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New range of mixing parameters and rare K decays

G. Bélanger
Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Québec, Canada H3C 3J7

C. Q. Geng
Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Québec, Canada H3C 3J7
and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
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The constraints on the elements of the quark mixing matrix are updated and are used to study rare kaon decays. The top-quark-mass-dependent limit on the mixing matrix from the measurement of the branching ratio $K_L \to \mu\bar{\mu}$ is also included, and the effects on other rare decays are presented.

I. INTRODUCTION

In the standard model, quarks of different flavors are mixed in the charged weak currents by an unitary matrix V, the Cabibbo-Kobayashi-Maskawa (CKM) matrix.[1] For three generations of quarks, the matrix contains three real angles and only one phase which is related to the observed CP violation.[2] The origin of the parameters in this matrix is unknown; it is therefore important to obtain information on these matrix elements from experimental data. The elements of the matrix V can be determined from direct and indirect measurements, which occur at the tree- and one-loop levels, respectively. The most recent direct measurement[3,4] is the $b \to u$ transition matrix element which gives a nonzero value of the ratio $|V_{ub}/V_{cb}|$. The current limit[5] from Fermilab on the t-quark mass, $m_t > 89$ GeV$/c^2$, implies that the virtual t-quark loop contribution to the processes involving indirect measurements of the CKM matrix element, such as B_d^0-$\bar{B}_d^0$ mixing[3,6] and the CP-violating parameter ϵ in the K^0-$\bar{K}^0$ system,[7] is becoming more important. This information on the elements of the CKM matrix can then be used to study the short-distance part of kaon decays.[8]

Recently there have been two measurements of the Dalitz decay $K_L \to e^+e^-\gamma$ at CERN[9] and BNL,[10] allowing the determination of the decay form factor which describes $\Delta S = 1$ nonleptonic weak transitions between pseudoscalar states $(K_L \to \pi, \eta, \eta' \to \gamma^*\gamma)$ and between vector states $[K_L \to K^*\gamma \to (\rho,\omega,\phi)\gamma \to \gamma^*\gamma]$. The measurement of this form factor sheds light on the structure of the $K_L \to \gamma^*\gamma$ vertex which could be important for determining the long-distance contributions in $K_L \to \mu\bar{\mu}$ decay.[11] With this information and the most recent data[12,13] on the branching ratio for $K_L \to \mu\bar{\mu}$, we can further limit the range of the CKM parameters and also the allowed values for the t-quark mass by studying the short-distance effect on the branching ratio. In fact a very heavy t quark implies a rate too large[14] for the decay $K_L \to \mu\bar{\mu}$. This constraint has, of course, important consequences for other rare K decays such as $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, and $K_L \to \pi^0 e^+e^-$.

The paper is organized as follows. In Sec. II we obtain improved bounds on the CKM matrix parameters from the recent measurements of $|V_{ub}/V_{cb}|$, the B_d^0-$\bar{B}_d^0$ mixing, and the CP-violating parameter ϵ in the K^0 system. In Sec. III we study various rare K decays. In particular, we extract newer constraints on the t-quark mass and the CKM parameters from the short-distance contributions to the decay $K_L \to \mu\bar{\mu}$ and discuss its implications on other rare kaon decays. Finally we give our concluding remarks in Sec. IV.

II. RANGE OF THE CKM MATRIX PARAMETERS

We use the Maiani-Wolfenstein parametrization[15] of the CKM matrix; this matrix is written as

$$V \simeq \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}, \quad (2.1)$$

where $\lambda = 0.22$ is the Cabibbo angle. From the B-meson lifetime[16] $[\tau_B = (1.18 \pm 0.14) \times 10^{-12}$ s] and the semileptonic decay rate to charmed final states $[(10.9 \pm 0.4)\%]$, the matrix element V_{cb} can be determined to be[16,17] 0.049 ± 0.005; this implies

$$A = 1.0 \pm 0.1 \ . \quad (2.2)$$

We now study constraints on the other two parameters of the matrix in Eq. (2.1), ρ and η, from the measurements of $|V_{ub}/V_{cb}|$, ϵ and B_d^0-$\bar{B}_d^0$ mixing. Recently, both the CLEO[3] and ARGUS[4] Collaborations have reported evidence for nonzero $b \to u$ transition in the semileptonic B decay. The ratio $|V_{ub}/V_{cb}|$ of the CKM matrix elements is obtained[17] to be 0.10 ± 0.05 by combining the experimental and theoretical uncertainties. This gives the constraint

$$\rho^2 + \eta^2 = (0.46 \pm 0.23)^2 \ . \quad (2.3)$$

The CP-violating parameter ϵ calculated from the box diagrams is given by

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}\Delta M_K} \frac{G_F^2 M_W^2}{12\pi^2} \times M_K f_K^2 B_K \mathrm{Im}[\eta_{cc}(V_{cs}V_{cd}^*)^2 B(x_c) + 2\eta_{ct}(V_{cs}V_{cd}^*)(V_{ts}V_{td}^*)B(x_c,x_t) + \eta_{tt}(V_{ts}V_{td}^*)^2 B(x_t)] , \tag{2.4}$$

which leads to

$$|\epsilon| = \frac{1}{\sqrt{2}\Delta M_K} \frac{G_F^2 M_W^2}{12\pi^2} \times M_K f_K^2 B_K 2A^2\lambda^6 \eta[-\eta_{cc}B(x_c) + \eta_{ct}B(x_c,x_t) + \eta_{tt}A^2\lambda^4(1-\rho)B(x_t)] , \tag{2.5}$$

where

$$B(x_i) = \frac{x_i}{4}\left[1 + \frac{3-9x_i}{(x_i-1)^2} + \frac{6x_i^2\ln(x_i)}{(x_i-1)^3}\right] \tag{2.6a}$$

and

$$B(x_i,x_j) = \frac{x_i x_j}{4}\left[\frac{x_j^2 - 8x_j + 4}{(x_j-1)^2(x_j-x_i)} - \frac{3}{2(1-x_i)(1-x_j)} + (x_i \leftrightarrow x_j)\right] \tag{2.6b}$$

with $x_i = m_i^2/M_W^2$, $i=c,t$. Here we use the values of the parameters quoted in Ref. 18: $|\epsilon| = (2.26\pm0.02)\times10^{-3}$, $G_F = 1.166\times10^{-5}$ GeV^{-2}, $M_K = 498$ MeV/c^2, $\Delta M_K = 3.52\times10^{-15}$ GeV/c^2, $M_W = 80$ GeV/c^2, $f_K \simeq 160$ MeV, $B_K = 0.7\pm0.2$, $\eta_{cc} = 0.76$, $\eta_{ct} = 0.36$, and $\eta_{tt} = 0.61$. The first two terms in the parentheses of Eq. (2.5) are mostly independent of the top-quark masses for the range of masses we are concerned with: $m_t > 89$ GeV/c^2. For this range, they can be replaced by a constant ($\sim 0.8\times10^{-3}$). Adding errors in quadrature we get the constraint equation from Eq. (2.5):

$$\eta = \frac{0.76\pm0.22}{A^2[0.8 + 1.43A^2(1-\rho)B(x_t)]} . \tag{2.7}$$

The B_d^0-$\bar{B}_d^0$ mixing parameter is defined as $x_d \equiv \Delta M/\Gamma$ which can be written as

$$x_d = \frac{G_F^2}{6\pi^2} M_B f_B^2 B_B \tau_B M_W^2 |V_{tb}V_{td}^*|^2 B(x_t) = \frac{G_F^2}{6\pi^2} M_B f_B^2 B_B \tau_B M_W^2 A^2\lambda^2[(1-\rho)^2 + \eta^2]B(x_t) \tag{2.8}$$

since the t-quark contribution is dominant in the box diagrams. The ratio r_d measured by experiments is given by[3,6]

$$r_d = \frac{x_d^2}{2 + x_d^2} = 0.18\pm0.05 , \tag{2.9}$$

which yields

$$x_d = 0.66\pm0.11 . \tag{2.10}$$

Using the value in Eq. (2.10) and the parameters[7,18] $M_B = 5.28$ GeV/c^2, $f_B = 140\pm25$ MeV, $B_B = 0.85\pm0.10$, $\eta_B = 0.85\pm0.05$, and $\tau_B = (1.18\pm0.14)\times10^{-12}$ s, we get the third constraint equation which also has a dependence on the t-quark mass:

$$(1-\rho)^2 + \eta^2 = \frac{3\pm1}{A^2 B(x_t)} . \tag{2.11}$$

In Figs. 1–3, the intersection of the three constraints, (2.3), (2.7), and (2.11), gives the allowed regions in the ρ-η parameter space for $A = 0.9$, 1.0, and 1.1, and for m_t of

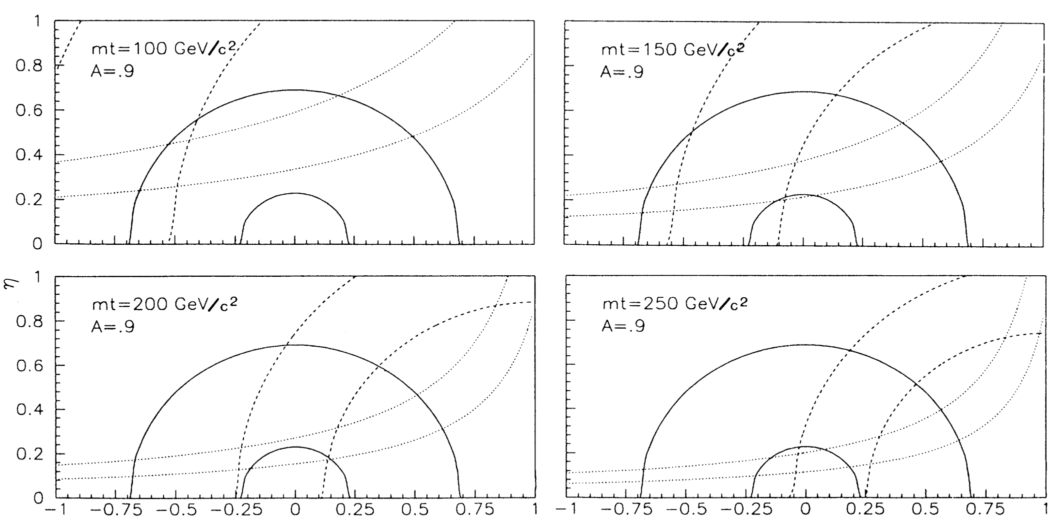

FIG. 1. Constraint in the ρ-η parameter space from $|V_{ub}/V_{cb}|$ (solid curves), the B_d^0-$\bar{B}_d^0$ mixing (dashed curves), and the CP-violating parameter ϵ (dotted curves) for $A = 0.9$ and $m_t = 100$, 150, 200, and 250 GeV/c^2, respectively.

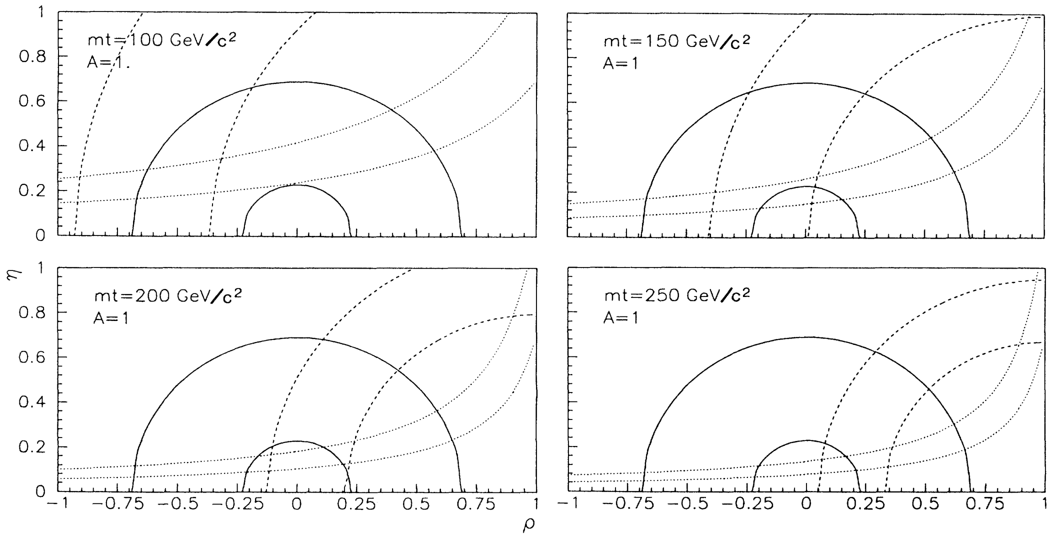

FIG. 2. Same as Fig. 1 but for $A=1.0$.

100, 150, 200, and 250 GeV/c^2. The x_d constraint forces ρ to become larger as m_t increases. For the larger values of A, there is an intermediate top-mass region where there are two sets of allowed values for ρ either positive or negative with a forbidden band in the middle. This can be seen, for example, for $A=1.1$ and $m_t=150$ GeV/c^2. This implies two bands of allowed values for the rare decays which will be shown in the next section.

Finally we remark that the mixing matrix parameter space can be further improved by more precise measurements[19–21] of the direct CP violation, ϵ'/ϵ, in $K\to\pi\pi$ decay.[22] This will be studied elsewhere.[23]

III. RARE KAON DECAYS

The study of kaon rare decays has played a pivotal role in formulating the standard model of electroweak interactions. It is now attracting renewed interest due to the prospect of significantly improved ongoing experiments and the possibility of a very large t-quark mass. The rare K decays such as the neutral decays $K_L\to\mu\bar{\mu}$, $K_L\to\pi^0e^+e^-$, and $K_L\to\pi^0\nu\bar{\nu}$ involving flavor-changing neutral-current interactions and the charged decay $K^+\to\pi^+\nu\bar{\nu}$ are forbidden in the lowest order but they can occur radiatively through the typical one-loop diagrams shown in Fig. 4, which are sensitive to the virtual t quark. The standard-model predictions on these rare K decays depend on the t-quark mass and also on the range of the CKM parameters. For each decay, we first obtain the prediction for the whole range of the parameter space and then show explicitly the effect of imposing the constraint from the short-distance contribution to the decay $K_L\to\mu\bar{\mu}$ in the light of the new experimental data on the

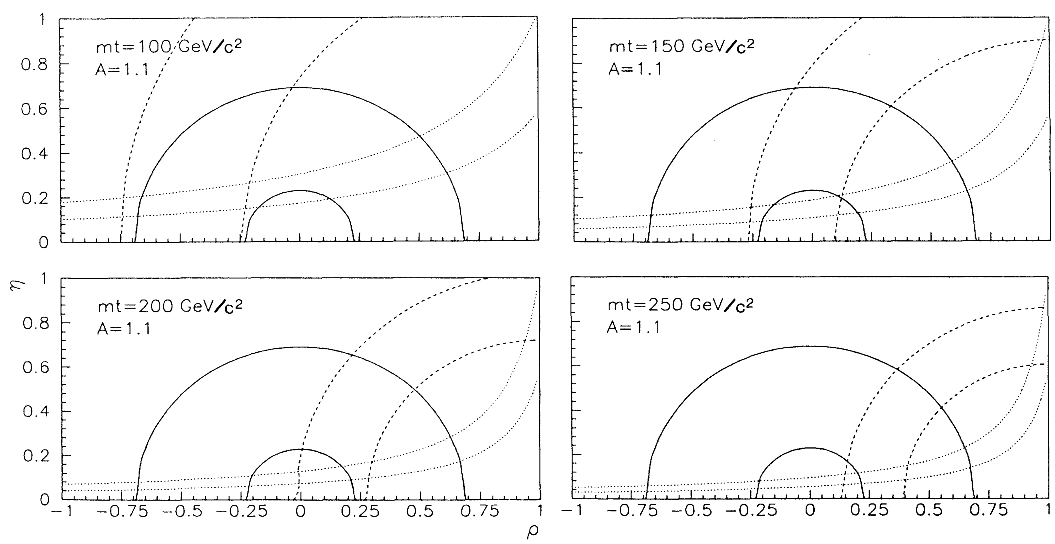

FIG. 3. Same as Fig. 1 but for $A=1.1$.

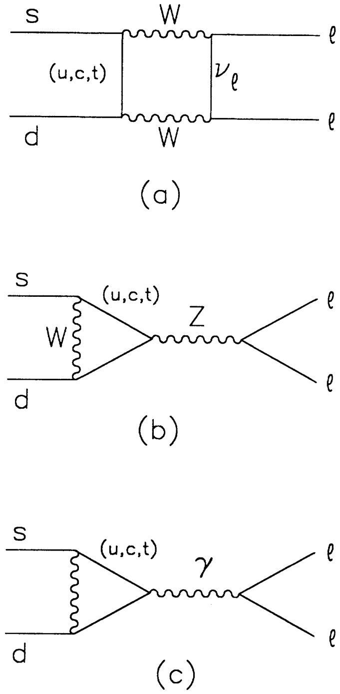

FIG. 4. One-loop diagrams for the short-distance contribution of the rare K decays from the standard model: (a) the W box; (b) the Z penguin; and (c) the electromagnetic penguin.

decay branching ratio[12,13] of $K_L \to \mu\bar{\mu}$ and the form factor[9,10] of the Dalitz decay $K_L \to e^+e^-\gamma$.

A. $K_L \to \mu\bar{\mu}$

Recently there have been several new measurements of $K_L \to \mu\bar{\mu}$ decay. The most recent data on the branching ratio are given by $(8.2\pm0.8\pm0.7)\times10^{-9}$ from 114 events in experiment E-137 at KEK[12] and $(5.8\pm0.6\pm0.4)\times10^{-9}$ and $(7.6\pm0.5\pm0.4)\times10^{-9}$ from runs of 87 and 286 events in E-791 at BNL.[13] These new results are lower than the previous value[7] of $(9.5^{+2.4}_{-1.5})\times10^{-9}$ in the Particle Data Group (PDG) compilation of 1988. Taking an average of all these measurements we find that

$$B(K_L \to \mu\bar{\mu}) \equiv \frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \text{all})} = (7.28^{+0.59}_{-0.57})\times10^{-9} . \quad (3.1)$$

If we denote the real (dispersive) and imaginary (absorptive) parts of the amplitude for the decay $K_L \to \mu\bar{\mu}$ by $\mathrm{Re}\,A$ and $\mathrm{Im}\,A$, respectively, we have

$$B(K_L \to \mu\bar{\mu}) \equiv |\mathrm{Im}\,A|^2 + |\mathrm{Re}\,A|^2 . \quad (3.2)$$

The decay receives contributions from both short- and long-distance effects. Although the long-distance effects cannot be calculated exactly, one can estimate the size of these effects and obtain a range of values for the short-distance part in which we are interested. The absorptive part of the amplitude $\mathrm{Im}\,A$ receives contributions from various intermediate states, the dominant one being the real one-photon ($\gamma\gamma$) intermediate state (see Fig. 5). It is known that other contributions from various intermediate states such as $\pi\pi$, $\pi\gamma$, etc., are all small[24] compared to $A_{2\gamma}$. The two-photon contribution to (3.1) denoted by $B(K_L \to \mu\bar{\mu})_{2\gamma}$ can be calculated[25] from the measured branching ratio[7] of $\Gamma(K_L \to \gamma\gamma)/\Gamma(K_L \to \text{all}) = (5.70\pm0.23)\times10^{-4}$ and gives the unitarity bound

$$\begin{aligned} B(K_L \to \mu\bar{\mu}) &\geq B(K_L \to \mu\bar{\mu})_{2\gamma} \\ &\equiv |\mathrm{Im}\,A_{2\gamma}|^2 \\ &= (6.83\pm0.28)\times10^{-9} . \end{aligned} \quad (3.3)$$

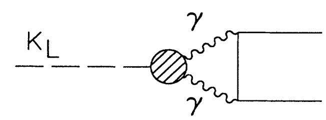

FIG. 5. Two-photon intermediate-state contribution to $K_L^0 \to \mu\bar{\mu}$ decay.

The difference between $B(K_L \to \mu\bar{\mu})$ in (3.1) and $B(K_L \to \mu\bar{\mu})_{2\gamma}$ in (3.3) is

$$\begin{aligned} \Delta B &\equiv B(K_L \to \mu\bar{\mu}) - B(K_L \to \mu\bar{\mu})_{2\gamma} \\ &= (0.45^{+0.65}_{-0.64})\times10^{-9} . \end{aligned} \quad (3.4)$$

While Eq. (3.4) indicates that the decay $K_L \to \mu\bar{\mu}$ is consistent with the unitarity bound, it also implies that other possible contributions must be small. Thus from (3.2) and (3.4) we have an allowed range for the real part of the amplitude:

$$\Delta B \simeq |\mathrm{Re}\,A|^2 = (0-1.1)\times10^{-9} . \quad (3.5)$$

The real part of the amplitude $\mathrm{Re}\,A$ can be written as

$$\mathrm{Re}\,A = \mathrm{Re}\,A_{\mathrm{LD}} + A_{\mathrm{SD}} , \quad (3.6)$$

where LD and SD refers to the long- and short-distance contributions, respectively. The long-distance part in Eq. (3.6) comes dominantly from the virtual two-photon ($\gamma^*\gamma^*$) intermediate state denoted as A_{em} which is poorly known theoretically.[11,26] However it may be determined by virtue of the recent experimental value of the form factor for the decay $K_L \to e^+e^-\gamma$, which involves the $K_L - \gamma - \gamma^*$ vertex. Following the model of Bergström, Massó, and Singer,[27] the form factor $A_{\gamma\gamma^*}$ for the Dalitz decay $K_L \to e^+e^-\gamma$ can be written as

$$\frac{A_{\gamma\gamma^*}(s)}{|A(K_L \to \gamma\gamma)_{\mathrm{expt}}|} = \frac{1}{1-0.418x} + \frac{2.5\alpha_K}{1-0.311x}\left[\frac{4}{3} - \frac{1}{1-0.418x} - \frac{1}{9(1-0.405x)} - \frac{2}{9(1-0.238x)}\right] \quad (3.7)$$

with $x = s/M_K^2$ and α_K a parameter to be determined. In the right-handed side of Eq. (3.7), the first and second terms correspond to $K_L \to \pi, \eta, \eta' \to \gamma^* \gamma$ and $K_L \to K^* \gamma \to (\rho, \omega, \phi)\gamma \to \gamma^* \gamma$ transitions, shown in Figs. 6(a) and 6(b), respectively. The recent measurements at CERN[10] and BNL[11] of this form factor give

$$\alpha_K = -0.28 \pm 0.13 \tag{3.8a}$$

and

$$\alpha_K = -0.280 \pm 0.083^{+0.053}_{-0.034} , \tag{3.8b}$$

respectively. This indicates that the weak transition of the vector mesons cannot be ignored. Using the central experimental value in Eq. (3.8) and the formulas given by Bergström *et al.* in Ref. 11 with the updated parameters[7] of $\Gamma(K^* \to K^0 \gamma) = 0.118$ MeV, $f_\rho = 4.99$, and $f_{K^*} = 5.78$, we find[28]

$$0.6 \times 10^{-5} \le |A_{\rm em}| \le 1.4 \times 10^{-5} , \tag{3.9}$$

where the maximal and minimal values arise from saturating one virtual photon by vector mesons (corresponding to a $PV\gamma$ vertex) and both virtual photons (PVV vertex), respectively. A more stringent value of $|A_{\rm em}|$ could be obtained by the measurements of the decays $K_L \to \mu^+ \mu^- \gamma$ and $K_L \to e^+ e^- e^+ e^-$. From the values $|A_{\rm em}|$ in Eq. (3.9) and ΔB in Eq. (3.4), a bound on the

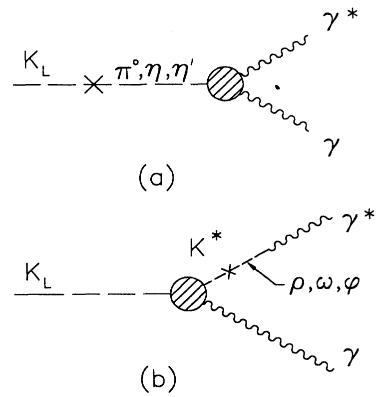

FIG. 6. Diagrams that contribute to $K_L \to \gamma^* \gamma$ through $\Delta S = 1$ nonleptonic weak transition between (a) pseudoscalar states and (b) vector states.

short-distance contribution can be obtained. For example, we get

$$0.1 \times 10^{-9} \le B(K_L \to \mu\bar{\mu})_{\rm SD} \equiv |A_{\rm SD}|^2 \le 1.2 \times 10^{-9} \tag{3.10a}$$

for $\Delta B = 0.45 \times 10^{-9}$ and

$$0.4 \times 10^{-9} \le B(K_L \to \mu\bar{\mu})_{\rm SD} \le 2.2 \times 10^{-9} \tag{3.10b}$$

for $\Delta B = 1.10 \times 10^{-9}$.

The short-distance contribution $A_{\rm SD}$ comes from one-loop diagrams depicted in Figs. 4(a) and 4(b) involving the exchange of virtual heavy quarks, in particular the t quark. The branching ratio due to this contribution is[29,30]

$$B(K_L \to \mu\bar{\mu})_{\rm SD} = \frac{\alpha^2}{4\pi^2 \sin^4\theta_W} \frac{\left[1 - \dfrac{4m_\mu^2}{M_K^2}\right]^{1/2}}{\left[1 - \dfrac{m_\mu^2}{M_K^2}\right]^2} \frac{\left|{\rm Re} \sum_{i=c,t} \eta_i V_{is}^* V_{id} C_\mu(x_i)\right|^2}{|V_{us}|^2} B(K^+ \to \mu^+ \nu_\mu) \frac{\tau(K_L)}{\tau(K^+)} , \tag{3.11}$$

where η_i are the QCD correction factors, $x_i = m_i^2/M_W^2$, $\tau(k_L) = 5.18 \times 10^{-8}$ s, $\tau(K^+) = 1.237 \times 10^{-8}$ s, $B(K^+ \to \mu^+ \nu_\mu) = 0.64$, and

$$C_\mu(x_i) = \frac{4x_i - x_i^2}{4(1-x_i)} + \frac{3x_i^2 \ln x_i}{4(1-x_i)^2} . \tag{3.12}$$

It is straightforward to show that the dominant contribution in Eq. (3.11) arises from the t-quark exchanges and the corresponding QCD correction is negligible,[31,32] i.e., $\eta_t \simeq 1$. We thus obtain

$$B(K_L \to \mu\bar{\mu})_{\rm SD} = 4.06 \times 10^{-10} A^4 |C_\mu(x_t)|^2 (1-\rho)^2 . \tag{3.13}$$

We plot $B(K_L \to \mu\bar{\mu})_{\rm SD}$ as a function of m_t, for the allowed range of the quark mixing matrix and for $A = 0.9$, 1.0, 1.1, and 0.9−1.1 in Figs. 7(a)−7(d), respectively. From Fig. 7 we see that the lower bounds in Eq. (3.10) do not give a useful lower limit for the mass of the t quark. For the central value of A, i.e., $A = 1.0$, the limits $B(K_L \to \mu\bar{\mu})_{\rm SD} \le 1.2 \times 10^{-9}$, and 2.2×10^{-9} in Eq. (3.10) put upper bounds on the top mass of 170 and 242 GeV/c^2, while the absolute upper bounds on m_t for the whole ranges of A values are 196 and 300 GeV/c^2, respectively. For each top mass, an upper bound on the branching ratio translates into a lower bound on the parameter ρ. This bound will show up as a straight vertical line in our previous results of Figs. 1−3. This lower bound on ρ also implies slightly higher values for the

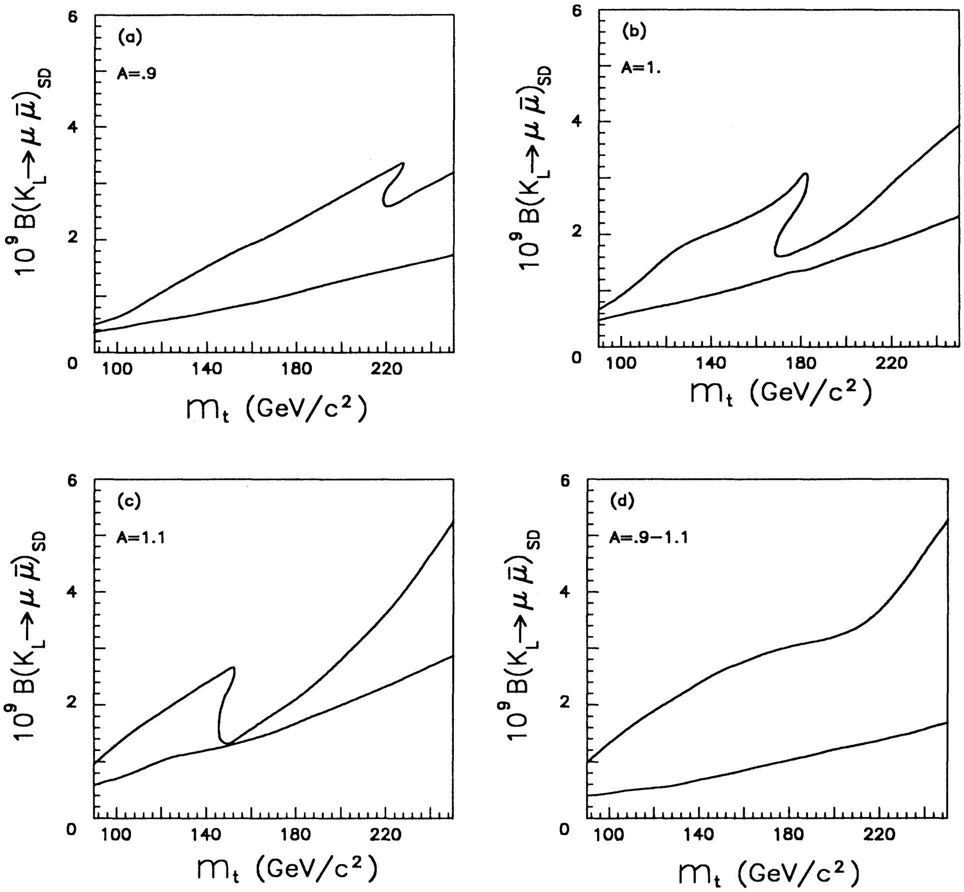

FIG. 7. The short-distance contribution to the branching ratio of $K_L^0 \to \mu\bar{\mu}$ decays as function of m_t with (a) $A = 0.9$; (b) $A = 1.0$; (c) $A = 1.1$; and (d) $A = 0.9 - 1.1$.

CP-violating parameter η. The limits on ρ and η from the $K_L \to \mu\bar{\mu}$ decay and Figs. 1–3 are summarized in Table I. One can see clearly that the parameter space is greatly reduced especially for the higher top-quark mass as expected. We also see that the parameter space from $B(K_L \to \mu\bar{\mu})_{SD} \leq 2.2 \times 10^{-9}$ in Eq. (3.10b) is only slightly reduced. In this case we shall not expect large effects on other decays.

TABLE I. The allowed ranges of ρ and η for different values of m_t for (a) no constraint on $B(K_L \to \mu\bar{\mu})_{SD}$, (b) $B(K_L \to \mu\bar{\mu})_{SD} \leq 1.2 \times 10^{-9}$, and (c) $B(K_L \to \mu\bar{\mu})_{SD} \leq 2.2 \times 10^{-9}$.

		(a)		(b)		(c)	
A	m_t (GeV/c^2)	ρ_{min}	η_{min}	ρ_{min}	η_{min}	ρ_{min}	η_{min}
0.9	100	−0.65	0.23	−0.65	0.23	−0.65	0.23
0.9	150	−0.54	0.15	−0.27	0.26	−0.54	0.15
0.9	200	−0.24	0.12			−0.10	0.18
0.9	250	0.08	0.12			0.23	0.14
1.0	100	−0.67	0.16	−0.67	0.16	−0.67	0.16
1.0	150	−0.39	0.11	−0.03	0.23	−0.40	0.11
1.0	200	0.12	0.12			0.12	0.12
1.0	250	0.17	0.09				
1.1	100	−0.68	0.11	−0.26	0.29	−0.68	0.11
1.1	150	−0.26	0.09			−0.15	0.19
1.1	200	0.17	0.08			0.27	0.09
1.1	250	0.19	0.07				

B. $K^+ \to \pi^+ \nu\bar{\nu}$

Unlike the previous decay there is no electromagnetic contribution involving photons to the rare decay $K^+ \to \pi^+ \nu\bar{\nu}$. The possible long-distance effects on this decay are estimated to be negligibly small[33] and thus the decay is expected to be dominated by short-distance physics. The loop diagrams are shown in Figs. 4(a) and 4(b) with $l \leftrightarrow \nu$. The branching ratio is given by[30,34]

$$B(K^- \to \pi^- \nu\bar{\nu}) = \frac{\alpha^2}{8\pi^2 \sin^4\theta_W} \frac{\left| \sum_{i=c,t} \eta_i V_{is}^* V_{id} C_\nu(x_i) \right|^2}{|V_{us}|^2} \times B(K^+ \to \pi^0 e^+ \nu) , \quad (3.14)$$

where

$$C_\nu(x_i) = \frac{x_i}{4} \left[\frac{3(x_i - 2)}{(x_i - 1)^2} \ln x_i + \frac{x_i + 2}{x_i - 1} \right] . \quad (3.15)$$

In this decay the c- and t-quark contributions are of the same order of magnitude, the t quark becoming more important for larger m_t. The QCD corrections have been calculated;[35,36] these corrections were found to be important for the c quark but negligible for the t quark when $m_t > m_W$. Using $B(K^+ \to \pi^0 e^+ \nu) = 0.048$ and $m_c = 1.5$ GeV/c^2, the branching ratio is expected in the range 10^{-10} and it is explicitly given by

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = 10^{-6} |C_\nu(x_c) + 3.3 \times 10^{-3} A^2 (1-\rho) C_\nu(x_t)|^2 + 1.08 \times 10^{-11} A^4 \eta^2 |C_\nu(x_t)|^2 . \quad (3.16)$$

The branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ as a function of the t-quark mass and for the three different values and whole range of the parameter A is shown in Fig. 8. From Fig. 8(d) we find $B(K^+ \to \pi^+ \nu\bar{\nu}) = (1.2 - 4.0) \times 10^{-10}$ for m_t ranging from 90 to 250 GeV/c^2. The constraint from $K_L \to \mu\bar{\mu}$ is superimposed on this, clearly ruling out the upper range of branching ratios which is now predicted to be $(1.2-2.4) \times 10^{-10}$ and $(1.2-2.9) \times 10^{-10}$ for $B(K_L \to \mu\bar{\mu})_{SD} \leq 1.2$ and 2.2×10^{-9}, respectively. Here the upper values correspond to a t-quark mass of about 95 and 130 GeV/c^2, respectively. Our results agree with the conclusions given in Ref. 14 where no accidental cancellation between the short- and long-distance contributions to the real part of the decay $K_L \to \mu\bar{\mu}$ amplitude was assumed.

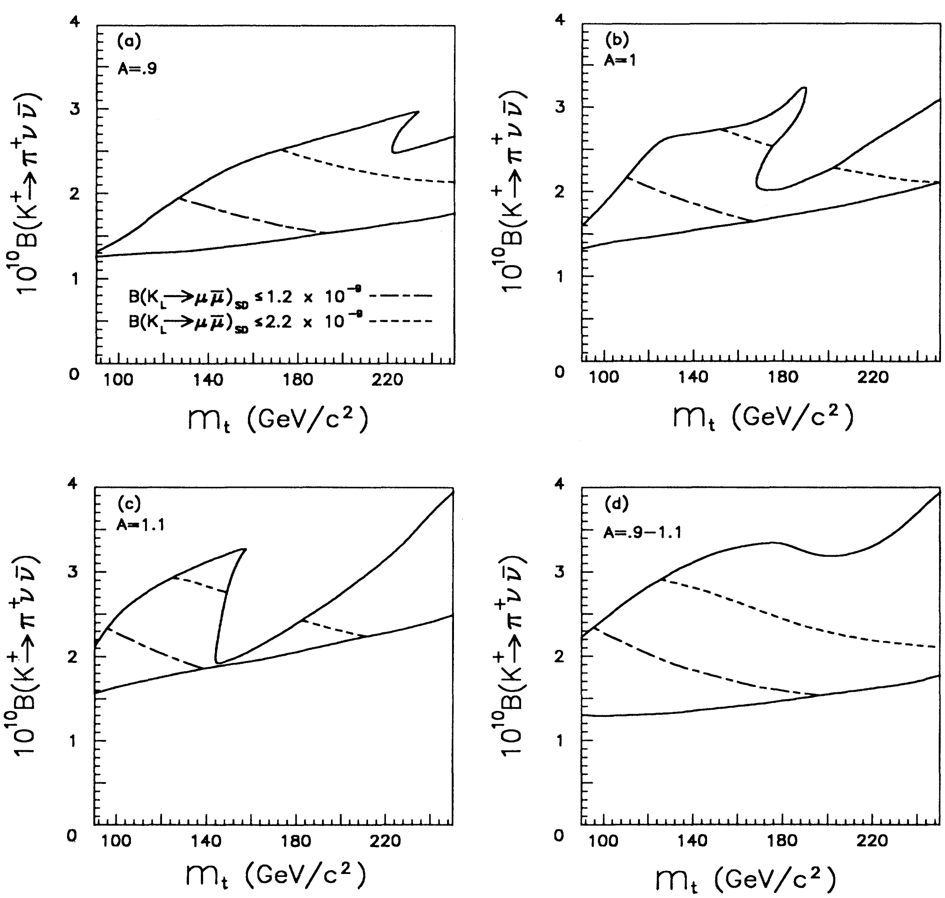

FIG. 8. Allowed branching ratio of $K^\pm \to \pi^\pm \nu\bar{\nu}$ as a function of m_t with (a) $A = 0.9$; (b) $A = 1.0$; (c) $A = 1.1$; and (d) $A = 0.9 - 1.1$. The dotted-dashed curves are $B(K_L \to \mu\bar{\mu})_{SD} \leq 1.2 \times 10^{-9}$ and the dashed curves are for $B(K_L \to \mu\bar{\mu})_{SD} \leq 2.2 \times 10^{-9}$.

C. $K_L \to \pi^0 e^+ e^-$

The rare decay $K_L \to \pi^0 e^+ e^-$ has recently attracted much attention theoretically and experimentally since it may directly test the mechanism of *CP* violation in the standard CKM model. We define K_1 and K_2 to be the *CP*-even and -odd states in the neutral K system, respectively, and then we can write $K_L \simeq K_2 + \epsilon K_1$. The decay $K_L \to \pi^0 e^+ e^-$ receives three types of contributions: (1) a *CP*-conserving one through a two-photon intermediate state, i.e., $K_2 \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$; (2) an indirect *CP*-nonconserving one induced by the mixing of $K^0, \bar{K}^0$ states characterized by ϵ, i.e., $K_1 \to \pi^0 V^* \to \pi^0 e^+ e^-$, where V^* is an effective $J=1$ *CP*-even state; and (3) through the direct *CP*-violating decay of $K_2 \to \pi^0 V^* \to \pi^0 e^+ e^-$.

The branching ratio for the *CP*-conserving part of $K_L \to \pi^0 e^+ e^-$, which depends on the decay $K_L \to \pi^0 \gamma\gamma$, is estimated to be order of 10^{-14} in chiral perturbation theory[37,38] (ChPT) or 10^{-11} in a vector-meson-dominance (VMD) model.[39,40] Recently, the NA-31 experiment at CERN[41] has indicated that the observed distribution of the invariant $\gamma\gamma$ mass in the decay $K_L \to \pi^0 \gamma\gamma$ favors models involving ChPT but the observed branching ratio $(2.1 \pm 0.6) \times 10^{-6}$ is somewhat higher than the predictions of ChPT. Moreover, the search for the decay $K_L \to \pi^0 \gamma\gamma$ in the E-731 experiment at Fermilab[42] gives only a limit for the branching rato, $< 2.7 \times 10^{-6}$. Clearly, to settle down the *CP*-conserving contribution to $K_L \to \pi^0 e^+ e^-$ decay, more efforts on the measurement of the decay $K_L \to \pi^0 \gamma\gamma$ are needed.

The associated branching ratio due to the indirect *CP*-violating contributions is given by[31,37,43]

$$B(K_L \to \pi^0 e^+ e^-)_{\rm ind} = B(K^+ \to \pi^+ e^+ e^-) \frac{\Gamma(K^+ \to \text{all})}{\Gamma(K_L \to \text{all})} \times \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \times \frac{\Gamma(K_L \to \pi^0 e^+ e^-)_{\rm ind}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \simeq 6 \times 10^{-12} R_e \,, \tag{3.17}$$

where we have used $B(K^+ \to \pi^+ e^+ e^-) = 2.7 \times 10^{-7}$ and $|\epsilon| = 2.26 \times 10^{-3}$ and $R_e \equiv \Gamma(K_1 \to \pi^0 e^+ e^-)/\Gamma(K^+ \to \pi^+ e^+ e^-)$. The value of R_e which depends on the unmeasured decay $K_S \to \pi^0 e^+ e^-$ can be calculated in ChPT[43] through the measured branching ratio of the decay $K^+ \to \pi^+ e^+ e^-$ and is found to be 0.25 or 2.52. This results in two possible branching ratios[43]

$$B(K_L \to \pi^0 e^+ e^-)_{\rm ind} = \begin{cases} 1.5 \times 10^{-12} \,, \\ 1.5 \times 10^{-11} \,. \end{cases} \tag{3.18}$$

This twofold ambiguity in ChPT will be resolved from the precise measurement of $K_S \to \pi^0 e^+ e^-$ or $K^+ \to \pi^+ \mu^+ \mu^-$.

Most of the direct *CP* contribution to the decay comes from the short-distance electroweak effects with the loop diagrams depicted in Fig. 4. The rate for this decay is directly proportional to η^2. Including QCD corrections one finds[32,44]

$$B(K_L \to \pi^0 e^+ e^-)_{\rm dir} = 2.6 \times 10^{-14} A^4 (C_V^2 + C_A^2) \eta^2 \tag{3.19}$$

with

$$C_V \simeq F_1(x_t) + \frac{1}{\sin^2\theta_W} [F_2(x_t) + (1 - 4\sin^2\theta_W) F_3(x_t)] \,, \quad C_A \simeq -\frac{1}{\sin^2\theta_W} [F_2(x_t) + F_3(x_t)] \,, \tag{3.20}$$

where $x_t = m_t^2 / M_W^2$ and

$$F_1(x_t) = -17 - \frac{2(25 - 19x_t)x_t^2}{9(1-x_t)^3} - \frac{4(3x_t^4 - 30x_t^3 + 54x_t^2 - 32x_t + 8)\ln x_t}{9(1-x_t)^4} \,,$$
$$F_2(x_t) = \frac{2x_t(1 - x_t + \ln x_t)}{(1-x_t)^2} \,, \tag{3.21}$$
$$F_3(x_t) = -\frac{x_t[(x_t - 6)(x_t - 1) + (3x_t + 2)\ln x_t]}{(1-x_t)^2} \,.$$

Here the m_t-dependent C_V and C_A functions are evaluated from Ref. 32 with $\Lambda_{\rm QCD} = 150$ MeV. Using $\sin^2\theta_W = 0.23$ and $A = 0.9 - 1.1$, the branching ratio of the direct *CP*-violating contribution to $K_L \to \pi^0 e^+ e^-$ is shown in Fig. 9 with the parameter η given in Sec. II as well as Table I. As can be seen from Fig. 9, the lower

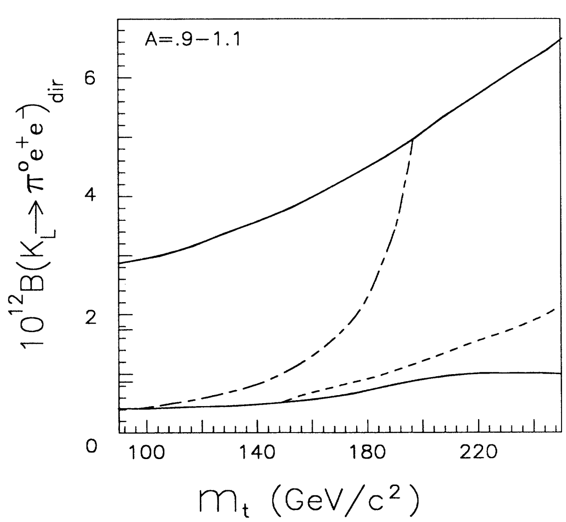

FIG. 9. Allowed branching ratio for the direct *CP*-violating contribution to $K_L \to \pi^0 e^+ e^-$ with the whole range of the parameter A, i.e., $A = 0.9 - 1.1$. The dashed and dotted-dashed curves are boundaries taking into account the $K_L \to \mu\bar{\mu}$ decay. Legend is the same as in Fig. 8.

bound of $\sim 0.4\times 10^{-12}$ depends very weakly on the t-quark mass, whereas the upper bound shows a stronger dependence with the maximum predicted $\sim 6.5\times 10^{-12}$ for $m_t \sim 250$ GeV/c^2. The lower limit on ρ obtained from $K_L \to \mu\bar{\mu}$ implies higher values for η and therefore eliminates lower values for this branching ratio especially for the heavier t quark. It can be seen also that a higher value for $B(K_L \to \mu\bar{\mu})_{\rm SD}$ has only minor effect on the predictions for this decay. The branching ratio from the direct CP-violating contribution is lower than previously thought.[32] Only if the CP-conserving and indirect CP-violating contribution turn out to be near their lower predicted values, will the direct CP-violating contribution become interesting. These issues can be settled by measuring $K_L \to \pi^0\gamma\gamma$, $K_S \to \pi^0 e^+ e^-$, or $K^+ \to \pi^+\mu^+\mu^-$.

D. $K_L \to \pi^0 \nu\bar{\nu}$

The decay $K_L \to \pi^0\nu\bar{\nu}$ is similar to the previous one but it only receives contributions from the CP-violating amplitudes since there is no two-photon intermediate CP-conserving process. This decay is extremely interesting because the direct CP-violating amplitude dominates over the indirect one as shown by Littenberg.[45]

By analogy with Eq. (3.17), the indirect CP-violating contribution to the decay $K_L \to \pi^0\nu\bar{\nu}$ can be estimated:

$$\begin{aligned} B(K_L \to \pi^0\nu\bar{\nu})_{\rm ind} &= B(K^+ \to \pi^+\nu\bar{\nu}) \frac{\Gamma(K^+ \to {\rm all})}{\Gamma(K_L \to {\rm all})} \frac{\Gamma(K_1 \to \pi^0\nu\bar{\nu})}{\Gamma(K^+ \to \pi^+\nu\bar{\nu})} \frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})_{\rm ind}}{\Gamma(K_1 \to \pi^0\nu\bar{\nu})} \\ &\simeq 2.1\times 10^{-5} B(K_L \to \pi^0\nu\bar{\nu}) R_\nu \sim 4\times 10^{-15} , \end{aligned} \tag{3.22}$$

where we have used $B(K^+ \to \pi^+\nu\bar{\nu}) \sim 2\times 10^{-10}$ and $R_\nu \equiv \Gamma(K_1 \to \pi^0\nu\bar{\nu})/\Gamma(K^+ \to \pi^+\nu\bar{\nu}) = 1$. For the direct CP-violating part, we have, from Figs. 4(a) and 4(b) with $l \leftrightarrow \nu$,

$$\begin{aligned} B(K_L \to \pi^0\nu\bar{\nu})_{\rm dir} &= \frac{\alpha^2}{8\pi^2 \sin^4\theta_W} \frac{\left| \sum_{i=c,t} {\rm Im}(V_{is}^* V_{id})\eta_i C_\nu(x_i) \right|^2}{|V_{us}|^2} B(K^+ \to \pi^0 e^+ \nu) \frac{\Gamma(K^+ \to {\rm all})}{\Gamma(K_L \to {\rm all})} \\ &\simeq 4.61\times 10^{-11} A^4 |C_\nu(x_t)|^2 \eta^2 , \end{aligned} \tag{3.23}$$

where the charm-quark contribution has been neglected. Since this decay has the same KM matrix elements dependence as $K_L \to \pi^0 e^+ e^-$, we get a similar curve which is shown in Fig. 10. Again the lower bound of $\sim 0.3\times 10^{-11}$ depends very weakly on the t-quark mass.

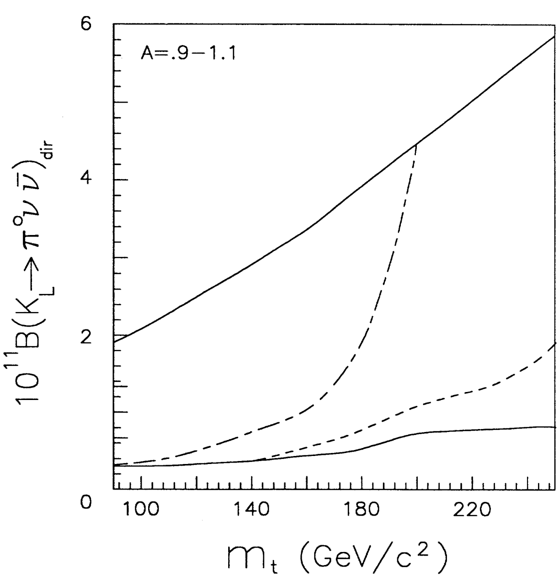

FIG. 10. Allowed branching ratio for the direct CP-violating contribution to $K_L \to \pi^0\nu\bar{\nu}$. Legend is the same as in Fig. 9.

The predicted upper bound is $\sim 5.5\times 10^{-11}$. For this decay also, the lower limits on η obtained from $K_L \to \mu\bar{\mu}$ pushes up the lower values for the branching ratios, especially for the heavier top quark.

Clearly, the branching ratio of the direct CP-violating contribution is much larger than that of the indirect one. Albeit difficult to measure, this decay provides a very clean test for CP violation in the standard model.

IV. CONCLUDING REMARKS

We have studied the ranges of the CKM mixing matrix parameters in the standard model from the measurements of $|V_{ub}/V_{cb}|$, the B_d^0-$\bar{B}_d^0$ mixing and the CP-violating parameter ϵ in the K^0 system. With these parameters we have calculated the short-distance effects on the K decays. In particular we get the following limits on the decay branching ratios:

$$\begin{aligned} &0.4\times 10^{-9} \le B(K_L \to \mu\bar{\mu})_{\rm SD} \le 5.3\times 10^{-9} , \\ &1.2\times 10^{-10} \le B(K^+ \to \pi^+\nu\bar{\nu}) \le 4.0\times 10^{-10} , \\ &0.4\times 10^{-12} \le B(K_L \to \pi^0 e^+ e^-)_{\rm dir} \le 6.5\times 10^{-12} , \\ &0.3\times 10^{-11} \le B(K_L \to \pi^0\nu\bar{\nu})_{\rm dir} \le 5.5\times 10^{-11} \end{aligned} \tag{4.1}$$

for $m_t \le 250$ GeV/c^2.

Incorporating the recent measurements on the decay branching ratio of $K_L \to \mu\bar{\mu}$ and the form factor of the Dalitz decay $K_L \to e^+ e^- \gamma$ we have extracted further constraints on the CKM mixing parameters and upper bounds on the t-quark mass, which are 196 and 300 GeV/c^2 for $B(K_L \to \mu\bar{\mu})_{SD} \leq 1.2$ and 2.2×10^{-9}, respectively. Taking into account these newer constraints, the branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$ is found to lie in the following tighter range:

$$1.2\times 10^{-10} \leq B(K^+ \to \pi^+ \nu\bar{\nu}) \leq (2.4 \text{ or } 2.9)\times 10^{-10} \quad (4.2)$$

for $B(K_L \to \mu\bar{\mu})_{SD} \leq 1.2$ or 2.2×10^{-9}. These constraints eliminate lower values for the direct CP-violating contributions to $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \nu\bar{\nu}$ especially for large m_t. Because of the higher branching ratio and less important effects from either indirect CP-nonconserving or CP-conserving contributions, the decay $K_L \to \pi^0 \nu\bar{\nu}$ is far more interesting than $K_L \to \pi^0 e^+ e^-$ for a study of direct CP violation.

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