Formation of primordial black holes by cosmic strings

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We calculate the density of primordial black holes formed by cosmic strings. When these black holes evaporate due to Hawking radiation, they produce γ bursts. We find that comparison with observations of the γ burst and γ background yields a new nontrivial upper limit for the string linear mass density. Our result is in good agreement with that found by Hawking. However, we use a quite different method to estimate the probability of black-hole formation. It should be mentioned that the adoption of a measure in the space of the parameter describing the form of loops is the key point. We present two reasonable choices, but the physical measure may be very different, so quantitative results may be changed, while qualitative conclusions remain valid.

I. INTRODUCTION

Cosmic strings are one-dimensional vacuum defects which appear naturally in a certain class of field theories.[1] Recently it has been realized that cosmic strings may play an important role in various scenarios of structure formation in the Universe.[2,3] Such scenarios require a string to have the linear mass density μ large enough to effectively interact with surrounding matter; that is, $G\mu/c^2$ should be of the order of 10^{-6}.[2]

In this paper we give the upper limit for μ which seems to contradict the demand $G\mu/c^2 \sim 10^{-6}$. The basic idea is that a cosmic-string loop may become a black hole during its evolution, when it falls under its gravitational horizon. The black hole may then evaporate due to the Hawking mechanism[4] emitting its energy mainly by γ photons. During the radiation-dominated era the density of black holes falls with time slower than the energy density of radiation. Thus, even if the density of primordial black holes was very small, the evaporating black holes may now significantly influence the γ spectrum. Combining the lower limit for the density of primordial black holes produced by cosmic strings with observations of γ-ray background and γ bursts we have obtained the upper bound for the string linear mass density.

Recently Hawking[5] has analyzed the same problem. However, he has estimated the probability of black-hole formation in a different way. Note that this probability is the key element of calculation. The idea of Hawking is to consider a string loop as a set of x parts, where x is a free parameter that can be expressed in terms of the correlation length s and the string lengths l: $x=l/s$. The approach presented in this paper is based on an exact solution for the evolution of cosmic strings. However, as will be shown later, we are also left with one unknown parameter that represents the uncertainty of the measure in the parameter space of the exact string solution.

II. THE MECHANISM OF BLACK-HOLE FORMATION BY COSMIC STRINGS

First we formulate the criterion for the creation of the black hole from a cosmic-string loop.

Let E be the energy of the loop in the center-of-mass frame. Its gravitational or Schwarzschild radius is equal to $(c=1)$

$$R_g = 2GE = 2G\mu l \ , \tag{2.1}$$

where $l=E/\mu$ is the fundamental (or invariant) length of the string loop and μ is the string linear mass density.

It is clear that the black hole is formed when the whole loop falls under its gravitational horizon. Thus one can simply follow the evolution of a string loop looking for an instant when the loop can be enclosed in a sphere of the radius $R \le R_g$. Certainly this condition establishes only the lower limit for the creation of black holes. For example, corrections to the equation of motion of the string due to its gravitational field can effectively result in increasing the radius R_g. It is also possible that only a part of the loop creates the black hole and this black hole draws in the rest of the string later. In the following, however, we neglect these effects because we are interested in the lower bound for the number density of primordial black holes formed by cosmic strings.

Notice, that due to one-dimensional nature of the string the ratio

$$f = \frac{R_g}{l} = 2G\mu \tag{2.2}$$

does not depend on the fundamental length l of the loop. Thus, if the scaling solution for the whole ensemble of strings exists,[6] then there is also the "scale-independent" production of black holes.

Now let us discuss how the gravitational radiation from the string loop[7] can modify this picture. We neglect the change of the shape of the string but we consider the change of the total string energy due to the gravitational radiation. This assumption is reasonable because, as will be shown later, black holes are formed mainly by strings which initially have an almost circular shape. It is easy to show that under this assumption the gravitational radiation does not alter the value of f. Hence our criterion for the black-hole formation remains unchanged.

III. SPECTRUM OF PRIMORDIAL BLACK HOLES

Let $n_{\rm str}(M)$ be the number density of strings with the mass M. The very small fraction p of them can form black holes, each of mass M. We neglect the change of the mass of the string due to gravitational radiation because the loop will radiate all its energy in time $t \sim M/(\gamma G\mu^2) \gg t_{\rm BH}$, where $t_{\rm BH}$ is the time of black-hole formation. Then the density of black holes with the mass M is

$$n_{\rm BH}(M) = p n_{\rm str}(M) . \tag{3.1}$$

The probability p of black-hole formation is equal to the probability that the string loop can be enclosed in a sphere of the radius R_g. Clearly, it can depend only on the ratio $R_g/l \sim G\mu$. As when R_g goes to zero, p must also go to zero, we parametrize the probability p, for small value of R_g/l, in the form

$$p = \kappa (G\mu)^{2+q} , \tag{3.2}$$

where κ and q are some coefficients.

We take into account only these black holes which are formed shortly after the string loops fall under the cosmological horizon. The number density of such loops, with sizes about the horizon scale, is equal to[1]

$$n_{\rm str}(M) = \nu' \frac{1}{R^3} = \nu \frac{\mu^3}{M^3} \tag{3.3}$$

and its fundamental length is about $l \sim 2\pi R$, where R is the distance to the horizon and ν and ν' are some numerical constants [if $l \simeq 2\pi R$ then $\nu \simeq (2\pi)^3 \nu'$].

The ratio of the energy density of black holes to the density of the matter at the instant $t = t_{\rm BH} \sim M/\mu$ when black holes of the mass M are formed, is equal to

$$\beta = \left. \frac{\rho_{\rm BH}}{\rho_{\rm mat}} \right|_{t=t_{\rm BH}} = \left. \frac{n_{\rm BH} M}{\rho_{\rm mat}} \right|_{t=t_{\rm BH}} = \frac{\nu\kappa (G\mu)^{2+q} \mu^3 M^{-2}}{\rho_{\rm mat}|_{t=t_{\rm BH}}} . \tag{3.4}$$

The density of the matter for $a \sim t^{1/2}$ (a is the scale factor) is

$$\rho_{\rm mat}|_{t=t_{\rm BH}} = \frac{3}{32\pi G t_{\rm BH}^2} \tag{3.5}$$

and finally we get

$$\beta = \frac{32\pi\nu\kappa}{3} (G\mu)^{3+q} . \tag{3.6}$$

The black hole with mass M will evaporate, due to Hawking radiation, after time

$$t_{\rm evp} = \left[\frac{M}{M_{\rm Pl}} \right]^3 t_{\rm Pl} , \tag{3.7}$$

where $t_{\rm Pl}$ and $M_{\rm Pl}$ are respectively Planck time and Planck mass.

The density of black holes at the moment of evaporation in the radiation-dominated era is

$$\alpha_{\rm evp} = \left. \frac{\rho_{\rm BH}}{\rho_{\rm mat}} \right|_{t=t_{\rm evp}} \sim \frac{a^{-3}}{a^{-4}} \sim t^{1/2} \tag{3.8}$$

so

$$\alpha_{\rm evp} = \left[\frac{t_{\rm evp}}{t_{\rm BH}} \right]^{1/2} \beta = (G\mu)^{1/2} \frac{M}{M_{\rm Pl}} \beta . \tag{3.9}$$

If the black hole evaporates in the matter-dominated era, the density of black holes is

$$\alpha_{\rm evp} = \left. \frac{\rho_{\rm BH}}{\rho_{\rm mat}} \right|_{t=t_{\rm evp}} \sim \frac{a^{-3}}{a^{-3}} \sim {\rm const} \tag{3.10}$$

and

$$\alpha_{\rm evp} = \alpha|_{t=t_{\rm eq}} = \left[\frac{t_{\rm eq}}{t_{\rm BH}} \right]^{1/2} \beta$$
$$= (G\mu)^{1/2} \left[\frac{t_{\rm eq}}{t_{\rm Pl}} \right]^{1/2} \left[\frac{M}{M_{\rm Pl}} \right]^{-1/2} \beta , \tag{3.11}$$

where $t_{\rm eq}$ is the instant when $\rho_{\rm mat} = \rho_{\rm radiation}$.

We can combine Eqs. (3.9) and (3.11) to obtain

$$\alpha_{\rm evp} = (G\mu)^{1/2} \beta \begin{cases} M/M_{\rm Pl} & \text{for } t_{\rm evp} < t_{\rm eq} , \\ (t_{\rm eq}/t_{\rm Pl})^{1/2} (M/M_{\rm Pl})^{-1/2} & \text{for } t_{\rm evp} > t_{\rm eq} . \end{cases} \tag{3.12}$$

Let us define the mass M_* equal to the mass of the black hole which evaporates at $t_{\rm evp} = t_{\rm eq}$:

$$M_* = \left[\frac{t_{\rm eq}}{t_{\rm Pl}} \right]^{1/3} M_{\rm Pl} \approx 10^{13}\ {\rm g} . \tag{3.13}$$

Substituting this to Eq. (3.12) we get, for $M > M_*$ [taking into account (3.6)],

$$\alpha_{\rm evp} = \frac{32\pi\nu\kappa}{3} (G\mu)^{7/2+q} (t_{\rm eq}/t_{\rm Pl})^{1/3} (M/M_*)^{-1/2} . \tag{3.14}$$

As discussed earlier, it gives us the lower limit for α.

IV. THE OBSERVATIONAL UPPER LIMIT

Now we are ready to compare our result with observations.

As the evaporating black holes with $M_{**} \approx 10^{15}$ g produce the γ burst, from observations of the γ background and γ bursts we can get the upper limit for the density of

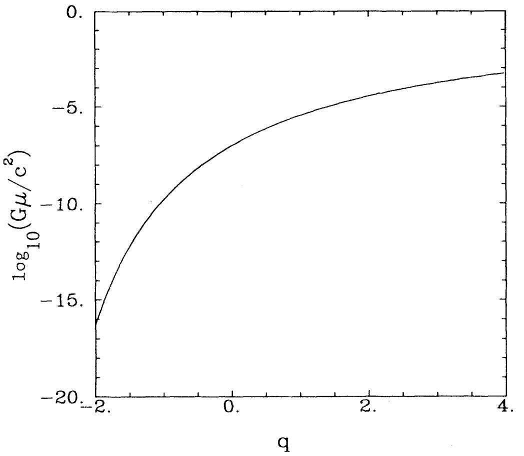

FIG. 1. The maximal value of $G\mu/c^2$ as a function of parameter q.

evaporating black holes:[8]

$$\alpha_{\rm evp} \lesssim \alpha_{\rm obs} \approx 3\times 10^{-8} \ . \qquad (4.1)$$

So we obtain

$$G\mu \lesssim \left[\frac{3}{32\pi\nu\kappa} \left[\frac{t_{\rm Pl}}{t_{\rm eq}} \right]^{1/3} \left[\frac{M_{**}}{M_*} \right]^{1/2} \right]^{2/7+2q}$$
$$\le [(G\mu)_{\max,q=0}]^{7/(7+2q)} \ , \qquad (4.2)$$

where $G(\mu)_{\max,q=0}$ is the upper limit for $G\mu$ corresponding to $q=0$. Taking into account that $(t_{\rm eq}/t_{\rm Pl})^{1/3}\simeq 10^{18}$, and $M_{**}/M_* \simeq 10^2$, we have

$$(G\mu)_{\max,q=0} = 10^{-7} \left[\frac{32\pi\kappa\nu}{3} \right]^{-2/7} < 10^{-7} \ . \qquad (4.3)$$

If $\nu\kappa > 3\times 10^{-2}$, this seems to be the case.

Our upper limit for $G\mu$, the fundamental value for cosmic-string theory, depends on the value of q. The form of this dependence is shown in Fig. 1. Note that for $q \le 1$ one gets the nontrivial, new upper limit on $G\mu$, which may contradict the requirement $G\mu \sim 10^{-6}$. It is therefore very important to find the realistic estimation for the parameter q.

V. BLACK HOLES FROM TWO-PARAMETER FAMILY

Now we give an example of calculation of the probability p and we discuss the meaning of q. In this section we deal with the two-parameter family of string loops found by Turok.[9]

Let $X^\mu(\tau,\sigma)$ be a four-vector describing the world sheet of a cosmic string loop. The equations of motion and constraints for gauge $\tau = X^0 \equiv t$, $X^\mu = (t, \mathbf{X})$ have a form

$$\ddot{\mathbf{X}} = \mathbf{X}'', \quad \dot{\mathbf{X}}^2 + \mathbf{X}'^2 = 1, \dot{\mathbf{X}}\cdot\mathbf{X}' = 0 \ , \qquad (5.1)$$

where an overdot represents a derivative with respect to τ, while a prime represents a derivative with respect to σ.

It is easy to check that the two-parameter family[9]

$$\mathbf{X} = \frac{1}{2} \begin{bmatrix} (1-\alpha)\sin\sigma^- + \frac{1}{3}\alpha\sin 3\sigma^- + \sin\sigma^+ \\ -(1-\alpha)\cos\sigma^- - \frac{1}{3}\alpha\cos 3\sigma^- - \cos\phi\cos\sigma^+ \\ -2\sqrt{\alpha(1-\alpha)}\cos\sigma^- - \sin\phi\cos\sigma^+ \end{bmatrix} \qquad (5.2)$$

($\sigma^\pm = \sigma \pm \tau$) satisfies all Eqs. (5.1).

Now we have to find the region in a parameter space of the family (5.2) for which the whole loop can, at some instant $t = t_0$, be enclosed in a sphere of the minimal radius $R \le R_g$. It is possible to show that this occurs at $t = \pi/2$ and

$$R^2|_{t=\pi/2} = [\sqrt{\alpha(1-\alpha)} - \sqrt{\beta(1-\beta)}]^2 + (\alpha/3 - \beta)^2 \ , \qquad (5.3)$$

where $\beta = \sin^2(\phi/2)$ (see Fig. 2). For $\alpha=\beta=0$ the minimum $R=0$ is reached. The case $\alpha=\beta=0$ represents the solution, for which the loop is a circle and at $t=\pi/2$ shrinks to a point.

To calculate the probability p it is necessary to perform the integration

$$p = \int_{R \le R_g} de \ , \qquad (5.4)$$

where de is the measure in parameter space. Unfortunately, we do not know this measure. If one naively chooses α and β to be the natural parameters (we call parameters x and y natural, if $de = dx\,dy$), $de = d\alpha\,d\beta$, he can evaluate the integral (5.4) and obtain

$$p_1 = \int_{R \le R_s} d\alpha\,d\beta = \lambda_1 (R_g/l)^{5/2} \ . \qquad (5.5)$$

Numerical investigation gives $\lambda_1 \simeq 1$; for example, if $R_g/l = 10^{-6}$ we get $p_1 \simeq 10^{-15}$. However, the other set of parameters, (η,ϕ), where $\alpha = \sin^2(\eta/2)$, $\beta = \sin^2(\phi/2)$,

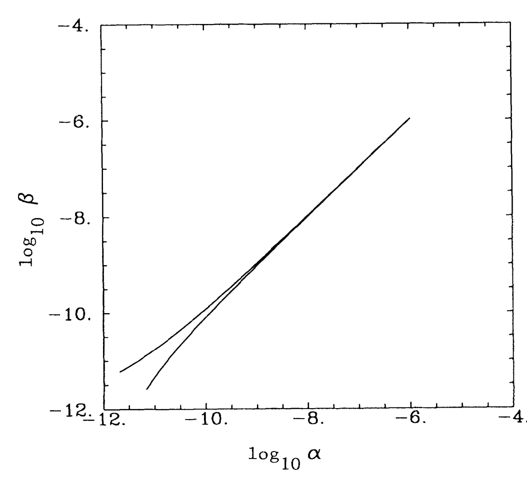

FIG. 2. Area in the parameter space of the Turok family of cosmic strings that leads to formation of black holes. The curve represents the solution of equation $R(\alpha,\beta) = R_g = 10^{-6} l$. In the region between the curves the minimal radius R is smaller than R_g, for $\alpha=\beta=0$ string loop shrinks to a point and $R=0$.

seem to be as "natural" as the previous one. It is not easy to say which set if any is "more natural." But $de = d\eta\, d\phi$ yields the quite different scaling

$$p_2 = \int_{R \le R_g} d\eta\, d\phi = \lambda_2 (R_g/l)^{3/2} , \quad (5.6)$$

where $\lambda_2 \sim 1$. For example, if $R_g/l = 10^{-6}$ then $p_2 \sim 10^{-9}$.

Comparing Eqs. (5.5) and (5.6) with (3.2) (recall that $R_g = 2G\mu l$) we conclude that the uncertainty of the choice of the proper measure de yields the uncertainty of the estimation of q.

VI. CONCLUSIONS

As we have seen in the previous section, the value of q crucially depends on the choice of the parametrization, that is, on the measure de of string space.

Another approach to the estimation of the probability p was given by Hawking.[5] He obtained

$$p \sim (G\mu)^{-2} (xG\mu)^{2x-2} , \quad (6.1)$$

where $x = l/s$ and s is the correlation length for a string loop. Comparing Eq. (6.1) with (3.2) we see the relationship between q and x:

$$q = 2x - 6 . \quad (6.2)$$

According to Hawking, x should lie in the range between 2 and 4, which corresponds to q in the range from -2 to 2. For $x = 3$ or $q = 0$ the upper limit for $G\mu$ obtained by Hawking[5] coincides with our result [see Eqs. (4.2) and (4.3)]

$$G\mu \lesssim 10^{-7} . \quad (6.3)$$

We have shown that comparison of the density of primordial black holes formed by cosmic strings with observational upper limits from the γ background and γ burst gives a nontrivial upper limit for $G\mu$. For reasonable values of q this contradicts the value of $G\mu$ required by the structure formation scenarios based on cosmic strings.[2] Therefore it is very important to calculate the value of q (or Hawking's x) more accurately or, equivalently, to introduce the measure in the parameter space of cosmic-string loops consistent with the dynamics of loop formation.

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