Cosmological models with a variable cosmological term

Marcelo Samuel Berman*
Department of Astronomy, University of Florida, 211 SSRB, Gainesville, Florida 32611
(Received 12 September 1990)

We discuss variable-Λ cosmological models, with G constant or variable. Our law of variation is $\Lambda \propto t^{-2}$ rather than the law recently suggested by Chen and Wu ($\Lambda \propto R^{-2}$).

I. INTRODUCTION

The "cosmological-constant problem" can be expressed as the discrepancies between the negligible value Λ has for the present Universe (as can be seen by the successes of Newton's theory of gravitation[1]) and the values 10^{50} times larger expected by the Glashow-Salam-Weinberg model[2] or by grand unified theory[3] (GUT) where it should be 10^{107} times larger.

Chen and Wu[4] have recently suggested that $\Lambda \propto R^{-2}$, where R is the scale factor in the Robertson-Walker metric. Abdel-Rahman[5] has recently considered a model with the same kind of variation.

Berman and Som[6] pointed out elsewhere that the relation $\Lambda \propto t^{-2}$ seems to play a major role in cosmology, and now I want to discuss some possibilities for realizing this hypothesis. In fact, Berman, Som, and Gomide[7] found this relation in Brans-Dicke static models; Berman[8] found it in a static universe with Endo-Fukui modified Brans-Dicke cosmology; Berman and Som[6] found it again in general Brans-Dicke models which obey the perfect-gas equation of state (see also Ref. 9); the same relation was found by Bertolami.[10,11] In what follows, I shall discuss a model obtained by augmenting the energy-momentum tensor of a perfect fluid by a term that represents a variable cosmological constant times the metric tensor, and afterwards I shall discuss a model with variable Λ and G, while the conservation law for the energy-momentum tensor is still valid.

II. VARIABLE COSMOLOGICAL TERM

Ozer and Taha[12,13] addressed cosmological models that arise from the assumption of a variable cosmological "constant" within Einstein's equations, by assuming that the energy-momentum tensor has an additional piece, interpreted as a vacuum contribution

$$T_{\mu\nu}^{(v)} = -\Lambda(t) g_{\mu\nu} , \tag{1}$$

where $\Lambda(t)$ is the cosmological term and $g_{\mu\nu}$ is the metric tensor. The quantity to be conserved is now

$$T_{\mu\nu} = p g_{\mu\nu} + (p+\rho) U_\mu U_\nu - \Lambda(t) g_{\mu\nu} , \tag{2}$$

where p and ρ stand for cosmic pressure and rest-energy density, respectively. For the Robertson-Walker (RW) metric, Einstein's field equation gives

$$H^2 = \frac{8\pi G}{3}(\rho+\Lambda) - kR^{-2} , \tag{3}$$

$$\frac{d(\rho R^3)}{dt} + p\frac{dR^3}{dt} + R^3 \frac{d\Lambda}{dt} = 0 , \tag{4}$$

where

$$H = \frac{\dot{R}}{R} . \tag{5}$$

Ozer and Taha examined the "critical density" case, and concluded that

$$k = 1 , \tag{6}$$

$$\Lambda(t) = \frac{3}{8\pi G R^2} . \tag{7}$$

We would like to point out a completely different solution to Eqs. (3) and (4). Suppose we have a perfect-gas equation of state,

$$p = \alpha\rho \quad (\alpha = \text{const}) , \tag{8}$$

and suppose that the deceleration parameter is constant:

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = m - 1 = \text{const} . \tag{9}$$

Then, according to Berman[14] and Berman and Gomide,[15]

$$R = (mDt)^{1/m} \quad (D = \text{const}) , \tag{10}$$

$$H = \frac{1}{mt} . \tag{11}$$

Now, postulate that

$$\Lambda(t) = Bt^{-2} \quad (B = \text{const}) , \tag{12}$$

$$\rho(t) = At^{-2} \quad (A = \text{const}) . \tag{13}$$

It is evident that we can solve our equations by simply imposing

$$m^{-2} = \frac{8\pi G}{3}(A+B) , \tag{14}$$

$$2B = (1+3\alpha)A , \tag{15}$$

$$k = 0 . \tag{16}$$

Now, we can work the different phases of the Universe: for radiation, put $\alpha = \frac{1}{3}$, for the present phase, $\alpha = 0$.

A problem with variable-Λ models is the creation of particles. For the pressure-free phase ($\alpha=0$), we would have, in our model

$$\rho R^3 \propto t^{3/m-2} ,$$

so that, in order not to have creation, we just impose

$$m=\tfrac{3}{2}, \quad \text{i.e.,} \quad q=\tfrac{1}{2} . \tag{17}$$

This yields the usual law for the present Universe:

$$R=(\tfrac{3}{2}Dt)^{2/3} . \tag{18}$$

It should be pointed out that the Ozer-Taha relation (7) is of the same kind as postulated by Chen and Wu.[4] In both cases, the creation of particles is required. In our models, this can be avoided, for the present phase, as discussed above.

This model can solve for the entropy problem. In standard general relativity,

$$T\,dS \equiv d(\rho V)+p\,dV=0 \tag{19}$$

while, here

$$\frac{T\,dS}{dt}=-\gamma R^3\frac{d\Lambda}{dt}=2\gamma(mDt)^{3/m}Bt^{-3} , \tag{20}$$

where

$$V=\gamma R^3 \quad (\gamma=\text{const}) .$$

We have $\dot{S}>0$ if $B>0$ and $m\neq 1$.

For the early Universe, we have a radiation phase $\alpha=\frac{1}{3}$. If we impose

$$m=1 \tag{21}$$

so that we solve the horizon problem, we have

$$A=B=\frac{3}{16\pi G} . \tag{22}$$

In fact, the horizon distance

$$d_H(t,t_0)=R(t)\int_{t_0}^{t}\frac{dt'}{R(t')}=t\ln(t/t_0)$$

and $\lim_{t_0\to 0} d_H \to \infty$ (the model has no horizon for $m=1$).

The time variation of Λ "explains" why Λ is, for the present Universe, negligible, but was very large in the very early phases. Further consequences of the models will be dealt in a subsequent article.

III. WHITROW-RANDALL RELATION AND VARIABLE GRAVITATIONAL AND COSMOLOGICAL "CONSTANTS"

Berman and Som[16] showed that the Whitrow-Randall relation[17,18]

$$\frac{GM}{R}\sim 1 , \tag{23}$$

where G is Newton's gravitational "constant," and M is the mass of the observable Universe with radius R, is compatible with several cosmological models in Brans-Dicke theory. When relation (23) is substituted by

$$G\rho\sim H^2 , \tag{24}$$

Berman[19] showed that it is compatible with general relativity and Brans-Dicke models with a constant deceleration parameter.

Berman[20] studied a case where G and Λ vary with time, when the field equations are given by

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R^{\lambda}_{\lambda}=8\pi G(t)T_{\mu\nu}+\Lambda(t)g_{\mu\nu} , \tag{25}$$

where $R_{\mu\nu}$ is the Ricci tensor, $\Lambda(t)$ is a variable gravitational constant, and the energy tensor is that of a perfect fluid:

$$T_{\mu\nu}=-pg_{\mu\nu}+(p+\rho)U_\mu U_\nu . \tag{26}$$

Abdel-Rahman[5] proposed a model with these equations, along with the conservation equations (see Beesham[21])

$$\rho\dot{G}=-\frac{\dot{\Lambda}}{8\pi} , \tag{27}$$

$$\dot{\rho}=-3\frac{\dot{R}}{R}(\rho+p) . \tag{28}$$

Here, p, ρ, and R stand for the cosmic pressure, rest-energy density, and the scale-factor of the Robertson-Walker's metric.

The solution found by Berman,[20] with Abdel-Rahman's theory, is that of a perfect gas-law equation of state, reading

$$p=\alpha\rho \quad (\alpha=\text{const}) , \tag{29}$$

$$R=(mDt)^{1/m} \quad (m,D=\text{const}) , \tag{30}$$

$$q=-\frac{\ddot{R}R}{\dot{R}^2}=m-1 , \tag{31}$$

$$H=\frac{1}{mt} , \tag{31$'$}$$

$$G=Kt^{B/4\pi A} , \tag{32}$$

$$\rho=\frac{A}{K}t^{-(2+B/4\pi A)} , \tag{33}$$

$$\alpha=\frac{1}{3}\left[m\left[2+\frac{B}{4\pi A}\right]-1\right] , \tag{34}$$

$$m=\left[\frac{3}{8\pi A+B}\right]^{1/2} , \tag{35}$$

$$\Lambda=Bt^{-2} . \tag{36}$$

Here, K, A, and B are three constants, which define all the variables in the theory, for a flat universe ($k=0$).

Whitrow and Randall showed that their relation (1) was equivalent to

$$\rho_M\sim\frac{3}{4\pi G}t^{-2} , \tag{37}$$

where ρ_M stands for the Machian rest-energy density. Following the method of Berman and Som,[16] let us impose

$$\rho_M=\frac{3}{4\pi G}t^{-2} \tag{38}$$

as the "Machian condition."

From (32) and (38), we find

$$\rho_M = \frac{3}{4\pi K} t^{-(2+B/4\pi A)} . \tag{39}$$

We compose now (37) with (33), and see that

$$A = \frac{3}{4\pi} . \tag{40}$$

If we further impose

$$m = 1 , \tag{41}$$

we find, from (35) and (40),

$$B = -3 . \tag{42}$$

From (34), we get

$$\alpha = 0 . \tag{43}$$

This is then the "dust" solution, valid for the present Universe. The calculation of the value of constant K is now an easy task. The known values for H_0^{-1} and G_0 are

$$H^{-1} = 1.8 \times 10^{10} \text{ yr} ,$$

$$G_0 = 6.7 \times 10^{-8} \text{ cm}^3 (\text{g sec}^2)^{-1} ;$$

therefore

$$K \cong 3.8 \times 10^{10} \text{ cm}^3/\text{g sec} .$$

It is interesting to point out that, in our framework, Mach's principle is not only valid for $\alpha = 0$ (which means $p = 0$). The density for the present Universe is half the "critical density."

I would like to point out that, although I showed in Ref. 20 how to "calculate" Planck's time for such a model, I would rather conclude now that, due to the variable nature of $G(t)$, the notion of Planck's time becomes useless, and meaningless, when compared with the standard interpretation. The same applies to the other Planck quantities.

Another topic of concern is the $\Lambda(t)$ dependence on t. Chen and Wu[4] suggested recently that we should have, from quantum cosmology, a dependence for Λ of the type

$$\Lambda \propto R^{-2} . \tag{44}$$

When $m = 1$, our present formulation is precisely of that form, but I suggest that the relation that is really important is

$$\Lambda \propto t^{-2} . \tag{45}$$

On the other hand, Abdel-Rahman[5] found

$$\Lambda = 3R^{-2} = 3(R_0^2 + t^2)^{-1} \tag{46}$$

but his model has $\Lambda = 0$ for a flat universe. In our model, on the other hand, we have (for the present Universe)

$$\Lambda = -3t^{-2} = -3D^2 R^{-2} . \tag{47}$$

It is now seen that the otherwise indetermined constant γ in Chen and Wu's paper,[4] defined by the relation

$$\Lambda(R) = \gamma R^{-2} , \tag{48}$$

is given here the value

$$\gamma = -3D^2 , \tag{49}$$

where $\dot{R} = D = \text{const}$. It is interesting also to acknowledge that the deceleration parameter is a null constant, for the present Universe, according to our results.

Let us now show that, for the radiation-dominated phase ($\alpha = \frac{1}{3}$), we now shall need

$$m = \tfrac{1}{2} \tag{50}$$

and

$$R = (\tfrac{1}{2} D t)^2 . \tag{51}$$

Indeed, from (34) we get, taking into account (40),

$$m = \frac{2}{2 + B/3} \tag{52}$$

while from (35) and (40) we have

$$m^2 = \frac{3}{6+B} . \tag{53}$$

From (52) and (53) we easily find relation (50), and thus, from (30), we get (51).

For the radiation phase, then

$$q_r = -\tfrac{1}{2} . \tag{54}$$

We now turn to the creation of particles per unit volume in the present phase r:

$$r = \frac{d}{R^3 dt}(\rho R^3) = 2\rho H \tag{55}$$

[we used (31′), (33), (41), (40), and (42)]. Our result (55) can be compared with steady-state cosmology, where

$$r_{\text{SS}} = 3\rho H . \tag{56}$$

r and r_{SS} are of the same order of magnitude, 10^{-41} g cm^{-3} s^{-1}, completely inaccessible to experimental verification.[22]

ACKNOWLEDGMENTS

I thank Professor James Ipser and Professor M. M. Som for discussing some points related to this work, and reading this manuscript. This work was supported by Conselho Nacional de Desenvolvimento Científico e Tenológico (Brazil).

---

*Permanent address: Departmento de Ciencias Exatas de Faculdade de Filsofia, Ciencias e Letras de FURJ, 89200, Joinville, SC, Brazil.

[1]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[2]E. S. Abers and B. W. Lee, Phys. Rep. **9**, 1 (1973).

[3]P. Langacker, Phys. Rep. **72**, 185 (1981).

[4]W. Chen and Y-S. Wu, Phys. Rev. D **41**, 695 (1990).

[5]A.-M. M. Abdel-Rahman, Gen. Relativ. Gravit. **22**, 655 (1990).
[6]M. S. Berman and M. M. Som, Int. J. Theor. Phys. (to be published).
[7]M. S. Berman, M. M. Som, and F. M. Gomide, Gen. Relativ. Gravit. **21**, 287 (1989).
[8]M. S. Berman, Int. J. Theor. Phys. **29**, 567 (1990).
[9]M. S. Berman, Int. J. Theor. Phys. (to be published).
[10]O. Bertolami, Nuovo Cimento B **93**, 36 (1986).
[11]O. Bertolami, Fortschr. Phys. **34**, 829 (1986).
[12]M. Ozer and M. O. Taha, Phys. Lett. B **171**, 363 (1986).
[13]M. Ozer and M. O. Taha, Nucl. Phys. **B287**, 776 (1987).
[14]M. S. Berman, Nuovo Cimento B **74**, 182 (1983).
[15]M. S. Berman and F. M. Gomide, Gen. Relativ. Gravit. **20**, 191 (1988).
[16]M. S. Berman and M. M. Som, Gen. Relativ. Gravit. **22**, 625 (1990).
[17]G. Whitrow, Nature **158**, 165 (1946).
[18]G. Whitrow and D. Randall, Mon. Not. R. Astron. Soc. **111**, 455 (1951).
[19]M. S. Berman, Int. J. Theor. Phys. **29**, 567 (1990).
[20]M. S. Berman (unpublished).
[21]A. Beesham, Int. J. Theor. Phys. **25**, 1295 (1986).
[22]J. B. Narlikar, *Introduction to Cosmology* (Jones and Bartlett, Boston, 1983).