$E(1420)$ meson as a $K\bar{K}\pi$ molecule

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(Received 18 January 1990)

In this article an experimental summary of the $E(1420)$ meson with $J^{PC}=1^{++}$ is presented and it is argued that the $E(1420)$ is not explained by QCD in terms of being a fundamental arrangement of quarks and gluons. We then develop a final-state rescattering mechanism based on one-particle-exchange Born terms. We sum these Born terms through a Fredholm integral equation and obtain a Fredholm determinant which shows an enhancement at the $E(1420)$ with $J^{PC}=1^{++}$. The subsequent sum of Born terms is analogous to a π orbiting in a p wave around an s-wave $K\bar{K}$ system. This represents the first example of a molecular state which is bound by color-singlet particle exchanges, as opposed to work by Weinstein and Isgur, which uses color forces to bind their molecule. A phenomenological analysis of all the latest $K\bar{K}\pi$ data arising from hadroproduction argues that the molecular picture for the $E(1420)$ is consistent. We show that if the molecular nature for the $E(1420)$ is generated by the above final-state Born terms, an exotic $K^+\bar{K}_0K^+$ $J^P=0^-$ molecule must exist at the $K\bar{K}K$ threshold.

In the simple quark model, mesons are quark-antiquark systems. Considering the general success of this model, especially in light of the discoveries of charmed and bottom particles, it is hard to imagine that its predicted spectrum of the low-lying meson would be grossly violated. With the advent of QCD as the fundamental force that binds quarks together, new types of mesons based on higher-order arrangements of quarks become possible. The addition of gluons into the game has expanded the types of mesons to include glueballs and hybrids. Therefore, if a new meson is discovered it becomes important to try to determine which type of meson has been found. To this end we have undertaken this article. We will shortly argue that all the above-mentioned mesons cannot explain the $E(1420)$ meson. We then turn to an old idea that the $E(1420)$ is generated by the successive interactions between a K, $\bar{K}$, and a π. We find that a molecular state is formed in which two kaons resonate at threshold in an s wave while the pion orbits around them in a p wave, and we identify this state with the E meson.

A SHORT HISTORY OF THE $E(1420)$

The $E(1420)$ with quantum numbers $J^{PC}=1^{++}$ was discovered in 1967[1] and confirmed in 1980[2] coming from the reaction $\pi^-p\rightarrow K\bar{K}\pi n$ at 4 GeV/c. However it became clear that the $E(1420)$ should be the $\eta(1420)$ with $J^{PC}=0^{-+}$ in 1985,[3] with a higher statistics analysis of $\pi^-p\rightarrow K\bar{K}\pi n$ at 8 GeV/c. This would have been the end of the $E(1420)$ with $J^{PC}=1^{++}$ if it had not been reconfirmed in the reaction $(\pi^+$ or $p)p\rightarrow(\pi^+$ or $p)\,K\bar{K}\pi p$ at 85 GeV/c (Ref. 4) at about the same time. A more detailed spin-parity analysis[5,6] of this data has confirmed the $J^{PC}=1^{++}$ assignment.

There is also evidence that the $E(1420)$ is present in $\gamma\gamma$ scattering from the reaction $e^+e^-\rightarrow e^+e^-K\bar{K}\pi$.[7,8,9] A peak is observed at a mass of 1417^{+13}_{-13} MeV with a width of 35^{+47}_{-20} MeV,[9] and is seen only in the high-Q^2 data sample. The Q^2 cut is applied to select events involving nearly real photons (low Q^2) and those well off mass shell (high Q^2). Because two real photons cannot produce a spin-one state, and since the peak is observed only for high-Q^2 data, the authors conclude that a spin-one object is being produced.

J/ψ decay into $\omega K\bar{K}\pi$ also shows the $E(1420)$.[10] The spin-parity analysis concludes that this state is a 1^+ resonance. The same experiment failed to observe the $E(1420)$ in the reaction $J/\psi\rightarrow\phi K\bar{K}\pi$, indicating the $E(1420)$ has little $s\bar{s}$ quark component. The conclusion that the $E(1420)$ has little strange-quark content was also confirmed by the nonobservation of the $E(1420)$ in the reaction $K^-p\rightarrow K\bar{K}\pi\Lambda$ at 11 GeV/c.[11,12]

POSSIBLE QCD-TYPE OBJECTS THAT COULD BE THE $E(1420)$

The first possibility one would consider is that the $E(1420)$ belongs to the axial-vector $J^{PC}=1^{++}$ nonet of the p-wave $q\bar{q}$ system. For many years this nonet was thought to consist of the $A_1(1270)$, $D(1285)$, $Q_A(1350)$ [the $Q_A(1350)$ and the $Q_B(1350)$ are mixtures of the $Q_1(1280)$ and the $Q_2(1400)$ which are mixed states of $J^{PC}=1^{++}$ and 1^{+-}], and the $E(1420)$. One of the clearest tests of this assignment would be a copious production of $E(1420)$ in the reaction $K^-p\rightarrow E(1420)\Lambda$. However, Refs. 11 and 12 which analyzed exactly this reaction did not see the $E(1420)$ at all, but saw two different 1^+ states, the $H'(1400)$ $J^{PC}=1^{+-}$ and the $D'(1530)$ $J^{PC}=1^{++}$. Therefore, it is clear that these two states are the $s\bar{s}$ members of the 1^{+-} and the 1^{++} nonets and the $E(1420)$ is now an extra member of the 1^{++} system.

Since we are dealing with a nonet with an extra member which is an isosinglet, it is tempting to attribute the presence of this extra member to a glueball. However, current glueball spectrum calculations suggest that the first 1^+ state is at a mass which is greater than 2 GeV and has the wrong charge conjugation $C=-1$.[13]

Next, let us consider the possibility that the $E(1420)$ is made up of a gluon plus a quark and an antiquark (hybrid state). Chanowitz suggested such an assignment in 1987, but alas it would require that $J^{PC}=1^{-+}$.[14]

The final possibility (we will see later an important ingredient) is a multiquark state. Jaffe predicted that there should be a $qq\bar{q}\bar{q}$ $J^{PC}=1^{++}$ state at around 1.6 GeV which decays into $K^*\bar{K}+K\bar{K}^*$ or $K\bar{K}\pi$.[15] Later, work by Jaffe and Low showed that such four-quark states are very broad and their masses are best defined using a method described as the P matrix.[16] Reference 17 showed that Jaffe's four-quark 1^{++} states were very important in understanding the $\rho\pi$ phase shifts. From the P matrix for the $\rho\pi$ system the authors of Ref. 17 were able to assign a mass of approximately 1.45 GeV to a broad 1^{++} state. This result is exactly what Jaffe predicted for the mass of the isovector arising from a 36-plet in his model. However, it is possible that one is observing the isovector member of the 18-plet which is predicted at a lower mass.

From the last paragraph we see that the phenomenology of the $qq\bar{q}\bar{q}$ state leads to broad resonances and is inconsistent with the $E(1420)$ whose width is about 50–60 MeV.

The $K\bar{K}\pi$ AS AN INTERACTING SYSTEM

We have seen from the preceding section that the $E(1420)$ does not have a simple QCD assignment and thus there is a real mystery as to what it could be. Rosner has suggested that the $E(1420)$ may be a molecule made out of a K, a $\bar{K}$, and a π.[18] In order to bind these three particles together, we need to develop a dynamical theory that cannot depend upon QCD color van der Waals forces[19] for binding, like atoms depend upon QED for van der Waals force to make molecules. This is opposed to work by Weinstein and Isgur, which uses color forces to bind their molecule.[20,21] Their theory generates a van der Waals force which is quite strong. However, they argue that this long-range force does not effect their derived spectrum.[21]

The phase-shift analysis of the $E(1420)$ depends on the isobar model,[22] thus the major requirement in understanding the interactions of these three particles is the construction of a unitary isobar model which will have the important long-range forces expected. We will assume that the only interaction among the particles occurs through one-particle exchange (OPE) between $K^*\bar{K}$, $\bar{K}^*K$, and $\delta\pi$ states [Fig. 1(a)]. We then choose as our dynamical framework the Blankenbecler-Sugar formalism[23] which yields sets of coupled integral equations for amplitudes $X(K^*\to K^*)$, $X(K^*\to\bar{K}^*)$, $X(K^*\to\delta)$, $X(\bar{K}^*\to K^*)$, $X(\bar{K}^*\to\bar{K}^*)$, $X(\bar{K}^*\to\delta)$, $X(\delta\to K^*)$, $X(\delta\to\bar{K}^*)$, and $X(\delta\to\delta)$. These amplitudes describe the quasi-two-body processes $K^*\bar{K}\to K^*\bar{K}$, $K^*\bar{K}\to\bar{K}^*K$, $K^*\bar{K}\to\delta\pi$, etc., whose solutions are Lorentz invariant, and satisfy two- and three-body unitarity, and the cluster property. In operator formalism these equations have the structure [a schematic representation is shown in Fig. 1(b)],

$$X_{ba}(W_E)=B_{ba}(W_E)+B_{bc}(W_E)G_c(W_E)X_{ca}(W_E)\ , \tag{1a}$$

$$a,b,c=K^*,\bar{K}^*,\delta\ . \tag{1b}$$

In Eqs. (1), W_E is the overall c.m. energy of the three-particle system. For isotopic spin $I=0$, momentum-space elements of various operators appearing in the above equation are now given.

B_{ba} are the OPE Born terms, where in Eq. (2) we write all the diagonal terms which are zero:

$$\langle \mathbf{p}'\lambda'|B^0_{aa}(W_E)|\mathbf{p}\lambda\rangle=0\ . \tag{2}$$

The $K^*\bar{K}^*$ OPE Born term is given by

$$\langle \mathbf{p}'\lambda'|B^0_{K^*\bar{K}^*}(W_E)|\mathbf{p}\lambda\rangle=\frac{X^0_{K^*\bar{K}^*}v_{K^*}(\mathbf{V}'^2)V'^*_{\lambda'}(\mathbf{p},\mathbf{p}')\overline{W}v_{\bar{K}^*}(\mathbf{V}^2)V_\lambda(\mathbf{p},\mathbf{p}')}{\omega_{\mathbf{p}+\mathbf{p}'}(\overline{W}^2-W_E^2)}\ . \tag{3}$$

The spectator energy for the K^* is

$$\omega_p=(p^2+m_p^2)$$

while the spectator energy for the $\bar{K}^*$ is

$$\omega_{p'}=(p'^2+m_p^2)$$

and the energy for the exchange particle is

$$\omega_{\mathbf{p}+\mathbf{p}'}=(p^2+p'^2+m_e^2+2\mathbf{p}\cdot\mathbf{p}')\ .$$

We also define

$$\begin{aligned}
&\overline{W}=\omega_p+\omega_{\mathbf{p}+\mathbf{p}'}+\omega_{p'}\ ,\\
&\mathbf{V}(\mathbf{p},\mathbf{p}')=\mathbf{V}'(\mathbf{p}',\mathbf{p})=\mathbf{p}'+\alpha(p^2,p'^2,\mathbf{p}\cdot\mathbf{p}')\mathbf{p}\ ,\\
&\alpha(p^2,p'^2,\mathbf{p}\cdot\mathbf{p}')=\tfrac{1}{2}-\frac{\tfrac{1}{2}p^2+\mathbf{p}\cdot\mathbf{p}'}{P_0(P_0+W)}\ ,\\
&P_0=\omega_{p'}+\omega_{\mathbf{p}+\mathbf{p}'}\ ,\\
&W^2=P_0^2-p^2\ .
\end{aligned} \tag{3a}$$

In Eq. (3), λ and λ' are initial and final z components of K^* and $\bar{K}^*$ spins. The off-shell polarization vector $\mathbf{V}$ is defined in Ref. 24; as pointed out in the latter work, V^2 is a Lorentz invariant and is equal to the square of the c.m. momentum of the π and $K(\bar{K})$ in the $K^*(\bar{K}^*)$ rest frame. The form factors v_{K^*} and $v_{\bar{K}^*}$ are required for convergence in our integral equation and are taken to be functions of V^2. Having defined all relevant quantities, we now display the last type of term:

$$\langle \mathbf{p}'|B^0_{\delta K^*}(W_E)|\mathbf{p}\lambda\rangle=\frac{X^0_{\delta K^*}v_\delta(\mathbf{V}'^2)\overline{W}v_{K^*}(\mathbf{V}^2)V_\lambda(\mathbf{p},\mathbf{p}')}{\omega_{\mathbf{p}+\mathbf{p}'}(\overline{W}^2-W_E^2)}\ . \tag{4}$$

The isospin-zero recoupling coefficient X^0_{ab} appearing in Eqs. (3) and (4) is given by

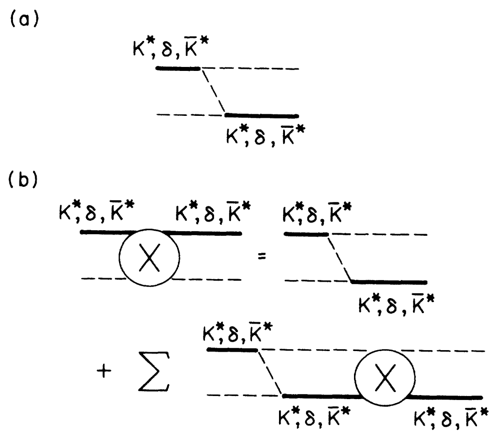

FIG. 1. (a) Long-range one-particle-exchange (OPE) mechanism. (b) Unitary sum of OPE diagrams in terms of coupled integral equations [Eq. (1)].

$$X_{ab}^{0} = -1 \ . \tag{5}$$

Finally, the propagator G_a in Eq. (1) has matrix element

$$\begin{gathered}\langle \mathbf{p}'|G_\delta(W_E)|\mathbf{p}\rangle = 2\omega_p(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')D_\delta^{-1}(p,W_E) \ , \\ D_\delta(p,W_E) = \sigma - M_\delta^{(0)2} - \frac{1}{(2\pi)^2}\int_0^\infty dq \frac{q^2 v_\delta^2(q)}{\omega_q(\sigma-4\omega_q^2)}\end{gathered} \tag{6}$$

or

$$\begin{gathered}\langle \mathbf{p}'|G_{K^*}(W_E)|\mathbf{p}\rangle = 2\omega_p(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')D_{K^*}^{-1}(p,W_E) \ , \\ D_{K^*}(p,W_E) = \sigma - M_{K^*}^{(0)2} - \frac{1}{3(2\pi)^3}\int_0^\infty dq \frac{q^4 V_{K^*}^2(q)}{\omega_q(\sigma-4\omega_q^2)} \ ,\end{gathered} \tag{7}$$

with

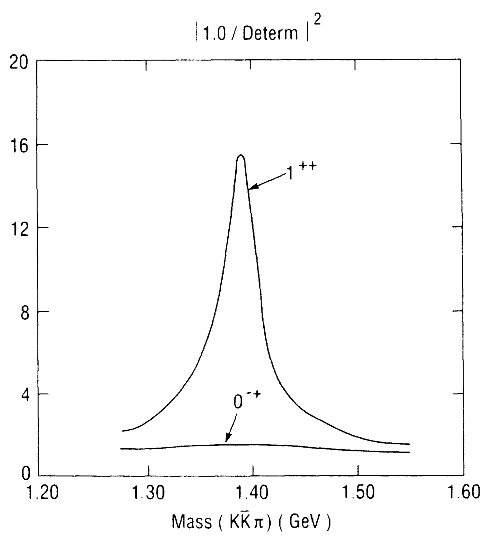

FIG. 2. The value of 1 over the Fredholm determinate squared for $J^{PC}=1^{++}$ and $J^{PC}=0^{-+}$ as a function of $K\bar{K}\pi$ mass (smooth curves).

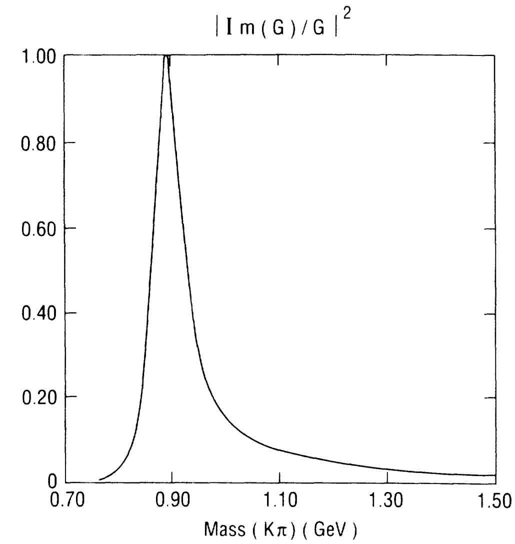

FIG. 3. The absolute value squared of the imaginary part of the $K\pi$ propagator for the $I=\frac{1}{2}$ and $J=1$ mode divided by the complete propagator, thus forming the square of the T-matrix scattering amplitude.

$$\sigma = W_E^2 - 2W_E\omega_p + m_p^2 \ . \tag{8}$$

As explained in Ref. 25, B_{ba} have the same residues at their pole as the corresponding Feynman Born term, but in addition have a cut structure which guarantees that all amplitudes satisfy two- and three-body unitarity. The propagators G_{K^*} and G_δ are essentially the two-body

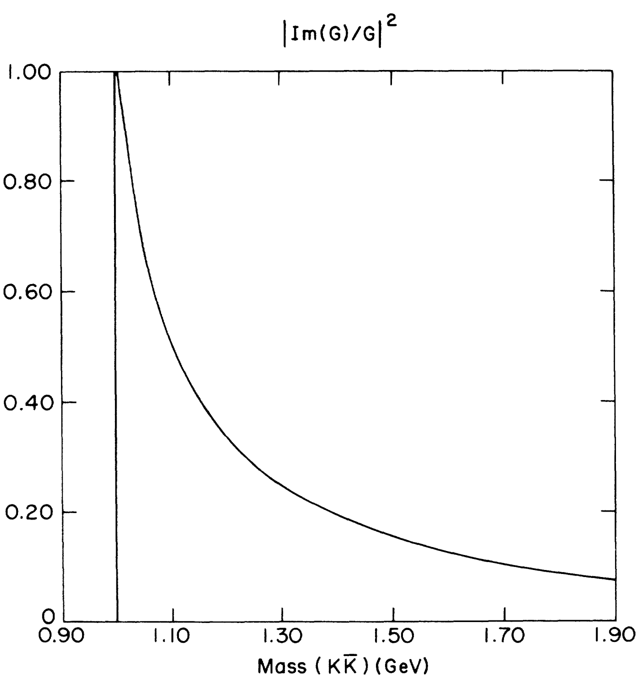

FIG. 4. The absolute value squared of the imaginary part of the $K\bar{K}$ propagator for the $I=1$ and $J=0$ mode divided by the complete propagator, thus forming the square of the T-matrix scattering amplitude.

scattering amplitudes and carry the phases of the $K\pi$ scattering in $I=\frac{1}{2}$, $J=1$ (K^* and $\bar{K}^*$), and $K\bar{K}$ scattering in $I=1$, $J=0$ (δ). Note that all Born terms and propagators are given terms of form factors v_δ and v_{K^*}; these are chosen to be smooth functions of the form

$$v_a(q^2)=g_a\beta_a^2(q^2+\beta_a^2)^{-1}, \quad a=\delta,K^* . \tag{9}$$

The values of g_a, β_a, and $M_a^{(0)}$ are determined from on-shell $K\pi$ and $K\bar{K}$ scattering. We have taken a form for the δ which gives a large coupling to $K\bar{K}$ compared to $\eta\pi$ as shown by Flatté.[26] This type of a δ makes the δ behave like a $K\bar{K}$ bound state whose quark structure is given by Jaffe.[15,16] The physics does not seem to depend on β_a, and we set all our β_a to be $50m_\pi^2$.

For application to the $E(1420)$ data analysis we require the angular momentum projections of Eq. (1). Partial-wave analysis yields the following set of matrix equations:

$$\langle p'Jl'|X_{ba}^0(W_E)|pJl\rangle=\langle p'Jl'|B_{ba}^0(W_E)|pJl\rangle +(2\pi)^{-3}\sum_{l'',c}\int_0^\infty p''^2dp''\langle p'Jl'|B_{bc}^0(W_E)|p''jl''\rangle D_c^{-1}(p'',W_E)\langle p''jl''|X_{ca}^0(W_E)|pJl\rangle . \tag{10}$$

In the above equation J is the total angular momentum, l the orbital angular momentum of the initial particle-isobar system, and l' the orbital angular momentum of the final particle-isobar system, where, for example,

$$\langle p'Jl'|X_{K^*\bar{K}^*}^0(W_E)|pJl\rangle=\sum_{\substack{M,\lambda\\ M'\lambda'}}\langle l'|m'\lambda'|JM\rangle\langle lJm\lambda|JM\rangle\int d\Omega_{\hat{p}'}\int d\Omega_{\hat{p}}\, Y_{l'm'}(\hat{p}\,')\langle \mathbf{p}'\lambda'|X_{K^*\bar{K}^*}^0(W_E)|\mathbf{p}\lambda\rangle Y_{lm}(\hat{p}) . \tag{11}$$

The partial-wave decomposition of the various Born amplitudes are now given. The $K^*\bar{K}^*$ term has the basic structure

$$\langle \mathbf{p}'\lambda'|B_{K^*\bar{K}^*}^0(W_E)|p\lambda\rangle\equiv a_{pp}p_\lambda p_{\lambda'}^* +a_{p'p'}p'_\lambda p'^*_{\lambda'} +a_{pp'}p_\lambda p'^*_{\lambda'} +a_{p'p}p'_\lambda p^*_{\lambda'} , \tag{12}$$

where a_{pp}, $a_{pp'}$, etc., are functions of p^2, p'^2, and $\mathbf{p}\cdot\mathbf{p}'$. Using Eq. (11), integrating over solid angles, and summing over Clebsch-Gordan coefficients, we obtain

$$\begin{aligned}\langle p'Jl'|B_{K^*\bar{K}^*}^0(W_E)|pJl\rangle=&[(2l'+1)(2l+1)]^{1/2}\\ &\times\Big[\langle l100|J0\rangle\langle l'100|J0\rangle(2J+1)^{-1}[p^2(a_{pp})_{l'}+p'^2(a_{p'p'})_l+p'p(a_{p'p})_J]pp'(-1)^{J-l-1}\\ &\times\sum_\Lambda\langle l100|\Lambda 0\rangle\langle l'100|\Lambda 0\rangle W(1\Lambda J1{:}ll')(a_{pp'})_\Lambda\Big] .\end{aligned} \tag{13}$$

In Eq. (13) above $W(|\Lambda J|;ll')$ is a Racah coefficient.[27] In going from Eqs. (12) to (13) we have expanded, for example,

$$a_{pp}(p^2,p'^2,\mathbf{p}\cdot\mathbf{p}')=\sum_{l,m}(a_{pp})_l Y_{lm}^*(\hat{p})Y_{lm}(\hat{p}\,')=\sum_{l,m}(a_{pp})_l Y_{lm}(\hat{p})Y_{lm}^*(\hat{p}\,') \tag{14}$$

and thus

$$(a_{pp})_l=2\pi\int_{-1}^{+1}d_z p_l(z)a_{pp} . \tag{15}$$

A similar expression may be obtained for the partial-wave projection of the $B_{\delta K^*}^0$ Born term which takes $\delta\pi\to K^*\bar{K}$:

$$\langle p'Jl'|B_{\delta K^*}^0(W_E)|pJl\rangle=\langle l100|l'0\rangle[(2l+1)/(2l'+1)]^{1/2}[p'(B_{\delta K^*}^0)_l+p(\alpha B_{\delta K^*}^0)_{l'}] \tag{16}$$

with α in Eq. (16) defined in Eq. (2a). All partial-wave projections in the above formulas use the normalization defined by Eqs. (14) and (15).

We have obtained a unitary and consistent formalism in terms of Born amplitudes generated by OPE. In Eq. (1), we have an integral equation which represents the successive rescattering of the three particles through OPE and two-body scattering amplitudes. By rewriting Eq. (1) we have

$$\sum_K(\delta_{ik}-M_{ik})X_{kj}=B_{ij}, \quad i,j,k=\delta,K^*,\bar{K}^* . \tag{17}$$

This Fredholm integral equation leads to a Fredholm determinant which at the mass of the $E(1420)$ gives us the result that

$$\mathrm{Det}(1-M)=0.258 \ . \tag{18}$$

As we can from this value of the determinant, the amplitude squared of the basic Born term is increased at the E mass by a factor 15 times or 1 over the absolute value of the determinant squared. Figure 2 shows the general mass dependence of this enhancement. Figure 3 shows $|\mathrm{Im}(G_{K^*})/G_{K^*}|^2$ for the K^* used in the above calculation, where Fig. 4 shows $|\mathrm{Im}(G_\delta)/G_\delta|^2$ for the δ. The most important Born term contributions to the higher-order rescattering are the K and the $\bar{K}$ exchanges going to or from the δ, while the π exchange to or from the K^* or $\bar{K}^*$ is unimportant. By way of comparison, earlier work for the 3π system for the A_1 showed that the Fredholm determinant was equal to 0.98.[28,29]

The kinematics in the K-exchange Born term takes the $\delta\pi$ system in an p wave to the $K^*\bar{K}$ system in an s wave. The c.m. momentum of K^* and the $\bar{K}^*$ is very small since we are near both the K^* and $\bar{K}^*$ thresholds, while the c.m. momentum of the π is near the maximum momentum possible. This means for the $\delta\pi$ system the p-wave barrier is of no consequence, while in the $K^*\bar{K}$ system there is no barrier (s wave). Thus, we achieve a picture of the K and the $\bar{K}$ resonating as a δ at the center of gravity, with the pion revolving around the center in a p wave and on each half-revolution forming a resonance as a K^* or a $\bar{K}^*$. In Fig. 2 we also show the value of 1 over the Fredholm determinant squared for the 0^{-+} $K\bar{K}\pi$ system. For these Born terms the $K^*\bar{K}$ system must be in a p wave, since the momentum is near zero the p-wave barrier suppresses these Born terms greatly. It is reassuring that no enhancement is seen in this wave.

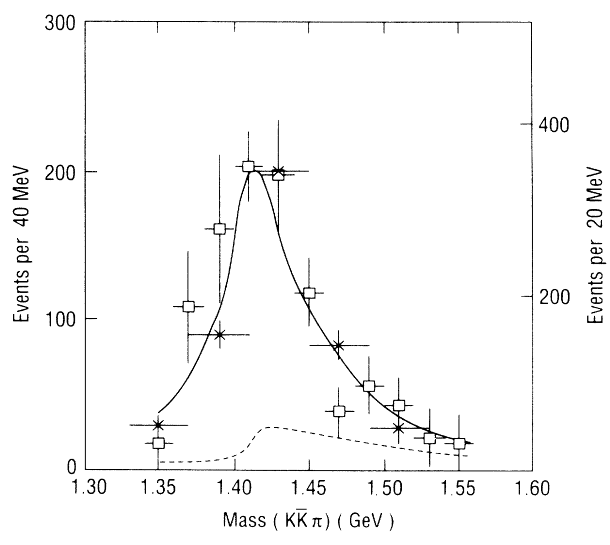

FIG. 5. The modulus squared of the $J^{PC}=1^{++}$ $K\bar{K}\pi$ amplitude from Ref. 6 (✱) events/0.04 GeV shown on the left-hand scale and from Ref. 30 (□) events /0.02 GeV shown on the right-hand scale. The smooth dashed curve is the Deck amplitude (described in text), while the solid curve is the Deck amplitude after the final-state interaction.

PHENOMENOLOGICAL FITS TO THE LATEST $K\bar{K}\pi$ DATA

The Brookhaven National Laboratory (BNL) held a Workshop on Glueballs, Hybrids, and Exotic Hadrons in 1988, where the latest $K\bar{K}\pi$ data was discussed. Kirk of the WA76 Collaboration[6] presented centrally produced $K_S^0K^\pm\pi^\mp$ events from a 300-GeV/c proton beam. They observed ≈ 400 events over an ≈ 300 event background. A partial-wave analysis (PWA) shows that all of the ≈ 400 events are consistent with a $J^{PC}=1^{++}$ $K^*\bar{K}+\bar{K}^*K$ decay mode (see Fig. 5). Blessing of BNL E771 (Ref. 30) presented a mass-dependent fit to a PWA of 52 222 events from the reaction $\pi^-p\to K^+K_S^0\pi^-n$ at 8 GeV/c. The $J^{PC}=1^{++}$ $K^*\bar{K}+\bar{K}^*K$ decay mode is also displayed in Fig. 5. One can see from Fig. 5 that the two data sets are quite consistent.

In order for the above-mentioned final-state interaction to create a molecule, one needs to first produce a $K^*\bar{K}+K\bar{K}^*$ system in an overall s wave. The production of such a system would be accomplished by a multiperipheral Deck mechanism[31] shown in Fig. 6(a) for $pp\to pK\bar{K}\pi p$, Fig. 6(b) for $\pi^-p\to K\bar{K}\pi n$, and Fig. 6(c) for $K^-p\to K\bar{K}\pi\Lambda$. We have found it very convenient to represent the above three production processes at a fixed energy by a sum of two Born terms for K^* and $\bar{K}^*$ production arising from off-shell $\pi\delta$ scattering [Eq. (4)].

We next perform a fit to the data displayed in Fig. 5.[6,30] At each mass point we derive the necessary Deck production strength which is the dashed curve in Fig. 5. This cross section is defined by

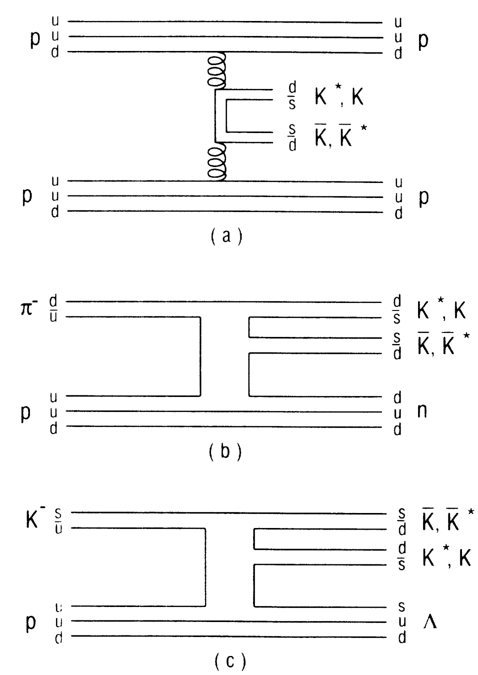

FIG. 6. Multiperipheral Deck mechanism diagrams showing the quark flow. (a) Double-Pomeron Deck mechanism for the reaction $pp\to pK\bar{K}\pi p$. (b) Charge-exchange Deck mechanism for the reaction $\pi^-p\to K\bar{K}\pi n$. (c) Strangeness-exchange Deck mechanism for the reaction $K^-p\to K\bar{K}\pi\Lambda$.

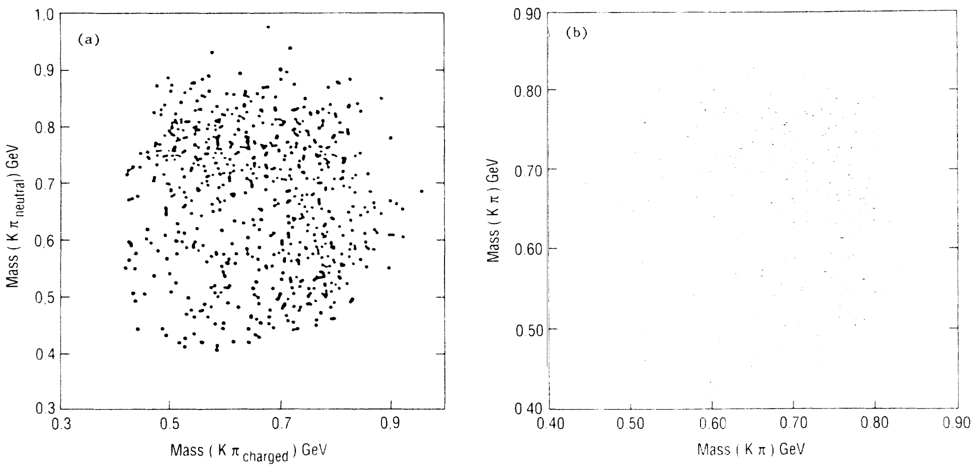

FIG. 7. (a) Dalitz plot of 753 $K^+K_S^0\pi^-$ or $K^-K_S^0\pi^+$ events as a function of $(K\pi)$ neutral and $(K\pi)$ charged masses from Ref. 6. (b) Dalitz plot of 7469 $K\bar{K}\pi$ events as a function of $K\pi$ and $\bar{K}\pi$ masses. Events generated as described in the text.

$$\sigma_{\rm Deck}(p,W_E)=\frac{\rm scale}{(2\pi)^3}\int_{\rm space}^{\rm phase}\frac{q'dq'q\,dq}{\omega_{q'}\omega_q}\left|\langle pJl''|B^0_{\delta K^*}|q'Jl\rangle\frac{\sqrt{{\rm Im}D_{K^*}(q')}}{D_{K^*}(q')}+\langle pJl''|B^0_{\delta\bar{K}^*}|qJl\rangle\frac{\sqrt{{\rm Im}D_{\bar{K}^*}(q)}}{D_{\bar{K}^*}(q)}\right|^2 . \tag{19}$$

The partial-wave Born terms are defined in Eq. (16). This curve represents the initial amount of $K^*\bar{K}+K\bar{K}^*$ production before the final-state interaction. The dashed curve is very reasonable for a Deck amplitude rising sharply at the K^* threshold and then slowly falling off with mass. The solid curve is the final $K\bar{K}\pi$ cross section after the molecule is formed. This cross section is given by

$$\sigma(p,W_E)=\frac{\rm scale}{(2\pi)^3}\int_{\rm space}^{\rm phase}\frac{q'dq'q\,dq}{\omega_{q'}\omega_q}\left|\langle pJl''|X_{\delta K^*}|q'Jl\rangle\frac{\sqrt{{\rm Im}D_{K^*}(q')}}{D_{K^*}(q')}+\langle pJl''|X_{\delta\bar{K}^*}|qJl\rangle\frac{\sqrt{{\rm Im}D_{\bar{K}^*}(q)}}{D_{\bar{K}^*}(q)}\right.$$
$$\left.+\langle pJl''|X_{\delta\delta}|q''Jl''\rangle\frac{\sqrt{{\rm Im}D_{\delta}(q'')}}{D_{\delta}(q'')}\right|^2 . \tag{20}$$

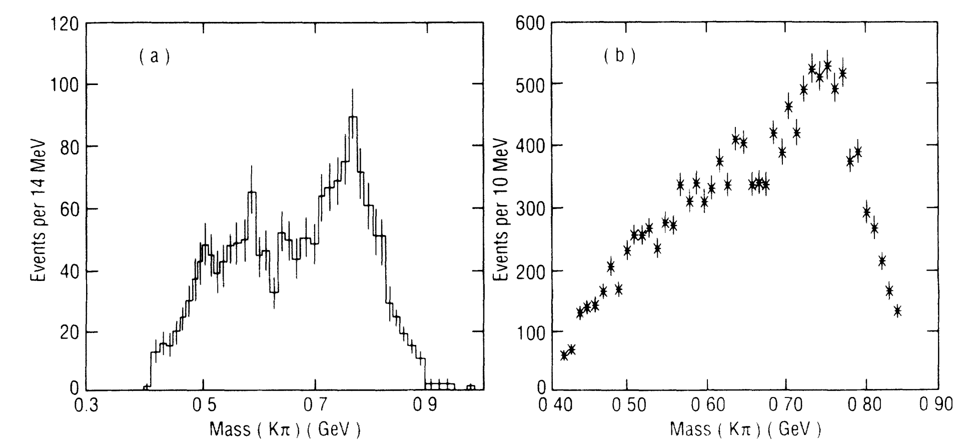

FIG. 8. (a) Dalitz-plot projection of 753 $K^+K_S^0\pi^-$ or $K^-K_S^0\pi^+$ events as a function of $K\pi$ mass from Ref. 6. (b) Dalitz-plot projection of 7469 $K\bar{K}\pi$ events as a function of $K\pi$ mass. Events generated as described in the text.

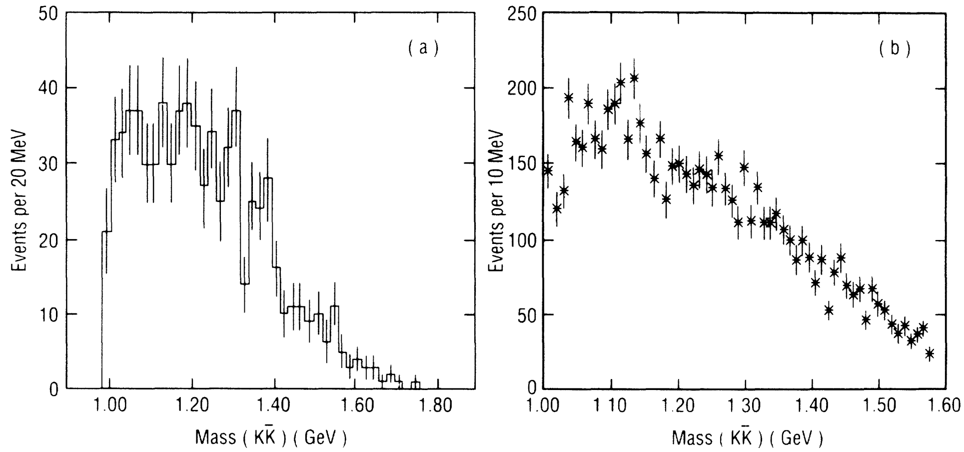

FIG. 9. (a) Dalitz-plot projection of 753 $K^+K_S^0\pi^-$ or $K^-K_S^0\pi^+$ events as a function of $K\bar{K}$ mass from Ref. 6. (b) Dalitz-plot projection of 7469 $K\bar{K}\pi$ events as a function of $K\bar{K}$ mass. Events generated as described in the text.

About 24% of this cross section is decaying by the $\delta\pi$ channel in the interference region with the overlapping K^* and $\bar{K}^*$ bands. It was pointed out in the preceding section that the δ formation is most crucial for the final-state interaction. This fact leads to an important $X_{\delta\delta}$ term. However, the final $\delta\pi$ cross section is not that important. The last term in Eq. (20), which represents the propagation and decay of the δ, suppresses the $X_{\delta\delta}$ contribution to the overall cross section.

This suppression of decays in the $\delta\pi$ final state is a general property and narrows the widths of the $f_1(1285)$ [$D(1285)$] and the η (1280). These states have isovector partners that are 15 to 20 times as wide.

Reference 6 has published a 753-event Dalitz plot in Fig. 7(a) which has taken $K\bar{K}\pi$ masses from 1.37–1.49 GeV. Figures 8(a) and 9(a) show the projections of the Dalitz plot onto the $K\pi$ and $K\bar{K}$ subsystems. In Figs. 7(b), 8(b), and 9(b), we show 7469 events coming from our molecule decaying plus background within the above mass cuts. It should be noted that 47% of the Dalitz plot is background and does not come from the E (1420).

We have also investigated the phase motion of the final-state interaction with respect to the initial phase of the production Deck amplitude. Figure 10 shows the phase motion as a function of mass. Unlike a resonance only about $\frac{1}{4}$ of 180° phase motion is observed. Reference 30 detected two important partial waves in the $E(1420)$ mass region, the first being a $J^{PC}=1^{++}$ and the second being a $J^{PC}=0^{-+}$. The 1^{++} is mainly $K^*\bar{K}$, while the 0^{-+} is mainly $\delta\pi$. Reference 30 reported the need for two resonances in the 0^{-+} channel while the 1^{++} is consistent with no phase motion even though they see a bump with a width of 60 MeV. However, they report that they need a 1^{++} resonance at around 1.540 GeV in order to flatten the relative phase motion [see Fig. 11(b)].

References 12 and 30 represent the highest-statistics hadroproduction experiments for the $K\bar{K}\pi$ system. No model for the $K\bar{K}\pi$ system could be considered correct if it does not explain the observations of the above two experiments. We have already fit the 1^{++} mass distribution of Ref. 30 (see Fig. 5). The analysis of Ref. 12 of the $K\bar{K}\pi$ system arising from a K^- beam found that the spectrum was consistent with a single resonance at 1.530 GeV with a width of $\Gamma\approx 100$ MeV. This state is the $s\bar{s}$ member of the 1^{++} nonet and is expected to dominate the K-induced reactions. This state is also seen in Ref. 30, but only at a very small rate, since it would be suppressed by the Okubo-Zweig-Iizuka (OZI) rule in the π-induced reaction. We see from Figs. 6(b) and 6(c) that the Deck amplitude could have very similar production strength in both K- and π-induced reactions. The usual OZI suppression in production is around a factor of 100, which implies a factor of 10 at the amplitude level. We have performed a fit to the 1^{++} $K^*\bar{K}$ of Ref. 12 and the

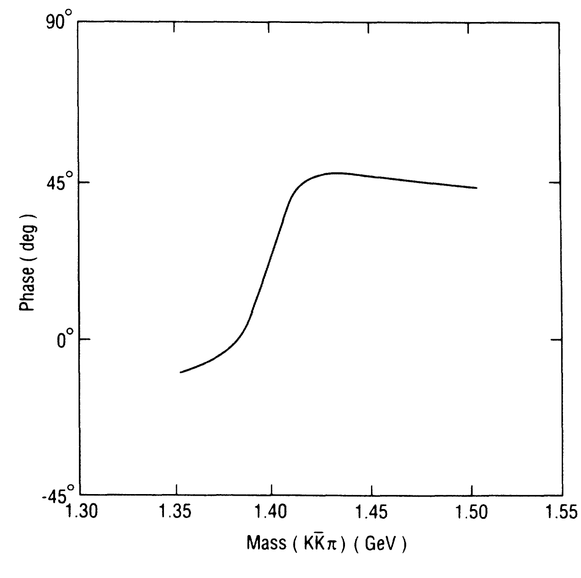

FIG. 10. Phase motion of the final-state $K^*\bar{K}$ amplitude with respect to the initial $K^*\bar{K}$ Deck amplitude as a function of $K\bar{K}\pi$ mass.

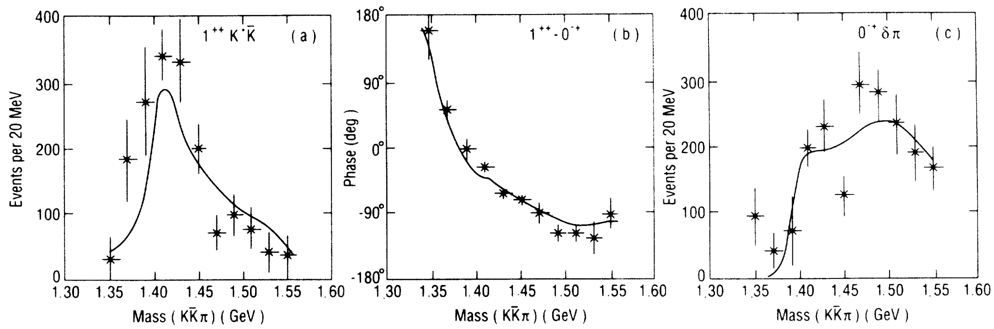

FIG. 11. (a) The modulus squared of the $J^{PC}=1^{++}$ $K^*\bar{K}$ amplitude from Ref. 30 (events/0.02 GeV). The smooth curve is from fit described in the text. (b) The relative phase (degrees) of the 0^{-+} $\delta\pi$ amplitude with respect to the 1^{++} $K^*\bar{K}$ amplitude from Ref. 30. The smooth curve is from fit described in the text. (c) The modulus squared of the 0^{-+} $\delta\pi$ amplitude from Ref. 30 (events/0.02 GeV). The smooth curve is from fit described in the text.

0^{-+} $\delta\pi$ and the 1^{++} $K^*\bar{K}$ of Ref. 30. In our fit we used the same 0^{-+} resonances and the one 1^{++} resonance of Ref. 30. We also used the Deck amplitude derived in Fig 5. We have made the assumption that the production ratio of the Deck amplitude compared to the 1^{++} resonance is a factor of 10 bigger in the π-induced reaction of Ref. 30 than in the K-induced reaction of Ref. 12, because of the OZI suppression of the $s\bar{s}$ state.

In Figs. 11 and 12, we show the results of the above fit. The fit is qualitatively a good fit with a systematic under prediction of the π-induced 1420 Deck peak [Fig. 11(a)]. The small 1420 Deck peak seen in the K-induced reaction Fig. 12 lies under a much larger $H'(1400)$ 1^{+-} peak seen in that experiment.[12] Figures 11(b) and 11(c), which show the relative phase between the 1^{++} and the 0^{-+} and the mass spectrum of the $0^{-+}\delta\pi$ wave, are reasonable and are of the same quality as Ref. 30. The mass and the widths that are obtained in this fit are 1.546 GeV with 80 MeV for the 1^{++} and 1.391 GeV with 42 MeV, 1.522 GeV with 200 MeV for the 0^{-+} states. This is to be compared with masses and widths of 1.546 GeV, with 84 MeV for the 1^{++} and 1.396 GeV, with 73 MeV, 1.515 GeV with 101 MeV for the same type of fit in Ref. 30. Reference 12 gets a 1.530-GeV mass and $\approx$100-MeV widths for the 1^{++} state. Finally, it should be noted that the fit depended on the reduced phase motion of the 1^{++} final-state interaction to give the observed backward relative phase motion.

THE $K\bar{K}K$ AS AN INTERACTING SYSTEM

We have seen in the preceding sections that the $K\bar{K}\pi$ system interacts to form a molecular state. This interac-

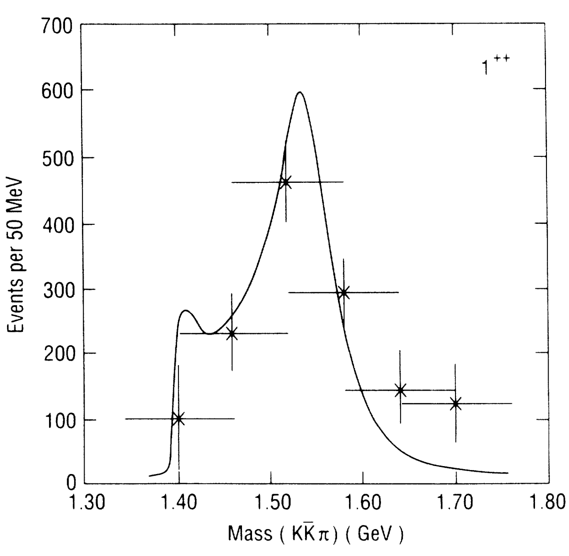

FIG. 12. The modulus squared of the $J^{PC}=1^{++}$ $K\bar{K}\pi$ amplitude from Ref. 12 (arbitrary units). The smooth curve is from fit described in the text.

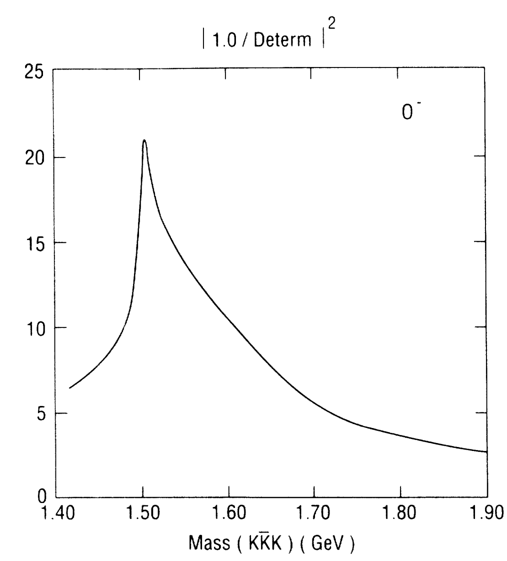

FIG. 13. The value of 1 over the Fredholm determinate squared for $J^P=0^-$ as a function of $K\bar{K}K$ mass (smooth curve).

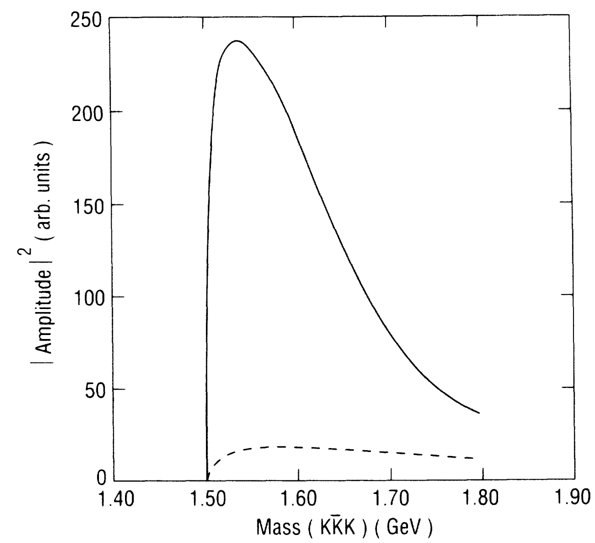

FIG. 14. The modulus squared of the $J^P=0^-$ δK amplitude after final-state interactions as a function of $K\bar{K}K$ mass (smooth curve). The smooth-dashed curve is the Deck amplitude for the $J^P=0^-$ δK system as a function of $K\bar{K}K$ mass before final-state interactions.

tion is driven by the attraction between the K and $\bar{K}$ system through the δ resonance. It seems only natural at this point to investigate the possibility that a three-K molecule might exist. However, the investigation of this system would only be worthwhile if it leads to the prediction of a molecule which has exotic quantum numbers. The only exotic quantum number which can be obtained from the $K\bar{K}K$ system is the isotopic spin. Thus, we will derive the same coupled equations as before for the case of the $K\bar{K}K$ system in an overall s wave with isotopic spin of $\frac{3}{2}$.

As before, we will assume that the only interaction among the particles occurs through OPE between the $\delta_1^+K_1^+$ and the $\delta_2^+K_2^+$ systems. In this case, there is only one type of Born term that links the $\delta_1^+K_1^+$ and the $\delta_2^+K_2^+$ systems. This OPE Born term is given by

$$\langle \mathbf{p}'|B_{\delta\delta}^{3/2}(W_{KKK})|\mathbf{p}\rangle = \frac{X_{\delta\delta}^{3/2} v_\delta(\mathbf{V}'^2)\overline{W} v_\delta(\mathbf{V}^2)}{\omega_{\mathbf{p}+\mathbf{p}'}(\overline{W}^2 - W_{KKK}^2)} , \quad (21)$$

where $X_{\delta\delta}^{3/2} = \frac{2}{5}$.

Using the above Born term, we can again set up a Fredholm integral equation which leads to a Fredholm determinant as a function of $K\bar{K}K$ mass whose inverse square is shown as a function of $K\bar{K}K$ mass in Fig. 13.

We next defined a Deck production amplitude in the same manner as before [Eq. (19)]. We use the s-wave projection of Eq. (21) and adjust the scale factor to give the very flat Deck amplitude shown in Fig. 14. When the final-state interaction is turned on, one sees a very large threshold enhancement with a width of approximately 200 MeV. This enhancement is a unique prediction of this model and if it does not exist then the final-state interaction mechanism is certainly not what is causing the $E(1420)$.

CONCLUSION

We have seen in the section on phenomenological fits to hadroproduction of the $K\bar{K}\pi$ system that the molecular state generated by successive rescatterings through K and $\bar{K}$ exchange gives a consistent account of the data. Let us also consider the $\gamma\gamma$ interaction. It is well known that $\gamma\gamma$ couples to two particles as calculated in great detail by Brodsky and LePage.[32] These two particle couplings show threshold enhancements (given in Ref. 32) which have similar mass dependencies as a Deck amplitude, which one can associate with Jaffe's four-quark states. Li and Liu have tried to make this connection for the $\rho\rho$ system produced by $\gamma\gamma$.[33] Since the $\gamma\gamma$ interaction happens at a small distance, the two particles are produced in an s wave. Therefore, $K^*\bar{K}$ s-wave production will become important as soon as Q^2 gets big enough for one of the γ's to be off shell. At this threshold the molecular final-state interaction will take over and the $E(1420)$ will be observed.

We have also shown that the molecular final-state interaction leads to the formation of an exotic $(I=\frac{3}{2})$ $K^+\bar{K}_0K^+$ threshold enhancement with a width of approximately 200 MeV. If this exotic molecule is not found, then it would be an absolute rejection of the calculations and mechanism presented in this paper. If, on the other hand, the $K\bar{K}K$ enhancement is seen, it would imply that the final-state interaction mechanism of this paper is probably correct. Furthermore, the strength of the final-state interaction is fundamentally driven by the strength of the $K\bar{K}$ binding in the δ meson. Thus, one is also predicting that the δ meson is a strongly bound $K\bar{K}$ system and not weakly bound like the deuteron.

ACKNOWLEDGMENTS

This research has been carried out under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

[1]O. I. Dahl *et al.*, Phys. Rev. **163**, 1377 (1967).
[2]C. Dionisi *et al.*, Nucl. Phys. **B169**, 1 (1980).
[3]S. U. Chung *et al.*, Phys. Rev. Lett. **55**, 779 (1985).
[4]T. Armstrong *et al.*, Phys. Lett. **146B**, 273 (1984).
[5]T. Armstrong *et al.*, Z. Phys. C **34**, 23 (1987).
[6]T. A. Armstrong *et al.*, in *Glueballs, Hybrids, and Exotic Hadrons,* proceedings of the BNL Workshop, Upton, New York, 1988, edited by S. U. Chung (AIP Conf. Proc. No. 185) (AIP, New York, 1989), p. 340.
[7]H. Aihara *et al.*, Phys. Rev. Lett. **57**, 2500 (1986).
[8]G. Gidal *et al.*, Phys. Rev. Lett. **59**, 2016 (1987).
[9]D. O. Caldwell, in *Glueballs, Hybrids, and Exotic Hadrons* (Ref. 6), p. 465.
[10]J. J. Becker *et al.*, Phys. Rev. Lett. **59**, 186 (1987).
[11]D. Aston *et al.*, in *Hadron '87,* proceedings of the 2nd International Conference on Hadron Spectroscopy, Tsukuba,

Japan, 1987, edited by Y. Oyanagi, K. Takamatsu, and T. Tsun (KEK Report No. 87-7 Tsukuba, 1987).
[12]D. Aston *et al.*, in *Glueballs, Hybrids, and Exotic Hadrons* (Ref. 6), p. 160.
[13]C. E. Carlson *et al.*, Phys. Rev. Lett. **30**, 1594 (1984).
[14]M. Chanowitz, Phys. Lett. B **187**, 409 (1987).
[15]R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
[16]R. L. Jaffe and F. E. Low, Phys. Rev. D **19**, 2105 (1979).
[17]R. Aaron and R. S. Longacre, Phys. Rev. D **24**, 1207 (1981).
[18]J. L. Rosner, in *Hadron '87* (Ref. 11).
[19]O. W. Greenberg and H. J. Lipkin, Nucl. Phys. **A370**, 349 (1981). (See this article for references on color van der Waals forces.)
[20]J. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982).
[21]J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).
[22]D. J. Herdon *et al.*, Phys. Rev. D **11**, 3165 (1975). See also B. H. Bransden and R. G. Moorhouse, *The Pion-Nucleon System* (Princeton University Press, Princeton, NJ, 1973). These authors suggest the following earlier references: R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. **123**, 333 (1961); D. Z. Freedman *et al.*, Nuovo Cimento **43A**, 218 (1966); M. C. Olsson and G. B. Yodh, Phys. Rev. **145**, 1309 (1966); J. M. Namylslowski *et al.*, *ibid.* **157**, 1323 (1967); D. Morgan, *ibid.* **166**, 1731 (1968).
[23]R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).
[24]R. Aaron *et al.*, Phys. Rev. **174**, 2022 (1968).
[25]R. Aaron, in *Modern Three-Hadron Physics*, edited by A. W. Thomas (Springer, Heidelberg, 1976), Chap. 5.
[26]S. M. Flatté, Phys. Lett. **63B**, 224 (1976).
[27]Our Clebsch-Gordan and Racah coefficients are defined in M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).
[28]R. L. Schult and H. W. Wyld, Phys. Rev. D **16**, 62 (1977).
[29]R. Aaron *et al.*, Ann. Phys. (N.Y.) **100**, 425 (1976).
[30]S. Blessing *et al.*, in *Glueballs, Hybrids, and Exotic Hadrons* (Ref. 6), p. 363.
[31]The Deck mechanism was first discussed by R. T. Deck, Phys. Rev. Lett. **13**, 169 (1964).
[32]S. Brodsky and P. LePage, Phys. Rev. D **24**, 1808 (1981).
[33]B. Li and K. Liu, Phys. Rev. Lett. **51**, 1510 (1983).