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Derivation of the two-component model for multiplicity distributions from a stochastic branching mechanism

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Multiplicity distributions in high-energy hadronic reactions are broader in the central rapidity region than in the fragmentation region. An explanation of this phenomenon in terms of a stochastic branching equation is given supporting the picture of two sources, one mainly chaotic and one essentially coherent.

I. INTRODUCTION

Multiplicity distributions in high-energy reactions are a subject of high current interest because of their potential role in elucidating the mechanism of particle production in "soft" strong interactions. In the last years the interesting observation has been made that the probability $P(n)$ to produce n particles depends, among other things, on the rapidity region.[1,2] This dependence is twofold.

(I) $P(n)$ broadens with the width Y of the rapidity region.

(II) $P(n)$ narrows with the shift of the position y of the rapidity interval considered from the central towards the fragmentation region.

For the plateau part of the rapidity region, where the hadronic fields may be assumed to be "stationary" in y, the Y dependence of $P(n)$ can be understood within a quantum-statistical (QS) formalism in terms of a source which generates a superposition of coherent and chaotic fields, with a finite rapidity coherence length.[3] Outside the plateau region, the applicability of the QS formalism used in Ref. 3 is not granted anymore.

To cope with this "nonstationary" region by using the same QS concepts of coherence and chaos two possibilities emerge. One could try to generalize the stationary QS formalism to include also nonstationary fields. Such an enterprise is at present in the process of completion[4] and is useful as long as the derivation from stationarity is small. Near the fragmentation region, however, where the hadronic fields change rapidly with y, another approach is necessary. Such an approach was suggested in Ref. 5 and led to a semiquantitative explanation of effect (II). In this approach there are two sources: one contributing mainly to the central rapidity region and characterized by a negative-binomial distribution $P_{\rm NB}$ and the other contributing to the whole rapidity region and characterized by a Poisson distribution P_P. The overall multiplicity distribution is then written as a convolution:

$$P(n)=\sum_{n_a,n_b} P_P(n_a)P_{\rm NB}(n_b)\delta(n-n_a-n_b)\ . \qquad (1)$$

When applying Eq. (1) to the experimentally observed multiplicity distributions in shifted rapidity regions at different energies $\sqrt{s}$,[2] in order to interpret effect (II), the following surprising result was found.[5]

The mean multiplicity $\langle n_a\rangle$ of P_P increased with $\sqrt{s}$ quite slowly, consistent with a logarithmic dependence, while the mean multiplicity $\langle n_b \rangle$ of $P_{\rm NB}$ increased faster, consistent with a dependence of the form $s^{1/4}$. This result suggested[5] the possibility of the existence of two sources: one source which is thermally equilibrated and which is acting in the central region and another one displaying characteristics of bremsstrahlung emission and contributing to the whole rapidity region. The possible candidates for these two sources were a gluon plasma and radiating quarks, respectively.[5] Given the important possible implications of this finding, a more microscopic derivation of the two-component (sources) model appears desirable. The purpose of this paper is to provide such a derivation in terms of a stochastic branching equation. Another result reported here is the improvement of the agreement between Eq. (1) and data for the whole rapidity region since the publication of results in Ref. 5. This improvement results from using in the branching equation initial conditions different from those initial conditions corresponding to Ref. 5.

II. STOCHASTIC BRANCHING EQUATION FOR RELATION (1)

We assume that a hadron participating in a collision shakes off or breaks up into two types of partons: Type (*a*) ("break-up coupling" μ_0) cannot cascade while type (*b*) ("break-up coupling λ_0) can (with "branching coupling" λ). An example for such a classification could be partons near mass shell (which do not cascade) and partons far off mass shell (which do cascade). Species (*a*) could also be associated with quarks and species (*b*) with gluons. The coupling between (*a*) and (*b*) is neglected mainly for simplicity reasons. The initial hadron may survive the collision process in the form of a leading particle. As in Ref. 6 we assume the the multiplicity distribution of hadrons is similar to that of the partons which generated them. One may further conjecture that, because of the degradation of momenta, hadrons which originate from cascades prefer to populate the central region.

Such a picture can be represented by the following stochastic branching equation:

$$\frac{\partial}{\partial t}P(n_a,n_b;t)=\mu_0[P(n_a-1,n_b;t)-P(n_a,n_b;t)]+\lambda_0[P(n_a,n_b-1;t)-P(n_a,n_b;t)]+\lambda[(n_b-1)P(n_a,n_b-1;t)-n_bP(n_a,n_b;t)]\ , \quad (2)$$

where μ_0, λ_0, and λ are production rates in the interval dt. Using the techniques of generating functions,[7-10] we find that Eq. (1) is one of its possible solutions. Equation (2) represents two independent emission processes, (*a*) and (*b*), followed by a cascade process (cf. Fig. 1). Given an infinitimal interval dt, the probability of producing a particle of type a from a virtual source is given by $\mu_0 dt$. Similarly the probability of producing a particle of type (b) from a virtual source is given by $\lambda_0 dt$. Finally the probability that particles of type (b) reproduce themselves (as in Fig. 1) is given by $\lambda\, dt$. The variable t is the evolution variable, which could be related to the rapidity[9] or to the incident-energy variable.[11] Let us define the following generating function:[10]

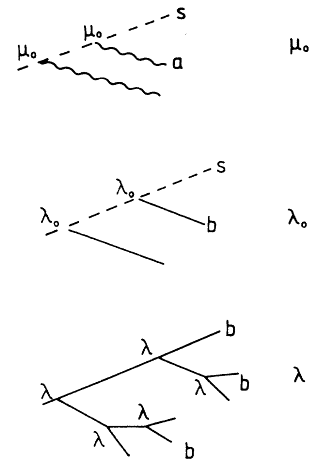

FIG. 1. Typical evolution processes where two types of particles (partons) a and b are produced by virtual source (s). The production vertices are characterized by couplings μ_0 and λ_0 respectively. The bottom process corresponds to the evolution (branching) of a single particle (parton) b. The corresponding production vertex is characterized by a coupling λ.

$$\Pi(u,v;t)=\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}P(n_a,n_b;t)u^{n_a}v^{n_b}\ . \quad (3)$$

From Eq. (3) and Eq. (2) we obtain the following partial differential equation:

$$\frac{\partial\Pi}{\partial t}=[\mu_0(u-1)+\lambda_0(v-1)]\Pi+\lambda v(v-1)\frac{\partial\Pi}{\partial v}\ . \quad (4)$$

The boundary condition for Eq. (4) follows from the initial condition $P(n_a,n_b;t=0)$ as

$$F(u,v)=\Pi(u,v;t=0)=\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}P(n_a,n_b;t=0)u^{n_a}v^{n_b}\ . \quad (5)$$

Then, Eq. (4) has the solution [10]

$$\Pi(u,v;t)=F(u,w)e^{\mu_0 t(u-1)}(v/w)^{-k}\ , \quad (6)$$

where $w=v[1-(e^{\lambda t}-1)(v-1)]$, and $k=\lambda_0/\lambda$. Notice that whereas in the QCD-like cascades it is expected that $k\simeq\frac{1}{2}$ (at least in the limit of large number of colors, $N_c\to\infty$, cf. Refs. 7, 9, and 10), in our case λ_0 should be interpreted as an effective coupling constant equal to λ_0 (QCD) times the number of quarklike jets originating from the colliding hadrons.[7] Therefore, in pp and $p\bar{p}$ collisions which we are dealing with (where we have $p+p\to 2[(q)+(qq)]$), $k=\frac{1}{2}\times 4=2$. This value will be then used throughout the paper. It is amusing to notice that it coincides with the value of the parameter k in the NB distribution $P_{\rm NB}(n_b)$ of Eq. (1) found in Ref. 5.

In the following we shall consider two types of initial conditions.

Initial condition (I). Poisson distribution times Kronecker's δ function:

$$P(n_a,n_b;t=0)=\frac{n_{a0}^{n_a}}{n_a!}e^{-n_{a0}}\delta_{n_b,0}\ . \tag{7}$$

The physical picture for (I) is given in Fig. 2(a). Combinging Eqs. (3) and (7), we have

$$\Pi^{(\mathrm{I})}(u,v;t=T)=e^{(\mu_0T+n_{a0})(u-1)}[1-(v-1)(e^{\lambda T}-1)]^{-k}\ , \tag{8}$$

where $k=\lambda_0/\lambda$ and T is the maximum value of t. On the other hand, the generating function of Eq. (1) is given by a similar formula:

$$\Pi(z)=\sum_{n=0}^{\infty}P(n)z^n =e^{\langle n_a\rangle(z-1)}\left[1-(z-1)\frac{\langle n_b\rangle}{k}\right]^{-k}\ . \tag{9}$$

Equation (9) coincides with Eq. (8) when $u=v=z$, provided that

$$\langle n_a\rangle=\mu_oT+n_{a0}\ , \tag{10}$$

$$\langle n_b\rangle=k(e^{\lambda T}-1)\ . \tag{11}$$

By writing $T=\gamma\ln\sqrt{s}-n_{a0}$, one obtains

$$\langle n_a\rangle=a+b\ln\sqrt{s}\ , \tag{12}$$

$$\langle n_b\rangle=c+ds^{f/2}\ , \tag{13}$$

where $\sqrt{s}$ in GeV and $a=n_{a0}(1-\mu_0)$, $b=\mu_0\gamma$, $c=-k$, $d=k\exp(-\lambda n_{a0})$ and $f=\lambda\gamma$ are suitable constants to be determined from data (cf. Fig. 3 for details). Notice that these equations coincide with the remarkable empirical relationships found in Ref. 5. Notice also that for any given T, the mean multiplicities $\langle n_a\rangle$ and $\langle n_b\rangle$ have to satisfy the following relation obtained by eliminating T from Eqs. (10) and (11):

$$\frac{\langle n_a\rangle-n_{a0}}{\ln(1+\langle n_b\rangle/k)}=\frac{\mu_0}{\lambda}\ . \tag{14}$$

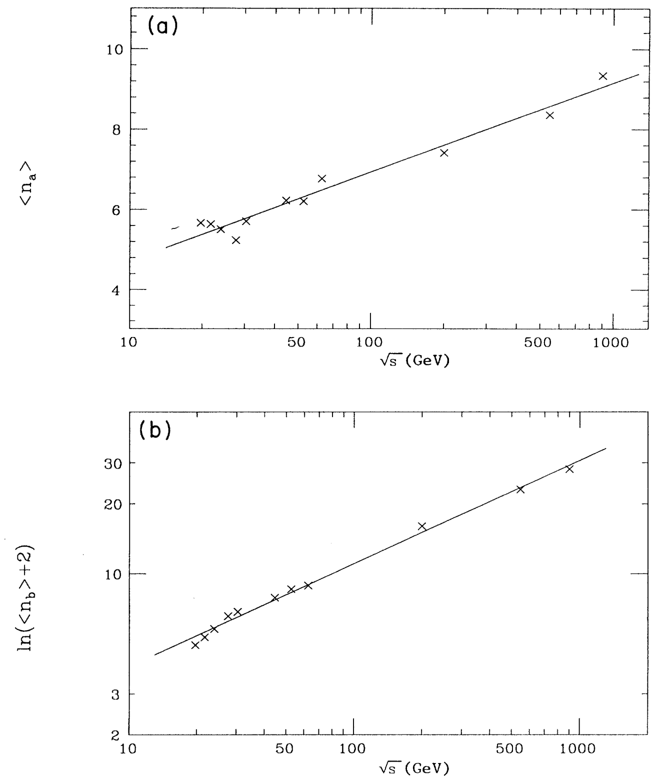

FIG. 3. Analyses of data (Ref. 23) by means of Eqs. (10)–(14) and (17) [initial condition (I)] with $k=2$. The solid line in (a): $\langle n_a\rangle=2.491+0.966\ln\sqrt{s}$, $\sqrt{s}$ in GeV) is determined by the method of linear regression with a correlation coefficient (cc) of 0.982. The solid line in (b): $\ln(\langle n_b\rangle+k)=0.344+0.446\ln\sqrt{s}$, is similarly determined with cc = 0.997. They correspond to $n_{a0}=3.25$, $\lambda=0.11$, $\gamma=4.15$, and $\mu_0=0.23$. Notice that $\lambda_0=2\lambda\simeq\mu_0$. It is straightforward to check that with these values of parameters Eq. (14) is indeed satisfied.

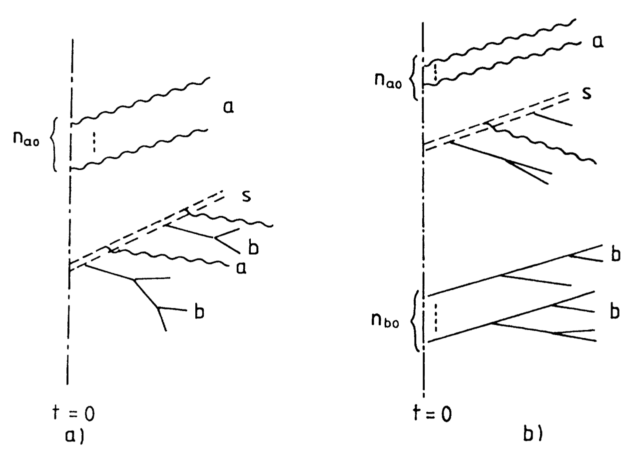

FIG. 2. Various initial conditions. n_{a0} and n_{b0} become zero whenever the Kronecker's δ functions are used. (a) corresponds to $n_{a0}=2$, $n_{b0}=0$, while (b) corresponds to $n_{a0}=2$, $n_{b0}=2$.

Had we used two Kronecker's δ functions ($\delta_{n_a,0},\delta_{n_b,0}$) in Eq. (7), we would have obtained $\langle n_a\rangle=\mu_0T$ in Eq. (10) and no further changes. The solution of Eq. (2) can now be written as $P(n_a,n_b)=P_P(n_a)P_{\mathrm{NB}}(n_b)$. Thus we have shown that Eq. (1) is equivalent to the solution of Eq. (2) with the initial condition (I) establishing therefore a link between the purely phenomenological approach of Ref. 5 and the parton model as represented by the stochastic equation (2).

The factorial moments of the total $P(n)$ are now given by[12]

$$F_q=\langle n(n-1)\cdots(n-l+1)\rangle\ ,$$

$$=\sum_{j=0}^{q}\binom{q}{j}\frac{\Gamma(k+j)}{\Gamma(k)}\left[\frac{p}{k}\right]^j(1-p)^{q-j}\langle n\rangle^q\ , \tag{15}$$

$$F_1=\langle n\rangle=\langle n_a\rangle+\langle n_b\rangle=(1-p)\langle n\rangle+p\langle n\rangle\ , \tag{16}$$

$$F_2=\langle n\rangle^2+p\frac{\langle n\rangle^2}{k}\ , \tag{17}$$

where $p=\langle n_b\rangle/\langle n\rangle$. Notice that in Ref. 5 the quantity p was interpreted as a measure of the amount of chaoticity in the system and was then determined through the relation

$$p=\left[k\left[C_2-1-\frac{1}{\langle n\rangle}\right]\right]^{1/2}\ , \tag{18}$$

where $C_2=F_2/\langle n\rangle^2+1/\langle n\rangle$. By making use of the inverse Poisson transform, we can also calculate the corresponding Koba-Nielsen-Olesen (KNO) scaling function:

$$\Psi_k\left[z=\frac{n}{\langle n\rangle}\right]=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\Pi^{(\mathrm{I})}\left[u=1-\frac{s}{\langle n\rangle},\ v=1-\frac{s}{\langle n\rangle}\right]\exp(sz)ds \tag{19}$$

$$=\frac{k}{p}\frac{1}{\Gamma(k)}\left[\frac{k}{p}[z=(1-p)]\right]^{k-1}e^{[z-(1-p)]k/p}\ \text{for } z\geq(1-p) \tag{20}$$

$$=0\quad\text{for } z<(1-p)\ . \tag{21}$$

The vanishing of the *KNO* scaling function below a certain z suggests the existence of a "threshold" in the multiplicity distribution[8] which may be observable at high energies.

Initial condition (II). Two Poisson distributions: Let us now consider another, more general, initial condition which, as we shall see, provides better agreement with data for higher C_q moments. We take

$$P(n_a,n_b;t=0)=\frac{n_{a0}^{n_a}}{n_a!}e^{-n_a}\frac{n_{b0}^{n_b}}{n_b!}e^{-n_{b0}}\ . \tag{22}$$

The physical picture for (II) is shown in Fig. 2(b). These Poisson distributions correspond to fluctuations[9] in n_a and n_b at $t=0$. It can be shown (cf. Appendix) that the solution of Eq. (2) with the initial condition (22) is

$$P(n)=\sum_{n_a,n_b}P_P(n_a)P_{\mathrm{PM}}(n_b)\delta(n-n_a-n_b)\ , \tag{23}$$

where P_{PM} is the generalized Glauber-Lachs[13] or the Perina-McGill distribution:[14]

$$P_{\mathrm{PM}}(n_b)=\left[\frac{A}{k}\right]^{n_b}\left[1+\frac{A}{k}\right]^{-n_b-k}\times\exp\left[-\frac{|\zeta|^2}{1+A/k}\right]L_{n_b}^{(k-1)}\times\left[-\frac{k|\zeta|^2}{A(1+A/k)}\right]\ . \tag{24}$$

Notice that whereas in (I) all partons of type (b) were supposed to be purely chaotic, now they can also be partially coherent. In Eq. (24) A and $|\zeta|^2$ denote their chaotic and coherent average multiplicities, respectively. Further details of this second case are discussed in the Appendix.

III. ANALYSES OF C_q MOMENTS

We shall first give our results with the initial condition (I) for the experimentally measured $C_q=\langle n^q\rangle/\langle n\rangle^q$ moments in the full rapidity range[15–24] using Eqs. (10)–(14) and (17). The $\sqrt{s}$ dependence of $\langle n_a\rangle$ and $\langle n_b\rangle$ is given in Fig. 3 and is of course, in agreement with Eqs. (12) and (13). A similar analysis is performed for the initial condition (II). For example, for the lower-order factorial moments

$$F_q=\langle n(n-1)\rangle\cdots\langle(n-q+1)\rangle\ , \tag{25}$$

the general solution of Eqs. (23 and 24) gives us (cf. Appendix)

$$F_1=\langle n_a\rangle+\langle n_b\rangle\ , \tag{26}$$

$$F_2=\langle n\rangle^2+p'(2-p')\langle n_b\rangle^2/k\ , \tag{27}$$

with

$$\langle n_a\rangle=\mu_0T+n_{a0}\ ,\quad \langle n_b\rangle=A+|\zeta|^2\ ,$$
$$A=k(e^{\lambda T}-1),\quad |\zeta|^2=n_{b0}e^{\lambda T}\ ,\quad p'=\frac{A}{\langle n_b\rangle}\ , \tag{28}$$

where n_{b0} and the "chaoticity content" p' of $\langle n_b\rangle$ are additional parameters as compared to the case of initial condition (I). A typical n_{b0} value yielding a reasonable description of the higher C_q moments is $n_{b0}=1.5$. The corresponding functional relationship between $\langle n_a\rangle$ and the energy $\sqrt{s}$ is given in the caption of Fig. 4. The general agreement with the data for initial condition (II) is now better than with initial condition (I). While Fig. 4 provides the details of the parametrization, Fig. 5 gives the overall comparison with the C_q moments. As an example, summing overall all energies, the total chi-square d value is $\chi_2=1.05$ for C_3, and $\chi^2=3.95$ for C_4.

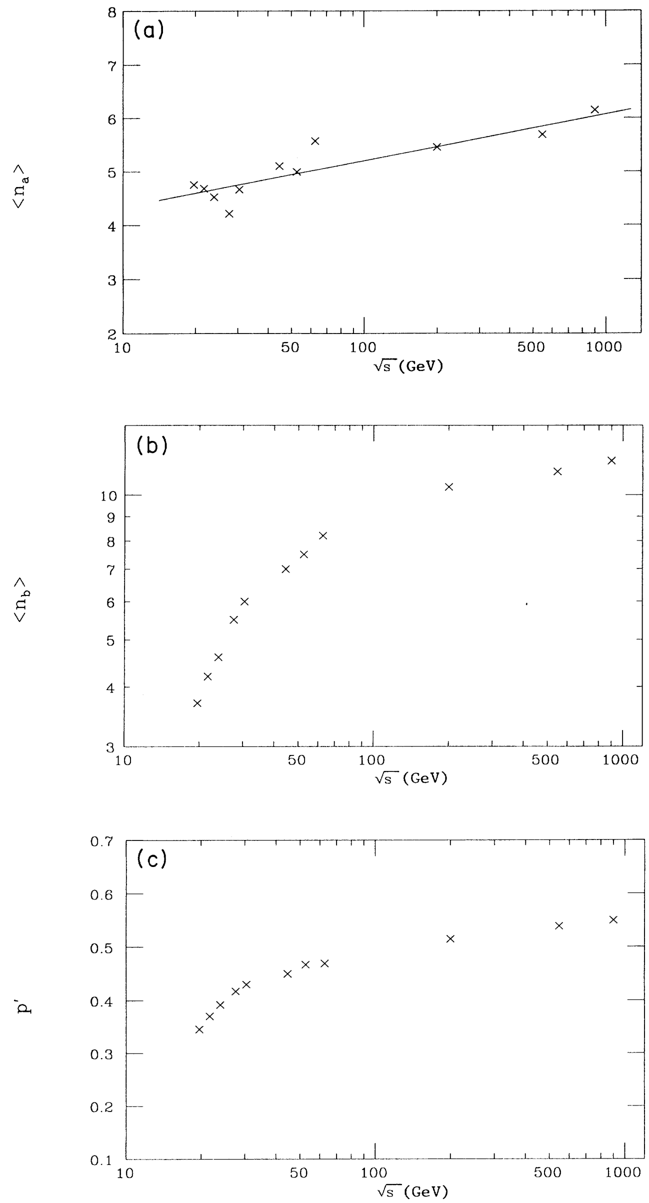

FIG. 4. Analyses of data (Ref. 23) by means of Eqs. (26) and (28) [initial condition (II)] for $k=2$ and $n_{b0}=1.5$. The solid line in (a) is obtained from $\langle n_a \rangle = 3.460 + 0.378 \ln\sqrt{s}$ ($\sqrt{s}$ in GeV, cc = 0.893, cf. Fig. 3). The corresponding average multiplicity $\langle n_b \rangle$ is given in (b) and its associated "chaoticity content" p' is given in (c).

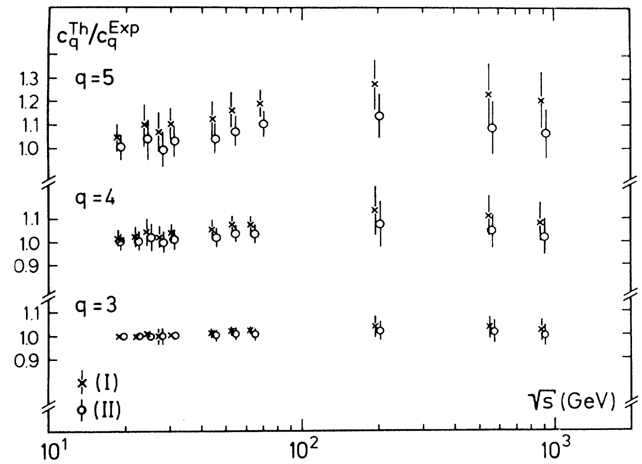

FIG. 5. Comparison (via ratios of experimental to theoretical values) of C_q moments ($q=3$ to 5) for various beam energies; the data are from Refs. 15–24.

IV. CONCLUDING REMARKS

We have demonstrated that Eq. (2) is a possible stochastic branching equation representing the two-component model, Eq. (1) of Ref. 5. Initial condition (I) provides a natural setting for the average multiplicities $\langle n_a \rangle$ and $\langle n_b \rangle$ to evolve with energy, as found in Ref. 5 when T is identified with $\sqrt{s}$. When the initial condition (II) is used, the agreement with experiment (especially for higher C_q moments) is even better, cf. Figs. 4 and 5. This indicates then the need for the presence of initial fluctuations in both types of emitting sources.

The stochastic branching equations discussed here provide a natural framework for incorporation of many other types of fluctuations and initial conditions not addressed here. But it is the relative simplicity of the present approach that permitted us to penetrate into the formidable complexity of the hadronization. Other initial conditions, applications of our formulas to data of rapidity window, KNO scaling problems, and the forward-backward correlation, and other types stochastic branching equations of two-component models will be discussed elsewhere.[25]

The derivation of the two-component model presented here, if taken literally, suggests that in the central region two processes take place.

(1) An independent-emission process consistent with bremsstrahlung, and which exists also in the fragmentation region.

(2) A cascade process.

Whether these two processes can lead to thermal equilibrium, as conjectured in Ref. 5, remains to be proved. It should be also clear that the two-component model presented here and in Ref. 5 does not yet contain a correlation length. This again is a task for the future.

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APPENDIX

By use of the same procedure as before for Eq. (7), we obtain the generating function

$$\Pi^{(\mathrm{II})}(u,v;t=T)=e^{(\mu_0 T+n_{a0})(u-1)}[1-(v-1)(e^{\lambda T}-1)]^{-k}\exp\left[n_{b0}\frac{e^{\lambda T}(v-1)}{1-(e^{\lambda T}-1)(v-1)}\right] . \tag{A1}$$

From Eq. (A1), we can calculate $P(n_a,n_b;t)$ and its moments. To study further Eq. (A1), let us replace in Eq. (1) the NB distribution by a more general distribution P_{PM}:

$$P(n)=\sum_{n_a,n_b} P_P(n_a)P_{\mathrm{PM}}(n_b)\delta(n-n_a-n_b) , \tag{A2}$$

where for P_{PM} we shall choose the generalized Glauber-Lachs,[13] or the Perina-McGill distribution:[14]

$$P_{\mathrm{PM}}(n_b)=\left[\frac{A}{k}\right]^{n_b}\left[1+\frac{A}{k}\right]^{-n_b-k}\exp\left[-\frac{|\zeta|^2}{1+A/k}\right]L_{n_b}^{(k-1)}\left[-\frac{k|\zeta|^2}{A(1+A/k)}\right] . \tag{A3}$$

In Eq. (A3) A and $|\zeta|^2$ denote the chaotic and coherent average multiplicities, respectively. An explicit expression of Eq. (A2) is given by

$$P(n)=\left[1+p'\frac{\langle n_b\rangle}{k}\right]^{-k}\exp\left[1-\frac{(1-p')\langle n_b\rangle}{1+p'\langle n_b\rangle/k}-\langle n_a\rangle\right]\sum_{j=0}^{n}\frac{\langle n_a\rangle^{n-j}}{(n-1)!}\left[1+\frac{k}{p'\langle n_b\rangle}\right]^{-j}L_j^{(k-1)}\left[\frac{k(1-p')}{p'(1+p'\langle n_b\rangle/k)}\right] , \tag{A4}$$

where $\langle n\rangle=\langle n_a\rangle+\langle n_b\rangle$, $\langle n_b\rangle=A+|\zeta|^2$, and $p'=A/\langle n_b\rangle$. Equation (A4) coincides with the Perina-Horak formula in quantum optics.[26] It was also used by Blazek in hadronic distributions.[27] The generating function of the Poisson distribution with the mean multiplicity $\langle n_a\rangle$ and that of Eq. (A3) lead to the expression

$$\tilde{\Pi}^{(\mathrm{II})}(u,v)=e^{\langle n_a\rangle(u-1)}[1-(v-1)A/k]^{-k}\exp\left[\frac{(v-1)|\zeta|^2}{1-(v-1)A/k}\right] . \tag{A5}$$

Equation (A5) with $u=v=z$ becomes the generating function of Eq. (A4). The generating functions $\Pi^{(\mathrm{II})}(u,v;T)$ and $\tilde{\Pi}^{(\mathrm{II})}(u,v)$ are equivalent to each other, provided that the following relations hold:

$$\langle n_a\rangle=\mu_0 T+n_{a0} , \tag{A6}$$

$$A=k(e^{\lambda T}-1) , \tag{A7}$$

$$|\zeta|^2=n_{b0}e^{\lambda T} . \tag{A8}$$

A second solution of Eq. (2) can now be written as $P(n_a,n_b)=P_P(n_a)P_{\mathrm{PM}}(n_b)$, showing that it is equivalent to Eq. (A4) with the initial condition (II). From Eq. (A1), we can obtain the factorial moments

$$F_q=\langle n(n-1)\cdots(n-q+1)\rangle=\sum_{j=0}^{q}\begin{bmatrix} q\\ j\end{bmatrix}\Gamma(j+1)\langle n_a\rangle^{q-j}\left[\frac{p'\langle n_b\rangle}{k}\right]^{j}L_j^{(k-1)}\left[\frac{k(1-p')}{p'}\right] , \tag{A9}$$

$$F_1=\langle n_a\rangle+A+|\zeta|^2=\langle n\rangle , \tag{A10}$$

$$F_2=\langle n\rangle^2+p'(1-p')\frac{\langle n_b\rangle^2}{k} , \tag{A11}$$

where $p'=A/\langle n_b\rangle$. Given p' and n_{b0}, which represent additional parameters as compared to the case of initial condition (I), we have

$$e^{\lambda T}=\frac{(k+n_{b0})+\sqrt{n_{b0}^2+(k+2n_{b0})\langle n\rangle^2(C_2-1-1/\langle n\rangle)}}{k+2n_{b0}} , \tag{A12}$$

where $C_2=F_2/\langle n\rangle^2+1/\langle n\rangle$. Notice that the experimental values of C_2 and $\langle n\rangle$ are used here as input.

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