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Pion-nucleon scattering in the Skyrme model and the *P*-wave Born amplitudes

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We treat fluctuating pion fields around a rotating Skyrmion by means of Dirac's quantization method. The rotational collective motion of the Skyrmion is described by collective coordinates, and conventional gauge-fixing conditions are imposed. Taking into account all the relevant terms at the tree level appearing in the Hamiltonian, we show that pion-nucleon scattering amplitudes exhibit the *P*-wave Born amplitudes attributed to the Yukawa coupling of order $\sqrt{N_c}$, which is consistent with the prediction of chiral symmetry such as the Adler-Weisberger relation. This resolves the difficulty that the Skyrme model predicts a wrong N_c dependence for the coupling of order $N_c^{-3/2}$.

I. INTRODUCTION

Recently, much attention has been attracted to the Skyrme model[1] of the nucleon since Witten[2] conjectured that baryons may appear as topological solitons in the large-N_c limit of QCD. The Skyrme model is essentially a nonlinear σ model of pions supplemented by the Skyrme term to stabilize solitons (Skyrmions), which is considered to be an effective Lagrangian of QCD at low energies. Static properties of the nucleon have been shown to be reproduced within about 30% in the model. It has been further shown[3,4] that the pion-nucleon scattering at higher partial waves is also well described in the model. The description, however, failed at lower partial waves such as *S*, *P*, and *D* waves. It was particularly disappointing to see that there appear no Born amplitudes in the *P* wave necessary to reproduce the Δ-isobar resonance, but a zero-energy pole term appears instead. These seem serious because the model is based on chiral symmetry and possesses the Δ isobar as the rotational excitation, and thus is expected to yield a good description of low-energy phenomena in the pion-nucleon system.

One of the main defects is known to be no appearance of linear pion coupling to the Skyrmion,[5] because the existence of stable soliton solutions means no linear coupling of fluctuating fields. However, zero modes appear, as is well known, because of the breaking of symmetries such as rotational and translational degrees of freedom. In a quantized case, linear coupling may not disappear completely, because there is mismatching of the rotating Skyrmion field with the equation of motion. Following a quantization procedure to eliminate zero modes properly, we can obtain a linear coupling term, but such a surviving coupling term[6,7] could not be of leading order in the $1/N_c$ expansion, where N_c is the number of colors. It is actually of order $N_c^{-3/2}$ (Ref. 6) or $N_c^{-1/2}$ (Ref. 7) depending on gauge-fixing conditions. This N_c scaling is higher by $1/N_c^2$ or $1/N_c$ than that of the πNN pseudovector coupling constant expected from the quark model and current algebra.[8] This discrepancy is serious and is called the Yukawa-coupling problem.

Recently, the amplitudes for a meson scattering off a soliton have been investigated by Uehara[9] in the case of $(1+1)$-dimensional scalar-meson theory. There, it was found that the $1/N_c^2$ amplitudes calculated from a two-meson vertex, the zero-mode pole terms of order N_c^0, and the Born amplitudes calculated from the above surviving Yukawa-coupling term add up to the correct Born amplitudes with the Yukawa-coupling term of order $\sqrt{N_c}$. This work was limited to the static case, but was recently relaxed to the moving soliton case by including the amplitudes of order N_c^{-1} by the present authors.[10] This finding is remarkable, because this shows that, although the linear meson coupling term is of order $N_c^{-3/2}$ under conventional gauge-fixing conditions, the correct Born amplitudes are obtained by including all the relevant terms at the tree level.

In this paper we discuss the quantization of the fluctuating fields around the Skyrmion: The rotational invariance is properly taken into account by introducing the collective coordinates, and the Dirac quantization method[11] is utilized to remove the zero modes. The translational motion of the Skyrmion is not treated in this paper. In a way parallel to the case of $(1+1)$-dimensional scalar-meson field theory,[10] we make use of the reduction formula to calculate the pion-nucleon scattering amplitudes. In Sec. II we show how to introduce the fluctuating fields around the Skyrmion and the constraint conditions imposed on the fluctuating fields to eliminate the zero modes. Further, the Hamiltonian and Dirac commutation relations are presented. Here the canonical momenta are properly symmetrized by following the prescription of Tomboulis.[12] In this way we obtain linear and quadratic coupling terms of pions. In Sec. III we derive the reduction formula to calculate the pion-nucleon scattering amplitudes. We retain the equal-time-commutator term, which remains only when the source functions depend on the canonical momenta. There appear such source functions in nonlinear field theories with solitons when collective quantization methods are applied. In Sec. IV we calculate the various source functions relevant in the tree approximation. In Sec. V we calculate the elastic-scattering amplitudes of the pion-nucleon system and show that the amplitudes agree with those resulting from the classical pion-nucleon form factor of order $N_c^{1/2}$. Conclusions and discussions are given in Sec. VI.

II. QUANTIZATION OF FLUCTUATION FIELDS AROUND THE SKYRMION

The Skyrme Lagrangian is given by

$$\mathcal{L}=\tfrac{1}{4}f_\pi^2\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)+\frac{1}{32e^2}\mathrm{Tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger)]^2\}+\tfrac{1}{4}f_\pi^2 m_\pi^2\mathrm{Tr}(U+U^\dagger-2)\ , \tag{2.1}$$

where U is an SU(2) matrix of the chiral field, f_π the pion decay constant, e the coupling constant of the Skyrme term which is necessary to stabilize the soliton solution, and m_π the pion mass. The static soliton solution is given by the hedgehog ansatz $U_0=\exp[iF(r)\boldsymbol{\tau}\cdot\hat{\mathbf{x}}]$, where $F(r)$ is the chiral angle. To remove the invariance under the global rotation $U\rightarrow AUA^\dagger$, we consider A as time-dependent collective coordinates, where A is an SU(2) matrix. By quantizing these degrees of freedom, we describe the nucleon and the Δ isobar as the rotational excitations of the Skyrmion.

We now represent the fluctuation around the nucleon as chiral perturbation[13] with $U=U_\pi AU_0A^\dagger U_\pi$, where U_π is given by $\exp(i\boldsymbol{\tau}\cdot\boldsymbol{\varphi}/2f_\pi)$ with the fluctuating fields $\varphi_a(\mathbf{x},t)$. Substituting this form into the Lagrangian in Eq. (2.1), we obtain

$$L=\frac{\lambda}{2}\omega_a^2+\int d^3x[\tfrac{1}{2}\dot{\varphi}_a(\mathbf{x},t)K_{ab}(\mathbf{x})\dot{\varphi}_b(\mathbf{x},t)+\dot{\varphi}_a(\mathbf{x},t)\tilde{\Phi}{}^a_b(\mathbf{x})\omega_b]-M_0-\tfrac{1}{2}\int d^3x\,\varphi_a(\mathbf{x},t)\Omega_{ab}\varphi_b(\mathbf{x},t)\ , \tag{2.2}$$

where M_0 is the static mass of the Skyrmion, λ the moment of inertia, and the ω_a's the angular velocities. We used the summation convention for repeated indices. Here we have neglected the terms higher than the second powers of the fluctuating fields. Denoting the adjoint matrix by

$$R_{ai}=\tfrac{1}{2}\mathrm{Tr}(A^\dagger\tau_a A\tau_i)\ , \tag{2.3}$$

we obtain the time derivative of the matrix as follows:

$$\dot{R}_{ai}=\epsilon_{abc}\omega_c R_{bi}\ . \tag{2.4}$$

Here we used the convention that $a,b,c,\ldots$ denote the isospin indices, and $i,j,k,\ldots$ those of the spin in the laboratory frame or the isospin in the soliton-fixed frame. In Eq. (2.2) K_{ab} is a metric, not involving the fluctuating fields, and Ω_{ab} involves second-order differential operators for the spatial variables. Further, $\tilde{\Phi}{}^a_b$ denotes the coupling between the fluctuation and rotation. Here we have used the fact that in the $1/N_c$ expansion the lowest-order term of the coupling is described by the time component of the axial-vector current $A_0^a=2f_\pi\tilde{\Phi}{}^a_b\omega_b$ with

$$\tilde{\Phi}{}^a_b(\mathbf{x})=K_{ac}(\mathbf{x})\Phi_b^c(\mathbf{x})\ , \tag{2.5}$$

where Φ_b^c is the zero-mode solution given by

$$\Phi_b^c=(iL_b\phi_s)^c\ , \tag{2.6a}$$

with

$$\phi_s^a(\mathbf{x})=f_\pi R_{ai}\hat{x}_i\tan F\ , \tag{2.6b}$$

and $(iL_a)^{bc}=\epsilon_{abc}$.

The canonical momenta conjugate to the Euler angles and to the fluctuating fields are given by

$$I_a=\frac{\partial L}{\partial\omega_a}=\lambda\omega_a+\int d^3x\,\dot{\varphi}_b(\mathbf{x},t)\tilde{\Phi}{}^b_a(\mathbf{x})\ , \tag{2.7a}$$

$$\pi_a=\frac{\partial L}{\partial\dot{\varphi}_a}=K_{ab}\dot{\varphi}_b+\tilde{\Phi}{}^a_b\omega_b\ . \tag{2.7b}$$

Note that the I_a's are the angular momenta in the body-fixed frame, which are identified with the isospin operators with the negative sign. The operators in the laboratory frame defined as $J_i=R_{ai}I_a$ are the spin of the Skyrmion. However, the momenta in Eqs. (2.7) are not independent and are constrained by the relations

$$\psi_a=I_a-\int d^3x\,\Phi_a^b(\mathbf{x})\pi_b(\mathbf{x},t)=0\ . \tag{2.8}$$

We can see that the above constraints are first-class ones. Then we impose the gauge-fixing conditions

$$\chi_a=\int d^3x\,\tilde{\Phi}{}^b_a(\mathbf{x})\varphi_b(\mathbf{x},t)=0\ . \tag{2.9}$$

The Poisson brackets between the ψ and χ conditions are

$$\{\psi_a,\chi_b\}_P=\lambda\delta_{ab}-\Xi_{ab}\ , \tag{2.10}$$

where Ξ_{ab} is given by

$$\Xi_{ab}=\int d^3x\,\xi_{ab}^c(\mathbf{x})\varphi_c(\mathbf{x},t)\ , \tag{2.11}$$

with

$$\xi_{ab}^c=-\{I_a,\tilde{\Phi}{}^c_b\}_P\ . \tag{2.12}$$

It can be easily seen that the antisymmetric part of Ξ_{ab} vanishes because of the constraint conditions in Eq. (2.9): namely,

$$\Xi_{ab}-\Xi_{ba}=-\int d^3x\,\tilde{\Phi}{}^d_c(\mathbf{x})\varphi_d(\mathbf{x},t)=0\ , \tag{2.13}$$

where a, b, and c are cyclic. We can therefore write $\Xi_{ab}=\Xi_{(ab)}$ with the definition

$$\Xi_{(ab)}=\tfrac{1}{2}(\Xi_{ab}+\Xi_{ba})\ . \tag{2.14}$$

We now decompose the momentum π_a in the form

$$\pi_a=\tilde{\pi}_a+\frac{1}{\lambda}\tilde{\Phi}{}^a_b I_b\ , \tag{2.15}$$

where the first term $\tilde{\pi}_a$ is transverse to and the second is longitudinal to the rotational collective motions. With this decomposition the ψ conditions turns out to be

$$\tilde{\psi}_a=\int d^3x\ \Phi_a^b(\mathbf{x})\tilde{\pi}_b(\mathbf{x},t)=0\ . \tag{2.16}$$

In this procedure the canonical momentum I_a is modified as

$$\tilde{I}_a=\left[1-\frac{1}{\lambda}\Xi\right]_{ab} I_b\ . \tag{2.17}$$

The Hamiltonian is written as

$$H=I_a\omega_a+\int \pi_a\dot{\varphi}_a-L$$
$$=M_0+\tfrac{1}{2}\int[\pi_a(K^{-1})_{ab}\pi_b+\varphi_a\Omega_{ab}\varphi_b]\ . \tag{2.18}$$

Following Tomboulis,[12] we symmetrize π_a as follows:

$$\pi_a=\tilde{\pi}_a+\frac{1}{\lambda}\tilde{\Phi}{}_b^a(\Lambda^{-1})_{bc}\tilde{I}_c$$
$$\rightarrow\tilde{\pi}_a+\frac{1}{2\lambda}[\tilde{\Phi}{}_b^a(\Lambda^{-1})_{bc}\tilde{I}_c+\tilde{I}_c(\Lambda^{-1})_{bc}\tilde{\Phi}{}_b^a]\ , \tag{2.19}$$

where $\Lambda=1-\Xi/\lambda$. Substituting this into the Hamiltonian [Eq. (2.18)], we find

$$H=M_0+\frac{1}{8\lambda}\{(\Lambda^{-1})_{ab},\tilde{I}_b\}_+\{(\Lambda^{-1})_{ac},\tilde{I}_c\}_++H_{\text{meson}}+H_{\text{qc}}\ , \tag{2.20}$$

where

$$H_{\text{meson}}=\tfrac{1}{2}\int[\tilde{\pi}_a(K^{-1})_{ab}\tilde{\pi}_b+\varphi_a\Omega_{ab}\varphi_b]\ , \tag{2.21}$$

$$H_{\text{qc}}=\frac{1}{4\lambda}\int[\tilde{\pi}_a(K^{-1})_{ab},[\tilde{I}_d,(\Lambda^{-1})_{cd}\tilde{\Phi}{}_c^b]]+\frac{1}{8\lambda^2}\int[(\Lambda^{-1})_{bc}[\tilde{\Phi}{}_b^a,\tilde{I}_c](K^{-1})_{ad}\tilde{\Phi}{}_e^d,\{(\Lambda^{-1})_{ef},\tilde{I}_f\}_+]$$
$$-\frac{1}{8\lambda^2}\int(\Lambda^{-1})_{bc}[\tilde{I}_c,\tilde{\Phi}{}_b^a](K^{-1})_{ad}[\tilde{I}_f,\tilde{\Phi}{}_e^d](\Lambda^{-1})_{ef}\ . \tag{2.22}$$

H_{qc} involve double commutators or the second multiple of single commutators. Hence these are of $O(\hbar^2)$ and are thus at the two-loop level. We shall ignore these in the following, because we want to calculate everything in the tree approximation. Noting Λ to involve the fluctuating fields, we may expand the collective part of the Hamiltonian in Eq. (2.20) in powers of Ξ as follows:

$$\frac{1}{8\lambda}\{(\Lambda^{-1})_{ab},\tilde{I}_{ab}\}_+\{(\Lambda^{-1})_{ac},\tilde{I}_c\}_+=\frac{1}{2\lambda}\tilde{I}{}_a^2+\frac{1}{4\lambda^2}\{\tilde{I}_{ab},\{\Xi_{ab},\tilde{I}_b\}_+\}_+$$
$$+\frac{1}{8\lambda^3}(\{\Xi_{ab},\tilde{I}_b\}_+\{\Xi_{ac},\tilde{I}_c\}_++2\{\tilde{I}_a,\{(\Xi^2)_{ab},\tilde{I}_b\}_+\}_+)+\cdots$$
$$=H_{\text{rot}}+H_1+H_2+\cdots\ , \tag{2.23}$$

where the first term denotes the rotational energy of the Skyrmion, the second one gives rise to the Yukawa coupling which is linear in the fluctuating fields, and the third one gives seagull terms. Note that H_{rot} is of order N_c^{-1}, H_1 order $N_c^{-3/2}$, H_2 order N_c^{-2}, because the moment of inertia λ is of order N_c, and Ξ order $N_c^{1/2}$. These N_c dependences are obtained by noting that f_π and the inverse of the coupling constant of the Skyrme term, $1/e$, are of order $N_c^{1/2}$. The N_c dependence of the meson part H_{meson} is of order N_c^0. It should be noted that we have obtained the Yukawa-coupling term by the quantization procedure, but it is only of order $N_c^{-3/2}$ in contrast with the expected term of order $N_c^{1/2}$.

Having obtained the Hamiltonian, we proceed now to quantize the system. It is well known that the constraint conditions in Eqs. (2.9) and (2.16) modify the Poisson brackets as the Dirac brackets[11] defined by

$$\{f,g\}_P^*=\{f,g\}_P+\frac{1}{\lambda}(\{f,\chi_a\}_P\{\tilde{\psi}_a,g\}_P-\{f,\tilde{\psi}_a\}_P\{\chi_a,g\}_P)\ , \tag{2.24}$$

where the brackets without the asterisks denote the naive ones. We assume that the naive Poisson brackets are given by

$$\begin{aligned}&\{\varphi_a(\mathbf{x},t),\tilde{\pi}_b(\mathbf{y},t)\}_P=\delta_{ab}\delta(\mathbf{x}-\mathbf{y})\ ,\\&\{\tilde{I}_a,\tilde{I}_b\}_P=-\epsilon_{abc}\tilde{I}_c,\\&\{\tilde{I}_a,R_{bk}\}_P=-\epsilon_{abc}R_{ck}\ ,\\&\text{others}=0\ .\end{aligned} \tag{2.25}$$

These lead to the Dirac brackets

$$\{\varphi_a(\mathbf{x},t),\tilde{\pi}_b(\mathbf{y},t)\}_P^*=\delta_{ab}\delta(\mathbf{x}-\mathbf{y})-\frac{1}{\lambda}\Phi_c^a(\mathbf{x})\tilde{\Phi}_c^b(\mathbf{y}) , \tag{2.26a}$$

$$\{\varphi_a(\mathbf{x},t),\varphi_b(\mathbf{y},t)\}_P^*=\{\tilde{\pi}_a(\mathbf{x},t),\tilde{\pi}_b(\mathbf{y},t)\}_P^*=0 , \tag{2.26b}$$

$$\{\tilde{I}_b,\varphi_a(\mathbf{x},t)\}_P^*=\frac{1}{\lambda}\Phi_c^a(\mathbf{x})\Xi_{bc} , \tag{2.26c}$$

$$\{\tilde{I}_b,\tilde{\pi}_a(\mathbf{x},t)\}_P^*=\frac{1}{\lambda}\tilde{\Phi}_c^a(\mathbf{x})\Theta_{bc} , \tag{2.26d}$$

$$\{\tilde{I}_a,\tilde{I}_b\}_P^*=-\epsilon_{abc}\tilde{I}_c-\frac{1}{\lambda}(\Xi_{ac}\Theta_{bc}-\Theta_{ac}\Xi_{bc}) , \tag{2.26e}$$

$$\{\tilde{I}_a,R_{bk}\}_P^*=-\epsilon_{abc}R_{ck} , \tag{2.26f}$$

$$\{\varphi_a(\mathbf{x},t),R_{bk}\}_P^*=\{\tilde{\pi}_a(\mathbf{x},t),R_{bk}\}_P^*=\{R_{ak},R_{bl}\}_P^*=0 , \tag{2.26g}$$

where we have defined

$$\Theta_{ab}=\int d^3x\, \zeta_{ab}^c(\mathbf{x})\tilde{\pi}_c(\mathbf{x},t) , \tag{2.27}$$

with

$$\zeta_{ab}^c=-\{\tilde{I}_a,\Phi_b^c\}_P . \tag{2.28}$$

In quantum theory commutators are given by Dirac brackets as follows:

$$[f,g]=i\{f,g\}_P^* . \tag{2.29}$$

In the following we calculate pion-nucleon scattering amplitudes using reduction formulas within the plane-wave approximation. In this approximation we may neglect the tensor part of the metric K_{ab} in H_{meson}: The explicit form of K_{ab} is given by

$$K_{ab}=K_{ab}^D+K_{ab}^N=R_{ai}(f\delta_{ij}+g\hat{x}_i\hat{x}_j)R_{bj} , \tag{2.30}$$

where f and g are functions of the radial coordinate. We then write the meson part of the Hamiltonian as follows:

$$\begin{aligned}H_{\text{meson}}&=H_{\text{meson}}^{(0)}+H_{\text{meson}}^{(1)}\\&=\tfrac{1}{2}\int[\tilde{\pi}_a(K_D^{-1})_{ab}\tilde{\pi}_b+\varphi_a\Omega_{ab}\varphi_b]\\&\quad+\tfrac{1}{2}\int\tilde{\pi}_a[(K^{-1})_{ab}-(K_D^{-1})_{ab}]\tilde{\pi}_b .\end{aligned} \tag{2.31}$$

We consider that it is necessary to improve on the plane-wave approximation if we take into account $H_{\text{meson}}^{(1)}$. Such an attempt will be considered in a future work.

In the approximation to neglect $H_{\text{meson}}^{(1)}$, it is convenient to redefine the fluctuating fields by

$$\sqrt{f}\,\varphi_a(\mathbf{x},t)\rightarrow\varphi_a(\mathbf{x},t) , \tag{2.32a}$$

$$\frac{1}{\sqrt{f}}\tilde{\pi}_a(\mathbf{x},t)\rightarrow\tilde{\pi}_a(\mathbf{x},t) . \tag{2.32b}$$

In this definition we can write

$$\tilde{\Phi}_b^a=\Phi_b^a\equiv(iL_b\hat{\phi}_s)^a , \tag{2.33}$$

with

$$\begin{aligned}\hat{\phi}_s^a(\mathbf{x})&\equiv\sqrt{f}\,\phi_s^a(\mathbf{x})\\&=f_\pi\sin F\left[1+\frac{1}{e^2f_\pi^2}\left[F'^2+\frac{\sin^2F}{r^2}\right]\right]^{1/2}R_{ai}\hat{x}_i ,\end{aligned} \tag{2.34}$$

and $\zeta_{ab}^c=\xi_{ab}^c$. Furthermore, we see that the commutation relations in Eqs. (2.26) are not altered. Finally, defining

$$\frac{1}{\sqrt{f}}\Omega\frac{1}{\sqrt{f}}\equiv-\nabla^2+m_\pi^2+V , \tag{2.35}$$

we find that the Hamiltonian does not depend on K_{ab}^D explicitly.

III. REDUCTION FORMULA

We want to write the S matrix for scattering of pions off the nucleon through the reduction formula for the fluctuating fields alone in the same way as in the (1+1) scalar-meson theory.[10]

We first define the in and out fields for the pions:

$$\begin{aligned}\varphi_a^{\text{in}}(\mathbf{x},t)&=\int d^3k\frac{1}{\sqrt{2\omega_k}\sqrt{(2\pi)^3}}(a_{\mathbf{k},a}^{\text{in}}e^{i\mathbf{k}\cdot\mathbf{x}}e^{-i\omega_kt}+\text{H.c.})\\&\equiv\int d^3k(a_{\mathbf{k},a}^{\text{in}}f_{\mathbf{k}}(\mathbf{x},t)+\text{H.c.}) ,\end{aligned} \tag{3.1}$$

and similarly for the out fields. $a_{\mathbf{k},a}^{\text{in/out}}$ ($a_{\mathbf{k},a}^{\text{in/out}\dagger}$) is the annihilation (creation) operator, which satisfies

$$[a_{\mathbf{k},a}^{\text{in/out}},a_{\mathbf{k}',b}^{\text{in/out}\dagger}]=\delta(\mathbf{k}-\mathbf{k}')\delta_{ab} . \tag{3.2}$$

We write the initial state for the pion with the momentum $\mathbf{k}$ onto the baryon state B as

$$\begin{aligned}|B,\mathbf{k}a\ \text{in}\rangle&=a_{\mathbf{k},a}^{\text{in}\dagger}|B\rangle\\&=-i\int d^3x\, f_{\mathbf{k}}(\mathbf{x},t)\overleftrightarrow{\partial}_0\varphi_a^{\text{in}}(\mathbf{x},t)|B\rangle .\end{aligned} \tag{3.3}$$

A similar formula is applied to the final state.

Prepared with the above, we now write the S matrix as follows:

$$S_{fi}=\langle N',\mathbf{k}'b \text{ out}|N,\mathbf{k}a \text{ in}\rangle \quad (3.4)$$

$$=\delta_{fi}+i\int d^3x\, dt\, f_{\mathbf{k}}(\mathbf{x},t)\langle N',\mathbf{k}'b \text{ out}|J_a(\mathbf{x},t)|N\rangle , \quad (3.5)$$

where J_a is the source function defined by

$$J_a=\left[\frac{\partial^2}{\partial t^2}-\nabla^2+m_\pi^2\right]\varphi_a \equiv \mathcal{D}\varphi_a . \quad (3.6)$$

In deriving the above single-reduction formula, we have assumed that the fluctuating fields are the interpolating fields

$$\lim_{t\to-\infty}\varphi_a(\mathbf{x},t)=\varphi_a^{\text{in}}(\mathbf{x},t) , \quad \lim_{t\to+\infty}\varphi_a(\mathbf{x},t)=\varphi_a^{\text{out}}(\mathbf{x},t) . \quad (3.7)$$

The same procedure can be applied to the out state to obtain the double-reduction formula

$$S_{fi}=\delta_{fi}+i^2\int d^3x'\, dt'\int d^3x\, dt\, f_{\mathbf{k}}(\mathbf{x},t)f_{\mathbf{k}'}^*(\mathbf{x}',t')\langle N'|\mathcal{D}'T[\varphi_b(\mathbf{x}',t')J_a(\mathbf{x},t)]|N\rangle , \quad (3.8)$$

where the prime on $\mathcal{D}$ means the variables are $\mathbf{x}',t'$, and T is the time-ordered product. Carrying out the time derivatives on the time-ordered product, we finally obtain

$$S_{fi}=\delta_{fi}+i^2\int d^3x'\, dt'\int d^3x\, dt\, f_{\mathbf{k}}(\mathbf{x},t)f_{\mathbf{k}'}^*(\mathbf{x}',t')\langle N'|\{-i\omega_{k'}\delta(t'-t)[\varphi_b(\mathbf{x}',t'),J_a(\mathbf{x},t)]+\delta(t'-t)[\dot\varphi_b(\mathbf{x}',t'),J_a(\mathbf{x},t)] + T[J_b(\mathbf{x}',t')J_a(\mathbf{x},t)]\}|N\rangle . \quad (3.9)$$

Here, to obtain the first term of the equal-time commutators, integration by parts was used. This term vanishes usually, but not in such a nonlinear field theory as the present one. The scattering amplitude is defined as

$$S_{fi}=\delta_{fi}+2\pi i\delta(\varepsilon_f-\varepsilon_i)\prod_{j=i,f}\left[\frac{1}{\sqrt{(2\pi)^3 2\omega_j}}\right]T_{fi} , \quad (3.10)$$

where $\varepsilon_{i,f}$ denotes the sum of the energies of the baryon and meson in the initial or final states.

IV. SOURCE FUNCTIONS

We will calculate scattering amplitudes by means of the reduction formula derived in the previous section. For this purpose we need time derivatives of the fluctuating fields in obtaining the source functions J_a's. These are calculated by means of the commutators with the Hamiltonian; for example,

$$\dot\varphi_a=i[H,\varphi_a] , \quad (4.1)$$

where H is given by Eqs. (2.20) and (2.23).

Noting that the Hamiltonian was expanded in powers of $N_c^{-1/2}$, we can also expand the source function in the same way:

$$J_a=J_a^{(0)}+J_a^{(1)}+J_a^{(3/2)}+J_a^{(2)}+L_a^{(2)}+K_a^{(2)}+\cdots , \quad (4.2)$$

where $J_a^{(n)}$'s mean that they are of order N_c^{-n}. Note that the terms higher than N_c^{-2} contribute only to loop corrections in the scattering amplitudes, because they involve more than the second power of the fluctuating fields. We therefore ignore them in the following.

The zeroth-order term $J_a^{(0)}$ is given as

$$J_a^{(0)}=-[H_{\text{meson}}^{(0)},[H_{\text{meson}}^{(0)},\varphi_a]]+(-\nabla^2+m_\pi^2)\varphi_a = -V_{ab}\varphi_b , \quad (4.3)$$

where V_{ab} is defined in Eq. (2.35).

The term of order N_c^{-1} is written as

$$J_a^{(1)}=-[H_{\text{meson}}^{(0)},[H_{\text{rot}},\varphi_a]]-[H_{\text{rot}},[H_{\text{meson}}^{(0)},\varphi_a]] = -\frac{1}{\lambda^2}\{\tilde I_b,\Phi_c^a\Theta_{(bc)}\}_+ . \quad (4.4)$$

Here the parentheses in the subscript of Θ mean symmetrization in the same way as in Eq. (2.14).

The Yukawa-coupling term H_1 in the Hamiltonian gives the next-order term

$$\begin{aligned}J_a^{(3/2)}=&-[H_1,[H_{\text{meson}}^{(0)},\varphi_a]]-[H_{\text{meson}}^{(0)},[H_1,\varphi_a]]\\ =&-\frac{1}{4\lambda^2}\{\tilde I_c,\{\tilde I_d,\xi_{dc}^a\}_+\}_+ +\frac{1}{4\lambda^3}\Phi_c^a[\{\tilde I_d,\{\Sigma_{(de)}^c,\tilde I_e\}_+\}_+\\ &-2\{\Theta_{(dc)},\{\Xi_{(de)},\tilde I_e\}_+\}_+ -2\{\tilde I_d,\{\Xi_{(de)},\Theta_{(ec)}\}_+\}_+\\ &-\{\tilde I_d,\{\Theta_{(de)},\Xi_{(ec)}\}_+\}_+ -\{\Xi_{(dc)},\{\Theta_{(de)},\tilde I_e\}_+\}_+] ,\end{aligned} \quad (4.5)$$

where

$$\Sigma_{ab}^c=\{\Xi_{ab},\tilde\psi_c\}_P=\int d^3x\,\xi_{ab}^d(\mathbf{x})\Phi_c^d(\mathbf{x}) , \quad (4.6)$$

with the naive Poisson brackets. It can be seen that the symmetrized $\Sigma^c_{(ab)}$ vanishes, because Σ^c_{ab} is antisymmetric in the subscripts as ϵ_{abc}. The other terms within the square brackets are of the second power of the fluctuating fields and give rise to only loop corrections to the reduction formula in Eq. (3.9).

Finally, we calculate the source functions of order N_c^{-2}, denoted by $J_a^{(2)}$, $L_a^{(2)}$, and $K_a^{(2)}$ in Eq. (4.2). Each term is written as follows. First, $J_a^{(2)}$ is defined by

$$\begin{aligned} J_a^{(2)} &= -[H_2,[H^{(0)}_{\text{meson}},\varphi_a]] \\ &= -\frac{1}{8\lambda^3}(\{\{\xi^a_{bc},\tilde{I}_c\}_+,\{\Xi_{bd},\tilde{I}_d\}_+\}_+ + 2\{\tilde{I}_b,\{\tilde{I}_c,\xi^a_{dc}\Xi_{bd}+\Xi_{dc}\xi^a_{bd}\}_+\}_+) + \cdots \ , \end{aligned} \tag{4.7}$$

where the ellipsis represents terms of higher power of the fluctuating fields and gives loop corrections. Second, $L_a^{(2)}$ is defined as

$$\begin{aligned} L_a^{(2)} &= -[H_{\text{meson}},[H_2,\varphi_a]] \\ &= O(\varphi^2\tilde{\pi}) \ . \end{aligned} \tag{4.8}$$

One finds that this contributes to loop corrections only in the S matrix. Third, $K_a^{(2)}$ is given by

$$\begin{aligned} K_a^{(2)} &= -[H_{\text{rot}},[H_{\text{rot}},\varphi_a]] \\ &= -\frac{1}{4\lambda^3}(\epsilon_{ebf}\{\tilde{I}_e,\{\tilde{I}_f,\Phi^a_c\Xi_{(bc)}\}_+\}_+ + \{\tilde{I}_e,\{\tilde{I}_b,(\xi^a_{ec}\Xi_{(bc)}+\Phi^a_c\Xi'_{e,(bc)})\}_+\}_+) + \cdots \ , \end{aligned} \tag{4.9}$$

where $\Xi'_{e,cb}$ was defined as the naive Poisson brackets

$$\Xi'_{e,bc} = \{\tilde{I}_e,\Xi_{bc}\}_P \ . \tag{4.10}$$

The ellipsis in Eq. (4.9) represents, again, terms of higher power giving loop corrections.

Now the time derivative of φ_a is written as

$$\begin{aligned} \dot{\varphi}_a &= \dot{\varphi}_a^{(0)} + \dot{\varphi}_a^{(1)} + \cdots \\ &= \tilde{\pi}_a - \frac{1}{2\lambda^2}\{\tilde{I}_b,\Phi^a_c\Xi_{(bc)}\}_+ + \cdots \ . \end{aligned} \tag{4.11}$$

Here the first term, which is the conjugate momenta to the fluctuating fields, is of order N_c^0, and the second term is of order N_c^{-1} and is linear in the fluctuating fields. The dots represent terms higher than or equal to the second power in the fluctuating fields. Therefore, these will contribute to the equal-time commutators in Eq. (3.9) as only loop corrections.

V. SCATTERING AMPLITUDES OF PIONS OFF THE SKYRMION

We have prepared all the machinery needed to calculate the scattering amplitudes of pions off the nucleon in terms of the reduction formula in Sec. III.

A. Yukawa-coupling term

First, we calculate the contribution of the Yukawa-coupling term, which yields $J_a^{(3/2)}$ in the source function, to the scattering amplitudes. Using the single-reduction formula similar to Eq. (3.5), the S matrix of a one-meson state to a zero-meson state is written as

$$\begin{aligned} \langle B;0\ \text{out}|N;\mathbf{k}a\ \text{in}\rangle &= i\int d^3x\, dt\, f_{\mathbf{k}}(\mathbf{x},t)\langle B|J_a^{(3/2)}(\mathbf{x},t)|N\rangle \\ &= -2\pi i\frac{1}{\sqrt{(2\pi)^3 2\omega_k}}\delta(E_B-E_N-\omega_k)\left[\frac{1}{2\lambda}\right]^2\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}}\langle B|\{\tilde{I}_c,\{\tilde{I}_d,\xi^a_{dc}(\mathbf{x})\}_+\}_+|N\rangle \ , \end{aligned} \tag{5.1}$$

where N and B denote the initial and final baryon states, respectively, and use has been made of Eq. (4.5). Making use of (A2b) in the Appendix, one can rewrite the above as

$$\begin{aligned} \langle B;0\ \text{out}|N;\mathbf{k}a\ \text{in}\rangle &= 2\pi i\frac{1}{\sqrt{(2\pi)^3 2\omega_k}}\delta(E_B-E_N-\omega_k)\left[\frac{1}{2\lambda}\right]^2\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}}\langle B|[\tilde{I}^2,[\tilde{I}^2,\hat{\phi}^a_s(\mathbf{x})]]|N\rangle \ , \\ &= 2\pi i\frac{1}{\sqrt{(2\pi)^3 2\omega_k}}\delta(E_B-E_N-\omega_k)(E_B-E_N)^2\langle B|\tilde{\phi}^a_s(\mathbf{k};\theta)|N\rangle \ , \end{aligned} \tag{5.2}$$

where E_B denotes the energy of the baryon state B described as $E_B = M_0 + I_B(I_b+1)/2\lambda$, with I_B its magnitude of the isospin, and we have defined

$$\tilde{\phi}_s^a(\mathbf{k};\theta) = \int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}} \hat{\phi}_s^a(\mathbf{x}) \; . \tag{5.3}$$

Here θ indicates the Euler-angle dependence of the function explicitly.

We now see that the time ordering of $J^{(3/2)}$ in the reduction formula of Eq. (3.9) yields

$$B_3 = \sum_B \left\{ \frac{[(E_B-E_N)^2\langle N'|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|B\rangle][(E_B-E_N)^2\langle B|\tilde{\phi}_s^a(\mathbf{k};\theta)|N\rangle]}{E_B-E_N-\omega_k} + \frac{[(E_B-E_N)^2\langle N'|\tilde{\phi}_s^a(\mathbf{k};\theta)|B\rangle][(E_B-E_N)^2\langle B|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|N\rangle]}{E_B-E_N+\omega_k} \right\} , \tag{5.4}$$

where $\mathbf{k}$ and $\mathbf{k}'$ denote the momenta of the incident and outgoing pions, respectively, and N and N' the initial and final nucleon states, respectively. Here we consider the elastic scattering so that $E_{N'} = E_N$. Note that the scattering amplitude B_3 is of order N_c^{-3}.

B. Equal-time-commutator term of φ_b with the source function J_a

Second, let us consider the equal-time-commutator term of the fluctuating field with the source function, the first term within the curly brackets in Eq. (3.9). We find, after some straightforward manipulation,

$$[\varphi_b(\mathbf{y},t), J_a(\mathbf{x},t)] = [\varphi_b(\mathbf{y},t), J_a^{(1)}(\mathbf{x},t)] + \text{higher-power terms} , \tag{5.5}$$

where higher-power terms mean that they are of the second or higher power of the fluctuating fields and therefore give only loop corrections in the scattering matrix. From now on we use "higher-power terms" in this meaning. Note that this equal-time commutator yields the term of order $1/N_c$. We find, for the right-hand side of Eq. (5.5),

$$[\varphi_b(\mathbf{y},t), J_a^{(1)}(\mathbf{x},t)] = -\frac{i}{\lambda^2}\{\tilde{I}_d, \Phi_c^a(\mathbf{x})\xi_{(dc)}^b(\mathbf{y})\}_+ + \text{higher-power terms} , \tag{5.6}$$

where use has been made of the vanishing property of the symmetrized $\Sigma_{(ab)}^c$ in Eq. (4.6).

Substituting Eq. (5.6) into the first term in the curly brackets in Eq. (3.9), we find the scattering amplitude of order N_c^{-1}, denoted as B_1, to be

$$B_1 = -i\frac{\omega_k}{\lambda^2}\int d^3x \int d^3y\, e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\mathbf{k}'\cdot\mathbf{y}} \langle B'|\{\tilde{I}_d, \Phi_c^a(\mathbf{x})\xi_{(cd)}^b(\mathbf{y})\}_+|B\rangle \; . \tag{5.7}$$

Using (A3b), one can show that

$$\{\tilde{I}_d, \Phi_c^a(\mathbf{x})\xi_{(cd)}^b(\mathbf{y})\}_+ = \frac{1}{4i}[\hat{\phi}_s^a(\mathbf{x}), [\tilde{I}^2, [\tilde{I}^2, \hat{\phi}_s^b(\mathbf{y})]]] + \text{higher-power terms} \; . \tag{5.8}$$

Here the commutators in the right-hand side are given by the naive ones, since the differences with the Dirac brackets yield terms of higher power of the fluctuating fields. We found in this case that the terms calculated from the constraint conditions in the definition of the Dirac brackets give rise to no contributions at the tree levels. Considering this and inserting the intermediate baryon states B, we find that B_1 in the tree approximation is

$$B_1 = -\omega_k \sum_B (E_B-E_N)^2[\langle N'|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|B\rangle\langle B|\tilde{\phi}_s^a(\mathbf{k};\theta)|N\rangle - \langle N'|\tilde{\phi}_s^a(\mathbf{k};\theta)|B\rangle\langle B|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|N\rangle] \; . \tag{5.9}$$

C. Equal-time-commutator term of $\dot{\varphi}_b$ with the source function J_a

Third, we calculate the equal-time-commutator term of the time derivative of the field with the source function in Eq. (3.9). In the tree approximation, we find

$$[\dot{\varphi}_b(\mathbf{y},t), J_a(\mathbf{x},t)] = [\dot{\varphi}_b^{(0)}(\mathbf{y},t), J_a^{(0)}(\mathbf{x},t)] + [\dot{\varphi}_b^{(0)}(\mathbf{y},t), J_a^{(2)}(\mathbf{x},t)] + [\dot{\varphi}_b^{(0)}(\mathbf{y},t), K_a^{(2)}(\mathbf{x},t)] + [\dot{\varphi}_b^{(1)}(\mathbf{y},t), J_a^{(1)}(\mathbf{x},t)] + \text{higher-power terms} , \tag{5.10}$$

where $\dot{\varphi}_a^{(0)}$ and $\dot{\varphi}_a^{(1)}$ denote the first and second terms in Eq. (4.11), respectively, and the superscripts mean they are of order N_c^0 and N_c^{-1}, respectively. $J_a^{(0)}$, etc., on the right-hand side have been defined in Sec. IV. Note that the equal-time commutators in Eq. (5.10) yield terms of order N_c^0 and N_c^{-2} in the tree approximation.

The term of order N_c^0 is calculated to be

$$[\dot{\varphi}_b^{(0)}(\mathbf{y},t),J_a^{(0)}(\mathbf{x},t)]=iV_{ac}(\mathbf{x})\left[\delta_{bc}\delta(\mathbf{x}-\mathbf{y})-\frac{1}{\lambda}\Phi_d^c(\mathbf{x})\Phi_d^b(\mathbf{y})\right] . \tag{5.11}$$

Substituting this into the reduction formula for the equal-time-commutator term, we find, for the scattering amplitude,

$$B_0=-\int d^3x\, e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{x}}V(\mathbf{x}) -\frac{1}{\lambda}\int d^3x\int d^3y\, e^{i\mathbf{k}\cdot\mathbf{x}}e^{-i\mathbf{k}'\cdot\mathbf{y}}\langle N'|\Phi_c^b(\mathbf{y})(-\nabla^2+m_\pi^2)\Phi_c^a(\mathbf{x})|N\rangle , \tag{5.12}$$

where we have used the fact that Φ_c^a is the zero-mode solution. Now using (A3a), we finally find

$$B_0=-\tilde{V}(\mathbf{k}'-\mathbf{k})-\omega_k^2\sum_B(E_B-E_N)[\langle N'|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|B\rangle\langle B|\tilde{\phi}_s^a(\mathbf{k};\theta)|N\rangle+\langle N'|\tilde{\phi}_s^a(\mathbf{k};\theta)|B\rangle\langle B|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|N\rangle] , \tag{5.13}$$

where $\tilde{V}$ denotes the Fourier transform. Note that we have also ignored loop corrections. We can see that the first term of Eq. (5.13) is the amplitude from the background scattering, and the second denotes the zero-mode pole terms as can be seen later on.

Let us consider the second term in Eq. (5.10), which yields the amplitude of order N_c^{-2}: The equal-time commutator is given by

$$[\dot{\varphi}_b^{(0)}(\mathbf{y},t),J_a^{(2)}(\mathbf{x},t)]=\left[\frac{i}{8\lambda^3}\right](\{\{\xi_{ce}^a(\mathbf{x}),\tilde{I}_c\}_+,\{\xi_{de}^b(\mathbf{y}),\tilde{I}_d\}_+\}_++2\{\tilde{I}_e,\{\tilde{I}_c,\xi_{(cd)}^a(\mathbf{x})\xi_{de}^b(\mathbf{y})+\xi_{(cd)}^b(\mathbf{y})\xi_{de}^a(\mathbf{x})\}_+\}_+) . \tag{5.14}$$

Using (A5), we find that the terms inside the small parentheses on the right-hand side are written as

$$(\cdots)=-\tfrac{1}{2}([\hat{\phi}_s^b(\mathbf{y}),[\tilde{I}^2,[\tilde{I}^2,[\tilde{I}^2,\hat{\phi}_s^a(\mathbf{x})]]]]+3[[\tilde{I}^2,\hat{\phi}_s^b(\mathbf{y})],[\tilde{I}^2,[\tilde{I}^2,\hat{\phi}_s^a(\mathbf{x})]]]) , \tag{5.15}$$

where the square brackets are again taken as the naive ones in the tree approximation. Substituting Eqs. (5.14) and (5.15) into the reduction formula in Eq. (3.9), and using the definition of the scattering amplitude in Eq. (3.10), we read the amplitude of order N_c^{-2} to be

$$B_2=-\sum_B(E_B-E_N)^3[\langle N'|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|B\rangle\langle B|\tilde{\phi}_s^a(\mathbf{k};\theta)|N\rangle+\langle N'|\tilde{\phi}_s^a(\mathbf{k};\theta)|B\rangle\langle B|\tilde{\phi}_s^{b*}(\mathbf{k}';\theta)|N\rangle] . \tag{5.16}$$

Let us now consider the third term in Eq. (5.10). Note that Φ_a^b and Ξ_{ab} are rewritten as

$$\Phi_a^b=i[\tilde{I}_a,\hat{\phi}_s^b] , \quad \Xi_{ab}=-\int[\tilde{I}_a,[\tilde{I}_b,\hat{\phi}_s^c]]\varphi_c , \tag{5.17}$$

provided that the brackets are taken as naive ones. We now derive

$$K_a^{(2)}=-\frac{1}{4\lambda_3}\int d^3y[\tilde{I}^2,\{\tilde{I}_c,[\tilde{I}_d,\hat{\phi}_s^a(\mathbf{x})][\tilde{I}_{(c},[\tilde{I}_{d)},\hat{\phi}_s^e(\mathbf{y})]]\}_+]\varphi_e(\mathbf{y},t)+O(\varphi^3) . \tag{5.18}$$

Then the amplitude calculated from the third term in Eq. (5.10) is proportional to the matrix element between the initial and final nucleon states of

$$[\tilde{I}^2,\{\tilde{I}_c,[\tilde{I}_d,\hat{\phi}_s^a][\tilde{I}_{(c},[\tilde{I}_{d)},\hat{\phi}_s^b]]\}_+]=\tfrac{1}{4}[\tilde{I}^2,[\hat{\phi}_s^a,[\tilde{I}^2,[\tilde{I}^2,\hat{\phi}_s^b]]]] . \tag{5.19}$$

This is apparently zero in elastic scattering. Ignored terms by taking naive commutators in the above yield loop corrections. We can therefore discard the contributions from the third term in Eq. (5.10) in the elastic-scattering amplitudes within the tree approximation.

Now let us consider the fourth term in Eq. (5.10). The relevant commutator is written, after some algebra, in lowest order, as

$$[\dot{\varphi}_b^{(1)}(\mathbf{y},t),J_a^{(1)}(\mathbf{x},t)]\sim i\frac{1}{16\lambda^3}(\{\tilde{I}_c,\{[\tilde{I}_d,\hat{\phi}_s^a(\mathbf{x})],\{\tilde{I}_c,[\tilde{I}_d,\hat{\phi}_s^b(\mathbf{y})]\}_+\}_+\}_++\{\tilde{I}_c,\{[\tilde{I}_c,\hat{\phi}_s^a(\mathbf{x})],\{\tilde{I}_d,[\tilde{I}_d,\hat{\phi}_s^b(\mathbf{y})]\}_+\}_+\}_+) , \tag{5.20}$$

where Eqs. (5.17) were used, and the commutator of Ξ and Θ was calculated as

$$[\Xi_{ab},\Theta_{cd}]=i\frac{\lambda}{2}(\delta_{ac}\delta_{bd}+\delta_{ab}\delta_{cd}) . \tag{5.21}$$

We find that the term in Eq. (5.20) is of higher order by $\hbar^2$ than the other amplitudes. This is easily seen, for example, by means of dimensional counting: The left-hand side of Eq. (5.20) has a dimension (mass)(length^{-4}) in the $c=1$ unit, while the right-hand side has a dimension (mass^2)(length^{-3}), so that the right-hand side has $\hbar^{-1}$ dependence. Let us now consider the case of the amplitude in B_2. The left-hand side of Eq. (5.14) has the same dimension as that of Eq. (5.20), while the right-hand side of Eq. (5.14), obtained by substituting Eq. (5.15) into Eq. (5.14), has a dimension (mass^4)(length^{-1}). It means that we must multiply $\hbar^{-3}$ onto the right-hand side. We similarly find that the amplitudes B_3, B_1, and the second term of Eq. (5.13) have the same $\hbar$ dependence as B_2. This confirms the above statement that Eq. (5.20) is higher order by $\hbar^2$. Hence the contribution can be neglected as that at the two-loop level. This situation is the same as that in (1+1) scalar-meson theory,[10] where the isospin operators are replaced with the canonical momentum conjugate to the coordinate of the center of the soliton.

D. Pion-nucleon scattering amplitudes

Summing up all the amplitudes calculated so far, we find

$$B=B_0+B_1+B_2+B_3$$
$$=-\tilde{V}(\mathbf{k}'-\mathbf{k})+\sum_B\left[\frac{\langle N'|\tilde{J}_s^{b*}(\mathbf{k}';\theta)|B\rangle\langle B|\tilde{J}_s^{a}(\mathbf{k};\theta)|N\rangle}{E_B-E_N-\omega_k}+\frac{\langle N'|\tilde{J}_s^{a}(\mathbf{k};\theta)|B\rangle\langle B|\tilde{J}_s^{b*}(\mathbf{k}';\theta)|N\rangle}{E_B-E_N+\omega_k}\right] . \tag{5.22}$$

In Eq. (5.22) we have defined the classical source function

$$(-\nabla^2+m_\pi^2)\hat{\phi}_s^a(\mathbf{x})=J_s^a(\mathbf{x};\theta) . \tag{5.23}$$

Then $\tilde{\phi}_s^a(\mathbf{k};\theta)$ is written as

$$\tilde{\phi}_s^a(\mathbf{k};\theta)=\tilde{J}_s^a(\mathbf{k};\theta)/\omega_k^2 . \tag{5.24}$$

where $\tilde{J}_s^a(\mathbf{k};\theta)$ is the Fourier transform of $J_s^a(\mathbf{x})$. Note that the amplitudes in Eq. (5.22) are just the desired Born amplitudes of order $\sqrt{N_c}$. If we note that $\langle B'|R_{ai}|B\rangle$ is given by $\Lambda_{B'B}(T_a)_{B'B}(S_i)_{B'B}$ with $\Lambda_{NN}=-\frac{1}{3}$, $\Lambda_{N\Delta}=1/\sqrt{2}$, and $\Lambda_{\Delta\Delta}=-\frac{1}{15}$, $\tilde{J}_s^a(\mathbf{k};\theta)$ is written as

$$\tilde{J}_s^a(\mathbf{k};\theta)=i\frac{G_{B'B\pi}(\mathbf{k}^2)}{2M_N}\mathbf{S}_{B'B}\cdot\mathbf{k}(T_a)_{B'B} , \tag{5.25}$$

where M_N is the nucleon mass and the $B'B\pi$ coupling form factor is given by

$$G_{B'B\pi}(\mathbf{k}^2)=-8\pi M_N f_\pi \Lambda_{B'B}\frac{m_\pi^2+\mathbf{k}^2}{|\mathbf{k}|}\int_0^\infty r^2dr\, j_1(kr)\sin F(r)E(r) , \tag{5.26}$$

with $E(r)=[1+(ef_\pi)^{-2}(F'^2+\sin^2F/r^2)]^{1/2}$. This expression of the $\pi BB'$ coupling agrees with its classical definition used by many authors[14] except for the factor $E(r)$ in the integrand on the right-hand side. It should be noted that this expression is not altered even if the fluctuations around the Skyrmion are introduced as $U=\exp\{i\tau_a[R_{ai}\hat{x}_iF(r)+\varphi_a/f_\pi]\}$.

Note that $F(r)\to(1+m_\pi r)e^{-m_\pi r}/r^2$ at large distance. From this we note that $G_{B'B\pi}$ is finite at $\mathbf{k}^2=-m_\pi^2$. The pseudoscalar coupling constant $g_{\pi NN}$ is given by $G_{B'B\pi}(\mathbf{k}^2=-m_\pi^2)$ and does not depend on the explicit form of $E(r)$ so far as $E(r)\to1$ as r goes to infinity. Therefore, the coupling constant agrees with that of Adkins, Nappi, and Witten.[15] In the case of massless pions, it satisfies the Goldberger-Treiman relation. Here we mention that the second term of Eq. (5.13) denotes the zero-mode pole terms; this can be seen using Eq. (5.24) and noting that the source function J_s^a is finite at $\mathbf{k}^2=-m_\pi^2$.

VI. CONCLUSION AND DISCUSSION

We have calculated the P-wave pion-nucleon scattering amplitudes in the Skyrme model. To describe the rotational invariance of the system, we have used the collective coordinate method and applied the Dirac quantization method to the fluctuating pion fields around the Skyrmion, the rotational zero modes being eliminated. To obtain the Hamiltonian we symmetrized the canonical momenta of the fluctuating fields following the prescription of Tomboulis.[12] The elimination of the zero modes induces a linear-coupling term of pions to the Skyrmion of order $N_c^{-3/2}$.

In describing the elastic pion-nucleon scattering, the reduction formula was used. The equal-time-commutator terms in the reduction formula give the amplitudes of order N_c^{-1} and N_c^{-2} in the tree approximation. Here the term of order N_c^{-1} is obtained from the commutator of the fields with source functions. Such a term usually vanishes because source functions do not involve canonical momenta, but it does not for the present case because of the nonlinear field theory with solitons. Note that Tomboulis' symmetrization prescription of the momenta in deriving the Hamiltonian is essential in obtaining the particular form of the amplitudes of order N_c^{-2} in Eq. (5.16). The amplitudes of order N_c^0 are obtained from the background scattering potential, in which the zero-mode pole terms are involved and are calculated from the equal-time-commutator terms. Now, calculating the time-ordering terms from the linear-coupling term of order

$N_c^{-3/2}$ and adding the above equal-time-commutator terms, we finally obtain the same amplitudes as the Born amplitudes calculated from the πNN coupling of order $\sqrt{N_c}$. It should be noted that the terms of order N_c do not appear because of the cancellation between the direct and crossed amplitudes. Consequently, we find that the soliton model is able to describe accurately the P-wave Born terms in pion-nucleon scattering.

We have calculated the amplitudes within the tree approximation and also disregarded the $\hbar^2$ terms which are considered as those at two loop levels. It is noted that the situation completely corresponds to the case of the $(1+1)$-dimensional meson field theory developed in Ref. 10. Therefore, our result is not special to the Skyrme model.

In deriving the expressions of pion-nucleon scattering amplitudes, we have not taken into account $H^{(1)}_{\text{meson}}$. We consider that this is appropriate in the plane-wave approximation for the scattered pion wave functions. If we transform the fluctuating fields φ_a to $\sqrt{K}_{ab}\varphi_b$, then we find that B_0 in Eq. (5.13) is not altered within the plane-wave approximation. However, the other amplitudes change their expressions. The nondiagonal terms of K_{ab} in $H^{(1)}_{\text{meson}}$ yield the effect of scattering to flip the isospin. Therefore, we must take into account the effect of the distortions of the incident and outgoing waves. This is not within the purpose of this paper.

We have not considered the translational invariance, which is important in the complete description of the P-wave pion-nucleon scattering. Because the zero modes for the translation are written as the sum of the S- and D-wave functions, the inclusion of the translational modes gives a large effect on the S and D waves. Furthermore, the translational modes couple to the rotational ones. The resulting Born amplitudes in the P wave are expected to be written in the center-of-mass system as

$$B=\sum_B\left[\frac{\langle N'|\tilde{J}_s^{b*}(\mathbf{k}';\theta)|B\rangle\langle B|\tilde{J}_s^{a}(\mathbf{k};\theta)|N\rangle}{E_B(\mathbf{p}+\mathbf{k})-E_N(\mathbf{p})-\omega_{\mathbf{k}}}+\frac{\langle N'|\tilde{J}_s^{a}(\mathbf{k};\theta)|B\rangle\langle B|\tilde{J}_s^{b*}(\mathbf{k}';\theta)|N\rangle}{E_B(\mathbf{p}+\mathbf{k})-E_N(\mathbf{p})+\omega_{\mathbf{k}}}\right], \tag{6.1}$$

where $\mathbf{p}$ is the momentum of the nucleon and $E_B(\mathbf{p})=\mathbf{p}^2/2M_N+I_B(I_B+1)/2\lambda$.

We would like to emphasize that our treatment is based on conventional gauge fixing, and that all the relevant terms are included to give the Born amplitudes. Other formalisms are considered to be possible such as the nonrigid gauge theory[7] and the nonconstraint approach.[16] Here we make a brief comment on these approaches within the context of the $(1+1)$-dimensional meson theory discussed in our previous papers[9,10] in order to avoid complications owing to the internal degrees of freedom.

Holzwarth[16] has recently given the argument that the desired P-wave Born amplitudes are obtained easily by ignoring the constraints to eliminate the redundant degrees of freedom coming from the introduction of the collective coordinates. His argument is as follows: The time derivative part of the original Lagrangian is written as

$$\tfrac{1}{2}\int dx\,\dot{\phi}^2(x,t)=\tfrac{1}{2}\int[\dot{\phi}_s^2(x-R)+2\dot{\phi}(x-R)\dot{\chi}(x,t)+\dot{\chi}^2(x,t)]\,, \tag{6.2}$$

where we have expanded the original field $\phi(x,t)$ into the sum of the soliton configuration $\phi_s(x-R)$ centered at $R(t)$ and the fluctuating field $\chi(x,t)$ around it. If we put simply

$$\dot{\phi}_s(x-R)=-\dot{R}\phi_s'(x-R) \quad\text{with } \dot{R}=\frac{P}{M_s}\,, \tag{6.3}$$

where M_s is the soliton mass and $\phi_s'(x)=d\phi_s(x)/dx$, we have, for Eq. (6.2),

$$\frac{P^2}{2M_s}-\frac{P}{M_s}\int dx\,\phi_s'(x-R)\dot{\chi}(x,t)+\tfrac{1}{2}\int dx\,\dot{\chi}^2\,, \tag{6.4}$$

where P is the canonical momentum of R. The interaction Hamiltonian then becomes

$$H_I=-\frac{P}{M_s}\int dx\,\phi_s'(x-R)\dot{\chi}(x,t)\,, \tag{6.5}$$

which corresponds to the interaction Hamiltonian $H_I=\int d^3x\,A_0^a\dot{\phi}_a/2f_\pi$ in the Skyrme model with A_0^a being the time component of the axial-vector current having isospin index a. In Eq. (6.5), $\dot{\chi}$ is replaced by the laboratory momentum field π_χ conjugate to χ at leading order in the expansion by $1/M_s$, where M_s plays the same role as N_c in the Skyrme model. If we transform π_χ into the soliton intrinsic field π as $\pi_\chi(x,t)=\pi(x-R,t)$, then H_I is written as

$$H_I=-\frac{1}{2M_s}\left\{P,\int dx\,\phi_s'(x)\pi(x,t)\right\}_+\,, \tag{6.6}$$

where we put $x-R$ by x after the symmetrization of P and $\phi_s'(x-R)$ is taken. This Hamiltonian of order $M_s^{-1/2}$ vanishes, if we put the constraint conditions $\int\pi\phi_s'=0$ to eliminate the redundant degree of freedom and the resultant interaction of order $N_c^{-3/2}$ at most. Nevertheless, if we take the plane-wave approximation for $\chi(x,t)$, we have fortunately the correct Born terms by combining them with the zero-mode pole terms.

We think that the nonconstraint approach may succeed or fail in obtaining a correct result depending on each case, because the quantization algorithm is not concretely formulated in this nonconstraint approach: If the commutation relations between the fluctuation fields and their canonical conjugate momenta are put equal to the canonical ones, the existence of zero-mode states would break the perturbation theory, and if one wants to eliminate the zero-mode states, one is required to put appropriate constraints on the collective coordinates and fluctuation fields.

In the nonrigid gauge condition theory,[7] as well as in the nonconstraint approach, the Yukawa interaction Hamiltonian of order $M_s^{-1/2}$ is written by $\dot{\chi}$ field as Eq. (6.5), but not by χ. Therefore, the source function of $M_s^{-1/2}$ relevant to the Yukawa interaction, defined as

$$i^2\{[H_{\rm meson},[H_I,\chi(x,t)]]+[H_I,[H_{\rm meson},\chi(x,t)]]\}\ ,$$

is written by terms such as χ^2 and $\chi\pi$ in the lowest power of the fluctuating fields, in contrast to our corresponding source function of order $M_s^{-3/2}$, where there appears a term not involving the fluctuating fields, as seen in the first term in Eq. (4.5). This means that we have to deal with the loop corrections from the first, when we want to construct the scattering amplitudes with them. This would be related to the fact that we need the loop correction even for the calculation of a singe soliton energy.[7] If this is the case, we might have no reason why we could discard other loop corrections.

APPENDIX

Here we derive several expressions used in Sec. V. For simplicity, we use the notations $D\equiv R_{ai}$ and $\tilde{D}\equiv R_{bj}$. Further, we write the commutators of these with the isospin operators as follows:

$$D_a\equiv[\tilde{I}_a,D]\ , \tag{A1a}$$

$$D_{abcd\cdots}\equiv[\tilde{I}_a,D_{bcd\cdots}]\ . \tag{A1b}$$

We then find

$$[\tilde{I}^2,D]=\{\tilde{I}_a,D_a\}_+\ , \tag{A2a}$$

$$[\tilde{I}^2,[\tilde{I}^2,D]]=\{\tilde{I}_a,\{\tilde{I}_b,D_{ba}\}_+\}_+\ , \tag{A2b}$$

$$[\tilde{I}^2,[\tilde{I}^2,[\tilde{I}^2,D]]]=\{\tilde{I}_a,\{\tilde{I}_b,\{\tilde{I}_c,D_{cba}\}_+\}_+\}_+\ . \tag{A2c}$$

We also find

$$[\tilde{D},[\tilde{I}^2,D]]=-2\tilde{D}_aD_a\ , \tag{A3a}$$

$$[\tilde{D},[\tilde{I}^2,[\tilde{I}^2,D]]=-2\{\tilde{I}_a,\tilde{D}_b(D_{ab}+D_{ba})\}_+\ , \tag{A3b}$$

where we used the following identity to derive (A3):

$$\{A,\{B,C\}_+\}_+=\{B,\{A,C\}_+\}_++[[A,B],C]\ . \tag{A4}$$

We now want to show that

$$-\tfrac{1}{2}([\tilde{D},[\tilde{I}^2,[\tilde{I}^2,[\tilde{I}^2,D]]]]+3[[\tilde{I}^2,\tilde{D}],[\tilde{I}^2,[\tilde{I}^2,D]]])$$
$$=\{\tilde{I}_a,\{\tilde{I}_b,\tilde{D}_{bc}D_{ca}+\tilde{D}_{cb}D_{ca}+\tilde{D}_{ca}D_{bc}+\tilde{D}_{ca}D_{cb}\}_+\}_++\{\{\tilde{I}_a,\tilde{D}_{ac}\}_+,\{\tilde{I}_b,D_{bc}\}_+\}_+\ . \tag{A5}$$

We first find that

$$[\tilde{D},[\tilde{I}^2,[\tilde{I}^2,[\tilde{I}^2,D]]]]=-2\{\tilde{I}_a,\{\tilde{I}_b,\tilde{D}_c(D_{bac}+D_{bca}+D_{cba})\}_+\}_+-2\tilde{D}_{cba}D_{cba}\ . \tag{A6}$$

Then, using relations such as

$$\begin{aligned}&D_{bca}=D_{cba}+i\epsilon_{bcd}D_{da}\ ,\\&\tilde{D}_cD_{bca}=\tilde{D}_cD_{cba}+(\tilde{D}_{db}-\tilde{D}_{bd})D_{da}\ ,\end{aligned} \tag{A7}$$

we find

$$\begin{aligned}[\tilde{D},[\tilde{I}^2,[\tilde{I}^2,[\tilde{I}^2,D]]]]=&-6\{\tilde{I}_a,\{\tilde{I}_b,\tilde{D}_cD_{cba}\}_+\}_+-4\{\tilde{I}_a,\{\tilde{I}_b,(\tilde{D}_{cb}-\tilde{D}_{bc})D_{ca}\}_+\}_+\\&-2\{\tilde{I}_a,\{\tilde{I}_b,(\tilde{D}_{ca}-\tilde{D}_{ac})D_{bc}\}_+\}_+-2\tilde{D}_{cba}D_{cba}\ .\end{aligned} \tag{A8}$$

Similarly, we find

$$[[\tilde{I}^2,\tilde{D}],[\tilde{I}^2,[\tilde{I}^2,D]]]=-\{\{\tilde{I}_a,\tilde{D}_{ac}\}_+,\{\tilde{I}_b,D_{bc}\}_+\}_+-2\{\tilde{I}_a,\{\tilde{I}_b,\tilde{D}_{bc}D_{ca}\}_+\}_++2\{\tilde{I}_a,\{\tilde{I}_b,\tilde{D}_cD_{cba}\}_+\}_+\ . \tag{A9}$$

Using (A8) and (A9), we finally obtain (A5).

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