pi$ is not chirally suppressed, the first term on the right-hand side (RHS) of Eq. (3.11) is expected to be smaller than $\Omega_{2\pi}^{\rm EWP}$. It is then straightforward, though somewhat involved, to apply the higher-derivative chiral realization of Q_8 to calculate the Q_8-induced $K\to3\pi$ amplitude via Fig. 1. The pole contributions are calculated using the lowest-order chiral Lagrangian for strong interactions,

$$\mathcal{L}_S=\frac{f_\pi^2}{8}\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)+\frac{f_\pi^2}{8}\mathrm{Tr}(MU^\dagger+UM^\dagger)\;, \tag{3.16}$$

where M is a meson mass matrix with the nonvanishing matrix elements

$$M_{11}=M_{22}=m_\pi^2,\quad M_{33}=2m_K^2-m_\pi^2\;. \tag{3.17}$$

We find

$$\langle \pi^+\pi^-\pi^0|Q_8|K^0\rangle=\frac{\sqrt{2}}{3}\frac{v^2}{\Lambda_\chi^2}m_K^2\;, \tag{3.18}$$

at the center of the Dalitz plot.

It is well known that the prediction of the constant term a in the Dalitz amplitude [Eq. (3.1)] by current algebra is too small by 18% when compared with experiment. In Ref. 24 (see also Ref. 26) we have shown that the inclusion of large-N higher-order Lagrangians can account for this discrepancy. At the origin of the Dalitz plot, the amplitude is given by [see Eq. (3.3)]

$$\mathrm{Re}A(K^0\to\pi^+\pi^-\pi^0)\cong-\frac{i}{3\sqrt{6}}\frac{1}{f_\pi}\frac{m_K^2}{m_K^2-m_\pi^2}(\mathrm{Re}A_0^{\mathrm{NL}})\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}. \tag{3.19}$$

Hence the first term of $\Omega_{+-0}^{\mathrm{EWP}}$ becomes

$$-\frac{1}{\sqrt{2}\omega}\frac{\mathrm{Im}A^{\mathrm{EWP}}(K^0\to\pi^+\pi^-\pi^0)}{\mathrm{Re}A(K^0\to\pi^+\pi^-\pi^0)}\frac{\mathrm{Re}A_0^{\mathrm{NL}}}{\mathrm{Im}A_0^{\mathrm{NL}}}=\frac{1}{2\sqrt{2}\omega}\frac{m_K^2}{m_K^2-m_\pi^2}\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\frac{y_7+3y_8}{3y_6}\;, \tag{3.20}$$

where $\mathrm{Im}A_0^{\mathrm{NL}}$ has been approximated by $y_6\langle Q_6\rangle_0$ and use of (2.21), (3.18), and (3.19) has been made. [As explained in Sec. II, the contribution of the Q_7 operator should be included because of the large coefficient y_7 relative to y_8.] When compared with $\Omega_{2\pi}^{\mathrm{EWP}}$ given by Eq. (2.22), it is evident that this contribution is suppressed by a factor of m_K^2/Λ_χ^2, as expected.

Equations (2.22), (3.11), and (3.20) lead to the final result for $\Omega_{+-0}^{\mathrm{EWP}}$:

$$\Omega_{+-0}^{\mathrm{EWP}}=-\frac{1}{2\sqrt{2}\omega}\frac{y_7+3y_8}{3y_6}\left\{\frac{\Lambda_\chi^2}{m_K^2-m_\pi^2}-\frac{m_K^2}{m_K^2-m_\pi^2}\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\right\}, \tag{3.21}$$

where the $\sqrt{2}\omega$ term in the numerator has been dropped. Hence

$$\Omega_{+-0}^{\mathrm{EWP}}=-27.1\frac{y_7+3y_8}{3y_6}\;. \tag{3.22}$$

As for $K^0\to 3\pi^0$, we note that the calculation can be simplified in view of the fact that the $K^0 3\pi^0$ matrix elements of Q_6 and Q_8 are related in the vacuum-insertion method via

$$\langle\pi^0\pi^0\pi^0|Q_8|K^0\rangle=-\tfrac{1}{2}\langle\pi^0\pi^0\pi^0|Q_6|K^0\rangle\;. \tag{3.23}$$

Using the chiral realization of Q_6 given by (2.20), we obtain

$$\langle\pi^0\pi^0\pi^0|Q_8|K^0\rangle=-4\frac{v^2}{\Lambda_\chi^2}m_K^2\;. \tag{3.24}$$

Since, at the center of the Dalitz plot,[24]

$$\mathrm{Re}A(K^0\to 3\pi^0)\cong-\frac{i}{\sqrt{6}}\frac{1}{f_\pi}\frac{m_K^2}{m_K^2-m_\pi^2}(\mathrm{Re}A_0^{\mathrm{NL}})\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}, \tag{3.25}$$

we find

$$\Omega_{000}^{\mathrm{EWP}}=-\frac{1}{2\sqrt{2}\omega}\frac{y_7+3y_8}{3y_6}\left\{\frac{\Lambda_\chi^2}{m_K^2-m_\pi^2}+2\sqrt{2}\frac{m_K^2}{m_K^2-m_\pi^2}\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\right\}, \tag{3.26}$$

or

$$\Omega_{000}^{\mathrm{EWP}}=-55.9\frac{y_7+3y_8}{3y_6}\;. \tag{3.27}$$

Evidently, the effect of electroweak penguins in $K^0\to 3\pi^0$ is 2 times as large as that in $K^0\to\pi^+\pi^-\pi^0$.

C. Isospin-breaking effect

When up- and down-quark masses are not equal, it is clear from Eq. (2.16) that the quark mass term

$$\mathcal{L}_m = -m_u \bar{u}u - m_d \bar{d}d - m_s \bar{s}s \tag{3.28}$$

can induce π^0-η-η' mixing

$$\mathcal{L}_{\rm IB} = B(\pi^0\eta_8 + \sqrt{2}\pi^0\eta_0) , \tag{3.29}$$

and $\eta_8, \eta_0 \to 3\pi$ transitions

$$-\frac{2}{3}\frac{B}{f_\pi^2}(\eta_8+\sqrt{2}\eta_0)(\pi^+\pi^-\pi^0+3\pi^0\pi^0\pi^0) , \tag{3.30}$$

with

$$B = \frac{1}{\sqrt{3}}v(m_d - m_u) \approx \frac{1}{\sqrt{3}}(m_K^2 - m_\pi^2)\frac{m_d - m_u}{m_s} . \tag{3.31}$$

There are five diagrams (see Fig. 2) contributing to the decay $K^0 \to 3\pi$ via π-η-η' mixing and $\eta, \eta' \to 3\pi$ transitions. However, as will be seen shortly, only two of them are relevant for our purposes.

To compute the amplitudes of Fig. 2, we shall use the lowest-order effective chiral Lagrangian for octet $\Delta S = 1$ weak interactions,

$$\mathcal{L}_W = g_8 \mathrm{Tr}(\lambda_6 \partial_\mu U \, \partial^\mu U^\dagger) , \tag{3.32}$$

to describe the K-η_8 and K-η_0 transitions

$$\begin{aligned} \langle \eta_8(k)|\mathcal{L}_W|K^0(k)\rangle &= -2\left[\frac{2}{3}\right]^{1/2}\frac{g_8}{f_\pi}k^2 , \\ \langle \eta_0(k)|\mathcal{L}_W|K^0(k)\rangle &= -2\sqrt{2}\langle \eta_8|\mathcal{L}_W|K^0\rangle . \end{aligned} \tag{3.33}$$

For simplicity in the ensuing discussion, we will not consider SU(3)- and nonet-symmetry breaking as parametrized in Eq. (2.13). Moreover, we will assume $m_\pi^2=0$ in order to simplify our calculation. It is easily seen that under this approximation Fig. 2(b) does not contribute to $K^0 \to 3\pi$. Figure 2(c) also vanishes because of the vanishing strong scattering $\eta\pi \to \eta\pi$ in the limit of $m_\pi^2=0$. Figure 2(e) contributes to $K \to 3\pi$ only at regions off the origin of the Dalitz plot. Consequently, when working at the center of the Dalitz plot, we have to consider only Figs. 2(a) and 2(d). The resulting amplitude is $A^{\rm IB}(K^0 \to \pi^+\pi^-\pi^0) = A(a) + A(d)$, with

$$\begin{aligned} A(a) &= -\frac{5}{\sqrt{6}}\frac{g_8}{f_\pi^4}B\left[\left[\cos\theta - \frac{2\sqrt{2}}{5}\sin\theta\right](\cos\theta - \sqrt{2}\sin\theta) + \left[\sin\theta + \frac{2\sqrt{2}}{5}\cos\theta\right](\sin\theta + \sqrt{2}\cos\theta)\frac{m_\eta^2}{m_{\eta'}^2}\right] , \\ A(d) &= 4\left[\frac{2}{3}\right]^{1/2}\frac{g_8}{f_\pi^4}B\left[(\cos\theta + 2\sqrt{2}\sin\theta)(\cos\theta - \sqrt{2}\sin\theta) + \frac{1}{4}(\sin\theta - 2\sqrt{2}\cos\theta)(\sin\theta + \sqrt{2}\cos\theta)\frac{m_\eta^2}{m_{\eta'}^2 - m_K^2}\right] , \end{aligned} \tag{3.34}$$

where use of the relation $3m_\eta^2 = 4m_K^2$ has been made. Recasting the isospin-breaking effect in terms of the isospin-zero $K \to 2\pi$ amplitude

$$A_0 = i4\sqrt{3}\frac{g_8}{f_\pi^3}(m_K^2 - m_\pi^2) , \tag{3.35}$$

the full isospin-breaking corrections to the $K^0 \to \pi^+\pi^-\pi^0$ amplitude are

$$A^{\rm IB}(K^0 \to \pi^+\pi^-\pi^0) = i\frac{\sqrt{2}}{6\sqrt{3}}\frac{A_0^{\rm NL}}{f_\pi}\left[\frac{m_d - m_u}{m_s}\right]\chi_3^{+-0} , \tag{3.36}$$

where

$$\begin{aligned} \chi_3^{+-0} = {} & (\cos\theta - \sqrt{2}\sin\theta)\left[2(\cos\theta + 2\sqrt{2}\sin\theta) - \frac{5}{4}\left[\cos\theta - \frac{2\sqrt{2}}{5}\sin\theta\right]\right] \\ & + (\sin\theta + \sqrt{2}\cos\theta)\left[\frac{1}{2}(\sin\theta - 2\sqrt{2}\cos\theta)\frac{m_\eta^2}{m_{\eta'}^2 - m_K^2} - \frac{5}{4}\left[\sin\theta + \frac{2\sqrt{2}}{5}\cos\theta\right]\frac{m_\eta^2}{m_{\eta'}^2}\right] . \end{aligned} \tag{3.37}$$

Our result is in agreement with Ref. 6.

In order to compute $\Omega^{\rm IB}_{+-0}$, we still need to know the $K^0\to 2\pi$ amplitude induced by π^0-η-η' mixing:[11,12]

$$A^{\rm IB}(K^0\to 2\pi)=-\frac{1}{2}\left[\frac{m_d-m_u}{m_s}\right]\chi_2 A(K^0\to 2\pi)\ , \tag{3.38}$$

with χ_2 being given by Eq. (2.12). Using the fact that the isospin-breaking effect does not modify the $K^0\to\pi^+\pi^-$ amplitude, we find

$$A_0^{\rm IB}=\frac{1}{\sqrt{3}}A^{\rm IB}(K^0\to\pi^0\pi^0)\ , \tag{3.39}$$

and, hence,

$$\frac{{\rm Im}A_0^{\rm IB}}{{\rm Im}A_0^{\rm NL}}=\frac{1}{6}\left[\frac{m_d-m_u}{m_s}\right]\chi_2\ . \tag{3.40}$$

From Eqs. (3.40), (3.36), (3.19), and (3.10), we are ready to write down the final result for $\Omega^{\rm IB}_{+-0}$:

$$\Omega^{\rm IB}_{+-0}=-\frac{1}{\sqrt{2}\omega}\left[\frac{m_d-m_u}{m_s}\right]\left[\chi_3^{+-0}-\frac{1}{6}\chi_2\right]\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\ . \tag{3.41}$$

Note that the isospin-breaking effect is independent of the top-quark mass. Numerically,

$$\Omega^{\rm IB}_{+-0}=0.71\ , \tag{3.42}$$

for $(m_d-m_u)/m_s=0.022$. This is to be compared with $\Omega^{\rm IB}\approx 0.25$ in the case of $K\to 2\pi$.

Repeating the same calculation for $K^0\to 3\pi^0$, we find

$$\Omega^{\rm IB}_{000}=-\frac{1}{\sqrt{2}\omega}\left[\frac{m_d-m_u}{m_s}\right]\left[\chi_3^{000}-\frac{1}{6}\chi_2\right]\left[1+\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\ , \tag{3.43}$$

with

$$\begin{aligned}\chi_3^{000}=&(\cos\theta-\sqrt{2}\sin\theta)\left[2(\cos\theta+2\sqrt{2}\sin\theta)-\frac{5}{12}\left[\cos\theta-\frac{2\sqrt{2}}{5}\sin\theta\right]\right]\\&+(\sin\theta+\sqrt{2}\cos\theta)\left[\frac{1}{2}(\sin\theta-2\sqrt{2}\cos\theta)\frac{m_\eta^2}{m_{\eta'}^2-m_K^2}-\frac{5}{12}\left[\sin\theta+\frac{2\sqrt{2}}{5}\cos\theta\right]\frac{m_\eta^2}{m_{\eta'}^2}\right]\ .\end{aligned} \tag{3.44}$$

Consequently,

$$\Omega^{\rm IB}_{000}=0.56\ . \tag{3.45}$$

The main uncertainty for $\Omega^{\rm IB}_{+-0}$ and $\Omega^{\rm IB}_{000}$ is due to the assumption of nonet symmetry and vanishing m_π^2.

D. Effect of higher-order chiral terms

It was conjectured in Ref. 5 that the Li-Wolfenstein relation (1.2) receives a large modification from the higher-order components of weak chiral Lagrangians, i.e., $\epsilon'_{+-0}\sim 10\epsilon'$. On the contrary, the authors of Ref. 6 claimed that in the large-N approach it is not modified by the inclusion of next-to-leading chiral terms. To resolve this issue, let us first review the argument presented in the latter reference. We will confine ourselves in this section to the nonleptonic part of the $\Delta S=1$ effective Hamiltonian given by Eq. (2.5).

As mentioned in Sec. II [cf. Eq. (2.21)], only the hadronic matrix elements of Q_1, Q_2, and Q_6 survive in the leading $1/N$ expansion. Therefore, Eq. (2.5) reduces to

$$\mathcal{H}_{\rm eff}^{\Delta S=1}(1/N)=\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*[b_1(Q_2-Q_1)+b_2(Q_2+2Q_1)+b_6Q_6]\ , \tag{3.46}$$

where $b_1=(2c_2-c_1)/3$, $b_2=(c_1+c_2)/3$, and $b_6=c_6$. The combination (Q_2-Q_1) is a $\Delta I=\frac{1}{2}$ four-quark operator which transforms as $(8_L,1_R)$ under chiral rotation, while (Q_2+2Q_1) is a 27-plet $\Delta I=\frac{3}{2}$ operator. The lowest-order chiral representation of (Q_2-Q_1) in the large-N limit reads[29]

$$Q_2-Q_1=-\frac{f_\pi^4}{4}{\rm Tr}(\lambda_6L_\mu L^\mu)\ , \tag{3.47}$$

where $L_\mu=(\partial_\mu U)U^\dagger$ is an $SU_R(3)$ singlet. It follows from Eq. (2.20) that

$$Q_6=-4\frac{v^2}{\Lambda_\chi^2}(Q_2-Q_1)\ , \tag{3.48}$$

to the first order in the chiral expansion. Since nonleptonic decay-amplitude *CP* violation is governed by the penguin operator Q_6, we have

$$\left.\frac{\text{Im}A(K^0\to 3\pi)}{\text{Re}A(K^0\to 3\pi)}\right|_{\text{LO}}=\frac{\text{Im}b_6\langle Q_6\rangle_3^{\text{LO}}}{\text{Re}b_1\langle Q_2-Q_1\rangle_3^{\text{LO}}+\text{Re}b_2\langle Q_2+2Q_1\rangle_3^{\text{LO}}+\text{Re}b_6\langle Q_6\rangle_3^{\text{LO}}}\ ,$$
$$=\frac{\text{Im}A(K^0\to 2\pi^0)}{\text{Re}A(K^0\to 2\pi^0)}\ , \tag{3.49}$$

where the last identity comes from current algebra or lowest-order weak chiral Lagrangian, and $\langle Q_i\rangle_3\equiv\langle 3\pi|Q_i|K^0\rangle$. In the large-$N$ approach, the higher-derivative chiral representation of (Q_2+2Q_1) is derivable. First, in the absence of gluonic corrections to all planar graphs, the nonanomalous higher-order strong chiral Lagrangians arise from the integration of nontopological chiral anomalies.[30] Hence bosonization of the quark currents can be done to the next-to-leading order in chiral expansion. This in conjunction with the validity of factorization in the limit of large N enables us to derive the higher-derivative chiral realization of (Q_2+2Q_1), as given by Eq. (3.9) of Ref. 29. We find

$$\frac{\langle Q_2+2Q_1\rangle_3^{\text{HO}}}{\langle Q_2+2Q_1\rangle_3^{\text{LO}}}=\frac{\langle Q_2-Q_1\rangle_3^{\text{HO}}}{\langle Q_2-Q_1\rangle_3^{\text{LO}}}\ , \tag{3.50}$$

at the center of the Dalitz plot [see Eqs. (13) and (14) of Ref. 24; the higher-order representation of (Q_2-Q_1) is given by Eq. (3.55)]. Now it becomes clear from Eqs. (3.46) and (3.48)–(3.50) that the $K^0\to 3\pi$ amplitude will still remain proportional to the $K^0\to 2\pi^0$ one, even in the presence of higher-order chiral corrections if the chiral realization of the penguin operator at higher order is the same as that of (Q_2-Q_1), as originally assumed in Ref. 6. Consequently, the Li-Wolfenstein relation does not get modified.

Crucial to the above argument is the assumption that the penguin operator Q_6 and the $\Delta I=\frac{1}{2}$ operator (Q_2-Q_1) have the same higher-derivative chiral realization. However, this is not the case; we are going to show that the ratio of

$$\frac{\langle Q_6\rangle_3^{\text{HO}}}{\langle Q_2-Q_1\rangle_3^{\text{HO}}}=-\beta\frac{v^2}{\Lambda_\chi^2} \tag{3.51}$$

has a parameter β different from four [cf. Eq. (3.48)]. That is, these two operators behave differently at higher order. Consequently, we have

$$\frac{\text{Im}A(K^0\to 3\pi)}{\text{Re}A(K^0\to 3\pi)}\simeq\frac{\text{Im}A(K^0\to 2\pi^0)}{\text{Re}A(K^0\to 2\pi^0)}\left[1-\frac{(4/\beta-1)\langle Q_6\rangle_3^{\text{HO}}}{\langle Q_6\rangle_3^{\text{LO}}+(4/\beta)\langle Q_6\rangle_3^{\text{HO}}}\right]\ , \tag{3.52}$$

where the term

$$b_6\langle Q_6\rangle_3^{\text{HO}}/(b_1\langle Q_2-Q_1\rangle_3^{\text{LO+HO}}+b_2\langle Q_2+2Q_1\rangle_3^{\text{LO+HO}}+b_6\langle Q_6\rangle_3^{\text{LO}}+b_6\frac{4}{\beta}\langle Q_6\rangle_3^{\text{HO}})$$

has been neglected.[31] It follows from Eq. (3.9) that

$$\Omega^{\text{HO}}=-\frac{i}{2\epsilon'_{\text{NL}}}\frac{\text{Im}A(K^0\to 2\pi^0)}{\text{Re}A(K^0\to 2\pi^0)}\left[\frac{4}{\beta}-1\right]\frac{\langle 3\pi|Q_6^{\text{HO}}|K^0\rangle}{\langle 3\pi|Q_6^{\text{LO}}+(4/\beta)Q_6^{\text{HO}}|K^0\rangle}\ ,$$
$$=\frac{1}{\sqrt{2}\omega}\left[\frac{4}{\beta}-1\right]\frac{\langle 3\pi|Q_6^{\text{HO}}|K^0\rangle}{\langle 3\pi|Q_6^{\text{LO}}+(4/\beta)Q_6^{\text{HO}}|K^0\rangle} \tag{3.53}$$

is nonzero when $\beta\neq 4$ and can be very large since it is not subject to the $\Delta I=\frac{3}{2}$ suppression.

One can employ either (3.53) or (3.10) to compute the higher-order effect Ω^{HO}. To determine the parameter β defined in Eq. (3.51), we need to know the higher-derivative chiral representation of Q_6, which can be obtained from Eqs. (2.6) and (2.16):

$$Q_6^{\text{HO}}=-\frac{1}{2}f_\pi^4\frac{v^2}{\Lambda_\chi^4}\text{Tr}(\lambda_6L_\mu L^\mu L_\nu L^\nu)\ . \tag{3.54}$$

Moreover, from Ref. 32 we have

$$(Q_2-Q_1)^{\text{HO}}=\frac{f_\pi^2}{4}\mathcal{O}^{\text{HO}}\ , \tag{3.55}$$

with

$$\mathcal{O}^{\rm HO}=h_1{\rm Tr}(\lambda_6L_\mu L^\mu L_\nu L^\nu)+h_2{\rm Tr}(\lambda_6L_\mu L_\nu L^\mu L^\nu)+h_3{\rm Tr}(\lambda_6L_\mu L_\nu L^\nu L^\mu)$$
$$+h_4{\rm Tr}(\lambda_6L_\mu L_\nu){\rm Tr}(L^\mu L^\nu)+h_5{\rm Tr}(\lambda_6\tilde{Y}\tilde{Y})+h_6{\rm Tr}([\lambda_6,\tilde{Y}]L_\mu L^\mu)+h_7{\rm Tr}([\lambda_6,\tilde{Y}_{\mu\nu}]L^\mu L^\nu)\ , \tag{3.56}$$

where[33]

$$3h_1=-h_2=h_4=3h_6=-3h_7=\frac{N}{8\pi^2},\quad h_3=h_5=0\ , \tag{3.57}$$

and $Y_{\mu\nu}=(\partial_\mu\partial_\nu U)U^\dagger$, $\tilde{Y}_{\mu\nu}=Y_{\mu\nu}-Y^\dagger_{\mu\nu}$, and $\tilde{Y}=g^{\mu\nu}\tilde{Y}_{\mu\nu}$. From Eqs. (3.54) and (3.55), it is obvious at this point that the structure of $Q_6^{\rm HO}$ and $(Q_2-Q_1)^{\rm HO}$ is not of the same form. After some manipulation, we find,[34] for $|3\pi\rangle=|\pi^+\pi^-\pi^0\rangle$ or $|\pi^0\pi^0\pi^0\rangle$,

$$\frac{\langle 3\pi|Q_6|K^0\rangle^{\rm HO}}{\langle 3\pi|Q_2-Q_1|K^0\rangle^{\rm HO}}=-\frac{v^2}{\Lambda_\chi^2},\quad \frac{\langle 3\pi|Q_6|K^0\rangle^{\rm HO}}{\langle 3\pi|Q_6|K^0\rangle^{\rm LO}}=\frac{1}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\ , \tag{3.58}$$

and hence $\beta=1$. Substituting (3.58) into (3.53) yields

$$\Omega^{\rm HO}=\frac{1}{\sqrt{2}\omega}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\left[1-\frac{2}{3}\frac{m_K^2}{\Lambda_\chi^2}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\right]^{-1}\ . \tag{3.59}$$

[Note that the pole contribution due to the four-derivative strong chiral Lagrangian should be taken into account in the denominator of (3.53).]

Care must be taken when Eq. (3.10) is employed to evaluate $\Omega^{\rm HO}$. Let us write $Q_6=-4(v^2/\Lambda_\chi^2)(Q_2-Q_1)+\tilde{Q}_6$. It follows from (3.54) and (3.55) that

$$\tilde{Q}_6=v^2\left[\frac{f_\pi^2}{\Lambda_\chi^2}\mathcal{O}^{\rm HO}-\frac{1}{2}\frac{f_\pi^4}{\Lambda_\chi^4}{\rm Tr}(\lambda_6L_\mu L^\mu L_\nu L^\nu)\right]\ . \tag{3.60}$$

As discussed before, the higher-order chiral terms of the first piece of Q_6 do not contribute to $\Omega^{\rm HO}$. Hence one should apply the higher-derivative operator $\tilde{Q}_6$ to compute the imaginary part of $A^{\rm HO}(K^0\to 3\pi)$. The $\tilde{Q}_6$-induced $K\to 3\pi$ transition is found to be

$$\langle 3\pi^0|\tilde{Q}_6|K^0\rangle=3\langle\pi^+\pi^-\pi^0|\tilde{Q}_6|K^0\rangle=\frac{2\sqrt{2}}{3}v^2\frac{m_K^4}{\Lambda_\chi^4}\left[1-3\frac{m_\pi^2}{m_K^2}\right]\ . \tag{3.61}$$

It is easy to check that this leads to the same result as (3.59).

Since, numerically,

$$\Omega^{\rm HO}_{+-0}=\Omega^{\rm HO}_{000}=3.66\ , \tag{3.62}$$

it is evident that the Li-Wolfenstein relation between $\epsilon'_{3\pi}$ and $\epsilon'_{2\pi}$ gets a very large modification from the higher-derivative chiral terms. Furthermore, the higher-derivative chiral correction dominates over the effect of electroweak penguins and isospin breaking, as expected.

E. Numerical results for $\epsilon'_{3\pi}/\epsilon$

We are now in position to present the numerical results for $\epsilon'_{3\pi}/\epsilon$. From Eqs. (3.9) and (2.8), we have

$$\frac{\epsilon'_{3\pi}}{\epsilon}=-G_F\frac{\omega}{{\rm Re}A_0|\epsilon|}({\rm Im}\lambda_t)y_6\langle Q_6\rangle_0(1-\Omega_{\rm oct}-\Omega_{27}-\Omega_p-\Omega^{\rm EWP}_{3\pi}-\Omega^{\rm IB}_{3\pi}-\Omega^{\rm HO}_{3\pi})$$
$$=4.82\times 10^{-2}y_6(1-\Omega^{\rm tot}_{3\pi})\ . \tag{3.63}$$

Hadronic matrix elements have been evaluated in the large-N approach in the previous sections (note that in this framework $\Omega_p=0$). Analytic expressions for $\Omega^{\rm EWP}$, $\Omega^{\rm IB}$, and $\Omega^{\rm HO}$ are given by (3.22), (3.41), and (3.59), respectively, for $K^0\to\pi^+\pi^-\pi^0$, and (3.26), (3.43), and (3.59), respectively, for $K^0\to 3\pi^0$. It is worth emphasizing that effects of isospin breaking and higher-order chiral terms are independent of the top-quark mass.

From Table II or Fig. 3 we see that direct CP violation in the $K^0\to 3\pi$ system behaves in a way drastically different from that in the $K\to 2\pi$ sector: $\epsilon'_{3\pi}/\epsilon$ is in general of order 10^{-2} and increases with the heavy-top-quark mass, contrary to the decreasing $\epsilon'_{2\pi}/\epsilon$ with m_t. As stressed in passing, the lowest-order relation between $\epsilon'_{3\pi}$ and $\epsilon'_{2\pi}$ can be significantly modified by the inclusion of the higher-order chiral effect because of the absence of

$\Delta I = \frac{3}{2}$ suppression for the latter. Effects of isospin breaking and Z^0 penguin diagrams, though not as dramatic as the higher-derivative chiral terms, are by no means less important than the counterparts in $K^0 \to 2\pi$ transition. The combined result of all those three effects leads to a large $\epsilon'_{3\pi}$ signal, which is an order of magnitude larger than previously estimated. Unlike a possible superweak behavior of $\epsilon'_{2\pi}/\epsilon$ in the standard model for very large m_t, a strong signal of $\epsilon'_{3\pi}/\epsilon$ is always expected, irrespective of the top-quark mass.

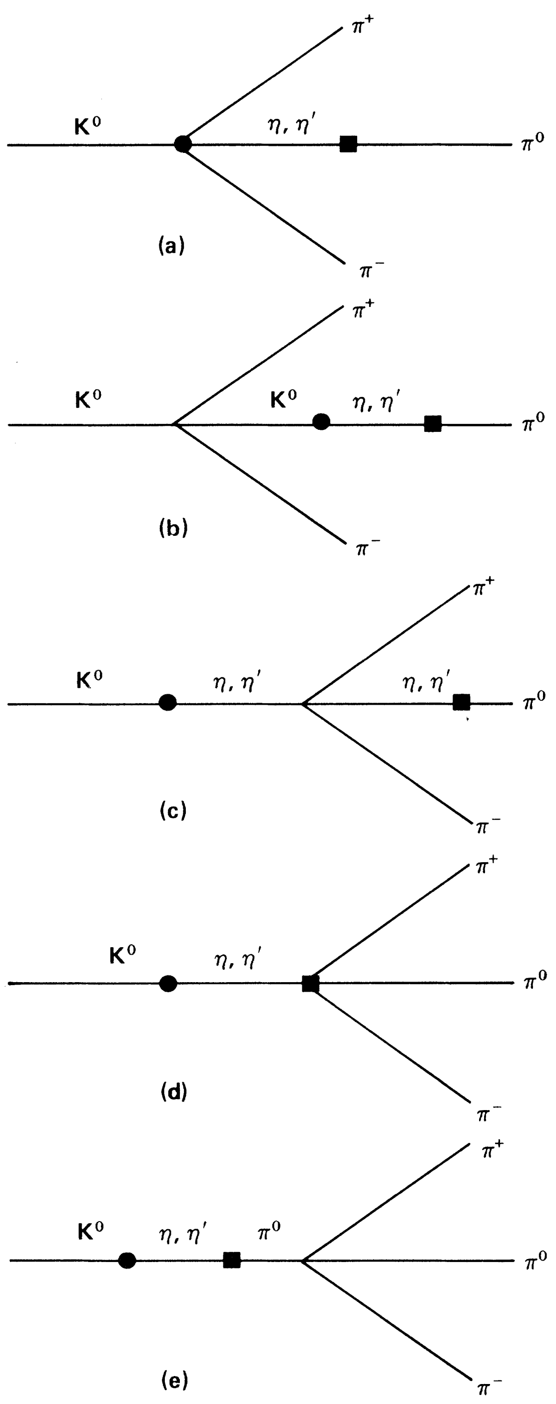

FIG. 2. Diagrams contributing to $K^0 \to \pi^+\pi^-\pi^0$ decay amplitudes via π^0-η-η' mixing and $\eta,\eta' \to 3\pi$ transitions. The black box represents the isospin-breaking interaction, and the black dot denotes a weak transition.

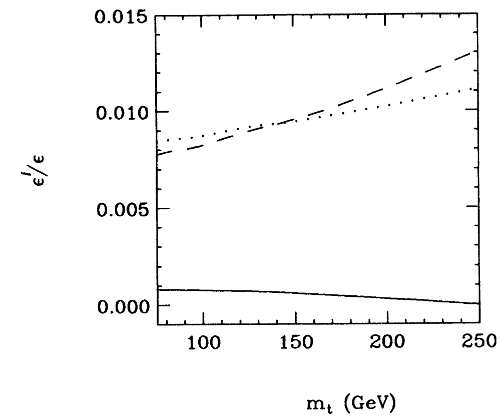

FIG. 3. Values of $\epsilon'_{2\pi}/\epsilon$ (solid curve), ϵ'_{+-0}/ϵ (dotted curve) and ϵ'_{000}/ϵ (dashed curve) vs the top-quark mass.

IV. SUMMARY AND CONCLUSIONS

It was realized recently that decay-amplitude CP violation in the $K \to 2\pi$ sector characterized by the parameter ϵ'/ϵ is strongly suppressed by the presence of Z^0 penguin diagrams for very large m_t. The standard KM model of CP noninvariance is milliweak in nature for $m_t \sim 100$ GeV, but it can behave like a superweak theory for $m_t \gtrsim 200$ GeV.

As for $K^0 \to 3\pi$ decay, direct CP violation $\epsilon'_{3\pi}$ is related to that in $K \to 2\pi$ via the current-algebra Li-Wolfenstein relation $\epsilon'_{3\pi} = -2\epsilon'_{2\pi}$ (final-state interactions being neglected). There are three important corrections to this relation: isospin breaking, electroweak penguins mediated by Z^0 and photon exchanges, and higher-order effective weak chiral Lagrangians. Corrections due to those three effects are potentially large because of the lack of $\Delta I = \frac{3}{2}$ suppression. However, owing to the smallness of the isospin-breaking effect and of the fine-structure constant, it is expected that the higher-derivative chiral effect dominates the modification to the Li-Wolfenstein relation as the chiral suppression is of order m_K^2/Λ_χ^2. The aforementioned three effects are studied in the $1/N_c$ approach. Especially, this method enables us to estimate the effects of higher-derivative chiral terms in the weak chiral Lagrangian. Since we work in the leading $1/N_c$ expansion, final-state interactions are consistently neglected.

Just as the QCD penguin, we find that, at the center of the Dalitz plot, contributions due to electroweak penguins are also *chirally suppressed.* This means that a chiral expansion of the electroweak-penguin operator to

TABLE II. Same as Table I except for $K_S^0\to\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$. The values of $\epsilon'_{3\pi}/\epsilon$ are obtained at the center of the Dalitz plot.

m_t (GeV)	y_6	$\Omega_{27}+\Omega_{\rm oct}$	$\Omega^{\rm HO}$	$\Omega^{\rm EWP}_{+-0}$	$\Omega^{\rm EWP}_{000}$	$\Omega^{\rm IB}_{+-0}$	$\Omega^{\rm IB}_{000}$	$\Omega^{\rm tot}_{+-0}$	$\Omega^{\rm tot}_{000}$	ϵ'_{+-0}/ϵ	ϵ'_{000}/ϵ
75	−0.054	−0.03	3.66	−0.10	−0.20	0.71	0.56	4.24	3.99	0.84×10^{-2}	0.78×10^{-2}
100	−0.055	−0.05	3.66	−0.03	−0.06	0.71	0.56	4.29	4.10	0.87×10^{-2}	0.82×10^{-2}
125	−0.057	−0.09	3.66	0.06	0.12	0.71	0.56	4.34	4.26	0.92×10^{-2}	0.89×10^{-2}
150	−0.057	−0.10	3.66	0.17	0.36	0.71	0.56	4.44	4.47	0.95×10^{-2}	0.95×10^{-2}
200	−0.058	−0.16	3.66	0.45	0.92	0.71	0.56	4.66	4.98	1.02×10^{-2}	1.11×10^{-2}
250	−0.059	−0.24	3.66	0.78	1.62	0.71	0.56	4.91	5.59	1.11×10^{-2}	1.31×10^{-2}

higher order is necessary in order to evaluate the effect of the Z^0 penguin diagram. The isospin-breaking effect is manifest in $K^0\to 3\pi$ via π^0-η-η' mixing and via $\eta,\eta'\to 3\pi$ transitions.

Since nonleptonic direct CP violation is governed by the QCD penguin operator Q_6, a higher chiral effect on $\epsilon'_{3\pi}$ can occur if Q_6 and the $\Delta I=\frac{1}{2}$ operator (Q_2-Q_1) behave differently at higher order. It is the assumption that the operators (Q_2-Q_1) and Q_6 have the same higher-derivative chiral realization that leads to the previous claim that the inclusion of the next-to-leading operators does not modify the Li-Wolfenstein relation in the isospin limit. We have shown in the present paper that this is not the case, especially for the CP-violating part. The different behavior of the above two operators at higher order actually accounts for the main bulk of the corrections to $\epsilon'_{3\pi}$.

We find a substantial enhancement of $\epsilon'_{3\pi}$ due to the higher-order chiral terms in the weak chiral Lagrangian and sizable corrections due to isospin-breaking effect and electroweak penguins. The overall effect is that $\epsilon'_{3\pi}/\epsilon$ appears to be of order 10^{-2} and increases (though not very sensitive) with m_t. Unlike $\epsilon'_{2\pi}/\epsilon$ in the case of $K\to 2\pi$, there is no "superweak" region for $\epsilon'_{3\pi}/\epsilon$ for any allowed region of m_t. Therefore, if the top-quark mass turns out to be larger than 200 GeV, a large signal of decay-amplitude CP violation is expected to be seen in the decay $K^0\to 3\pi$ rather than in $K^0\to 2\pi$.

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is negligible at the renormalization scale around 750 MeV.

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$$A_0 = ig_8(4\sqrt{3}/f_\pi^3)(m_K^2 - m_\pi^2) ,$$
$$A_2 = ig_{27}(4\sqrt{6}/f_\pi^3)(m_K^2 - m_\pi^2) .$$

As pointed out in Ref. 26, there is a small discrepancy between current algebra and the effective-Lagrangian approach for the relations between $K \to 3\pi$ and $K \to 2\pi$. For example, the term proportional to $(9/2\sqrt{2})z$ in Eq. (3.3) does not appear in the current-algebra analysis.

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$$\langle \pi^+|Q_8|K^+\rangle = -12[\tfrac{2}{3}\langle \pi^+|\bar{u}_R d_L|0\rangle\langle 0|\bar{s}_L u_R|K^+\rangle - \tfrac{1}{6}\langle \pi^+|\bar{s}d|K^+\rangle(\langle \bar{d}d\rangle + \langle \bar{s}s\rangle)] .$$

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[34]The result of (3.58) can be found in Ref. 24 or 26.