PHYSICAL REVIEW D VOLUME 42, NUMBER 3 1 AUGUST 1990

Unity of quarks and leptons at the TeV scale?

R. Foot*
Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706

H. Lew
Research Center for High Energy Physics, School of Physics, University of Melbourne, Parkville 3052, Australia
(Received 25 January 1990)

The gauge group $[\mathrm{SU}(3)]^2 \otimes [\mathrm{SU}(2)]^2 \otimes [\mathrm{U}(1)_{Y'}]^3$ supplemented by quark-lepton, left-right, and generation discrete symmetries represents a new approach to the understanding of the particle content of the standard model. In particular, as a result of the large number of symmetries, the fermion sector of the model is very simple. After symmetry breaking, the standard model can be shown to emerge from this highly symmetric model at low energies.

One of the puzzling features of the phenomenologically successful standard model of elementary-particle physics is the structure of the fermion representations. The electroweak interactions provide us with the well-known pattern in which each generation of quarks and leptons transform under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as left-handed doublets and right-handed singlets along with their peculiar values of hypercharges. Quarks are distinguished from leptons in that they transform as triplets under $\mathrm{SU}(3)_c$ whereas leptons transform as singlets. As far as we know there are three generations and the only distinguishing feature between different generations is their masses. From such a pattern it is tempting to speculate whether there exists fundamental symmetries in nature connecting quarks with leptons, left-handed with right-handed fields, and different generations; all of which are, of course, broken at low energies.

In this paper we propose a model which accommodates quark-lepton, left-right, and generation symmetries. To motivate this we postulate that the basic building blocks of the Lagrangian in particle physics should be simple functions which are consistent with the gauge symmetries. The resulting Lagrangian with a more complicated structure can then be generated by replicating these simple functions. This can be easily implemented by using discrete symmetries. One can think of the Lagrangian as a wallpaper pattern built up from a basic pattern which itself is dictated by the gauge symmetries.

As a concrete example of such a viewpoint consider a model with the gauge group given by[1]

$$\mathrm{SU}(3)_q \otimes \mathrm{SU}(3)_l \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes [\mathrm{U}(1)_{Y'}]^3 . \quad (1)$$

In addition there are three sets of discrete symmetries: (i) QL, a Z_2 discrete symmetry which connects the quark and lepton fields, (ii) LR, a Z_2 discrete symmetry which connects the left- and right-handed fields, and (iii) G, a cyclic symmetry group Z_3, which connects the different generations labeled by the $\mathrm{U}(1)_{Y'}$ quantum numbers. As an illustration of how these discrete symmetries act consider the first-generation quark multiplet $Q_{1L} \sim (3,1,2,1)(y,0,0)$ and its transformation under the three discrete symmetries:

$$\begin{aligned} &Q_{1L} \overset{\mathrm{QL}}{\leftrightarrow} (l_{1L})^c \sim (1,3,2,1)(y,0,0) , \\ &Q_{1L} \overset{\mathrm{LR}}{\leftrightarrow} Q_{1R} \sim (3,1,1,2)(y,0,0) , \\ &Q_{1L} \overset{\mathrm{G}}{\rightarrow} Q_{2L} \sim (3,1,2,1)(0,y,0) . \end{aligned} \quad (2)$$

In fact the complete set of fermions in the model can be neatly represented by a three-dimensional lattice as shown in Fig. 1(a). [The gauge bosons of the model are also related by the discrete symmetries and are shown in Fig. 1(b).] At this stage, the classical discrete symmetries have been used to fix the hypercharge degrees of freedom in terms of a single continuous parameter y. Also notice that in this model anomaly cancellation is an automatic consequence of the classical symmetries of the Lagrangian.

To obtain the standard model at low energies, it is sufficient to introduce the following Higgs bosons into the model in order to break the highly symmetric gauge group of Eq. (1). We list here only the scalar multiplets necessary to establish the notation. All other multiplets can be generated by the discrete symmetries as shown in Fig. 2:

$$\begin{aligned} &\Delta'_{1L} \sim (1,6^*,3,1)(-2y,0,0) , \\ &\chi_1^1 \sim (1,3,1,1)(0,-y,-y) , \\ &\phi_1 \sim (1,1,2^*,2)(0,0,0) . \end{aligned} \quad (3)$$

The Higgs fields Δ, χ, and ϕ then couple to the fermion fields as follows:

$$L_{\mathrm{Yuk}} = L_1 + L_2 + L_3 + \mathrm{H.c.} , \quad (4)$$

where

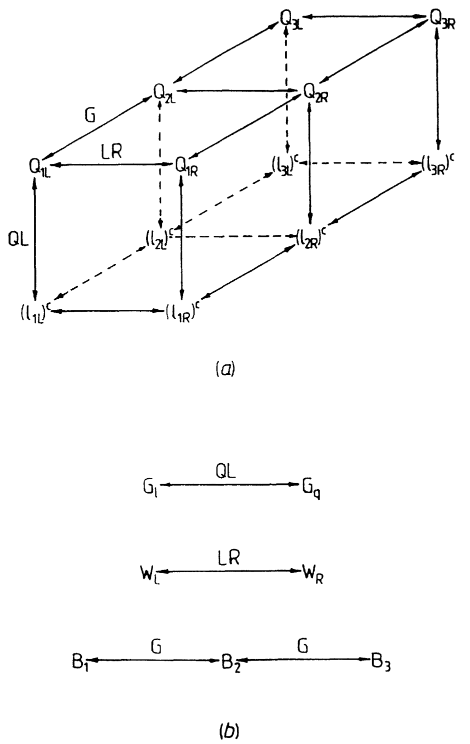

FIG. 1. (a) A three-dimensional lattice representation of the discrete symmetry transformations connecting the fermion fields of the model. (b) The gauge bosons and their transformation under the discrete symmetries. $G_{q,l}$ are the $SU(3)_{q,l}$, $W_{L,R}$ are the $SU(2)_{L,R}$, and B_i (for $i=1,2,3$) are the corresponding $U(1)_{Y_i'}$ gauge bosons, respectively.

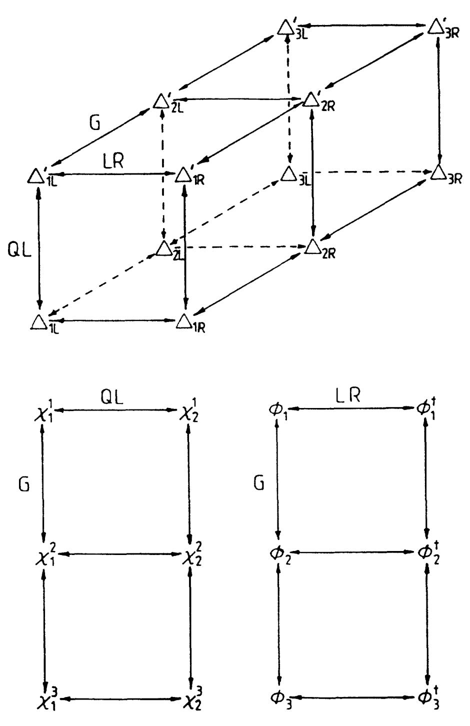

FIG. 2. The Higgs bosons and their transformations under the discrete symmetries.

$$L_1=\lambda_1 \sum_{i=1}^{3} [\bar{l}_{iL}\Delta'_{iL} i\tau_2^*(l_{iL})^c+\bar{l}_{iR}\Delta'_{iR} i\tau_2^*(l_{iR})^c + \bar{Q}_{iL} i\tau_2\Delta'^{\dagger}_{iL}(Q_{iL})^c+\bar{Q}_{iR} i\tau_2\Delta'^{\dagger}_{iR}(Q_{iR})^c] ,$$

$$L_2=\lambda_2[\bar{l}_{2L}\chi_1^1(l_{3L})^c+\bar{l}_{3L}\chi_1^2(l_{1L})^c+\bar{l}_{1L}\chi_1^3(l_{2L})^c + \bar{Q}_{2L}\chi_2^{1\dagger}(Q_{3L})^c+\bar{Q}_{3L}\chi_2^{2\dagger}(Q_{1L})^c + \bar{Q}_{1L}\chi_2^{3\dagger}(Q_{2L})^c]+L\to R , \quad (5)$$

$$L_3=\sum_{i=1,j}^{3}[\lambda'_{3ij}\bar{l}_{iL}\phi_j l_{iR}+\lambda_{3ij}\bar{l}_{iL}\phi_j^c l_{iR} + \lambda_{3ij}\bar{Q}_{iR}\phi_j^{\dagger}Q_{iL}+\lambda'_{3ij}\bar{Q}_{iR}\phi_j^{c\dagger}Q_{iL}]+L\to R ,$$

and $\phi_i^c=\tau_2\phi_i^*\tau_2$ for $i=1,2,3$. The gauge symmetry is then broken spontaneously by the vacuum

$$\langle\Delta'_{iL,R}\rangle=v_{iL,R}\begin{bmatrix}1&0\\0&0\end{bmatrix}\otimes\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},\quad \langle\Delta_{iL,R}\rangle=0 ,$$

$$\langle\chi_1^i\rangle=\begin{bmatrix}0&0&0\\0&0&w_i\\0&-w_i&0\end{bmatrix},\quad \langle\chi_2^i\rangle=0 , \quad (6)$$

$$\langle\phi_i\rangle=\begin{bmatrix}\kappa_i&0\\0&\kappa_i'e^{i\alpha}\end{bmatrix} .$$

This vacuum implies that the above Yukawa terms have the following properties: L_1 gives small Majorana masses to the ordinary neutrinos and large Majorana masses to the right-handed neutrinos. L_2 gives mass to the exotic leptonic degrees of freedom and L_3 is used to give mass to the ordinary quarks and leptons.[2] Below the scale of $\langle\chi_1^i\rangle$ and $\langle\Delta'_{iR}\rangle$ there is an effective field theory which contains the standard model. The symmetry-breaking pattern is

$$\begin{aligned}&SU(3)_q\otimes SU(3)_l\otimes SU(2)_L\otimes SU(2)_R\otimes[U(1)_{Y'}]^3\\ &\qquad\downarrow\langle\chi_1^i\rangle,\langle\Delta'_{iR}\rangle\\ &SU(3)_q\otimes SU(2)'\otimes SU(2)_L\otimes U(1)_Y\\ &\qquad\downarrow\langle\phi_i\rangle,\langle\Delta'_{iL}\rangle \qquad (7)\\ &SU(3)_q\otimes SU(2)'\otimes U(1)_Q ,\end{aligned}$$

where $SU(3)_q$ is identified as the color group and $U(1)_Q$ is the Abelian group of electromagnetism. By using Eq. (6) and the fact that the vacuum is invariant under $U(1)_Q$, one can show that the unbroken electric charge generator is given by

$$Q=I_{3L}+I_{3R}+\frac{1}{6y}(Y_1'+Y_2'+Y_3')-\tfrac{1}{6}T , \quad (8)$$

where T is the $SU(3)_l$ generator with its fundamental representation normalized as

$$T=\mathrm{Diag}(-2,1,1) . \quad (9)$$

One can also easily verify that Eq. (8) gives the correct electric charges for all the known fermions [the exotic heavy fermions have charges $\pm\frac{1}{2}$, and are expected to be confined by the unbroken-SU(2)′ force].

We have hypothesized that the standard model is a remnant of a theory which contains exact discrete symmetries between the fermions so that essentially only one fermion multiplet is required to generate the whole fermion spectrum of the model. The low-energy-symmetry violations, such as the different electric charges and masses of the quarks and leptons may be a result of spontaneous symmetry breaking.[3] Furthermore, since we have proposed that the symmetry connecting quarks and leptons is discrete, there are no gauge bosons coupling quarks to leptons and hence the unification of quarks and leptons may occur at relatively low energies, e.g., in the TeV energy region.

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*Present address: Physics Department, The University, Southampton, SO9 5NH, UK.

[1]This model is related to the usual left-right-symmetric model. For a review, see R. N. Mohapatra, *Unification and Supersymmetry* (Springer, Berlin, 1986). The model is also related to recent work on quark-lepton symmetry; see R. Foot and H. Lew, Phys. Rev. D **41**, 3502 (1990); Nuovo Cimento A (to be published).

[2]Observe that the Higgs-boson content of the model implies that the mass matrices for the ordinary quarks and leptons are diagonal. The off-diagonal elements in these mass matrices are not protected by any symmetry. The off-diagonal masses will be induced radiatively. Also note that a consequence of the quark-lepton symmetry in the Yukawa sector is that the tree-level mass matrices of the ordinary e-type leptons and u-type quarks are equal. However this equality only holds above the quark-lepton symmetry-breaking scale. Renormalization effects will typically increase the quark masses relative to the lepton masses. Such effects are very dependent on the β function in the unprobed energy region below the quark-lepton unification scale. Consequently, it is not clear whether the Higgs sector will have to be modified. We believe that the pattern of quark and lepton masses is suggestive of some underlying symmetry between the quarks and the leptons.

[3]A possibly important characteristic of the model is that all of the scaler bosons are defined in terms of the Yukawa Lagrangian. This suggests that the Nambu–Jona-Lasinio [Phys. Rev. **122**, 345 (1961)] idea may be applied to this model in order to eliminate all of the elementary scalar bosons (and perhaps eliminate the elementary gauge bosons as well).