Impact of new $|V_{ub}/V_{cb}|$ and ϵ'/ϵ measurements on weak mixing angles

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The value of the weak mixing parameter $|V_{ub}|$ has recently been determined to be of order $0.1|V_{cb}|$. A recent determination of ϵ'/ϵ in CP-violating kaon decays gives $(-0.5\pm1.5)\times10^{-3}$, in contrast to an earlier measurement of $(3.3\pm1.1)\times10^{-3}$. The implications of these results for the magnitudes and phases of the Kobayashi-Maskawa matrix elements and for the top-quark mass are explored.

I. INTRODUCTION

Our present understanding of *CP* violation is based on the three-family Kobayashi-Maskawa (KM) model[1] of quarks, some of whose charged-current couplings have phases. Over the past decade, new data have allowed us to refine our knowledge about parameters of this matrix. The most recent progress in this respect has been the determination by two experimental groups[2,3] that the $b\to u$ transition matrix element is nonzero.

Preliminary results of an experiment searching for the difference between *CP*-violating decays of kaons to pairs of neutral and charged pions have been presented.[4] The result of this experiment (Fermilab E731) is that the ratio ϵ'/ϵ governing this difference is $(-0.5\pm1.5)\times10^{-3}$, to be compared with a previous result from CERN (Experiment NA31)[5] of $(3.3\pm1.1)\times10^{-3}$. It is the purpose of the present paper to analyze and compare the implications of these two experiments with respect to the parameters of the KM matrix.

The top quark enters into several constraints on KM parameters through loop diagrams, so that such an analysis necessarily implies a favored range of top-quark masses. We shall show that the E731 result favors a heavy top quark (with mass above 200 GeV/c^2), while the NA31 result favors a top quark in the mass range 80–160 GeV/c^2. These two results correspond to two disjoint regions of parameters for the KM element V_{ub} governing the $b\to u$ transition: one with $\mathrm{Re}(V_{ub})>0$ and the other with $\mathrm{Re}(V_{ub})<0$ (in a phase convention which we shall specify). We shall suggest ways of resolving this ambiguity within the context of experiments sensitive to KM parameters. The comparison of electroweak radiative corrections[6] with data on W and Z masses and neutral-current processes would favor[7] a top-quark mass below 200 GeV/c^2, but there are small loopholes (which we shall mention) in such arguments.

Our analysis is patterned on one by Schubert.[8] In that work, the value for ϵ'/ϵ was taken to be the nonzero NA31 result,[5] and only the solution with $\mathrm{Re}(V_{ub})<0$ was found. Examples of previous analyses (by no means a complete list) may be found in Refs. 9–14, including a recent study[14] whose main emphasis is *CP* violation in *B*-meson decays, and which uses somewhat broader ranges of parameters than the present work. The twofold ambiguity in present solutions has been noted by others.[15,16] A preliminary account of our work has appeared in Ref. 17.

Concurrently with our preparation of the present manuscript, the extensive analysis of Ref. 16 appeared, taking into account corrections[18] which lower ϵ'/ϵ by a considerable amount, which grows with increasing m_t, from the prediction employed in Ref. 8. The investigation of Ref. 16 concludes that a negative value of ϵ'/ϵ, corresponding to the central value reported in Ref. 4, is compatible with a top-quark mass slightly above 200 GeV/c^2. We shall compare results for KM parameters and m_t obtained with the old[8] and new[16] relation between ϵ'/ϵ and these other quantities. Surprisingly, qualitative conclusions regarding m_t and KM parameters are affected much less than we might have expected by the new relations, which serve mainly to express a preference for the E731 data[4] over the earlier result[5] from the NA31 Collaboration.

If the two most recent results[4,5] for ϵ'/ϵ are averaged, one finds a solution qualitatively similar to that based on NA31 data alone, with a top-quark mass below 170 GeV/c^2 and $\mathrm{Re}(V_{ub})<0$. If *no* data on ϵ'/ϵ are used, one obtains two solutions. The one with $\mathrm{Re}(V_{ub})<0$ is remarkably similar to that based on the NA31 or the averaged data, while that with $\mathrm{Re}(V_{ub})>0$ resembles the one favored by the E731 data.

In Sec. II we define a parametrization of the three-generation KM matrix and review briefly what is known about its elements. The components of our fits, and the constraints they apply, are described in Sec. III. Section

IV is devoted to the results, for various choices of input data on ϵ'/ϵ. A discussion of the twofold ambiguity in parameters, and possible means of resolving it, occupies Sec. V. Section VI concludes.

II. PARAMETRIZATION AND KNOWN ELEMENTS

The weak charge-changing transitions may be described by a unitary matrix V ($V^\dagger V=1$) in the interaction term

$$\mathcal{L}_{\rm int}\sim \bar{U}_L\gamma_\mu V D_L W^{+\mu}+{\rm H.c.} \quad (2.1)$$

We shall assume here that there are only three generations of quarks, so that U_L and D_L are column vectors denoting $(u,c,t)_L$ and $(d,s,b)_L$, respectively. Then the matrix V (the KM matrix[1]) may be parametrized[19] as

$$V=\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{bmatrix} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1\end{bmatrix} . \quad (2.2)$$

The condition $V^\dagger V=1$ implies, for example, that

$$V_{ub}^* V_{ud}+V_{cb}^* V_{cd}+V_{tb}^* V_{td}=0 . \quad (2.3)$$

We adopt the convention[20] that KM elements along the diagonal and just above the diagonal are real and positive, and neglect small complex phases in the elements V_{cd} and V_{ts}. Since $V_{ud}\approx V_{tb}\approx 1$, we may write

$$V_{ub}^*+V_{td}\approx\lambda V_{cb}\approx A\lambda^3 . \quad (2.4)$$

The elements V_{ub}^*, V_{td}, and λV_{cb} then form a triangle in the complex plane,[20] as shown in Fig. 1. The angles ϕ_3, ϕ_1, and ϕ_2 lie opposite these respective sides. [The phase δ as originally defined in Ref. 1 is equal to $\arg(V_{ub}^*)-\arg(V_{td})=\phi_1+\phi_3=\pi-\phi_2$. For a discussion of the relation between the original KM phase convention and that adopted here, see Chau and Keung[20] or the Appendix of Ref. 21.] In a parametrization adopted by the Particle Data Group,[22] the angle ϕ_1 of Fig. 1 is known as δ_{13}.

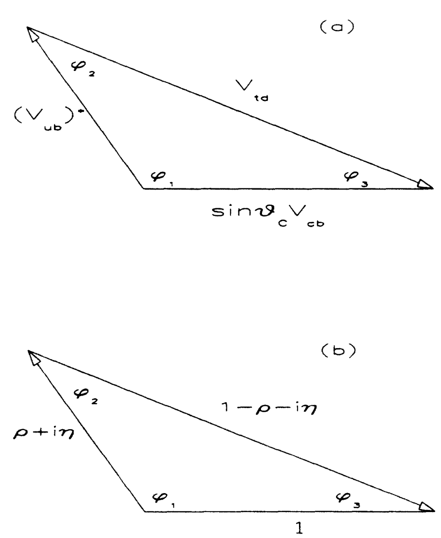

FIG. 1. Triangles formed by Kobayashi-Maskawa (KM) matrix elements in the complex plane as a result of the unitarity relation (2.3). (a) Matrix elements. Note the definitions of the angles ϕ_1, ϕ_2, and ϕ_3, also used in Ref. 30. (b) Matrix elements divided by the quantity $A\lambda^3=\sin\theta_C V_{cb}$.

There are four parameters in Eq. (2.2): λ, A, ρ, and η. We review the discussion of Ref. 21 regarding their values. One learns $\lambda=0.22$ ($=V_{us}=\sin\theta_C$) from strange-particle decays.[23] Nuclear beta decays and charm decays confirm the predictions of approximate unitarity for *two* quark generations that $V_{ud}=V_{cs}\approx 1-\lambda^2/2$ (see Ref. 24). Charm production with neutrinos and semileptonic decays of charmed particles to nonstrange final states are consistent[8,24] with $|V_{cd}|=\lambda$. We ignore any small errors in λ. The remaining parameters, which we discuss next, have uncertainties which are substantial enough to be taken into account in our fits.

III. CONSTRAINTS FROM DATA USED IN FITS

A. Magnitude of V_{cb}

The b-quark lifetime and semileptonic branching ratio to charmed final states govern the magnitude of the matrix element $V_{cb}=A\lambda^2$. In Ref. 8, the value $\tau_B=(1.18\pm0.14)\times10^{-12}$ s was used, corresponding to a weighted average of B^0, B^+, and B_s decay rates. The semileptonic decay rate to charmed final states was taken there to be $(10.9\pm0.4)\%$. An average of methods using exclusive and inclusive final states leads to the value[8,25] $V_{cb}=0.049\pm0.005$, or

$$A=(1.0\pm0.1) , \quad (3.1)$$

which we take in what follows.

B. Magnitude of V_{ub}

Recent data from the CLEO and ARGUS Collaborations,[2,3] based on the analysis of leptons with high center-of-mass momentum in B-meson decays, indicate that the matrix element $V_{ub}=A\lambda^3(\rho-i\eta)$ is nonzero. The complex phase of V_{ub} is very important for the successful description of CP violation within the framework of the KM matrix.

The errors on $|V_{ub}|$ are dominated by systematic effects,[25] including uncertainties on continuum subtraction [leptons not due to B decays from the $\Upsilon(4S)$ resonance] and on model dependence for the $b\to u$ and $b\to c$ transitions. For present purposes we have taken the range of possible values quoted for $|V_{ub}|$ to imply

$$|V_{ub}/V_{cb}|=0.10\pm0.05 \;\text{ or }\; (\rho^2+\eta^2)^{1/2}=0.46\pm0.23 . \quad (3.2)$$

The constraint thus restricts ρ and η to lie between two circles with origin (0,0) in the (ρ,η) plane.

C. B-$\bar{B}$ mixing

A new CLEO value[2] for mixing of neutral nonstrange B mesons has been combined with previous world data to obtain the new average

$$r \equiv \frac{(\Delta m/\Gamma)^2}{2+(\Delta m/\Gamma)^2} = 0.18 \pm 0.05 \tag{3.3}$$

or

$$\Delta m/\Gamma = 0.66 \pm 0.11 . \tag{3.4}$$

At present we believe this quantity to arise primarily as a result of box diagrams with top quarks in internal lines, and hence to probe the magnitude of the KM element $V_{td} = A\lambda^3(1-\rho-i\eta)$. In the limit in which the top quark dominates the mixing, one has[8]

$$\frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2} |V_{td}|^2 m_t^2 F\left[\frac{m_t^2}{M_W^2}\right] m_B f_B^2 B_B \eta_B \tau_B . \tag{3.5}$$

Here $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi decay constant, $m_B = 5.28$ GeV/c^2, and we take Schubert's values for the following parameters: $f_B = 140 \pm 25$ MeV, $B_B = 0.85 \pm 0.10$, η_B (a QCD correction) $= 0.85 \pm 0.05$, and τ_B as given earlier. The function $F(x)$ is 1 for $x=0$, $\frac{3}{4}$ for $x=1$, and $\frac{1}{4}$ for large x, and is explicitly equal to [26]

$$F(x) \equiv \frac{1}{4}\left[1 + \frac{3-9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^3}\right] . \tag{3.6}$$

The calculation in Ref. 27 of the QCD correction is equivalent to taking a coefficient η_B about $\frac{1}{2}$ to $\frac{2}{3}$ of that quoted above.

With the new experimental value (3.4), the assumption $M_W = 80$ GeV/c^2, and the parameters listed above, one obtains the relation

$$m_t |V_{td}| \sqrt{F(m_t^2/M_W^2)} = (1.47 \pm 0.32) \text{ GeV}/c^2 \tag{3.7a}$$

or

$$m_t A[(1-\rho)^2 + \eta^2]^{1/2} \sqrt{F(m_t^2/M_W^2)} = (138 \pm 30) \text{GeV}/c^2 . \tag{3.7b}$$

The error in Eqs. (3.7) is dominated by that in f_B. Theoretical estimates (see, e.g., the compilation in Ref. 21) tend to have a somewhat greater spread, so the error may be underestimated. A measurement of the decay $B \to \tau\nu$ could provide direct information on f_B and help to settle the question.[21,28] The coefficient on the right-hand side of Eq. (3.7) should be multiplied by about 1.2–1.4 if the QCD correction of Ref. 27 is taken into account; the top-quark mass would then increase correspondingly.

For any fixed value of m_t, Eq. (3.7b) requires (ρ, η) to lie between two circles with origin at the point (1,0).

D. CP violation in kaon decay

The parameters ϵ and ϵ' describe CP violation in neutral-kaon decay:

$$\frac{\langle \pi^+\pi^- | H_{\rm wk} | K_L \rangle}{\langle \pi^+\pi^- | H_{\rm wk} | K_S \rangle} \equiv \eta_{+-} = \epsilon + \epsilon' , \tag{3.8a}$$

$$\frac{\langle \pi^0\pi^0 | H_{\rm wk} | K_L \rangle}{\langle \pi^0\pi^0 | H_{\rm wk} | K_S \rangle} \equiv \eta_{00} = \epsilon - 2\epsilon' . \tag{3.8b}$$

Both of them receive important contributions from loops involving top quarks.

(1) *The parameter* $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ (see Ref. 29) describes the gross features of CP violation in the kaon system. It may be expressed in terms of KM elements and the parameter B_K denoting the connection between a free-quark estimate and the actual value of the $\Delta S = 2$ matrix element describing K-$\bar{K}$ mixing. We use the relation[30]

$$|\epsilon| = 4.33 A^2 B_K \eta [\eta_3 S(x_c, x_t) - \eta_1 S(x_c) + \eta_2 A^2 \lambda^4 (1-\rho) S(x_t)] , \tag{3.9}$$

where the first term on the right-hand side denotes the contribution of a loop diagram with one virtual charmed quark and one virtual top quark, the second comes from two charmed quarks, and the third from two top quarks. The QCD correction factors are taken[8,30,31] to be $\eta_3 = 0.36$, $\eta_1 = 0.85$, $\eta_2 = 0.61$, while $x_c \equiv m_c^2/M_W^2$, $x_t \equiv m_t^2/M_W^2$, with $m_c = 1.5$ GeV/c^2. We have taken[32] $B_K = \frac{2}{3} \pm \frac{1}{6}$. The functions on the right-hand side of Eq. (3.9) are

$$S(x) \equiv x F(x) \tag{3.10}$$

and

$$S(x_i, x_j) \equiv x_i x_j \left[\left[\frac{1}{4} + \frac{3}{2(1-x_j)} - \frac{3}{4(1-x_j)^2} \right] \frac{\ln x_j}{x_j - x_i} + (x_j \leftrightarrow x_i) - \frac{3}{4(1-x_i)(1-x_j)} \right] . \tag{3.11}$$

In the limit of large top-quark mass, the dominant term on the right-hand side of Eq. (3.9) is the third one. For fixed top-quark mass, the relation (3.9) then constrains the product $\eta(1-\rho)$, so that ρ and η must lie between two hyperbolas whose focus is the point $(\rho, \eta) = (1,0)$.

(2) *The parameter* ϵ'/ϵ has been measured most recently by two experimental groups (see Ref. 4 for a compilation of earlier values). The difference between the two values is just large enough to be interesting:

$$\text{Re}(\epsilon'/\epsilon) = (3.3 \pm 1.1) \times 10^{-3} \text{ (NA31, Ref. 5)} , \tag{3.12}$$

$$\text{Re}(\epsilon'/\epsilon) = (-0.5 \pm 1.4 \pm 0.6) \times 10^{-3} \text{ (E731, Ref. 4)} . \tag{3.13}$$

If averaged, these give a value of

$$\text{Re}(\epsilon'/\epsilon) = (2.0 \pm 0.9) \times 10^{-3} \text{ (NA31-E731 average)} . \tag{3.14}$$

In what follows, we shall assume, in accord with theoretical expectations, that ϵ'/ϵ is real. We shall also consider a situation in which no ϵ'/ϵ data are used in determining KM parameters, and will find that the remaining data lead to a prediction for the range of values of this parameter.

The estimate of Ref. 33 related ϵ'/ϵ directly to the parameter η:

$$\epsilon'/\epsilon \approx 1.1\times 10^{-2}\eta \left[\frac{150\ \mathrm{MeV}/c^2}{m_s(1\ \mathrm{GeV})}\right]^2 \frac{10^{-12}\ \mathrm{s}}{\tau_B} . \quad (3.15)$$

The quantities in this expression were evaluated in Ref. 8 and assigned probable errors. The result is equivalent to the relation

$$\epsilon'/\epsilon = (0.81\pm 0.27)\times 10^{-2} A^2 \eta . \quad (3.16)$$

This relation was used in the preliminary analysis of Ref. 17, and its implications will be compared with those of another relation based on the more complete work of Ref. 16.

One often sees pictures of ϵ'/ϵ as a decreasing function of top-quark mass (see, e.g., Ref. 34). It is important to note that these are consistent with the behavior in Eqs. (3.15) and (3.16), as will be seen presently.

The importance of electroweak penguin contributions has been stressed recently in Ref. 18. Corrections of this sort have been included in the analysis of Ref. 16. They lead to an explicit m_t dependence in the relation between ϵ'/ϵ and η, whose form depends on m_s and Λ_{QCD}. The relation obtained in Ref. 16 then may be parametrized approximately for $75 \le m_t \le 250\ \mathrm{GeV}/c^2$ as

$$\epsilon'/\epsilon = (0.19\pm 0.09)\times 10^{-2} A^2 \eta \times \left[1 - 0.46\left[\frac{m_t}{100\ \mathrm{GeV}/c^2} - 0.7\right]^2\right] . \quad (3.17)$$

The error ± 0.09 comes from assuming that $125 \le m_s \le 200\ \mathrm{MeV}/c^2$ and $100 \le \Lambda_{\mathrm{QCD}} \le 300$ MeV, with $m_s = 175\ \mathrm{MeV}/c^2$ and $\Lambda_{\mathrm{QCD}} = 200$ MeV taken as central values. The predicted value[18] of ϵ'/ϵ scales approximately as $(175\ \mathrm{MeV}/c^2/m_s)^2\,(\Lambda_{\mathrm{QCD}}/200\ \mathrm{MeV})^{1/2}$. Although with Eq. (3.17), ϵ'/ϵ becomes negative for $m_t > 217\ \mathrm{GeV}/c^2$, uncertainties in hadronic matrix elements could drastically shift this value in either direction.

We shall use Eq. (3.17) to construct an analysis parallel to that based on Eq. (3.16) and reported in Ref. 17. We regard (3.16) as a representative "high" estimate for ϵ'/ϵ (almost certainly invalid for $m_t \gg 100\ \mathrm{GeV}/c^2$), and (3.17) as a representative "low" estimate [which, however, is likely to be more reliable than (3.16) for top-quark masses in the range of 150–250 GeV/c^2.] We shall find that the determination of KM parameters and m_t is surprisingly insensitive to which of these two constraints is chosen.

E. Graphic depiction of constraints

Figure 2 describes contours of relations between parameters ρ and η in the KM element V_{ub} following from

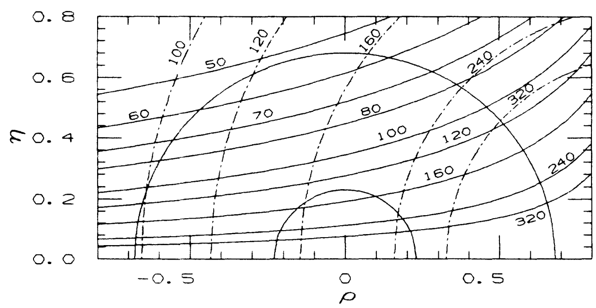

FIG. 2. Regions of parameters ρ, η in the KM element V_{ub}, as constrained by various experiments (see text).

the constraints just enumerated. Solid semicircles with center at $(\rho,\eta)=(0,0)$ correspond to 1σ limits [Eq. (3.2)] on $|V_{ub}/V_{cb}|$. Dotted-dashed circular arcs with centers at (1,0), labeled by top-quark masses in GeV/c^2, correspond to central values of $|1-\rho-i\eta|$ favored by B-$\bar{B}$ mixing [the constraint (3.7b)]. Hyperbolas labeled by top-quark masses, of approximate form $\eta(1-\rho)m_t^2 \sim$ const, are based on assuming that the CP-violating parameter ϵ is described by box-diagram contributions [Eq. (3.9)] in the three-generation KM model. Horizontal lines (not shown) would describe constraints on η based on Eq. (3.16) or (3.17) following from fixed values of ϵ'/ϵ.

IV. RESULTS OF FITS

We used the data described in Sec. III in conjunction with the χ^2 minimization program MINUIT. Attempts were made to search for both local and global minima. The results are shown in Tables I and II. To within less than a percent, all fits imply central values of $A=1$. The errors quoted are those corresponding to an increase of χ^2 by one unit above the local minimum. "World" data on ϵ'/ϵ refer to the average (3.14).

No useful 90% confidence level upper limits are obtained on the top-quark mass. The predicted values of ϵ'/ϵ are based on the relation (3.16) or (3.17), with errors in this relation and in η (from Table I or II) added in quadrature. The number of degrees of freedom denotes the difference between the number of pieces of data (four or five, depending on whether one uses data on ϵ'/ϵ) and the number of parameters (four, consisting of A, ρ, η, and m_t).

When data on ϵ'/ϵ are omitted, two exact solutions are obtained, with vanishing χ^2. These are shown in both tables for convenience. Many solutions incorporating ϵ'/ϵ data exhibit two local minima, vestiges of these exact solutions. We shall refer to these two local minima as region 1 [corresponding to $\mathrm{Re}(V_{ub}) \propto \rho < 0$] and region 2 [corresponding to $\mathrm{Re}(V_{ub}) \propto \rho > 0$]. In preliminary results of the fits shown in Table I not all local minima were found.[17] Only one set of local minima for NA31 and world ϵ'/ϵ data are found if the relation (3.17) is used (Table II).

Figures 3 and 4 describe the contour plots in ρ and η for values of χ^2 equal to one standard deviation (inner contours) and 90% confidence level limits (1.64σ, outer

TABLE I. Parameters involved in fits to data constraining the KM matrix. The relation (3.16) between ϵ'/ϵ and η has been used in these fits. Regions 1 and 2 correspond to two distinct local minima of χ^2. The lower bounds on m_t are 90%-confidence-level limits with respect to the local χ^2 minima, and the errors on parameters correspond to ranges which lead to a change in χ^2 by one unit with respect to these minima. N_{DF} denotes the number of degrees of freedom.

ϵ'/ϵ data	m_t (GeV/c^2)	m_t^{min} (GeV/c^2)	ρ	η	Predicted ϵ'/ϵ (10^{-3})	χ^2/N_{DF}
NA31 (1)	107^{+47}_{-31}	61	$-0.41^{+0.30}_{-0.24}$	$0.27^{+0.15}_{-0.10}$	2.2 ± 1.2	0.77/1
NA31 (2)	$259^{+289}_{\cdots}$ [a]	... [b]	$0.32^{+0.26}_{-0.38}$	$0.16^{+0.17}_{-0.09}$	1.3 ± 1.1	2.31/1
E731 (1)	138^{+57}_{-38}	82	$-0.38^{+0.28}_{-0.24}$	$0.17^{+0.09}_{-0.06}$	1.4 ± 0.8	1.79/1
E731 (2)	452^{+448}_{-197}	... [b]	$0.48^{+0.23}_{-0.24}$	$0.08^{+0.07}_{-0.04}$	0.7 ± 0.5	0.64/1
World (1)	119^{+49}_{-33}	71	$-0.40^{+0.29}_{-0.24}$	$0.23^{+0.11}_{-0.08}$	1.8 ± 1.0	0.02/1
World (2)	$309^{+287}_{\cdots}$ [a]	... [b]	$0.37^{+0.23}_{-0.35}$	$0.13^{+0.14}_{-0.07}$	1.1 ± 1.0	0.82/1
None (1)	122^{+54}_{-37}	66	$-0.40^{+0.28}_{-0.24}$	$0.22^{+0.14}_{-0.08}$	1.8 ± 1.1	0.00/0
None (2)	401^{+398}_{-192}	... [b]	$0.44^{+0.23}_{-0.27}$	$0.10^{+0.10}_{-0.05}$	0.8 ± 0.6	0.00/0

[a] No 1σ lower limit for solutions in region 2.
[b] No 90%-C.L. lower limit for solutions in region 2.

contours) above the *global* minima. Here, A and m_t are allowed to range over all possible values so as to minimize χ^2 at each value of (ρ,η). Individual sets of data on ϵ'/ϵ can lead to a preference for solutions of region 1 or 2 at most at the 1σ level, but not yet at 90% confidence level.

The apices of the triangles are located at the values of ρ,η listed in Table I (for Figs. 3) or Table II (for Figs. 4), corresponding to local minima in χ^2. Note that the horizontal scales in Figs. 3 and 4 are the same as the vertical scales, so the angles of the triangles can be estimated visually from these plots.

Figures 5 and 6 depict the corresponding contour plots in m_t and η. At each point, ρ and A are chosen to minimize χ^2. The local minima for lower top-quark masses correspond to parameters of region 1, while those of higher m_t correspond to region 2. As in Figs. 3 and 4, it is not yet possible to distinguish between solutions of region 1 and those of region 2 at the 90% confidence level using any combinations of ϵ'/ϵ data.

We now describe the nature of fits for each choice of input data on ϵ'/ϵ.

A. NA31 data alone

The 90% confidence level lower bounds on the top-quark mass shown in Tables I and II are not as stringent as those based on direct searches at the Fermilab Colliding Detector Facility[35] (CDF). The present limits, however, do not make assumptions about decay modes of the top quark, whereas the direct searches are predicated on a conventional top decay. The 1σ upper bounds are within the reach of extended searches that could be performed at the Fermilab Tevatron in the next few years.[36] A slightly smaller value of ϵ'/ϵ than the input (experimental) value is predicted on the basis of Eq. (3.16), while the newer relation (3.17) favors a much smaller value. The discrepancy between experiment and prediction is the source of the relatively large χ^2 value in Table II. The allowed regions of parameters are shown in Figs. 3(a), 4(a), 5(a), and 6(a). The real part of the KM element V_{ub}, proportional to the parameter ρ, is predicted to be negative.

The angles ϕ_1, ϕ_2, and ϕ_3 were defined in Fig. 1 and discussed in Sec. II. In the fits to NA31 data corre-

TABLE II. Parameters involved in fits to data constraining the KM matrix. The relation (3.17) among ϵ'/ϵ, η, and m_t has been used in these fits. Regions 1 and 2 correspond to two distinct local minima of χ^2. The lower bounds on m_t are 90%-confidence-level limits with respect to the local χ^2 minima, and the errors on parameters correspond to ranges which lead to a change in χ^2 by one unit with respect to these minima. N_{DF} denotes the number of degrees of freedom.

ϵ'/ϵ data	m_t (GeV/c^2)	m_t^{min} (GeV/c^2)	ρ	η	Predicted ϵ'/ϵ (10^{-3})	χ^2/N_{DF}
NA31 (1)	115^{+50}_{-35}	62	$-0.41^{+0.28}_{-0.24}$	$0.24^{+0.15}_{-0.09}$	0.4 ± 0.3	2.32/1
E731 (1)	125^{+55}_{-38}	69	$-0.39^{+0.29}_{-0.24}$	$0.21^{+0.13}_{-0.08}$	0.3 ± 0.2	0.31/1
E731 (2)	383^{+249}_{-167}	... [a]	$0.43^{+0.18}_{-0.24}$	$0.10^{+0.09}_{-0.05}$	-0.7 ± 0.6	0.02/1
World (1)	113^{+49}_{-35}	61	$-0.41^{+0.28}_{-0.24}$	$0.24^{+0.16}_{-0.09}$	0.4 ± 0.3	1.50/1
None (1)	122^{+54}_{-37}	66	$-0.40^{+0.28}_{-0.24}$	$0.22^{+0.14}_{-0.08}$	0.4 ± 0.3	0.00/0
None (2)	401^{+398}_{-192}	... [a]	$0.44^{+0.23}_{-0.27}$	$0.10^{+0.10}_{-0.05}$	-0.7 ± 0.7	0.00/0

[a] No 90%-C.L. lower limit for solutions in region 2.

sponding to the global χ^2 minimum, which occurs in region 1, ϕ_1 ranges between $\sim 110°$ and $\sim 160°$, with central value $\sim 150°$, while ϕ_2 ranges between $\sim 50°$ and $\sim 10°$, with central value $\sim 20°$. The angle ϕ_3 is about 10°. One finds $|V_{td}|/\lambda^3 = 1.4 \pm 0.3$. Almost identical results were obtained in Ref. 8.

The local χ^2 minimum in region 2 depicted in Figs. 3(a) and 5(a) is extremely shallow, only 0.1 unit of χ^2 lower than the saddle point separating it from region 1. No local minimum corresponding to region 2 was found for the cases corresponding to Figs. 4(a) and 6(a).

B. E731 data alone

The two regions of parameter space corresponding to local χ^2 minima are more cleanly separated than in the previous case. These regions are shown in Figs. 3(b), 4(b), 5(b), and 6(b).

A local (but not global) χ^2 minimum occurs for parameters in the same general region (region 1) as for the fit using NA31 data, though η is slightly smaller and m_t slightly larger. A lower bound on the top-quark mass $m_t > (82, 69)$ GeV/c^2 at 90% confidence level is obtained if one uses the relation (3.16) and (3.17). These 90%-C.L. bounds correspond to values of $\Delta\chi^2 = (1.64)^2$ above those for the *local* minima in region 1. The corresponding 90%-C.L. bounds with respect to the *global* minima are (95, 72) GeV/c^3 for (3.16) and (3.17), respectively. The 1σ upper bounds on m_t (with respect to the local minima in region 1) are nearly (200,180) GeV/c^2 for (3.16) and (3.17). The parameter ρ is negative in region 1. The values of ϕ_1, ϕ_2, ϕ_3, and $|V_{td}|$ are rather similar to those for the NA31 data.

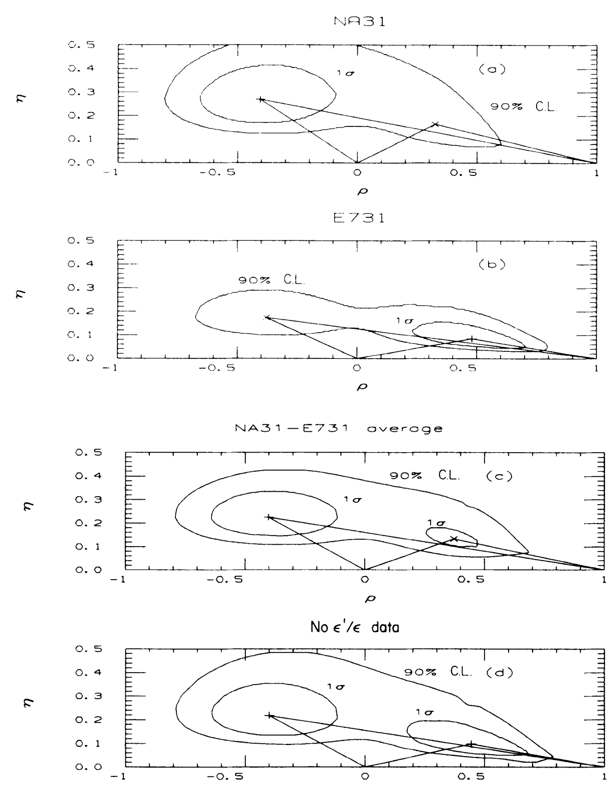

FIG. 3. Regions of parameters ρ, η allowed by fits when A and m_t are allowed to vary so as to minimize χ^2. Inner contours denote 1σ limits with respect to global minima (symbols +); outer contours denote 90%-confidence-level limits with respect to these minima. Points marked with the symbol $\times$ denote local but not global minima. Here the relation (3.16) has been used for ϵ'/ϵ. Inputs for ϵ'/ϵ: (a) NA31; (b) E731; (c) average of NA31 and E731 results; (d) none.

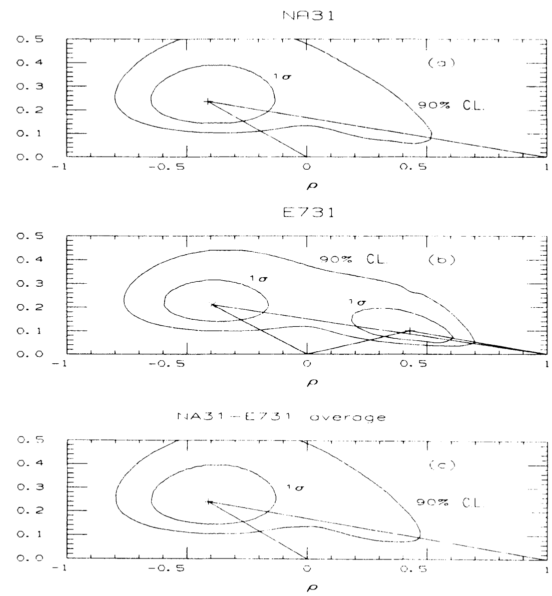

FIG. 4. Same as Fig. 3, but with (3.17) used for ϵ'/ϵ. (d) is omitted since it is identical to Fig. 3(d).

A deeper χ^2 minimum is found for region 2. The difference in χ^2 between regions 1 and 2 is not significant, however, if the input (3.17) is used. In region 2, the 1σ lower limit on the top-quark mass is about 250 GeV/c^2 for (3.16) and about 200 GeV/c^2 for (3.17), and the parameter ρ is expected to be positive. The large values of m_t could pose a problem for conventional analyses of radiative corrections to electroweak processes.[6,7] We shall discuss in Sec. V G possible ways of circumventing such limits. A large top-quark mass is considerably more likely to have some role in the generation of electroweak symmetry breaking through the formation of a $t\bar{t}$ condensate.[37–39] For some mass of the top quark above 200 GeV/c^2, its Yukawa coupling to the Higgs boson becomes so strong that perturbation theory ceases to make sense, and the problem must be treated by strong-coupling methods. The fact that the new E731 data favor larger m_t values than the older NA31 data has been stressed elsewhere.[4,39–41]

Notice that the shape of the unitarity triangle is very different for the two regions of parameters. For region 2, $\phi_1 \approx 10°$, $\phi_2 \approx 160°$, while $|V_{td}|/\lambda^3$ ranges from ~ 0.3 to ~ 0.8. The angle ϕ_3 is nearly identical ($\approx 10°$) for the two cases. This feature has been noted in previous discussions.[12,13,42] In Sec. V E we shall discuss some consequences of this very different shape of the triangle for B physics.

C. Combined NA31 and E731 data

If we use the "world" average (that of the two most recent experiments) for ϵ'/ϵ, and the relation (3.16), the χ^2 of the fit improves with respect to a fit using either set of data alone. With the relation (3.17), the χ^2 for the "world" average is worse than that for the E731 result.

One region of parameters ("region 1") is favored by the combined data, as illustrated in Figs. 3(c), 4(c), 5(c), and 6(c). The negative real part of V_{ub}, the relatively low top-quark masses, and the shape of the unitarity triangle all are reminiscent of the fits to NA31 data alone. One finds $\phi_1 \approx 150°$, $\phi_2 \approx 20°$, and $|V_{td}|/\lambda^3 = 1.4 \pm 0.3$. The second region corresponds to a rather shallow χ^2 minimum, about 0.4 unit below the saddle point separating it from region 1, on the assumption that (3.16) holds, while no local minimum was found if (3.17) is used. The 90%-C.L. contours in Figs. 4(c) and 6(c) do extend into ranges of parameters corresponding to region 2, however.

The 90%-C.L. lower bounds on the top-quark mass in these fits are (71,61) GeV/c^2 for (3.16) and (3.17). The 1σ upper limit on m_t for parameters in region 1 is about 165 GeV/c^2, probably within the search capabilities of a suitably upgraded Fermilab Tevatron. Values of higher top-quark mass are unlikely (at the 1σ level) on the basis of (3.17), which we regard as more reliable for heavy top.

D. No ϵ'/ϵ data

One can use all the information discussed in Sec. III aside from ϵ'/ϵ to predict a favored set of KM parameters. There is just enough information to allow for exact solution of the constraints, and two solutions [referred to as (1) and (2) in Table I] are obtained. These are illustrated in Figs. 3(d) and 5(d). The twofold ambiguity is described in more detail in Sec. V A.

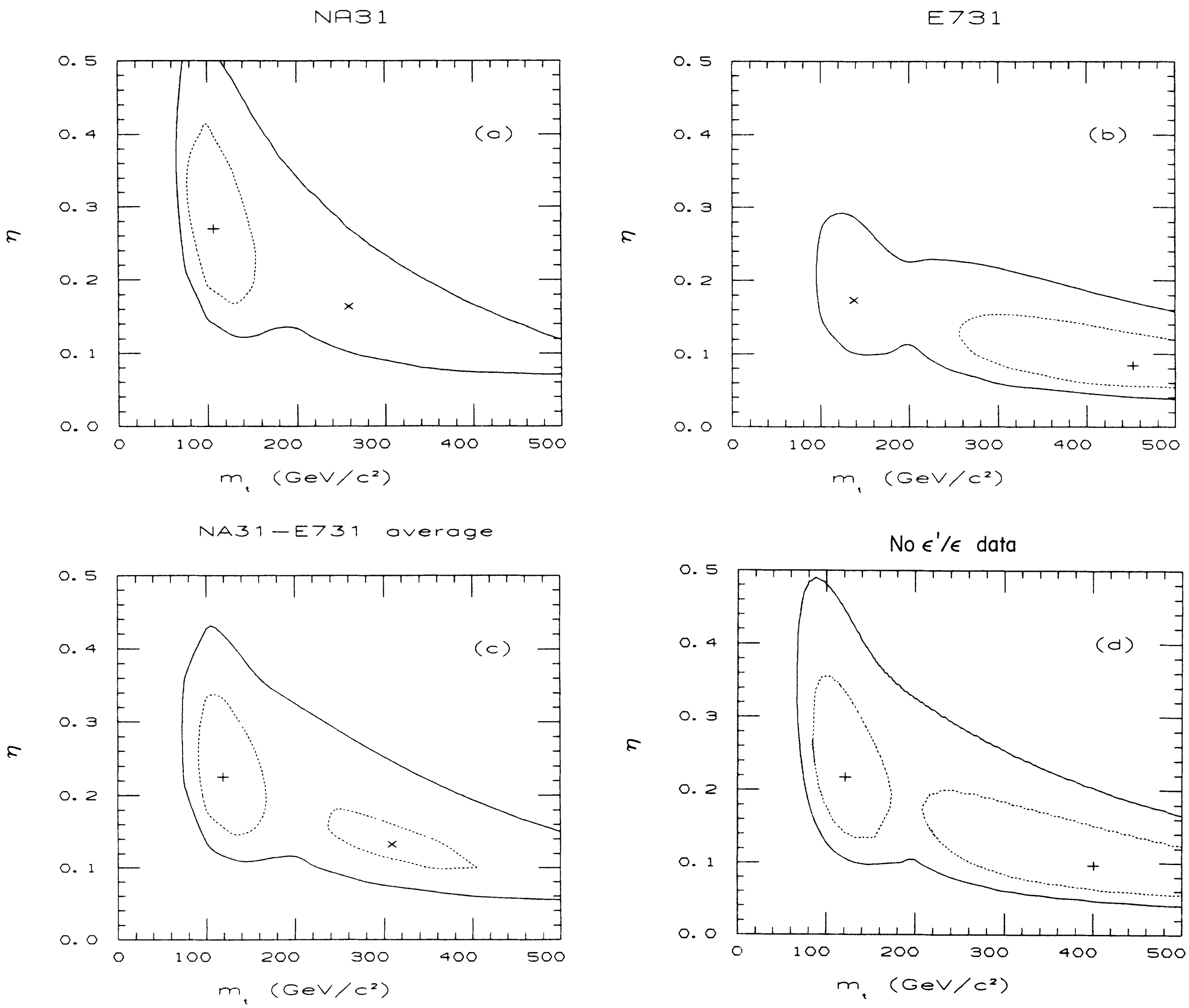

FIG. 5. Regions of parameters η and m_t allowed by fits when A and ρ are allowed to vary so as to minimize χ^2. Dotted contours denote 1σ limits with respect to global minima (symbols +); solid contours denote 90%-confidence-level limits with respect to these minima. See caption to Fig. 3 for other details. Here Eq. (3.16) has been used for ϵ'/ϵ.

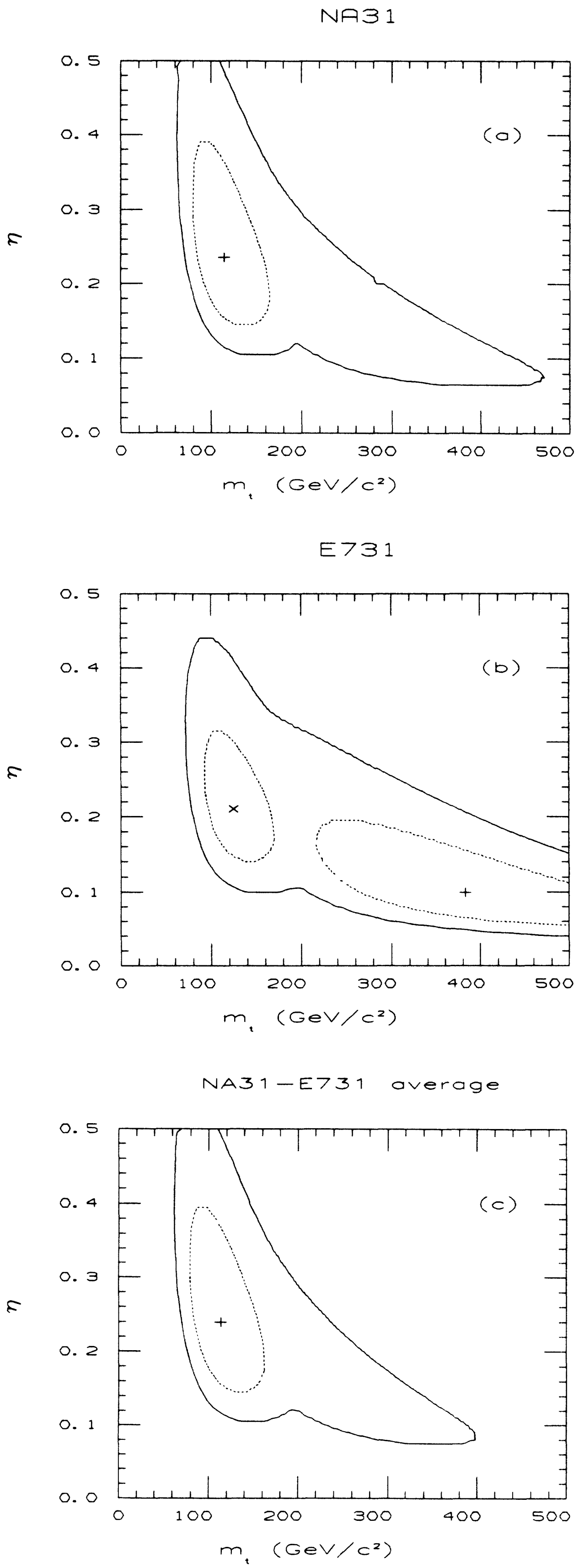

FIG. 6. Same as Fig. 5, but with (3.17) used for ϵ'/ϵ. (d) is omitted since it is the same as Fig. 5(d).

The solution corresponding to region 1 has a relatively low top-quark mass, and that corresponding to region 2 has a much higher range of top-quark mass values. The triangles are of very different shapes, but one of the angles (ϕ_3 as illustrated in Fig. 1) is nearly the same ($\approx 10°$) for either solution. In region 1, $\phi_1 \approx 150°$, $\phi_2 \approx 20°$, and $|V_{td}|/\lambda^3 = 1.4 \pm 0.3$, while in region 2, $\phi_1 \approx 10°$, $\phi_2 \approx 160°$, and $|V_{td}|/\lambda^3$ ranges from ~ 0.3 to ~ 0.8.

E. Dependence on top-quark mass

We wish to discuss the results of Figs. 5 and 6 in more detail.

(1) As the top-quark mass increases, one sees from the general shape of the contours that η (and hence ϵ'/ϵ) decreases. This behavior is familiar from other discussions,[34] and occurs for both (3.16) (which contains no explicit m_t dependence) and for (3.17) (which does).

(2) The two local minima (which are both exact solutions to the constraints when no ϵ'/ϵ data are used) correspond roughly to $m_t < 200$ GeV/c^2 (region 1) and to $m_t > 200$ GeV/c^2 (region 2). Top-quark masses around 200 GeV/c^2 are slightly less likely than either higher or lower values. A region of "less likely" top-quark masses, $m_t = 150 \pm 20$ GeV/c^2, is also found in Ref. 16; a larger value of f_B used in Eq. (3.5) may account for the difference.

(3) Using NA31 data [Figs. 5(a) and 6(a)] or averaged ϵ'/ϵ data [Figs. 5(c) and 6(c)] one finds a preference for the low top-quark-mass region (below about 200 GeV/c^2), while using E731 data [Figs. 5(b) and 6(b)] one has a preference for the high top-quark-mass region (above about 200 GeV/c^2). This preference is very slight when the corrected relation (3.17) is used, however.

(4) Since the two regions are never separated at the 90% confidence level, there is still in fact a continuum of possible KM phases joining the two solutions. The two regions of parameters are separated because of the constraint (3.2) due to the nonzero value of $|V_{ub}|$, as we shall see presently. Thus, to accentuate this separation, one useful means would be to reduce the experimental errors on $|V_{ub}|$.

(5) Values of ϵ'/ϵ above 2×10^{-3} are possible only for relatively light top-quark masses, and only when the other parameters discussed in Ref. 16 as affecting this quantity (such as $\Lambda_{\rm QCD}$ and m_s) take on their extreme values.

V. RESOLUTION OF AMBIGUITY

A. Twofold nature of the solution without ϵ'/ϵ data

We can understand the presence of two different solutions for KM parameters in a simplified limit in which both B-$\bar{B}$ mixing and ϵ are dominated by loops involving top quarks. We ignore error limits, assuming all quantities except ϵ'/ϵ are known precisely. Similar arguments have been presented in Refs. 12, 14, and 42.

The ratio of $b \to u$ to $b \to c$ decay rates fixes the ratio

$$\alpha \equiv |V_{ub}/V_{cb}|^2/\lambda^2 = \rho^2 + \eta^2 \ . \qquad (5.1)$$

The magnitude of B-$\bar{B}$ mixing fixes the combination

$$\beta \equiv m_t^2 F(m_t^2/M_W^2)[(1-\rho)^2+\eta^2] \; , \tag{5.2}$$

where F is the function defined in Eq. (3.6). Finally, the magnitude of ϵ specifies

$$\gamma \equiv m_t^2 F(m_t^2/M_W^2)\eta(1-\rho) \; . \tag{5.3}$$

We can divide Eq. (5.3) by Eq. (5.2) to obtain an equation relating $1-\rho$ and η. This equation has the solution $\eta/(1-\rho)=c=\tan\phi_3=-\tan[\arg(V_{td})]$, where c is the solution of $(1+c^2)/c=\beta/\gamma$. Although two solutions (with $c \leftrightarrow 1/c$) are possible in principle, the condition (5.1) is consistent only with the solution with $c<1$. The argument of V_{td} thus is specified uniquely. One might have anticipated this more directly since $|\epsilon|$ constraints $\mathrm{Im}(V_{td}^*)^2$, while B-$\bar{B}$ mixing constrains $|V_{td}|^2$. The coefficient multiplying the two quantities is the same function of m_t in the large top-quark-mass limit.

The intersection of the line $\eta=c(1-\rho)$ and the circle (5.1) gives an equation for the parameter ρ alone. This equation has two solutions, one for negative ρ and the other for positive ρ. These two solutions correspond to the last two lines of Tables I and II.

Whereas the slope of the line $\eta=c(1-\rho)$ (and hence the angle ϕ_3) is fairly well specified, the radius of the circle (5.1) is still quite uncertain. Thus, the exact solutions correspond to two large regions of parameters at the 1σ level, and these regions actually merge at the 90% confidence level. Improvements on the precision with which $|V_{ub}|$ is known thus will help to separate the two regions of parameters more decisively.

From Eq. (5.2) one sees that the solution with positive ρ corresponds to a much larger value of m_t than that with negative ρ. The high top-quark-mass solution is usually rejected because it conflicts with constraints from radiative corrections to electroweak processes. We shall comment further on this point in Sec. V G.

B. Further measurements of ϵ'/ϵ

The question of whether ϵ'/ϵ is closer to the NA31 value or the E731 value must be answered through analysis of the remaining 80% of the E731 data sample. Results from this analysis are expected within the next year or two. If the statistical error in Eq. (3.13) is reduced by a factor of $\sqrt{5}$, the overall error on ϵ'/ϵ will be less than 0.9×10^{-3}. In fact, one can anticipate doing somewhat better by lowering the systematic error as well.

C. Theoretical corrections to ϵ'/ϵ

Theorists may have to calculate the hadronic matrix elements to greater accuracy. One would like to know B_K better, for example, as well as hadronic matrix elements of operators contributing to ϵ'/ϵ. An extensive program to learn these quantities through lattice-gauge-theory calculations is under way.[43,44]

Electroweak penguin contributions[16,18] can lower the value of ϵ'/ϵ significantly for $m_t > 100$ GeV/c^2. In order to evaluate their effects reliably, as well as those of other operators transforming as a 27-plet of flavor SU(3) which can *raise* ϵ'/ϵ, one requires the matrix elements of a number of different operators. An extensive analysis of such effects has appeared very recently,[16] but uncertainties in hadronic matrix elements are still large enough[18] that we doubt the question is closed yet. It looks as if a value of ϵ'/ϵ below 10^{-3} will not, by itself, be able to demonstrate a conflict with the standard KM picture unless supplemented by other experiments or further theoretical calculations. Such experiments might, for example, consist of the determination of angles in the unitarity triangle through studies of B mesons, which we discuss below.

D. Bounds from rare kaon decays and K-$\bar{K}$ mixing

(1) *The decay* $K_L \to \mu^+\mu^-$ is expected to occur at a rate bounded from below by the unitarity limit associated with the two-photon intermediate state:[45]

$$B(K_L\to\mu^+\mu^-) \ge (1.2\times 10^{-5}) B(K_L\to\gamma\gamma) \approx 7\times 10^{-9} \; , \tag{5.4}$$

where $B(\cdots)$ denotes a branching ratio, and we have used recent $K_L\to\gamma\gamma$ data[46] as well as those quoted in Ref. 29. Experimental results for $B(K_L\to\mu^+\mu^-)$ are very close to this lower bound: $(8.4\pm1.1)\times10^{-9}$ (Ref. 47) and $(5.8\pm0.6\pm0.4)\times10^{-9}$ (Ref. 48). There is very little room for an additional dispersive contribution.

The dispersive contribution to $K_L\to\mu^+\mu^-$ can come both from the two-photon state and from short-distance (electroweak) effects. Theoretical estimates of the dispersive two-photon contribution suggest that it is small, less than 0.5 in amplitude of the absorptive contribution.[49] However, one can check the ratio of dispersive and absorptive two-photon contributions directly in the decay $\eta\to\mu^+\mu^-$, where it is found[50] that the dispersive two-photon contribution to the branching ratio can be at most of the same order of magnitude as the absorptive contribution. [Here we use the most recent value quoted in Ref. 29 for $B(\eta\to\mu^+\mu^-)$.] The dispersive short-distance contribution can attain its maximum value consistent with experiment when it interferes destructively with the dispersive two-photon contribution, and we then arrive at the estimate

$$B(K_L\to\mu^+\mu^-)|_{\mathrm{SD}} \le 7\times10^{-9} \; , \tag{5.5}$$

where the subscript refers to "short distance." Retracing the discussion in Refs. 49 and 51, we then arrive at the bound

$$F_1(m_t^2/M_W^2)|\mathrm{Re}(V_{ts}^* V_{td})| m_t^2 \le 13.5\ (\mathrm{GeV}/c^2)^2 \times \left[\frac{B(K_L\to\mu^+\mu^-)|_{\mathrm{SD}}}{7\times10^{-9}}\right]^{1/2} \; , \tag{5.6}$$

where[26]

$$F_1(x) \equiv \frac{1}{1-x} - \frac{x}{4(1-x)} + \frac{3x\ln x}{4(1-x)^2} \; . \tag{5.7}$$

We have explicitly exhibited the dependence on the short-distance contribution to $K_L\to\mu^+\mu^-$. The effects of u- and c-quark loops are negligible in comparison with those of t quark loops for the ranges of m_t we are considering.

The constraint (3.7) from B-$\bar{B}$ mixing provides an independent estimate of $|V_{td}|$:

$$m_t|V_{td}|\sqrt{F(m_t^2/M_W^2)}\approx 1.5\ \mathrm{GeV}/c^2\left[\frac{140\ \mathrm{MeV}}{f_B}\right], \quad (5.8)$$

where the function F was defined in Eq. (3.6). For the phases within the solution sets considered here, it is satisfactory to take $\mathrm{Re}(V_{td})=\cos\phi_3|V_{td}|\approx|V_{td}|$ in Eq. (5.6), since ϕ_3 is only about 10° (see Fig. 1). We can then take the quotient of (5.6) and (5.8) to obtain an upper bound on the top-quark mass:

$$m_t\frac{F_1(m_t^2/M_W^2)}{\sqrt{F(m_t^2/M_W^2)}}\le 190\ \mathrm{GeV}/c^2\left[\frac{f_B}{140\ \mathrm{MeV}}\right]\times\left[\frac{B(K_L\to\mu^+\mu^-)|_{\mathrm{SD}}}{7\times 10^{-9}}\right]^{1/2}, \quad (5.9)$$

which is satisfied as long as $m_t\le 350\ \mathrm{GeV}/c^2$ for the values of f_B and $B(K_L\to\mu^+\mu^-)|_{\mathrm{SD}}$ assumed here. Smaller values of these parameters will lead to tighter bounds, of course. Very large values of top-quark masses thus can be ruled out by this bound, independently of the arguments (to be cited in Sec. V G) based on electroweak corrections.

(2) *The decay* $K^+\to\pi^+\nu\bar{\nu}$ has been analyzed in some detail in Refs. 10 and 52, and more recently in Ref. 53. According to the limits set in Ref. 53, the upper limits on the branching ratio increase as the top-quark mass increases, approaching about 2.5×10^{-10} per neutrino flavor for $m_t=200\ \mathrm{GeV}/c^2$, but values as low as 0.2×10^{-10} are also possible, independent of m_t. Our analysis (based on expressions quoted in Ref. 10) gives a somewhat narrower range of branching ratios, ranging from 1.8 to 2.9×10^{-10} (for three neutrino flavors) as m_t varies between 100 and 200 GeV/c^2.

(3) *The mixing between* K and $\bar{K}$ due to loop diagrams involving the top quark[54] appears to be satisfactorily small for all values of m_t satisfying the bound (5.9), as mentioned in Ref. 10.

E. Studies of B mesons

Beauty holds the promise of a rich experimental program, culminating in the likely observation of *CP*-violating asymmetries.[11–14,21,55,56] Several quantities associated with B mesons must be better determined to improve the present analysis. In addition to $|V_{ub}/V_{cb}|$, one needs to know $|V_{cb}|$ itself (i.e., the parameter A) more precisely.[57,58] It may be possible[21,28] to learn about f_B, which governs B-$\bar{B}$ mixing, by searching for the decay $B\to\tau\nu$ at the predicted branching ratio of $\sim 10^{-4}$. One would thereby learn the useful combination $f_B|V_{ub}|$. One would also like to know B_B better.

(1) *Decay modes* of the b quark involving the $b\to u$ transition should be visible now that we have indirect evidence for nonzero $|V_{ub}/V_{cb}|$ from inclusive semileptonic decays.[2,3] According to a recent estimate,[59] one expects $B(B^0\to\pi^-e^+\nu)=(9-15)|V_{ub}|^2$ and $B(B^0\to\pi^+\pi^-)=(0.7-1.2)|V_{ub}|^2$, or about 2×10^{-5}, give or take a factor of 4. The predicted rate for the purely leptonic decay $B\to\tau\nu$ (see, e.g., Ref. 21 and 28) is

$$\Gamma(B^+\to\tau^+\nu_\tau)=\frac{G_F^2f_B^2m_\tau^2m_B}{8\pi}\left[1-\frac{m_\tau^2}{m_B^2}\right]^2|V_{ub}|^2\ . \quad (5.10)$$

The predicted branching ratio, taking account of the B lifetime quoted earlier, is then

$$B(B^+\to\tau^+\nu_\tau)=6.8\times 10^{-5}\left[\frac{f_B}{140\ \mathrm{MeV}}\right]^2\left|\frac{V_{ub}}{0.0049}\right|^2 . \quad (5.11)$$

(2) *Mixing* of the nonstrange neutral B mesons appears well established, but it would be useful to check the prediction that

$$\left[\frac{\Delta m}{\Gamma}\right]_{B_s}=\left[\frac{f_{B_s}}{f_B}\right]^2\left|\frac{V_{ts}}{V_{td}}\right|^2\left[\frac{\Delta m}{\Gamma}\right]_B . \quad (5.12)$$

Lattice estimates give a range[28] of 1.2–2 for $(f_{B_s}/f_B)^2$. The fits to KM parameters presented above suggest $|V_{ts}/V_{td}|^2$ is about 10 for solutions of region 1, but it could be considerably larger if the solutions of region 2 hold. We expect $|V_{ts}|\approx\lambda^2$, while $|V_{td}|\approx(1.1-1.7)\lambda^3$ in region 1 and (0.3–0.8)λ^3 in region 2. The central value of $\Delta m/\Gamma$ for B_s is then predicted to be 11 ± 6 for region 1, but at least 20 (more likely 65) for region 2.

The measuring of mixing in the B_s system thus holds the prospect of resolving the ambiguity between solutions of region 1 and those of region 2. As a result of the likely saturation of B_s mixing,[58] the measurement of time-dependent effects is of great importance, and many suggestions have been made[12,42,60,61] for observing them.

The ratio of f_{B_s} and f_B in Eq. (5.12) may be more accurately determined theoretically (e.g., via a lattice calculation) than either parameter alone. A test of the corresponding prediction for charmed particles might be possible, since both f_{D_s} and f_D could be measured via $D_s\to\tau\nu$ and $D^+\to\mu\nu$ decays with modest improvements in present data.[21]

(3) *CP violation* in the B system at predicted levels would provide a dramatic confirmation that the Kobayashi-Maskawa phases are the source of that effect for the kaons.

One favored state for observing an asymmetry between B^0 and $\bar{B}^0$ decays is $J/\psi K_S$,[62] for which the expected asymmetry[12,61] measures the angle ϕ_3 opposite V_{ub}^* in the unitarity triangle of Fig. 1.[12,32] This quantity is expected[12,13,42,58] to be nearly independent of present uncertainties about the shape of the unitarity triangle, as we see from the fits described in Sec. IV.

The time-integrated partial rate asymmetry for decays of states B^0_{phys} and $\bar{B}^0_{\mathrm{phys}}$ (i.e., states which have these corresponding identities at $t=0$) to a final state f is defined to be

$$C_f \equiv \frac{\Gamma(B^0_{\rm phys} \to f) - \Gamma(\bar{B}^0_{\rm phy} \to f)}{\Gamma(B^0_{\rm phys} \to f) + \Gamma(\bar{B}^0_{\rm phys} \to f)} \; . \quad (5.13)$$

For the $J/\psi K_S$ final state, one finds[61]

$$C_f = \frac{\Delta m/\Gamma}{1+(\Delta m/\Gamma)^2} \sin(2\phi_3) \; , \quad (5.14)$$

or about 0.16 for $\phi_3 \approx 10°$ and $\Delta m/\Gamma = 0.66$ [see Eq. (3.4)]. Here and below we neglect $\Delta\Gamma/\Gamma$ for nonstrange B mesons.

The asymmetry for the $\pi^+\pi^-$ final state is expected to depend on the magnitude of $|V_{ub}|$ in such a way that the total number of $B\bar{B}$ pairs needed to see the asymmetry remains relatively constant.[12] The asymmetry is proportional to $\sin 2\phi_2$, where ϕ_2 is defined in Fig. 1, so that its sign provides a clear-cut distinction between solutions of region 1 and those of region 2. Specifically, one finds (see Ref. 63 for corrections due to penguin graphs)

$$C_f = -\frac{\Delta m/\Gamma}{1+(\Delta m/\Gamma)^2} \sin(2\phi_2)$$
$$\approx \begin{cases} -0.3 \text{ for } \phi_2 = 20°, \\ \;\; 0.3 \text{ for } \phi_2 = 160° \; , \end{cases} \quad (5.15)$$

Taking account of reconstruction efficiencies, the authors of Ref. 12 find that with 10^8 $B\bar{B}$ pairs at the $\Upsilon(4S)$, one could see such an asymmetry at the 2.5σ level.

Experiments which can make a start on addressing CP violation in B mesons include CLEO, ARGUS, Fermilab E-771 and 789, CDF (with a vertex detector), the proposed Bottom Collider Detector[64] (BCD), a forward spectrometer proposed for[60] CERN, and a comprehensive program at[65] UNK.

F. Uncertainties associated with f_B and B_K

Note added. We have been urged by colleagues to explore the dependence of our conclusions upon the rather tight limits assumed for f_B and B_K in Sec. III. This question assumes particular urgency when applying Eq. (5.14) to an estimate for prospective CP asymmetries in the decay of neutral B mesons to $J/\psi K_S$.

In the limit of large top-quark mass, Eq. (3.9) can be expressed in terms of the contribution of the box diagram with internal t quarks alone:

$$|\epsilon| \approx \frac{1}{\sqrt{2}} \frac{G_F^2}{12\pi^2 \Delta m_K} |V_{ts}|^2 \mathrm{Im}(V_{td}^2) m_t^2$$
$$\times F\left[\frac{m_t^2}{M_W^2}\right] m_K f_K^2 B_K \eta_2 \; , \quad (5.16)$$

where we have assumed $\arg(\epsilon) \approx \arctan(2\Delta m_K/\Gamma_S) \approx 45°$. Here we use the experimental value of $\Delta m_K = 3.52 \times 10^{-15}$ GeV, and $f_K = 160$ MeV. For other parameters, see Sec. III. This approximation provides about (70%, 80%) of the total contribution to $|\epsilon|$ for $m_t = (80, 120)$ GeV/c^2, and improves for larger m_t.

Dividing (5.16) by (3.5) and noting that $\sin(2\phi_3) = \mathrm{Im}(V_{td}^2)/|V_{td}|^2$, we find

$$\sin(2\phi_3) \approx \frac{2\sqrt{2}|\epsilon|}{|V_{ts}|^2} \frac{\Delta m_K}{\Delta m_B} \frac{m_B}{m_K} \frac{B_B}{B_K} \frac{\eta_B}{\eta_t} \left[\frac{f_B}{f_K}\right]^2 . \quad (5.17)$$

The use of the more exact expression (3.9) for $|\epsilon|$ would reduce the inferred values of $\sin(2\phi_3)$ slightly for the smallest values of m_t allowed by present experimental lower limits.[35] One can see this effect in Figs. 3 and 4, in which the angle ϕ_3 is slightly smaller for the (lower-m_t) solutions of region 1 than for the (higher-m_t) solutions of region 2.

By using the value of r in Eq. (3.3) and the observed average B lifetime quoted in Sec. III we would have $\Delta m_B = (3.68 \pm 0.75) \times 10^{-13}$ GeV. Here we shall use a more recent average,[66] which quotes $r = 0.20 \pm 0.06$, or equivalently $\Delta m_B/\Gamma = 0.71 \pm 0.13$, and hence implies $\Delta m_B = (3.96 \pm 0.86) \times 10^{-13}$ GeV. Numerically Eq. (5.17) then implies

$$\sin(2\phi_3) = (0.35 \pm 0.09) \frac{2/3}{B_K} \left[\frac{f_B}{140 \text{ MeV}}\right]^2 . \quad (5.18)$$

Contours of equal values of ϕ_3 based on the central value for the coefficient in this relation are plotted in Figs. 7. The smaller ellipses correspond to the range of f_B and B_K assumed in the present paper. The larger ellipses correspond to $f_B = 150 \pm 50$ MeV, as advocated in Ref. 14, and (a) $B_K = \frac{2}{3} \pm \frac{1}{3}$, the range suggested in Ref. 14, or (b) $B_K = 0.8 \pm 0.2$, a range suggested by more recent calculations.[67] Values of ϕ_3 exceeding 45°, although permitted by Eq. (5.18), are excluded by the upper bound on $|V_{ub}|$, as one can see from Fig. 2.

We show in Figs. 8 and 9 the 1σ and 90%-C.L. contours for the fits corresponding to the cases illustrated in Figs. 3(d) and 5(d) (no ϵ'/ϵ constraints) but with the values of f_B and B_K just mentioned. As in Figs. 3–6, the expression (3.9) has been used for $|\epsilon|$. One still sees vestiges of two regions, but considerable overlap can occur when ϕ_3 is a good deal larger than its central value. The lower bounds on m_t shown in Figs. 9 are poorer than in Fig. 5(d), as a result of the larger value of f_B which has been allowed.

Our conclusion in the present paper (also reached earlier in Ref. 12) has been that ϕ_3 is rather small. This result remains valid as long as f_B is not close to 200 MeV or B_K close to $\frac{1}{3}$. Indirect information on f_B may be forthcoming via improvements on present measurements[68] of f_D and the use of the scaling relation[69,70] $f_B/f_D = \sqrt{m_D/m_B} \approx 0.6$. The present bound[68] $f_D < 290$ MeV already implies $f_B < 170$ MeV. The most recent quoted range for B_K (0.9–1.0), when combined with the bound on f_B and the error limits in Eq. (5.18), implies that ϕ_3 cannot exceed about 15°.

The conclusion that there are two main regions of parameters (which we have called "region 1" and "region 2") depends rather sensitively on the upper bound on ϕ_3. To see this, return to the discussion in Sec. V A. In the ρ-η plane, a line with fixed ϕ_3 corresponds to a line $\eta = c(1-\rho)$. We argued that such a line intersected the

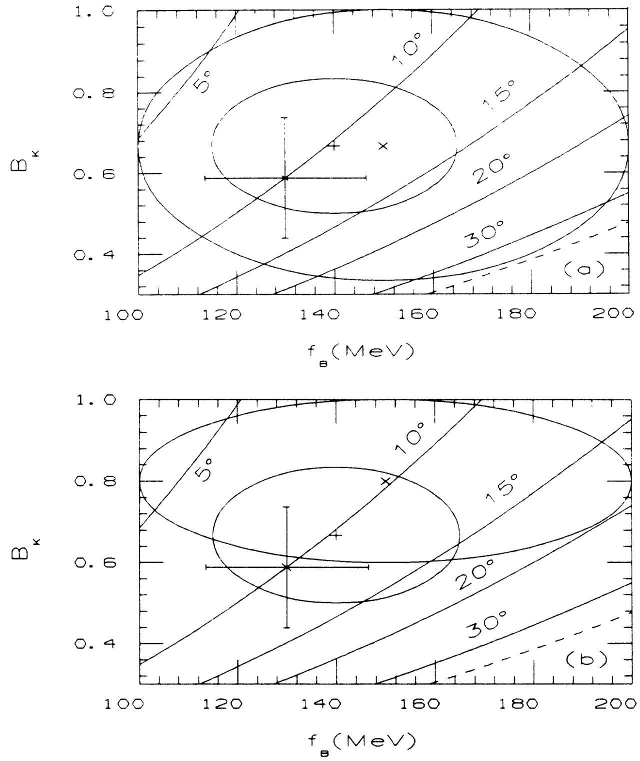

FIG. 7. Contours of constant values for the angle ϕ_3. The dashed lines correspond to the boundary of the physical region of parameters, with $\phi_3=45°$. Contours for ϕ_3 and $90°-\phi_3$ are identical. Error limits on the contour for $\phi_3=10°$ correspond to the errors on the coefficient in Eq. (5.18). The ellipses with vertical crosses at the center correspond to the range of f_B and B_K assumed in the present paper. The ellipses with diagonal crosses at the center correspond to $f_B=150\pm50$ MeV, and (a) $B_K=\frac{2}{3}\pm\frac{1}{3}$, or (b) $B_K=0.8\pm0.2$.

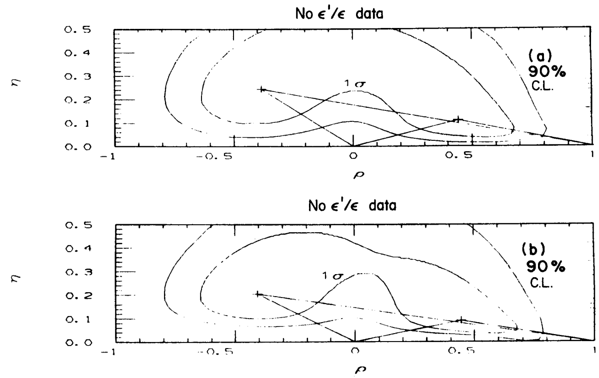

FIG. 8. Same as Fig. 3(d), but with $f_B=150\pm50$ MeV and (a) $B_K=\frac{2}{3}\pm\frac{1}{3}$; (b) $B_K=0.8\pm0.2$.

circle $\rho^2+\eta^2=\alpha$ [due to the $|V_{ub}|$ constraint; see Eq. (5.1)] at two points. Uncertainty on the radius of the circle turned these points into regions. However, uncertainty on the slope c can cause these two regions to overlap.

There is a critical angle $\phi_3^{\text{crit}}=\arcsin\sqrt{\alpha_{\text{min}}}$ above which the two regions of parameter space will overlap. In the present work we have taken $\sqrt{\alpha_{\text{min}}}=0.23$ [see Eq. (3.2)], which corresponds to $\phi_3^{\text{crit}}=13°$. The present CLEO and ARGUS data have been interpreted in one recent model[57] to give a somewhat more restrictive range of $|V_{ub}/V_{cb}|$ than taken in the present paper: between 0.07 and 0.12 instead of 0.05 and 0.15. The more restrictive limits lead to the conclusion $\sqrt{\alpha_{\text{min}}}=0.32$, $\phi_3^{\text{crit}}=19°$.

To summarize this subsection, recent information suggests that our conclusion of a relatively small value of ϕ_3 remains robust. This conclusion entails an upper limit on the expected asymmetry in the $J/\psi K_S$ decays of neutral B mesons, and the existence of two distinct regions of parameter space for KM elements and top-quark masses. We have indicated ways in which further information on f_B, B_K, and $|V_{ub}/V_{cb}|$ could enhance the reliability of this conclusion. The discovery of a top quark corresponding to the solutions of region 1 [i.e., lying below about 200 GeV$\times$(140 MeV$/f_B$)] would, of course, render the question of two regions moot. In the next subsection we discuss the prospects for such a discovery.

G. Top-quark searches

The arguments for a top quark of *some* mass or other are indirect but compelling, and include the absence of exotic-b-quark decays and the observation of a forward-backward asymmetry in $e^+e^-\to b\bar{b}$ whose value agrees with standard-model predictions if the left-handed b quark is in a weak isodoublet.[71]

(1) *Production estimates* indicate that a top quark of up to about 200 GeV$/c^2$ should be accessible at Fermilab if a suitable luminosity upgrade is performed.[36] Beyond about 160 GeV$/c^2$, the presence of a W^+W^- background of comparable magnitude to the signal requires special care in choosing decay signatures.

(2) *Signatures* in hadronic production include the channels leptons + jets, $e+\mu$, dileptons of the same type, possible b tagging to reduce backgrounds, and multijet final states. There have been some suggestions regarding signatures of very heavy top quarks in e^+e^- interactions.[72]

(3) *Searches at present hadron colliders*[35] have ruled out "standard" top quarks up to nearly 80 GeV$/c^2$, with the indirect analyses based on the KM matrix mentioned above placing a slightly weaker lower bound. (*Note added.* The lower bounds on a "standard" top quark have now been improved to 89 GeV$/c^2$; see Ref. 73.) A light top quark is still possible if a light charged Higgs boson dominates B-$\bar{B}$ mixing,[74] but this scenario is soon to be ruled out by more precise measurements of Γ_Z/Γ_W and Γ_Z. [*Note added.* Such measurements have now been performed; see Ref. 75. The lower bounds on the top-quark mass are 41 (35) GeV$/c^2$ at the 90% (95%) confidence level, independent of the decay modes of the top quark.]

(4) *Upper limits to the top-quark mass* at present depend mainly on the validity of analyses of deep-inelastic neutrino scattering.[7,17,39] A lingering uncertainty in these analyses is concerned with the role of the charmed-quark

mass in the charged-current cross section. Proposals have been made for reducing this uncertainty.[76]

Now that the Z mass is known quite precisely,[77] measurements of the W mass to within 300 MeV/c^2 will begin to constrain the top-quark mass as closely as the deep-inelastic neutrino scattering experiments just noted. The average of present values[78] for the W mass is uncertain to about 600 MeV/c^2 and does not provide much information, though values of the top-quark mass above about 300 GeV/c^2 are probably excluded. (Slightly more restrictive bounds are quoted in Ref. 79.) As mentioned previously, our perturbative one-loop calculations must fail for sufficiently large top-quark mass, so we hesitate to quote firm upper limits.

It may be possible to cancel out the effects of a very heavy top quark in electroweak radiative corrections with a suitably chosen Higgs triplet.[17,80]

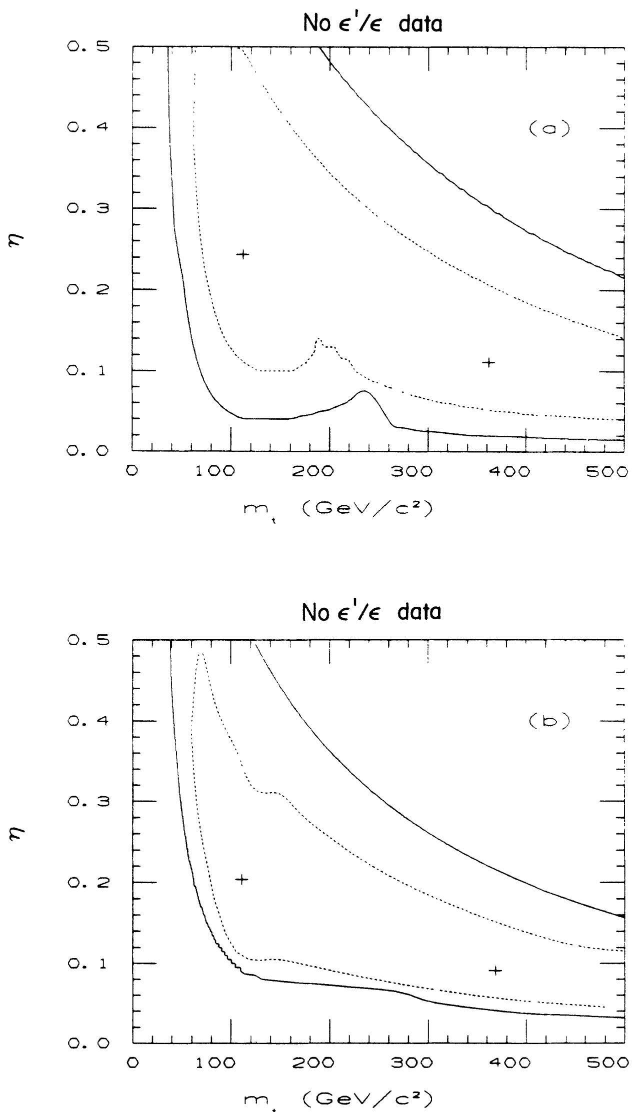

FIG. 9. Same as Fig. 5(d), but with $f_B = 150 \pm 50$ MeV and (a) $B_K = \frac{2}{3} \pm \frac{1}{3}$; (b) $B_K = 0.8 \pm 0.2$.

H. Beyond the standard model

(1) *Right-handed W*'s could be responsible for *CP* violation in the kaon system. Direct searches can be performed, both for them and for their neutral partners in extended versions of the electroweak theory. Heavy gauge bosons are accessible to mass ranges limited by luminosity, energy, and types of coupling constants. The reach for "standard" W and Z bosons has been discussed in many places.[81] For example, Fermilab can probably exclude right-handed W's up to several hundred GeV/c^2 with present data. However, it may require substantially higher energies [even, perhaps, beyond those of the Superconducting Super Collider (SSC)], to see the sort of right-handed W's that could lead to *CP* violation in the kaon system.[82]

(2) *An extended Higgs* sector is still a possible source of the observed *CP* violation.[83] It could lead to a neutron electric dipole moment close to present experimental upper limits,[84] in contrast with the schemes mentioned above.

(3) *The superweak model* of *CP* violation in the kaon system[85] implies $\epsilon' = 0$. It is excluded at the 3σ level by NA31 data, but only by slightly above 2σ if these data are combined with the partial E731 results. Suppose the areas of the triangles in Figs. 3 and 4 were really zero, and all the angles were zero or π. Could we know this by measuring the magnitudes of the sides sufficiently precisely? We would then discard the information on ϵ, relying on precise determinations of $|V_{ub}|$, $|V_{td}|$, and $|V_{cb}|$. We need the top-quark mass in order to extract $|V_{td}|$ from B-$\bar{B}$ mixing. It would take measurements of the sides to within a couple of percent—a daunting prospect—to distinguish the triangles in Figs. 3 and 4 from ones with vanishing area.

The superweak model would not be expected to lead to observable *CP* violation in the B system, for which the mass differences and lifetimes are about a factor of 100 smaller than for the short-lived kaon.

VI. CONCLUSIONS

In the past sixteen years since the six-quark model for charged-current interactions was proposed, it has fared remarkably well, surviving numerous experimental tests. Present data are becoming precise enough that nearly all the elements of the Kobayashi-Maskawa matrix are known or inferred. The major remaining uncertainties include an error of about 50% on $|V_{ub}|$, and a considerably greater uncertainty in $\arg(V_{ub}^*)$ and $|V_{td}|$. This uncertainty would be considerably reduced if $|V_{ub}|$ were better known and if one had independent information on the shape of the unitarity triangle illustrated in Figs. 1, 3, and 4. At present there remains a discrete ambiguity between solutions of the general form

$$\text{region 1: } \arg(V_{ub}^*)\approx 150°, \quad |V_{td}|\approx 1.4\sin\theta_C|V_{cb}|, \quad m_t\le 200\ \text{GeV}/c^2\ ,$$

$$\text{region 2: } \arg(V_{ub}^*)\approx 10°, \quad |V_{td}|\approx 0.6\sin\theta_C|V_{cb}|, \quad m_t\ge 200\ \text{GeV}/c^2\ .$$

In both solutions, $\arg(V_{td})\approx -10°$. (Here we have used a specific phase convention for KM elements.) Values of $m_t\approx 200\ \text{GeV}/c^2\times(140\ \text{MeV}/f_B)$ are slightly less likely, on the basis of our analysis, than either lower or higher values. The "ridge" separating the two regions of parameter space is eroded by very large values of f_B or very small values of B_K, both of which we regard as unlikely (see Sec. V F).

Efforts to resolve the ambiguity between the two solutions discussed here will consist in the near term of more precise ϵ'/ϵ meaasurements, including analysis of the remaining E731 data. A value of ϵ'/ϵ above 2×10^{-3} would favor parameters of the low-m_t solution, while a value below this would be subject to further uncertainties in theoretical interpretation, as we have seen. In the longer term, more precise theoretical calculations, studies of rare-kaon decays and B mesons, improved experiments testing radiative corrections to electroweak processes, and discovery of the top quark will provide additional information.

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