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Dynamical mass generation at finite temperature

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The dynamical generation of fermion mass at finite temperature is studied in the Gross-Neveu model and quantum chromodynamics, in a generalization of the approach of Nambu and Jona-Lasinio. The dynamical quark mass behaves, in leading-logarithm approximation, in exactly the same way as the dynamical fermion mass in the Gross-Neveu model. Chiral symmetry is restored in quantum chromodynamics via a second-order phase transition at $T=1.10\Lambda$.

I. INTRODUCTION

In this paper, we investigate the dynamical generation of fermion mass in the Gross-Neveu model[1] and in quantum chromodynamics[2] (QCD) at finite temperature T (Ref. 3). We are interested in these problems because of the possibility that spontaneously broken chiral symmetries may be restored in these models by thermal effects.[4,5] The phase diagram of the asymptotically free[1,6] Gross-Neveu model has been extensively studied,[7–9] as has the question of chiral-symmetry restoration in QCD at finite temperature.[10–12] The aim of this paper is to show that a recent study[13] of a generalization of the Nambu–Jona-Lasinio (NJL) technique[14] of dynamical mass generation to the renormalizable theory of $T=0$ QCD can be extended to provide an elementary derivation of dynamical fermion mass when $T\neq 0$. The present work has been directly motivated by worthwhile initial attempts[15–17] to extend the analysis of Ref. 13, and we hope that it will lead to an improved understanding of the dynamics underlying the breaking of chiral symmetry in the Gross-Neveu model and QCD.

The Gross-Neveu model is an appropriate subject for our investigation because it is an adaptation, in two space-time dimensions, of the NJL model, which in turn was constructed in analogy with superconductivity. Part of our aim has recently been satisfied in the case of the Gross-Neveu model by an analysis[9] that used the imaginary-time formalism of quantum field theory at finite temperature and density.[3] In Sec. II, we will repeat part of the analysis of Ref. 9 using the alternative real-time formalism[3] in order to gain confidence in our approach to dynamical mass generation[13,14] in a simple setting that permits direct comparison of results obtained in the two formalisms. The Gross-Neveu model has not been studied in detail in the real-time formalism, and it is instructive to see the parallels between our real-time calculations and earlier work carried out in the imaginary-time formalism.[7,9] However, our main reason for using the real-time formalism is its simplicity, especially in the more complicated study of the dynamical quark mass in $T\neq 0$ QCD. In Sec. III, we will find that the dynamical quark mass exhibits, in leading-logarithm approximation, exactly the same behavior at finite T as the fermion mass in the Gross-Neveu model,[7–9] and that chiral symmetry is restored in QCD via a second-order phase transition at a phenomenologically reasonable critical temperature.[11,18] In part because of its simplicity, our analysis usefully complements the qualitative variational calculations of Ref. 12, in which the QCD effective potential[19,20] was studied in the imaginary-time formalism; we expect that the relationship between our work and a fully quantitative QCD effective potential calculation may be similar to that between the analysis of Ref. 9 and the complementary effective potential calculations for the Gross-Neveu model,[7,8] and we are encouraged in this belief by the observed similarity between the phase diagrams of the two models.[12]

II. GROSS-NEVEU MODEL

We will first consider dynamical mass generation in the Gross-Neveu model at finite T in the infinite-N limit, where N is the number of fermion flavors.[1,7–9] The Lagrangian of the Gross-Neveu model is given by[1]

$$L=i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-\tfrac{1}{2}\sigma^{2}-g\bar{\psi}\psi\sigma \ , \tag{1}$$

where σ is a nonpropagating auxiliary field. Following NJL,[13,14] we rewrite the Lagrangian (1) as

$$L=L_{\text{pert}}+L_{\text{ct}} \ , \tag{2}$$

where

$$L_{\text{pert}}=L-m\bar{\psi}\psi, \quad L_{\text{ct}}=(\delta m)\bar{\psi}\psi \ .$$

As an intermediate step, we will evaluate radiative corrections in perturbation theory to one-loop order using the Lagrangian L_{pert} in (2). Next, we will set $\delta m=m$ and impose the condition that the radiative correction to the dynamical mass m, Σ_m, must vanish. We will then obtain a "gap equation" for the dynamical mass m, which, because of the renormalizability of the theory, will be a renormalization-group (RG) invariant.[13,21]

In the infinite-N limit, only the tadpole graph contributes to the fermion self-energy $\Sigma(p)$, and we have

$$\Sigma(p)=g^{2}N\delta^{ab}\int\frac{d^{2}k}{(2\pi)^{2}}\mathrm{Tr}S_{\beta}(k) \ , \tag{3}$$

where[3]

$$S_\beta(p)=\frac{i}{\not{p}-m+i\epsilon}-\frac{2\pi(\not{p}+m)}{e^{\beta E}+1}\delta(p^2-m^2)\ ,$$

$E=|p_0|$, $\beta=T^{-1}$, and a,b are flavor indices.

The divergent self-energy (3) may be renormalized in the modified minimal subtraction ($\overline{\text{MS}}$) scheme using a $T=0$ counterterm,[21,22] and we obtain

$$\Sigma(p)=\frac{1}{\pi}g^2N\delta^{ab}m\,[\ln(\mu/m)-2I(\beta m)]\ ,$$

where μ is an arbitrary mass scale and

$$I(a)=\int_0^\infty\frac{(x^2+1)^{-1/2}dx}{\exp[a(x^2+1)^{1/2}]+1}\ .$$

We write the renormalized inverse fermion propagator as

$$\delta^{ab}(\not{p}-m+\delta m)-\Sigma(p)=\delta^{ab}Z^{-1}(\not{p}-m+\Sigma_m)\ ,$$

where Z^{-1} is a finite wave-function renormalization, and note that $Z^{-1}=1$ at one-loop order. The nontrivial solution of the conditions $\delta m=m, \Sigma_m=0$ is found to be[1,7,9]

$$m(T)=m(0)\exp[-2I(\beta m)]\ ,$$

and we readily obtain[5] the critical temperature[7–9] $T_c=\pi^{-1}e^\gamma m(0)\approx 0.5669m(0)$ at which the discrete chiral symmetry of the Gross-Neveu model is restored via a second-order phase transition, where γ is Euler's constant. We may expand the integrand of $I(\beta m)$ in a geometric series and use the integral representation for the modified Bessel function $K_0(x)$ (Ref. 23, p. 959) to obtain[7,9]

$$m(T)=m(0)\exp\left[2\sum_{n=1}^\infty(-1)^nK_0(\beta mn)\right]\ . \tag{4}$$

The evaluation of a class of series that includes the series in (4) as a limiting case has recently been studied in Refs. 9 and 24 [see also Ref. 3, Appendix (A2)], and we may be confident that we fully understand the application of the approach of Ref. 13 in the Gross-Neveu model using the real-time formalism of $T\neq0$ field theory.

III. QUANTUM CHROMODYNAMICS

We will now consider the dynamical generation of quark mass in QCD at $T\neq0$. We will repeat the NJL approach[13,14] with the fermionic part of the QCD Lagrangian, specified by

$$L_{\text{quark}}=i\bar\psi\gamma^\mu D_\mu\psi\ ,$$

where $D_\mu=\partial_\mu+igA_\mu$; the usual gauge-field (A_μ), gauge-fixing, and ghost contributions are understood.[2] The one-loop quark self-energy is given by

$$-i\Sigma(p)=(ig)^2C_f\int\frac{d^4k}{(2\pi)^4}\gamma_\mu S_\beta(p-k)\gamma_\nu D_\beta(k)_{\mu\nu}\ ,$$

where the gluon propagator is given by[3]

$$D_\beta(p)_{\mu\nu}=-\left[\frac{i}{p^2+i\epsilon}+\frac{2\pi}{e^{\beta E}-1}\delta(p^2)\right]\times\left[g_{\mu\nu}-(1-\alpha)\frac{p_\mu p_\nu}{p^2+i\epsilon}\right]\ ,$$

$E=|p_0|$, C_f is a group theory factor,[2] and α is the covariant Lorentz gauge parameter. We renormalize $\Sigma(p)$ in the $\overline{\text{MS}}$ scheme by adding a $T=0$ counterterm. In the rest frame of the quark ($p_0=m$), we obtain[3,15,25]

$$-i\Sigma(p)=im(\gamma_0A+B)\ ,$$

where

$$(\lambda C_f)^{-1}A=G(\alpha)-8J(\beta m)-\frac{4}{3}\left[\frac{\pi}{\beta m}\right]^2\ ,$$

$$(\lambda C_f)^{-1}B=-G(\alpha)+16I(\beta m)-2[2-3\ln(m/\mu)]\ ,$$

$$G(\alpha)=2\alpha[1-\ln(m/\mu)]+4(1-\alpha)I(\beta m)\ ,$$

$$J(a)=\frac{1}{a^2}\int_0^\infty\frac{(x^2+a^2)^{1/2}dx}{\exp[(x^2+a^2)^{1/2}]+1}\ ,$$

$\lambda=g^2/(16\pi^2)$, and μ is an arbitrary mass scale. We note for later convenience that we may write[16] $J(a)\equiv I(a)+a^{-2}K(a)$, where $K(a)$ satisfies the relation

$$\frac{dK(a)}{da^2}=-\tfrac{1}{2}I(a)\ .$$

The approach of Ref. 13 involves a subtle departure, in its intermediate steps, from the usual perturbative study of $T\neq0$ mass corrections.[25] We write the one-loop renormalized inverse quark propagator as[13]

$$\not{p}-m+\delta m-\Sigma(p)=Z^{-1}(\not{p}-m+\Sigma_m)\ ,$$

where $Z^{-1}=1+A$ is a temperature-dependent finite wave-function renormalization. We obtain

$$(1+A)\Sigma_m=m(A+B)+\delta m\ . \tag{5}$$

In the usual perturbative study with a preexisting tree-level mass m, the δm term in (5) is absent, $-\Sigma_m=0(\lambda)$ is interpreted as the mass shift, and we obtain the manifestly gauge-independent one-loop result[25]

$$(1+A)\Sigma_m\approx\Sigma_m=m(A+B)\ . \tag{6}$$

In the approach of Ref. 13, we impose the condition $\Sigma_m=0$ to obtain a "gap equation" for the dynamical mass m, but before we do this we must note that $\Sigma_m\neq0(\lambda)$ since Σ_m now contains a term of $O(\lambda^0)$ [cf. (5)]. We must therefore be careful to impose the condition $\Sigma_m=0$ in a self-consistent manner at the one-loop level. Setting $\delta m=m$, we may divide (5) by $1+A$ to obtain

$$\Sigma_m\approx m(1-A)(1+A+B)\approx m(1+B)\ . \tag{7}$$

Alternatively, we may solve iteratively for Σ_m, obtaining

$$\Sigma_m=m(1+A+B)-A[m(1+A+B)-A(\cdots)]\ ;$$

this solution illustrates most clearly the need to exercise caution since $A\Sigma_m=Am+O(\lambda^2)$, and we again obtain

(7) as the expression for Σ_m that correctly enumerates all the terms of $O(\lambda^0)$ and $O(\lambda)$ that are needed for the self-consistent solution of the one-loop condition $\Sigma_m=0$ in the approach of Ref. 13. Unfortunately, we now find that Σ_m contains an explicit gauge dependence, but we can appeal to complementary studies of the Dyson-Schwinger equation (see, e.g., Ref. 26) to justify our expectation[13,20] that the choice of Landau gauge ($\alpha=0$) will specify the correct gauge-independent dynamical quark mass; in the following, we will evaluate the dynamical quark mass in leading-logarithm approximation.

We impose the condition $\Sigma_m=0$ in the form[2,13]

$$\Sigma_m=\Sigma_0(\lambda/\lambda_0)^{h/b}=0 \ , \tag{8}$$

where Σ_m and λ satisfy the one-loop RG equations

$$\mu\frac{d\Sigma_m}{d\mu}=-h\lambda\Sigma_m \ ,$$

$$\mu\frac{\partial\lambda}{\partial\mu}=-b\lambda^2 \ ,$$

$h=6C_f$ and $b=22-\frac{4}{3}N_f$. We match (7) and (8) at the one-loop level[13] to obtain the "gap equation"

$$\Sigma_m=m\left[1+\frac{b}{h}B\right]^{h/b}=0 \ ,$$

which has the nontrivial solution

$$m(T)=m(0)\exp[-2I(\beta m)] \ ,$$

where

$$m(0)=\mu\exp\left[\frac{2}{3}-\frac{1}{b\lambda}\right]\equiv e^{2/3}\Lambda$$

and Λ is the one-loop RG-invariant scale parameter of QCD in the $\overline{\text{MS}}$ scheme.[27,28] We find that $m(T)$ has exactly the same behavior as the $T\neq0$ fermion mass in the Gross-Neveu model. In particular, we find a second-order chiral-symmetry-restoring phase transition at the phenomenologically acceptable[11,18] critical temperature T_c given by

$$T_c=\frac{1}{\pi}\exp(\tfrac{2}{3}+\gamma)\Lambda\approx1.10\Lambda \ .$$

Finally, we note that[5,23]

$$A\approx\lambda C_f\left[1-\frac{2\pi^2}{\beta^2m^2}\right]$$

for $\beta m\ll1$, and that consequently the perturbative mass shift of Ref. 25 is infrared (IR) divergent in the limit $m\to0$ [cf. (6); in massless scalar field theory, it is the two-loop contribution to the $T\neq0$ mass shift that is IR divergent[29]]. In our approach,[13] we are fortunate to be able to avoid this IR divergence by using the renormalization group to shunt it into the wave-function renormalization $Z^{-1}=1+A$, where it does not worry us. This explains why we are able to obtain a second-order phase transition in $T\neq0$ QCD. It would be interesting to try to understand this "harmless" IR divergence in terms of the IR divergences that appear to plague perturbative thermal QCD;[30] clearly it would also be desirable to extend our analysis to the two-loop (next-to-leading-logarithm) level.[3,13] It is worth pointing out that the work of Ref. 13 treated $(\delta m)\bar{\psi}\psi$ as an $O(\lambda)$ counterterm; then, if we take $\delta m=O(\lambda)$ in (5), we obtain a gauge-independent result that does not lead straightforwardly, if at all, to a phase transition.[15,31] This might seem natural because at first sight it appears from (5) that $\delta m=O(A+B)=O(\lambda)$; however, the $\lambda\ln(m/\mu)$ term in B may be seen to be of $O(\lambda^0)$ once the functional dependence of m/μ on λ is self-consistently taken into account. Consequently, we have treated $(\delta m)\bar{\psi}\psi$ as a tree-level $O(\lambda^0)$ "spectator" term in the Lagrangian, thereby obtaining the self-consistent result (7). We also point out that our one-loop dynamical mass is not RG invariant unless $\alpha=0$, but that we expect use of (8) to be reliable in this case because of the invariance of the Landau gauge under renormalization.[22,32]

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