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Final-state interactions and *CP* violation in weak decays

Lincoln Wolfenstein
Department of Physics, Carnegie Mellon University, Schenley Park, Pittsburgh, Pennsylvania 15213
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The *CP*-violating difference between the partial decay rates of a particle and antiparticle depends on final-state interactions. A general formalism is presented for calculating this difference based on *CPT* invariance and unitarity. Applications are given to *B* decays and the formalism is compared to the standard method using penguin graphs.

I. INTRODUCTION

Final-state interactions play an important role in tests for *CP* and *T* violation in weak decays. Triple correlations such as those of the form $\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2$ can occur either as a result of *T* violation or final-state interactions. In order to use such correlations as a test of *T* violation the final-state-interaction effect must be negligible or calculable. A test for *CP* violation is a comparison of the partial decay rates of a particle and its antiparticle. In this case final-state interactions are necessary since in their absence the partial decay rates are equal from *CPT* invariance even if *CP* is violated. In this paper we will be interested in the evaluation of the final-state-interaction effects for this case.

Early examples of the calculation of final-state interactions involved semileptonic decays. For nuclear β decay the electromagnetic interaction between the electron and the residual nucleus may be included by a second-order perturbation calculation (weak plus electromagnetic) which contains an absorptive cut.[1] Alternatively one can do a partial-wave analysis of the final state and adjoin final-state scattering phase shifts to the weak decay amplitudes.[2] Similar considerations have been applied to the semileptonic $K_{\mu 3}$ decay.[2,3] In these cases the results are relatively unambiguous.

In this paper we will be concerned with nonleptonic decays and final-state strong interactions. In a simple example such as $K\rightarrow 2\pi$ it is possible to take account of the final-state scattering using phase shifts deduced from experiments. In the case of *B* decay there are many final states and the language of phase shifts is not useful. The general formalism for dealing with such cases is described in Sec. II. The application to the simple case of *K* decays is given in Sec. III.

The major part of this paper is concerned with *B* decays. The examples of greatest interest are those involving one-loop or penguin graphs. In this case the final-state interaction enters into the standard calculation[4] as the absorptive part of the penguin graph. If one follows this method without special care[5] it is easily possible to violate the constraints of *CPT* invariance and unitarity. This point has been emphasized by Gérard and Hou.[6] In Sec. IV we use the formalism of Sec. II to analyze the example of semi-inclusive *B* decays considered in Ref. 6. In Sec. V we use our formalism to critically analyze the standard use of penguin graphs. In Sec. VI we make some comments on the more difficult problem of exclusive *B* decays.

II. GENERAL FORMALISM

The requirements of *CPT* invariance and unitarity provide a relationship[7] between the weak decay amplitudes of a meson *P* and its antiparticle $\bar{P}$:

$$\langle \bar{F}|T|\bar{P}\rangle^* = \sum_{F'} \langle F|S^\dagger|F'\rangle\langle F'|T|P\rangle \ , \qquad (1)$$

where *T* is the transition amplitude calculated to lowest order in the weak interaction and *S* is the strong plus electromagnetic interaction scattering matrix connecting different final states *F*. Note the *C* invariance of strong interactions means that

$$\langle F|S|F'\rangle = \langle \bar{F}|S|\bar{F}'\rangle \ .$$

Our main interest is in the rate difference

$$\Delta_F = \Gamma(\bar{P}\rightarrow\bar{F}) - \Gamma(P\rightarrow F) \ .$$

In order for such a rate difference to be nonzero it is necessary that *CP* be violated and to have significant final-state interactions. If we consider $|F\rangle$ and $|\bar{F}\rangle$, which are eigenstates $|I\rangle$ and $|\bar{I}\rangle$ of *S*, then Eq. (1) yields

$$\begin{aligned} \langle \bar{I}|T|\bar{P}\rangle &= A_I e^{i\delta_I} \ , \\ \langle I|T|P\rangle &= A_I^* e^{i\delta_I} \ , \end{aligned} \qquad (2)$$

where δ_I are the strong-interaction phase shifts. Even though a nonreal A_I indicates *CP* violation, the rate difference Δ_I is seen to vanish when *I* is an eigenstate. To get a nonzero Δ_F it is necessary to consider a final state (or set of states) that is not an eigenstate and the important *S*-matrix elements are the off-diagonal elements connecting this state to other possible final states.

For our applications the *S* matrix may be considered as block diagonal with a block defined by the flavor of its states. For any one such block we divide the states into two sets *A* and *B*. Then it follows from *CPT* invariance that

$$\Delta_A = -\Delta_B \ . \qquad (3)$$

We write the *S* matrix as

$$S=S_0+S_1 ,$$

where S_0 connects A states to A states and B to B, whereas S_1 connects A to B. We choose as a basis the eigenstates A_α, B_β of S_0. If we treat S_1 perturbatively then to first order in S_1 unitarity and time reversal require

$$S_{0\,\rho\rho}=e^{2i\delta_\rho} \quad (\rho=\alpha \ \text{ or } \beta) , \tag{4a}$$

$$S_{1\,\alpha\beta}=2it_{\alpha\beta}e^{i(\delta_\alpha+\delta_\beta)} , \tag{4b}$$

$$t_{\beta\alpha}=t_{\alpha\beta} , \tag{5}$$

with $t_{\alpha\beta}$ real.

To first order in S_1 the solution to Eq. (1) for the weak transition is given by

$$\begin{aligned}\langle \bar{A}_\alpha|T|\bar{P}\rangle &= e^{i\delta_\alpha}\left[T_\alpha+\sum_\beta it_{\alpha\beta}T_\beta\right] ,\\ \langle \bar{B}_\beta|T|\bar{P}\rangle &= e^{i\delta_\beta}\left[T_\beta+\sum_\alpha it_{\alpha\beta}T_\alpha\right] ,\end{aligned} \tag{6}$$

where the replacement of $\bar{P}$ by P corresponds to changing T_α, T_β to T_α^*, T_β^*. Then

$$\begin{aligned}\Delta_\alpha &= 4\sum_\beta \operatorname{Im}(T_\alpha^* T_\beta)t_{\alpha\beta} ,\\ \Delta_\beta &= 4\sum_\alpha \operatorname{Im}(T_\beta^* T_\alpha)t_{\alpha\beta} .\end{aligned} \tag{7}$$

For the set of states A and B we have

$$\Delta_A=-\Delta_B=4\sum_{\alpha\beta}\operatorname{Im}(T_\alpha^* T_\beta)t_{\alpha\beta} . \tag{8}$$

An important point to note is that the "diagonal" phase shifts $\delta_\alpha, \delta_\beta$ do not enter the answer for the rate difference. Our result is limited by the approximation that only the first order in S_1 is considered. In the case when there are only two states (such as the K^0 system considered in the next section) Eq. (4b) holds in general and the second order in S_1 simply results in multiplying the right-hand side (RHS) of Eq. (4a) by a factor less than unity. The results Eqs. (6)–(8) are unchanged. In the case of B decays discussed in Sec. IV it is usually assumed that the relevant final-state scattering occurs at high enough energy that it can be treated perturbatively.

It is easy to generalize the result to more than two sets of states, A and B. Let us label the sets $I,J,K,\ldots$. Again we define the matrix S_1 to interconnect members of different sets and treat S_1 perturbatively. Then defining the eigenstates of S_0 as I_α, J_β, K_γ, etc., we find

$$\Delta_{IJ}=\sum_\alpha\sum_\beta \operatorname{Im}(T_\alpha^* T_\beta)t_{\alpha\beta} ,$$

$$\Delta_I=\sum_{J\neq I}\Delta_{IJ} ,$$

$$\Delta_{JI}=-\Delta_{IJ} .$$

The CPT requirement is automatically satisfied; that is,

$$\sum_I \Delta_I=0 .$$

III. K DECAYS

The decay $K^0\to 2\pi$ can be analyzed in terms of final states of definite isospin if electromagnetic interactions (and other isospin violations) are neglected; the results are given by Eq. (2) with $I=0$ or 2. Translating this into the $\pi^+\pi^-$ and $\pi^0\pi^0$ states we have

$$\langle \pi^+\pi^-|T|K^0\rangle=(\tfrac{2}{3})^{1/2}A_0e^{i\delta_0}+(\tfrac{1}{3})^{1/2}A_2e^{i\delta_2} ,$$

$$\langle \pi^+\pi^-|T|\bar{K}^0\rangle=(\tfrac{2}{3})^{1/2}A_0^*e^{i\delta_0}+(\tfrac{1}{3})^{1/2}A_2^*e^{i\delta_2} .$$

There is then a rate difference

$$\begin{aligned}\Delta_K &= \Gamma(\bar{K}^0\to\pi^+\pi^-)-\Gamma(K^0\to\pi^+\pi^-)\\ &= \Gamma(K^0\to\pi^0\pi^0)-\Gamma(\bar{K}^0\to\pi^0\pi^0)\\ &= (4\sqrt{2}/3)\sin(\delta_0-\delta_2)\operatorname{Im}A_2^*A_0 .\end{aligned} \tag{9}$$

The CP-violating asymmetry is

$$a=\Delta_K/[\Gamma(\bar{K}^0\to\pi^+\pi^-)+\Gamma(K^0\to\pi^+\pi^-)]\approx -2\operatorname{Re}\epsilon' ,$$

where the last equality follows from standard equations[8] for the usual parameter ϵ' with the approximation that $|\epsilon'|\ll 1$. As expected the result requires CP violation (A_2 and A_0 are not both real) and final-state interactions which interconnect the observed final states $\pi^+\pi^-$ and $\pi^0\pi^0$. It may be worth noting that one does not measure ϵ' in practice by measuring the asymmetry a, but rather $\operatorname{Re}(\epsilon'/\epsilon)$ is measured taking advantage of K^0-$\bar{K}^0$ mixing.

The result of Eq. (9) can be derived from Eq. (7) where α corresponds to $\pi^+\pi^-$ and β to $\pi^0\pi^0$ and there is no summation, if one writes

$$T_\alpha=(\tfrac{2}{3})^{1/2}A_0+(\tfrac{1}{3})^{1/2}A_2 ,$$

$$T_\beta=(\tfrac{1}{3})^{1/2}A_0-(\tfrac{2}{3})^{1/2}A_2 ,$$

$$\operatorname{Im}(T_\alpha^*T_\beta)=\operatorname{Im}A_2^*A_0 ,$$

$$t_{\alpha\beta}=(\sqrt{2}/3)\sin(\delta_2-\delta_0) .$$

The last equation follows from writing the strong S matrix in the $\pi^+\pi^- - \pi^0\pi^0$ representation.

IV. SEMI-INCLUSIVE B DECAYS

For B decays, we first consider an example that has been discussed a great deal in the literature because of the role of penguin graphs. We consider two classes of final states with strangeness $S=-1$, C and U. The states C contain two charmed particles plus a strange particle while the states U contain a strange particle but no charm. The CP-violating rate difference is

$$\begin{aligned}\Delta_C &= \Gamma(\bar{B}\to\bar{C})-\Gamma(B\to C)\\ &= \Gamma(B\to U)-\Gamma(\bar{B}-\bar{U})=-\Delta_U .\end{aligned} \tag{10}$$

We can now use the formalism of Sec. II where (A,B) are replaced by (C,U) and the part of the S matrix called S_1, which is treated perturbatively, is that connecting U states to C states.

In the first approximation the weak transition amplitudes result from tree-level diagrams. For the U states these arise from the quark transition

$$b \to u + \bar{u} + s \ , \tag{11a}$$

while the states C come from

$$b \to c + \bar{c} + s \ . \tag{11b}$$

At this point we neglect the one-loop graphs called penguin graphs. Neglecting S_1 and considering the eigenstates U_α and C_β of S_0 we have

$$\langle \bar{U}_\alpha | T | \bar{B} \rangle = e^{i\delta_\alpha} v_\mu A_\alpha \ , \tag{12a}$$

$$\langle \bar{C}_\beta | T | \bar{B} \rangle = e^{i\delta_\beta} v_c A_\beta \ , \tag{12b}$$

where $v_i = U_{bi} U_{si}^*$ are products of Kobayashi-Maskawa elements and $\delta_\alpha, \delta_\beta$ are final-state phase shifts. The amplitudes A_α, A_β are real since we have factored out the CP-violating part of the weak interaction in terms of v_i. We now include the effect of S_1 perturbatively as in Eq. (6) yielding

$$\langle \bar{U}_\alpha | T | \bar{B} \rangle = e^{i\delta_\alpha} \left[v_u A_\alpha + i v_c \sum_\beta A_\beta t_{\alpha\beta} \right] , \tag{13a}$$

$$\langle \bar{C}_\beta | T | \bar{B} \rangle = e^{i\delta_\beta} \left[v_c A_\beta + i v_u \sum_\alpha A_\alpha t_{\alpha\beta} \right] . \tag{13b}$$

The corresponding equations for B to U_α and B to C_β are given by changing (v_c, v_u) to (v_c^*, v_u^*).

The CP-violating rate difference is then given as in Eq. (8):

$$\Delta_C = -\Delta_U = 4\,\mathrm{Im}(v_u^* v_c) \sum_\alpha \sum_\beta A_\alpha A_\beta t_{\alpha\beta} \ . \tag{14}$$

The standard method of calculation for our example is to consider that U and C are given by the final quark configurations of Eqs. (11a) and (11b) plus the spectator quark. The matrix t is then evaluated using the one-gluon-exchange process

$$u + \bar{u} \rightleftarrows c + \bar{c} \ . \tag{15}$$

Adjoining this on-shell one-gluon-exchange graph to the tree graph of Eq. (5) corresponds to the calculation of an absorptive part of a one-loop (penguin) graph (Fig. 1). We return to this point later. On the other hand the absorptive parts associated with the "diagonal" process

$$u + \bar{u} \to u + \bar{u} \ , \tag{16a}$$

$$c + \bar{c} \to c + \bar{c} \tag{16b}$$

enter our calculation as the phases $(\delta_\alpha, \delta_\beta)$ and do not affect the final result Eq. (14).

So far we have only considered the tree-level amplitudes A_α, A_β. Much of the interest in the $B \to U$ transition lies in the probability that this process is dominated by one-loop penguin graphs. In the Appendix we carry out our calculation including penguin graphs. Following our general formalism we first add the *dispersive* part of the penguin graphs to the RHS of Eqs. (12). Then as in Eqs. (13) we add the final-state-interaction effect of t perturbatively.

Once we consider penguins two more final states S and

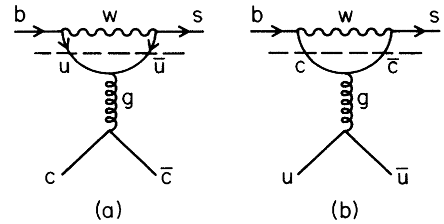

FIG. 1. Absorptive part of penguin graphs that contribute to Δ_C and Δ_U.

D can arise from the transitions

$$b \to s + \bar{s} + s \ , \tag{17a}$$

$$b \to d + \bar{d} + s \ , \tag{17b}$$

respectively. To be clear we now label the set $(U+D+S)$ as N, namely all states with $S=-1$ and no charm, so that

$$\Delta_C = -\Delta_N \ .$$

The somewhat surprising conclusion in the Appendix is that, except for corrections of order α_s, Δ_C and thus $(-\Delta_N)$ is still given by Eq. (14). Thus in calculating Δ_N we can simply ignore penguin graphs except insofar as we represent $t_{\alpha\beta}$ by the absorptive parts of the graphs of Fig. 1. In calculating the total rate for the decay to states N, on the other hand, we expect penguin graphs to dominate because they are proportional to v_c, which is about 40 times larger than v_u, the coefficient of the tree amplitude. However, Δ must be proportional to $v_u v_c^*$ in order to have CP violation, so that the large value of v_c^2 does not contribute to Δ.

As noted at the end of Sec. II it is easy to extend our results to considering separately the four sets U, C, S, and D provided we include in S_1, all the S-matrix elements connecting members of different sets. The quantity previously labeled Δ_C should now be called

$$\Delta_{CU} = -\Delta_{UC} \ ,$$

and is still given by Eq. (14). Labeling the eigenstates of S_0 as U_α, C_β, S_γ, and D_δ we have in addition to Eqs. (12) the amplitudes

$$\langle S_\gamma | T | \bar{B} \rangle = e^{i\delta_\gamma} v_c P_\gamma \ , \tag{18a}$$

$$\langle D_\delta | T | \bar{B} \rangle = e^{i\delta_\delta} v_c P_\delta \ . \tag{18b}$$

These are the leading penguin amplitudes (dispersive part only) responsible for the transitions (17a) and (17b). We then find as before

$$\Delta_{SU} = -\Delta_{US} = 4\,\mathrm{Im}(v_u^* v_c) \sum_{\alpha\gamma} A_\alpha P_\gamma t_{\alpha\gamma} \ , \tag{19a}$$

$$\Delta_{DU} = -\Delta_{UD} = 4\,\mathrm{Im}(v_u^* v_c) \sum_{\alpha\delta} A_\alpha P_\delta t_{\alpha\beta} \ . \tag{19b}$$

If we consider t as of order α_s these terms are of order α_s^2 in contrast with Δ_{CU} of Eq. (14) which is of order α_s. However, the calculations of Hou and Gérard[6] indicate that Δ_{SU} and Δ_{CU} have similar magnitudes because $t_{\alpha\beta}$ of Eq. (14) is suppressed by the threshold behavior of the reaction (15).

If we ignore Δ_{CS}, Δ_{CD}, and Δ_{SD}, all of which are suppressed relative to the terms we include

$$\begin{aligned} \Delta_U &= -\Delta_{CU} - \Delta_{SU} - \Delta_{DU} , \\ \Delta_S &= \Delta_{SU} , \\ \Delta_D &= \Delta_{DU} , \\ \Delta_C &= \Delta_{CU} . \end{aligned} \tag{20}$$

All of the above can be repeated replacing the final s quark by a d quark. All of the rate differences $\Delta_A \ldots$ are now found by replacing the v_i by $E_i = U_{bi} U_{di}^*$. Since

$$v_u^* v_c = -E_u^* E_c$$

all the rate differences are equal but opposite when s is replaced by d. The calculation of the total rates is very different for these two cases. For $b \to u + \bar{u} + d$ one expects penguin diagrams are relatively unimportant whereas one expects them to dominate $b \to u + \bar{u} + s$. Nevertheless there is a complete correspondence when one calculates the rate differences Δ. This can be verified from the numerical results given by Gérard and Hou.[5,6]

V. ON THE USE OF PENGUIN GRAPHS

The standard analysis of the problem we have considered is simply to add penguin graphs to tree graphs. Final-state interactions are automatically included via the absorptive part of the penguin graphs.[4] There are several possible problems with this approach. One, which has been emphasized by Hou and Gérard,[6] is that if one is not careful one may violate the constraints of CPT and unitarity. We believe that the formalism we have presented serves to elucidate this problem as discussed below. In addition our formalism identifies those final-state interactions which are relevant to the calculation of the CP-violating asymmetry Δ. One then can address the question whether it is a good approximation to treat these particular final-state interactions using the absorptive part of penguin graphs.

The standard attack on the semi-inclusive asymmetry Δ_{UC} discussed in Sec. IV is to consider U and C as the three-quark states of Eq. (11). Then

$$\langle \bar{U} | T | \bar{B} \rangle = v_u T_u + v_u P_1 e^{i\alpha_1} + v_c P_2 e^{i\alpha_2} , \tag{21a}$$

$$\langle \bar{C} | T | \bar{B} \rangle = v_c T_c + v_u P_i' e^{i\alpha_1'} + v_c P_2' e^{i\alpha_2'} . \tag{21b}$$

Here T_u, T_c come from the tree graphs and the P's come from the penguin graphs. The phases in the penguin terms arise from the absorptive parts. The quantity v_t has been eliminated using the unitarity of the Kobayashi-Maskawa (KM) matrix

$$v_t = -(v_u + v_c) .$$

The final-state-interaction effects proportional to $\sin\alpha_2$ and $\sin\alpha_1'$ correspond to those included in Eqs. (13a) and (13b) inside the large parentheses arising from Fig. 1. The final-state-interaction effects proportional to $\sin\alpha_1$, and $\sin\alpha_2'$ on the other hand, correspond to the phases δ_α and δ_β, respectively, in Eqs. (13a) and (13b).

From Eqs. (21) one derives the rate difference for the U ($\bar{U}$) final states as

$$\Delta_{UC} = 4\,\mathrm{Im}(v_u^* v_c)[-T_u P_2 \sin\alpha_2 + P_1 P_2 \sin(\alpha_1 - \alpha_2)] , \tag{22a}$$

while for the C ($\bar{C}$) final states it is

$$\Delta_{CU} = 4\,\mathrm{Im}(v_u^* v_c)[T_c P_1' \sin\alpha_1' + P_1' P_2' \sin(\alpha_2' - \alpha_1')] . \tag{22b}$$

The first terms in the square brackets of Eqs. (22) correspond to the result given in Eq. (14) and indeed can be shown to be equal and opposite. However, the second terms in the square brackets depend on α_1, and α_2', which arise from the absorptive parts of the "diagonal" scattering (S_0 in our notation). We have argued in general in Sec. II that the answer should be independent of these. Thus we conclude that Eqs. (22) are wrong and that Eqs. (21) are inadequate. Gérard and Hou[6] have reached the same conclusion by noting that if m_c were greater than ($m_b/2$) then $\langle \bar{C} | T | \bar{B} \rangle$ and Δ_{CU} vanish obviously, α_2 vanishes, but Eq. (18a) still gives a nonzero Δ_{UC} because of α_1. Thus the unitarity relation of Eq. (10) is violated.

To analyze this problem from our perspective let us assume $m_c > m_b/2$ and ignore the states D and S and reexamine Eq. (21a). The first two terms can be combined; to first order in α_1 we can write

$$v_u T_u + v_u P_1 e^{i\alpha_1} + v_c P_2 = (v_u T_u) e^{i\delta_\alpha} + v_u P_1 + v_c P_2 .$$

The phase δ_α is equivalent to that in Eq. (13a). The problem is that the rescattering phase δ_α does not occur in the last two terms. To get this Gérard and Hou find it is necessary to add the absorptive part of two-loop diagrams as shown in Fig. 2. This has the effect of multiplying the dispersive parts of the penguins by $e^{i\delta\alpha}$. Thus $e^{i\delta_\alpha}$ can be factored out as in the analysis of Sec. IV and the asymmetry vanishes. From the point of view of Gérard and Hou the asymmetry vanishes because of a "cancella-

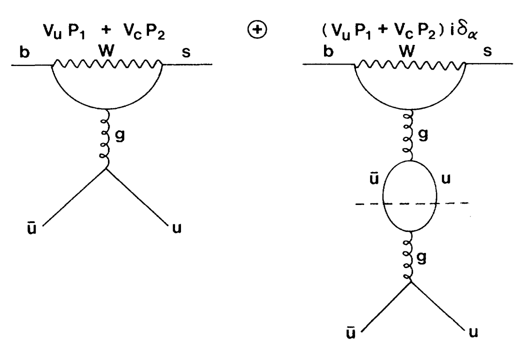

FIG. 2. Illustration of the combination of the dispersive part of the penguin diagram with the absorptive part of a two-loop diagram to give the rescattering phase shift.

tion" between two terms:

(absorptive part of penguin $v_u P_1 \alpha_1$)$\times$(penguin $v_c P_2$) ,

(absorptive part of two-loop proportional to v_c)

$\times$(tree $v_u T_u$) .

This appears to be a hard way to get rid of terms that never should have been included in the first place. Note also that in our treatment it is not necessary to treat the interactions responsible for δ_α perturbatively as it is in the penguin approach.

Questions have been raised[9] as to the accuracy of using the absorptive part of penguin graphs to calculate final-state interactions; that is, using the one-gluon-exchange processes to describe the final-state S matrix. We will not pursue this question here. However, it is useful to note that the process (16a) is irrelevant for the calculation of the semi-inclusive asymmetry and only process (15) matters. It is easier to defend the use of one-gluon exchange for the necessarily hard process (15) than it may be for (16a). Note also that the use of a single absorptive cut to describe the final-state interactions is equivalent to our assumption that only the lowest order in S_1 is included.

Let us turn to the penguin graph approach to Δ_{US} and Δ_{SU}. We focus on the states S and U, neglect the state D, and let $m_c > m_b/2$ so as to avoid involving Δ_{UC} and Δ_{SC}. For the U state we start with Eq. (21a) with $\alpha_2=0$ while for S we have

$$\langle \bar{S}|T|\bar{B}\rangle = v_u P_{S1} e^{i\alpha_S} + v_c P_{S2} \ .$$

Then

$$\Delta_{SU} = 4\,\mathrm{Im}(v_u^* v_c) P_{S1} P_{S2} \sin\alpha_S \ . \tag{23}$$

To obtain Δ_{US} [which must equal $(-\Delta_{SU})$] we must now add to Eq. (21a) the absorptive part of a two-loop graph (Fig. 3) involving an $\bar{s}s$ loop. Thus Δ_{SU} appears as

(absorptive part of penguin $v_u P_{S1} \alpha_5$)$\times$(penguin $v_c P_{S2}$) ,

whereas Δ_{US} appears as

(tree $v_u T_u$)$\times$(absorptive part of two-loop graph) .

Both of these are described by the one Eq. (19a) provided $t_{\alpha\gamma}$ is calculated from the one-gluon-exchange process $u+\bar{u} \rightleftarrows s+\bar{s}$. The correspondence is explained in Fig. 3.

VI. EXCLUSIVE B DECAYS

From an experimental view it is more interesting to consider exclusive rather than inclusive decays. Many papers (see, for example, Refs. 10–13) have considered these. Corresponding to the U states of Sec. V one may consider the specific transitions $B^+ \to K^+\pi^0$ or $B^+ \to K^+\rho^0$. Similarly for the transitions $b \to u+\bar{u}+d$ one may consider $B^+ \to \pi^+\rho^0$ or $B^+ \to p\bar{p}\pi^+$. In general the analysis of such exclusive decays involves additional uncertainties.

To extend our approach we label the exclusive state U_1

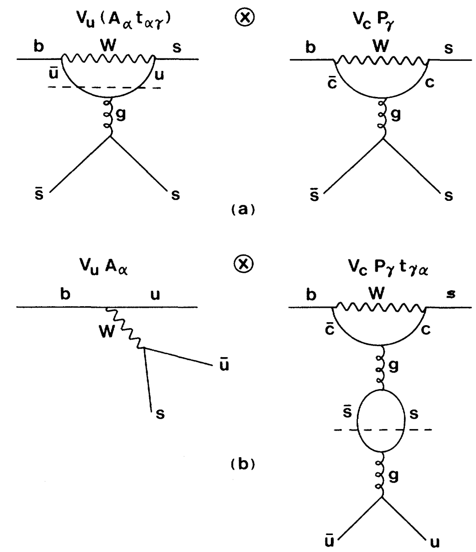

FIG. 3. Two ways of looking at Eq. (19a) in terms of penguin graphs. In calculating the rate difference Δ_{SU} for the final-state S it appears as (a) the interference between two penguin graphs. In calculating Δ_{US} for the final-state U it appears as (b) the interference between a tree amplitude and a two-loop graph.

and the other U states U_α ($\alpha \neq 1$). In addition there are the states D, S, and C. For multiparticle states such as $K^+\pi^-\pi^+$ the distinction between U and D states is not really meaningful and it may be more useful to lump the U_α and D states together. This will not affect most of our discussion. To go further we must once again treat all off-diagonal S matrices perturbatively, which may be considerably less justified than for the semi-inclusive case. The rate difference for U_1 is given by

$$\begin{aligned} \Delta_1 &= \Gamma(\bar{B} \to \bar{U}_1) - \Gamma(B \to U_1) \\ &= \Delta_{1U} + \Delta_{1D} + \Delta_{1S} + \Delta_{1C} \ , \\ \Delta_{1U} &= \sum_{\alpha \neq 1} \Delta_{1\alpha} \ . \end{aligned} \tag{24}$$

The quantities $\Delta_{1D}, \Delta_{1S}, \Delta_{1C}$ can be deduced from Eqs. (14) by setting $\alpha=1$ and omitting the sum over α. The term Δ_{1U}, however, does not have an analog in the semi-inclusive result. Nevertheless we might expect K^+-π^0 scattering primarily yields U_α states such as $K^*+\rho$ or $K+n\pi$ rather than S or C states. Thus Δ_{1U} might be a dominant contribution to Δ_1.

Writing explicitly only the terms needed for Δ_{1U} the transition amplitude to $\bar{U}_1$ is

$$\langle 1|T|\bar{B}\rangle = e^{i\delta_1}\left[v_u T_1 + i v_u \sum_{\alpha\neq 1} T_\alpha t_{1\alpha} + v_c P_1 + i v_c \sum_{\alpha \neq 1} P_\alpha t_{1\alpha} + \cdots \right] . \tag{25}$$

Any penguin term proportional to v_u has been absorbed in T_1. With similar equations for final U_α states one calculates

$$\Delta_{1U}=\mathrm{Im}(v_u^* v_c)\sum_{\alpha\neq 1} t_{1\alpha}(P_1 T_\alpha - P_\alpha T_1)\ . \tag{26}$$

From a naive quark point of view one might imagine $t_{1\alpha}$ arises from the quark scattering (16a). The first term in Eq. (26) corresponds to the interference between the real and absorptive parts of penguin diagrams whereas the second term corresponds to the interference between a tree and the absorptive part of a two-loop diagram. As discussed in Sec. V these terms cancel for the semi-inclusive case. It is seen that Δ_{1U} vanishes only if

$$P_1/T_1=P_\alpha/T_\alpha \tag{27}$$

independent of α. Thus one can consider, in some sense, that a nonzero Δ_{1U} requires scattering from U states that are more "treelike" to U states that are more "penguin-like". One problem in trying to calculate Δ_{1U} is that soft physics may give large terms in $t_{1\alpha}$ corresponding to soft π emission. On the other hand it is possible that for states so connected Eq. (27) holds and there is no contribution to Δ_{1U}.

In fact even from the quark point of view $t_{1\alpha}$ is not well represented by the $\bar{u}u$ quark scattering (16a). When K^+ scatters from π^0, for example, there are also $\bar{s}\,\bar{u}$, $\bar{s}u$, uu, $\bar{s}\,\bar{d}$, $\bar{s}d$, $u\bar{d}$, and ud scatterings. Some of these may be included when all absorptive parts of QCD corrections to order α_s^2 are taken into account.[14] In any case the whole question of the evaluation of Δ_{1U} deserves more serious attention.

In most of the previous analyses only the term Δ_{1C} is calculated. This is the case for the $K\pi$ final state as discussed in Refs. 11 and 12 and the $\bar{p}p\pi$ state discussed in Ref. 13. In contrast Gérard and Hou suggest that the Δ_{1D} and Δ_{1S} terms may dominate the case of the $K\pi$ final state in which case the asymmetry has the opposite sign. There seems to be no serious consideration of Δ_{1U} in any of the papers.

Our conclusion is that no published quantitative results for the asymmetries in exclusive B decays can be trusted.

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APPENDIX

We extend the calculation of Sec. IV to include the amplitudes from penguin graphs. We first neglect the strong-interaction matrix t so that we add the dispersive parts of the penguin to Eqs. (12a) yielding

$$\langle \bar{N}_\alpha|T|\bar{B}\rangle = e^{i\delta_\alpha}[v_u(A_\alpha+\alpha_s P_\alpha)+v_c\alpha_s P'_\alpha]\ , \tag{A1a}$$

$$\langle \bar{C}_\beta|T|\bar{B}\rangle = e^{i\delta_\beta}[v_c(A_\beta+\alpha_s P_\beta)+v_u\alpha_s P'_\beta]\ . \tag{A1b}$$

As noted in the text we have replaced U_α by $N_\alpha \equiv (U+D+S)_\alpha$. We have replaced v_t by (v_u+v_c) via the unitarity of the KM matrix and explicitly shown the strong-coupling constant α_s dependence. The quantities P and P' are smaller than A_α, A_β in general by the factor $[\ln(m_t/m_b)/6\pi]$ so that it is only the term $v_c\alpha_s P'_\alpha$ that makes a significant new contribution because $v_c \gg v_u$.

We now once again include t perturbatively. In place of Eq. (13a) we have

$$\langle \bar{N}_\alpha|T|\bar{B}\rangle = e^{i\delta_\alpha}\left[v_u\left[A_\alpha+\alpha_s P_\alpha + i\alpha_s\sum_\beta P'_\beta t_{\alpha\beta}\right] + v_c\left[\alpha_s P'_\alpha + i\sum_\beta (A_\beta+\alpha_s P_\beta)t_{\alpha\beta}\right]\right] \tag{A2}$$

plus a similar equation replacing (13b). The CP-violating rate difference is now given by

$$\Delta_c = -\Delta_N = 4\,\mathrm{Im}(v_u^* v_c) \times \sum_\alpha\sum_\beta t_{\alpha\beta}[A_\alpha A_\beta + \alpha_s(P_\alpha A_\beta + P_\beta A_\alpha) + \alpha_s^2(P_\alpha P_\beta - P'_\alpha P'_\beta)]\ . \tag{A3}$$

Since we expect the various P's to be less than the A's we see that Eq. (A3) differs from Eq. (14) at most by a term of order α_s. Given the large uncertainty in calculating $t_{\alpha\beta}$ our previous result is quite adequate. Thus we conclude that in calculating Δ we can simply ignore one-loop penguin graphs except insofar as we represent $t_{\alpha\beta}$ by the absorptive graphs of Fig. 1. A detailed analysis[14] shows that the α_s correction in (A3) is about 30%.

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