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Weakly interacting massive particle densities in decaying-particle-dominated cosmology

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We consider the present cosmological density of weakly interacting massive particles (WIMP's) in the context of models where the energy density is dominated by decaying massive particles, such that WIMP's freeze out of chemical equilibrium during the massive-particle-dominated era. An expression for the density of WIMP's is derived by a generalization of the Lee-Weinberg argument for the density of heavy neutrinos in the standard cosmological model.

I. INTRODUCTION

A possible feature of many particle-physics models is the existence of heavy particles which decay in the early Universe at low temperatures ~ 1 GeV or less. For example, in the context of supersymmetric models,[1] it is possible for the gravitino to decay at temperatures of ~ 1 MeV to ~ 1 GeV, if its mass is in the range 10^4-10^6 GeV.[2,3] (The 1-MeV lower bound on the temperature at the end of the decaying-particle-dominated period is necessary in order to not interfere with nucleosynthesis.) Also, symmetry breaking along flat directions in the scalar potential of supersymmetry (SUSY) models, such as must occur in SUSY axion models[4] or models with enlarged symmetry groups such as $SU(5)\times U(1)$,[5] will result in a coherently oscillating scalar field equivalent to a density of massive scalar particles.[6,7] These particles will be long lived, such that they typically decay at temperatures in the range 1 MeV to 1 GeV. It is therefore of interest for these and possibly other models to consider generally the consequences of a period of decaying-particle-dominated cosmology at temperatures of ~ 1 MeV to ~ 1 GeV in the early Universe.

In this paper we consider the question of densities of weakly interacting massive particles (WIMP's) in the cosmological scenario where the energy density of the Universe is dominated by decaying massive particles at temperatures 1 MeV $\lesssim T \lesssim$ 1 GeV. During the decaying-particle-dominated period of interest to us here the radiation energy density in the Universe is primarily that coming from the decay of the particles.[4,7] As a result, the Universe expansion as a function of temperature is more rapid than is the case with a standard radiation-dominated cosmology ($R \propto T^{-8/3}$ compared to $R \propto T^{-1}$). This unorthodox cosmology necessitates the generalization of the calculation of WIMP densities due to freezing out of chemical equilibrium, originally done by Lee and Weinberg,[8] to the case where freeze-out occurs during the decaying-particle-dominated era.

The question of the present density of WIMP's in the Universe is relevant to the problem of the nature of dark matter. Dark matter is expected theoretically from the inflationary universe scenario,[9] which predicts $\Omega=1$, while the dynamically inferred density is $\Omega=0.1-0.3$.[10] Studies of galactic rotation curves and of galactic clusters indicate that around 90% of the mass on every mass scale is dark matter, which if $\Omega>0.15$ cannot be baryonic.[11,12] It is important therefore to consider the value of $\Omega_{\rm WIMP}$ in a decaying-particle-dominated cosmology.

The paper is organized as follows. In Sec. II we briefly review the cosmological scenario of interest. In Sec. III we discuss for a general WIMP the density of WIMP's in the Universe at present which arises from freezing out of chemical equilibrium. In Sec. IV we illustrate these results by considering the case where the WIMP corresponds to a light photino in SUSY models. Section V contains our conclusions.

II. EVOLUTION OF THE UNIVERSE DOMINATED BY DECAYING MASSIVE PARTICLES

In this section we briefly review the cosmological scenario of interest, in the context of which we wish to discuss constraints on WIMP's coming from the present observed matter density in the Universe.

At an initial temperature T_0 and corresponding time t_0, it is assumed that the energy density in the massive decaying particles, which we label X, dominates the total energy density of the Universe. For $t>t_0$ the energy density in X particles is

$$\rho_x(t)=\rho_{x_0}\left[\frac{R(t_0)}{R(t)}\right]^3 e^{-\Gamma(t-t_0)} , \tag{2.1}$$

where ρ_{x_0} is the energy density at $t=t_0$, Γ is the decay rate of the X particles, and the scale factor for $\Gamma(t-t_0)\ll 1$ is

$$R(t)=[6\pi G\rho_{x_0}R(t_0)^3]^{1/3}[t-t_0+(6\pi G\rho_{x_0})^{-1/2}]^{2/3} , \tag{2.2}$$

where G is Newton's constant. In most cases of interest one can take $t-t_0$ to be large compared with $(6\pi G\rho_{x_0})^{-1/2}$. Equation (2.1) is then given by

$$\rho_x(t)=\frac{1}{6\pi G(t-t_0)^2}\ . \tag{2.3}$$

We denote by t_f the time at which $t-t_0=1/\Gamma$ and by $t_=$ the time at which the radiation energy from X decays dominates the primordial radiation energy density existing before X decays.[4,7]

$t_=$ is given by

$$t_=-t_0=\left[\frac{5}{3}\frac{\rho_{r_0}}{(6\pi G)^{1/3}\Gamma\rho_{x_0}^{4/3}}\right]^{3/5}, \tag{2.4}$$

where ρ_{r_0} is the initial radiation density at t_0. The time-temperature relation is then (for $\Gamma t<1$) (Ref. 4)

$$\frac{\pi^3}{3}g(T)T^4(t-t_0)=\frac{\Gamma}{G}\ , \tag{2.5}$$

where $g(T)=g_B+\frac{7}{8}g_f$ ($g_B=2$ for the photon) counts the number of spin degrees of freedom. Thus during the period $t>t_=$, one has from (2.5) and (2.2) that $R\propto T^{-8/3}$ compared to $R\propto T^{-1}$ when entropy is conserved. The X field decays away and the Universe becomes radiation dominated again once $t>t_f$, with a corresponding temperature T_f given by[4]

$$\Gamma^2=\tfrac{2}{9}\pi^3 g(T_f)GT_f^4\ . \tag{2.6}$$

Because of the much increased expansion between any two tempeatures when $T_=>T>T_f$ (during which the expansion rate is proportional to T^4 rather than T^2 as for standard radiation-dominated cosmology with RT $=$const) the discussion of relic WIMP densities arising from freezing out of chemical equilibrium must be reconsidered if freeze-out occurs at a temperature greater than T_f.

III. WIMP DENSITIES IN DECAYING-MASSIVE-PARTICLE-DOMINATED COSMOLOGY

In this section we consider the stable WIMP density in the Universe today due to the freezing out of an equilibrium density of WIMP's. This requires a generalization of the mass bounds obtained in Refs. 8, 13, and 14 for the case of a standard radiation-dominated cosmology to the case of a decaying-particle-dominated cosmology and a general WIMP. The evolution of a density of stable WIMP's can be studied via the rate equation[8] (we use units with $\hbar=c=k_B=1$)

$$\frac{dn}{dt}=-\frac{3\dot{R}}{R}n+(n_0^2-n^2)\langle\sigma v\rangle\ . \tag{3.1}$$

Here $n(t)$ is the WIMP number density, $\langle\sigma v\rangle$ is the thermal average value of the cross section for WIMP annihilation times the relative WIMP velocity, and $n_0(T)$ is the number of WIMP's in chemical equilibrium,

$$n_0(T)=\frac{N}{(2\pi)^3}\int_0^\infty 4\pi p^2dp\left[\exp\left[\frac{(m^2+p^2)^{1/2}}{T}\right]\pm 1\right]^{-1} \tag{3.2}$$

with $+$ ($-$) for fermions (bosons) and N is the number of spin degrees of freedom ($N=2$ for fermions and 1 for scalars).

An approximate analytic solution of (3.1) for the WIMP density can be obtained by a method analogous to that of Ref. 8. For $T>T_f$ (3.1) can be rewritten as

$$\frac{dh}{dT}=\frac{12\langle\sigma v\rangle}{\left[g(T)\pi^3\frac{G}{\Gamma}\right]}(h^2-h_0^2)T^3\ , \tag{3.3}$$

where

$$h(T)=\frac{n(T)}{T^8}\ .$$

An approximate solution is obtained by using the fact that the rate of annihilation is fast enough to maintain $h^2\approx h_0^2$ down to a freeze-out temperature $T_{\rm fr}$, at which h_0 changes with T more rapidly than the rate of annihilations. Below $T_{\rm fr}$, the evolution of h is found by solving (3.1) in the absence of h_0 on the right-hand side (RHS) and with $h(T_{\rm fr})=h_0(T_{\rm fr})$ as the initial value for $h(T)$. (It is shown in Appendix A that for typical examples the approximate analytic solution is no more than 20% larger than the exact numerical solution, becoming increasingly accurate as $T_f/T_{\rm fr}$ is reduced.) The freeze-out temperature is found from

$$\left.\frac{dh_0}{dT}\right|_{T=T_{\rm fr}}=\frac{12\langle\sigma v\rangle}{\left[g(T)\pi^3\frac{G}{\Gamma}\right]}h_0^2T_{\rm fr}^3 \tag{3.4}$$

with $h_0(T)$ obtained from (3.2). In the nonrelativistic limit $T<m$ one has

$$h_0(T)=\frac{2}{T^5}\left[\frac{m}{2\pi T}\right]^{3/2}e^{-m/T}\ . \tag{3.5}$$

(In general $T_{\rm fr}$ will be small compared with m.) In order to calculate $T_{\rm fr}$ one requires an expression for $\langle\sigma v\rangle$ for the case of WIMP annihilations. For nonrelativistic WIMP's this has the general form

$$\langle\sigma v\rangle=a+\frac{bT}{m}\ . \tag{3.6}$$

In general a and b may be m dependent. For the case of Majorana fermions a is m independent while b is proportional to m^2. For massive Dirac neutrinos $b=0$, $a \propto m^2$, and for sneutrinos $b=0$ and a is m independent.

From (3.4) one obtains the condition for $T_{\rm fr}$:

$$\left[\frac{T}{m}\right]^{3/2} e^{m/T} = \frac{4T_f^2\langle \sigma v\rangle}{\pi^3 m\,[g(T_{\rm fr})G]^{1/2}} . \tag{3.7}$$

From this one obtains

$$T_{\rm fr} = \frac{m}{\ln\left[\dfrac{4T_f^2 m^{1/2}\left[a+\dfrac{bT_{\rm fr}}{m}\right]}{\pi^3[g(T_{\rm fr})G]^{1/2}T_{\rm fr}^{3/2}}\right]} . \tag{3.8}$$

From $T_{\rm fr}$ down to T_f one solves (3.3) with $h_0=0$ on the RHS and with $h(T_{\rm fr})=h_0(T_{\rm fr})$. The solution is

$$\eta(T) \equiv \frac{h(T)}{h(T_{\rm fr})} = \frac{1}{1+\dfrac{8T_f^2 h(T_{\rm fr})}{[2g(T_{\rm fr})\pi^3 G]^{1/2}}\left[\dfrac{b}{5m}(T_{\rm fr}^5-T^5)+\dfrac{a}{4}(T_{\rm fr}^4-T^4)\right]} . \tag{3.9}$$

Using (3.8) and (3.5) one has

$$h(T_{\rm fr}) = \frac{\delta[2g(T_{\rm fr})\pi^3 G]^{1/2}}{8T_f^2\left[a+\dfrac{b}{\delta}\right]T_{\rm fr}^4} , \tag{3.10}$$

where $\delta = m/T_{\rm fr}$. As seen from (3.8), δ depends only logarithmically on the parameters of the model. From (3.9) one obtains the relic WIMP density at T_f, $n(T_f)$. $\eta(T)$ accounts for the effect of annihilations at $T<T_{\rm fr}$. Note that $\eta(T)$ tends quickly to a constant as T decreases from $T_{\rm fr}$, i.e., annihilations are significant only for T close to $T_{\rm fr}$. Then one can write

$$h(T_f) = \left[\frac{g(T_f)}{g(T_{\rm fr})}\right]^2 \eta(T_f)h(T_{\rm fr}) . \tag{3.11}$$

The factor $[g(T_f)/g(T_{\rm fr})]^2$ in (3.11) accounts for the heating of the photon temperature due to annihilations of particles with masses between $T_{\rm fr}$ and T_f. (This is discussed in Appendix B.)

The number density of WIMP's in the Universe at present is then

$$n(T_\gamma) = \frac{[27g(T_{\rm fr})]^{1/2}}{[2\pi^3 G g(T_f)^3]^{1/4}}\left[\frac{g(T_\gamma)}{g(T_f)}\right]\left[\frac{g(T_f)}{g(T_{\rm fr})}\right]^2 \times \frac{\eta(T_f)T_\gamma^3\Gamma^{3/2}\delta^5}{8(a+b/\delta)m^4} , \tag{3.12}$$

where T_γ is the photon temperature at present and the factor $g(T_\gamma)/g(T_f)$ accounts for the heating of the photons due to annihilations of particles with masses between T_f and T_γ. Note that $n(T_\gamma)$ depends only on the decaying particle lifetime and not on its mass or initial density. From (3.12) we can write down the value of Ω for WIMP's of mass m:

$$\Omega_{\rm WIMP} \equiv \frac{\rho_{\rm WIMP}}{\rho_c} = \frac{mn(T_\gamma)}{\rho_c} , \tag{3.13}$$

where $\rho_c = 2.0\times10^{-47}h_{1/2}^2$ GeV^4 [$h_{1/2}=(1$ to $2)$] is the critical density at present ($h_{1/2}$ parametrizes the uncertainty in the present value of H). From (3.12) and (3.13) a lower bound on the WIMP mass can be obtained by requiring that $\Omega_{\rm WIMP}\le 1$. From (3.12) and (3.13) we see that the value of $\Omega_{\rm WIMP}$ in the decaying particle cosmology is sensitive both to the WIMP mass and to the decay rate of the decaying particle:

$$\Omega_{\rm WIMP} \propto \frac{\Gamma^{3/2}}{m^3(a+b/\delta)} . \tag{3.14}$$

In this we ignore the logarithmic dependence of δ on m. This may be contrasted with the case of standard radiation-dominated cosmology, where $\Omega_{\rm WIMP}$ depends on m only through a and b:

$$\Omega_{\rm WIMP} \propto \frac{1}{a+b/\delta} . \tag{3.15}$$

The decay rate Γ will be determined in most cases by model parameters relating to physics at a scale large compared with M_W. Typically one finds

$$\Gamma \approx K_d\frac{M_X^3}{M_I^2} , \tag{3.16}$$

where M_X is the mass scale of the decaying particle and M_I is a large mass scale ($M_X \ll M_I$) which serves to make the decaying particle long lived, for example, a large spontaneous symmetry-breaking scale.[4,6] K_d represents the factors from coupling constants and loop integration ($K_d \ll 1$). From (2.6) we find $T_f \propto K_d^{1/2}M_X^{3/2}/M_I$. Thus a wide range of values for the mass scale M_I will give rise to values of T_f in the range of interest (1 MeV $\lesssim T_f \lesssim$ 1 GeV). For example, with $M_X=100$ GeV (typical of SUSY models[6]) and $K_d=10^{-2}$, one finds T_f in the above range if 10^{11} GeV $\lesssim M_I \lesssim 10^{14}$ GeV. We illustrate these results in the following section by considering the example of a light photino freezing out during the decaying-particle-dominated era.

IV. RELIC LIGHT PHOTINO DENSITY IN DECAYING-PARTICLE-DOMINATED COSMOLOGY

We apply the analysis of the preceding section to the case of a light photino. In the limit where the photino does not mix significantly with the other neutralinos [i.e., where the $SU(2)_L$ and $U(1)_y$ SUSY-breaking gaugino masses are small compared with m_z] the main contribution to the annihilation cross section is from the exchange of squarks and sleptons (Fig. 1). This gives[13,14]

$$\langle \sigma v \rangle = \sum_i \frac{Q_i^4 4\pi\alpha^2}{(m_{\tilde{i}}^2+m_\gamma^2)^2}\left[2Tm_\gamma + \frac{(m_\gamma^2-m_i^2)^{1/2}}{m_\gamma}m_i^2\right] , \tag{4.1}$$

where α is the fine-structure constant, m_γ is the photino mass, Q_i the electric charge of the quark or lepton species i produced by the annihilation (with mass m_i), and $m_{\tilde{i}}$ is the mass of the corresponding squark or slepton. [In writing (4.1) it has been assumed that left-handed and right-handed sparticles have the same mass.]

Since in most SUSY models the slepton masses are smaller than the squark masses,[1] in the following we consider the example of photino annihilation mainly arising from exchange of sleptons of a common mass $m_{\tilde{l}}=90$ GeV. One then has

$$\langle \sigma v \rangle = \frac{4\pi\alpha^2}{m_{\tilde{l}}^4}\left[6Tm_\gamma + \frac{(m_\gamma^2-m_\tau^2)^{1/2}}{m_\gamma}m_\tau^2\right] . \tag{4.2}$$

(If m_γ is less than m_τ, then in the second term m_τ is replaced by m_μ.) From this we obtain, for coefficients a and b of (3.6),

$$a = \frac{4\pi\alpha^2}{m_{\tilde{l}}^4}\frac{(m_\gamma^2-m_\tau^2)^{1/2}m_\tau^2}{m_\gamma} , \tag{4.3a}$$

$$b = \frac{24\pi\alpha^2}{m_{\tilde{l}}^4}m_\gamma^2 . \tag{4.3b}$$

As a specific example we calculate the relic photino density for a 10-GeV photino, under the assumption that the slepton mass is $m_{\tilde{l}}=90$ GeV. The temperature T_f at which the decaying-particle-dominated period ends should be less than $T_{\rm fr}$ in the cases of interest to us here.

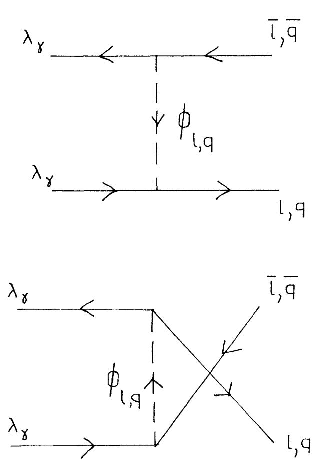

FIG. 1. Diagrams giving the main contribution to the photino annihilation cross section via squark and slepton exchange.

Since $T_{\rm fr}$ is typically $O(m/20)$ for a WIMP of mass m, one expects $T_{\rm fr} \lesssim 1$ GeV for light WIMP's. Thus T_f should be in the range 1 MeV $\lesssim T_f \lesssim 1$ GeV. As a typical example we consider $T_f = 100$ MeV. From (4.3) one obtains

$$a = 3.6\times10^{-11}\ \mathrm{GeV}^{-2} , \tag{4.4a}$$

$$b = 7.0\times10^{-9}\ \mathrm{GeV}^{-2} . \tag{4.4b}$$

Then from (3.8) one obtains the value of $T_{\rm fr}$:

$$T_{\rm fr} = 649\ \mathrm{MeV} . \tag{4.5}$$

In obtaining this one varies $T_{\rm fr}$ on both sides of (3.8) until they are equal. We assume that $T_{\rm fr}$ is greater than the temperature of the quark-hadron phase transition, in which case $g(T_{\rm fr})=\frac{247}{7}$. Thus $\delta \equiv m_\gamma/T_{\rm fr} = 15.4$. From (3.12) and (3.13) we finally obtain the relic photino density

$$\Omega_{\rm WIMP} = \frac{5.7\times10^{-3}}{h_{1/2}^2} , \tag{4.6}$$

where $g(T_f)=\frac{247}{7}$ and $g(T_\gamma)=2$ have been used, and from (3.9) $\eta(T_f)=0.24$. We see that a 10-GeV photino in this scenario would contribute only a small amount of dark matter. In fact we find that (for the case $h_{1/2}=1$) $\Omega=1$ occurs when the photino mass is $m_\gamma=2.6$ GeV with $\delta=12.3$. This is small compared with the value for the standard radiation-dominated cosmology, $m_\gamma=10.4$ GeV.

V. CONCLUSIONS

In this paper we have discussed, in the context of models where the energy density at low temperatures is dominated by decaying massive particles, the cosmological relic density of WIMP's which arises when WIMP's freeze out of equilibrium during this period. A general expression for the relic density in terms of the WIMP nonrelativistic annihilation cross section was derived by generalizing the Lee-Weinberg calculation of the density of heavy neutral leptons. It was shown that the lower bound on the WIMP mass from the requirement $\Omega < 1$ is in general reduced compared with the standard cosmology case.

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TABLE I. δ and $T_{\rm fr}$ for a 30-GeV photino for various values of T_f.

T_f (GeV)	δ	$T_{\rm fr}$ (GeV)
0.01	11.7	2.6
0.1	16.5	1.8
0.5	19.8	1.5
0.8	20.8	1.4

TABLE II. Numerical vs analytical approximation results for $h(T)$ in case (a) $T_f=0.01$ GeV and case (b) $T_f=0.8$ GeV.

(a)

T (GeV)	$h(T)$ (numerical)	$h(T)$ (analytical)	$h_0(T)$
3.2	9.3×10^{-7}	9.2×10^{-7}	9.2×10^{-7}
3.0	7.6×10^{-7}	7.5×10^{-7}	7.5×10^{-7}
2.9	6.8×10^{-7}	6.6×10^{-7}	6.6×10^{-7}
2.7	5.2×10^{-7}	4.9×10^{-7}	4.9×10^{-7}
2.5	3.8×10^{-7}	3.0×10^{-7}	3.3×10^{-7}
2.3	2.7×10^{-7}	1.9×10^{-7}	2.0×10^{-7}
2.0	1.8×10^{-7}	1.4×10^{-7}	7.1×10^{-8}
1.5	1.3×10^{-7}	1.2×10^{-7}	3.1×10^{-9}
1.0	1.2×10^{-7}	1.1×10^{-7}	2.0×10^{-11}
0.7	1.16×10^{-7}	1.11×10^{-7}	5.2×10^{-16}
0.5	1.16×10^{-7}	1.11×10^{-7}	1.7×10^{-22}
0.3	1.16×10^{-7}	1.11×10^{-7}	1.9×10^{-38}

(b)

T (GeV)	$h(T)$ (numerical)	$h(T)$ (analytical)	$h_0(T)$
2.2	1.5×10^{-7}	1.5×10^{-7}	1.5×10^{-7}
2.0	7.1×10^{-8}	7.1×10^{-8}	7.1×10^{-8}
1.8	2.7×10^{-8}	2.6×10^{-8}	2.6×10^{-8}
1.6	7.3×10^{-9}	7.1×10^{-9}	7.1×10^{-9}
1.4	1.5×10^{-9}	1.2×10^{-9}	1.2×10^{-9}
1.2	5.1×10^{-10}	5.2×10^{-10}	8.9×10^{-10}
1.0	3.5×10^{-10}	4.1×10^{-10}	2.0×10^{-12}
0.9	3.2×10^{-10}	3.8×10^{-10}	1.4×10^{-13}
0.8	3.1×10^{-10}	3.7×10^{-10}	4.6×10^{-15}

APPENDIX A: COMPARISON OF ANALYTIC AND NUMERICAL SOLUTIONS OF THE RATE EQUATION

In this appendix we compare for some typical examples the numerical solution of (3.1) with the analytical approximate solution (3.9). We consider the case of a photino of mass $m_\gamma=30$ GeV, for various values of T_f. The form of $\langle\sigma v\rangle$ for this case is given by (4.1), which for a 30-GeV photino is dominated by the T-dependent term. [We take $g(T_f)=g(T_{\rm fr})=60$ throughout.]

In Table I we give the analytic approximation values of $\delta\equiv m_\gamma/T_{\rm fr}$ and $T_{\rm fr}$, for various values of T_f. In Table II we give, for cases $T_f=0.01$ GeV and $T_f=0.8$ GeV, the analytical and numerical values of $h(T)$ for a range of values of T.

For the purposes of calculating the relic density of photinos at present the most significant results are those of Table III, which show the analytic approximation and numerical results for the limiting value of $h(T)$ at temperatures small compared with the freeze-out temperature. It is seen that the limiting value of $h(T)$ calculated analytically is at most 20% larger than the numerical value, with the error decreasing as T_f is reduced.

APPENDIX B: EFFECT OF ANNIHILATION OF PARTICLES ON THE PHOTON TEMPERATURE IN DECAYING-PARTICLE-DOMINATED COSMOLOGY

In this appendix we discuss the photon-heating factors which result from the temperature of the Universe dropping below the mass of particles which are lighter than $T_{\rm fr}$. In the standard cosmology, the Universe expands adiabatically, with total entropy conserved. Therefore,

$$g(T_a)T_a^3R_a^3=g(T_b)T_b^3R_b^3\ , \tag{B1}$$

where T_a and T_b are arbitrary temperatures and $g(T)=g_B+\frac{7}{8}g_F$ is the number of effectively light degrees of freedom in thermal equilibrium at temperature T. Thus one has, for a density of nonrelativistic effectively conserved particles between T_f and T_γ,

$$\frac{n(T_f)}{n(T_\gamma)}=\frac{R(T_\gamma)^3}{R(T_f)^3}=\frac{g(T_f)}{g(T_\gamma)}\left[\frac{T_f}{T_\gamma}\right]^3 . \tag{B2}$$

Between $T_{\rm fr}$ and T_f radiation from decaying particles dominates the total radiation density and entropy is not

TABLE III. Limiting values of $h(T)$ for T small compared with $T_{\rm fr}$.

T_f (GeV)	$h_{\rm limit}$ (numerical)	$h_{\rm limit}$ (analytical)
0.01	1.16×10^{-7}	1.11×10^{-7}
0.1	6.0×10^{-9}	6.8×10^{-9}
0.5	5.9×10^{-10}	6.9×10^{-10}
0.8	3.1×10^{-10}	3.7×10^{-10}

conserved, preventing one from using (B1). From (2.2) and (2.5) one has, for temperatures T_a, $T_b > T_f$,

$$\left[\frac{R(T_a)}{R(T_b)}\right]^3 = \frac{g(T_b)^2 T_b^8}{g(T_a)^2 T_a^8} \; . \tag{B3}$$

Therefore between $T_{\rm fr}$ and T_f one has (in the absence of WIMP annihilations)

$$\frac{n(T_{\rm fr})}{n(T_f)} = \frac{g(T_{\rm fr})^2}{g(T_f)^2}\left[\frac{T_{\rm fr}}{T_f}\right]^8 \; . \tag{B4}$$

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