Unitarity constraints on *CP* nonconservation in Higgs-boson exchange

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Bounds are derived for the dimensionless factors that enter in the violation of *CP* invariance in Higgs-boson exchange. In particular, for the class of theories with two scalar $SU(2)\times U(1)$ doublets [plus any other scalars whose expectation values do not break $SU(2)\times U(1)$], the magnitude of the parameter Im Z_2, which appears in the dominant neutral-Higgs-boson-exchange contribution to the neutron electric dipole moment, is bounded by $\frac{1}{2}(r^2+r^4)^{1/2}$, where r is a ratio of the magnitudes of scalar vacuum expectation values. It is shown that this bound can actually be reached in realistic models.

I. INTRODUCTION

It was recently pointed out[1] that there is a mechanism in all but the simplest versions of the standard model that can produce an observable value for the neutron electric dipole moment even if the Higgs particles are quite heavy. This is because integrating out the heavy particles (such as the top quark and neutral or charged[2] Higgs boson) in such theories can produce a dimension-six *CP*- and *P*-nonconserving term in the effective Lagrangian,

$$\Delta\mathcal{L}=-Cf_{abc}G_{a\mu}{}^{\rho}G_{b\rho\nu}G_{c\sigma\eta}\epsilon^{\mu\nu\sigma\eta}\ . \tag{1}$$

[Here $G_{a\mu\nu}$ is the gluon field-strength tensor, $\epsilon^{\mu\rho\sigma\eta}$ is the usual totally antisymmetric tensor with $\epsilon^{0123}=+1$, and f_{abc} is the totally antisymmetric Gell-Mann SU(3) tensor.] Unlike other *CP*- and *P*-nonconserving operators that arise in this way, the coefficient C is suppressed by only two inverse factors of heavy-particle masses, and not at all by factors of light-quark masses or small mixing angles. At first it was thought[3] that the effects of this operator at low energy are enhanced by QCD renormalization effects, and it was concluded on this basis that if *CP* and *P* were maximally violated in Higgs-boson exchange, then the contribution of (1) to the neutron electric dipole moment would be 4–5 orders of magnitude greater than present experimental bounds.[4] Similar remarks were made regarding the effects of possible new gauge bosons,[5] sparticles,[6] or singlet quarks.[7] Since then several independent calculations[8] have shown that apart from the running of the strong coupling-constant g_s in the factor g_s^3 in C, what had been thought to be an enhancement is actually a suppression. Questions have also been raised[9] regarding the validity of such one-loop calculations of the QCD correlation factor. It now seems clear that one cannot use present limits[4] on the neutron electric dipole moment to rule out a maximal *CP* nonconservation in scalar propagators for Higgs-boson masses in the range of several hundred GeV, but that such a mechanism acting through the operators (1) can produce a neutron electric dipole moment at a level that could show up in the next round of experiments. Although QCD correction factors have not yet been calculated for all competing operators, it seems that the largest contribution to the neutron electric diple moment from Higgs-boson exchange still arises from the operator (1), because this is the only operator that is only suppressed by two factors of heavy-particle masses.

Of course, the coefficient C in any model will be proportional to whatever imaginary parts of amplitudes produce the *CP* and *P* violation in the heavy-particle exchange. For instance, in Ref. 1 it was found that the contribution to C of the neutral-Higgs-boson-exchange diagram of Fig. 1 is proportional to an unknown dimensionless quantity ImZ_2. Therefore, in order to say what we mean by "maximal *CP* violation" and to get an idea of the likely value of the neutron electric dipole moment produced by these heavy-particle exchanges, we need to ask what is the best upper bound that can be set on quantities such as $|\mathrm{Im}Z_2|$.

We shall first discuss scalar exchange in a general context, and will then turn to the special case of theories with two scalar $SU(2)\times U(1)$ doublets, and with arbitrary numbers of scalar singlest and/or scalars with zero vacuum expectation values. In such theories, *CP* nonconservation can occur in the propagators only of the neutral Higgs bosons. Actually, the contribution to C from charged-Higgs-boson exchange is less suppressed[10] by QCD-correction factors than for neutral-Higgs-boson exchange, so the two-doublet models are not those that produce the largest neutron electric dipole moments. However, the two-doublet model is still interesting as an example of *CP* nonconservation. For instance, it has re-

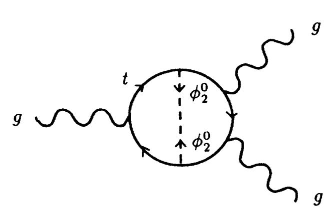

FIG. 1. Contribution of neutral-Higgs-boson exchange to the three-gluon *CP*-violating operator (1), involving ImA_2.

cently been pointed out that, through effects analogous to (1), neutral-Higgs-boson exchange can produce surprisingly large contributions to the electric dipole moment of the electron.[11] The results for the two-doublet model obtained here can be usefully applied to these effects. In any case, the methods described here can serve as a guide to the analysis of more general models.

II. SCALAR EXCHANGE: THE GENERAL CASE

We start with a general $SU(2)\times U(1)$ electroweak theory, involving an arbitrary number of scalar multiplets belonging to arbitrary $SU(2)\times U(1)$ representations. In order to prevent excessive rates of $K^0,\bar{K}^0$ or $D^0,\bar{D}^0$ oscillation in a natural way, we assume[12] that there is some discrete symmetry that only allows all charge $-\frac{1}{3}$ quarks to get their masses from interaction with just one scalar doublet, say ϕ_1, and that likewise only allows all charge $\frac{2}{3}$ quarks to get their masses from just one scalar doublet, say ϕ_2. The interactions of quarks with scalars is then

$$\mathcal{L}_Y = -\frac{1}{\lambda_1^*}(\bar{D}_R m_D V^\dagger U_L)\phi_1^{+*} - \frac{1}{\lambda_1^*}(\bar{D}_R m_D D_L)\phi_1^{0*} - \frac{1}{\lambda_2}(\bar{U}_R m_U U_L)\phi_2^0 + \frac{1}{\lambda_2}(\bar{U}_R m_U V D_L)\phi_2^+ + \text{c.c.} , \tag{2}$$

where U and D are the quark triplets (u,c,t) and (d,s,b), respectively; m_U and m_D are the corresponding real diagonal quark mass matrices; V is the unitary Kobayashi-Maskawa matrix; and λ_i are the scalar vacuum expectation values

$$\lambda_i \equiv \langle \phi_i^0 \rangle_{\text{vac}} . \tag{3}$$

At this point, we leave it an open question whether ϕ_1 and ϕ_2 are proportional or independent scalar doublets.

With this interaction between quarks and scalars, CP violation will show up in scalar exchange between quarks through imaginary terms in one or more of the quantities

$$\frac{1}{\lambda_1^*\lambda_2}\langle \phi_2^+ \phi_1^{+*} \rangle_q \equiv A(q^2) , \tag{4}$$

$$\frac{1}{\lambda_1^*\lambda_2}\langle \phi_2^0 \phi_1^{0*} \rangle_q \equiv A_0(q^2) , \tag{5}$$

$$\frac{1}{\lambda_1\lambda_2}\langle \phi_2^0 \phi_1^{0} \rangle_q \equiv \tilde{A}_0(q^2) , \tag{6}$$

$$\frac{1}{(\lambda_1)^2}\langle \phi_1^0 \phi_1^{0} \rangle_q \equiv A_1(q^2) , \tag{7}$$

$$\frac{1}{(\lambda_2)^2}\langle \phi_2^0 \phi_2^{0} \rangle_q \equiv A_2(q^2) , \tag{8}$$

where $\langle \chi\eta \rangle_q$ is for any pair of scalar fields χ,η an abbreviation for the momentum-dependent quantity

$$\langle \chi\eta \rangle_q \equiv \int d^4x \langle T[\chi(x)\eta(0)] \rangle_{\text{vac}} e^{-iq\cdot x} \tag{9}$$

evaluated in the zeroth order of perturbation theory. For instance, the Feynman diagram of Fig. 2 gives a contribution to the coefficient of the quark electric dipole moment operator involving $\text{Im}\,A(q^2)$, while the Feynman diagrams[13] of Figs. 3–6 yield contributions to the coefficient of the four-gluon operator $GGG\tilde{G}$ involving $\text{Im}\,A_0(q^2)$, $\text{Im}\,\tilde{A}_0(q^2)$, $\text{Im}\,A_1(q^2)$, and $\text{Im}\,A_2(a^2)$, respectively, and, of course, $\text{Im}\,A_2(q^2)$ also enters into the coefficient of the three-gluon operator (1) through the Feynman diagram of Fig. 1.

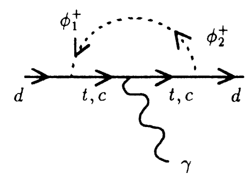

FIG. 2. Contribution of neutral-Higgs-boson exchange to the quark electric dipole moment operator, involving $\text{Im}\,A$.

We see immediately that if ϕ_1 and ϕ_2 are proportional, then the amplitudes A and A_0 are real, while the other amplitudes are all equal:

$$\text{Im}\,A = \text{Im}\,A_0 = 0 , \tag{10}$$

$$\tilde{A}_0 = A_1 = A_2 . \tag{11}$$

In this case, charged-Higgs-boson exchange automatically conserves CP, but CP and P nonconservation can still arise in neutral-Higgs-boson exchange through imaginary terms in the amplitude $\tilde{A}_0$. On the other hand, if ϕ_1 and ϕ_2 are independent, then in general CP and P nonconservation can arise in charged- and/or neutral-scalar exchange, through imaginary terms in any or all of the five amplitudes (4)–(8).

The tree-approximation amplitudes (4)–(8) may in general be expressed as sums over mass eigenstates:

$$A(q^2) = \sum_n \frac{\sqrt{2}G_F Z_n}{q^2 + m_{H'n}^2} , \tag{12}$$

$$A_0(q^2) = \sum_n \frac{\sqrt{2}G_F Z_{0n}}{q^2 + m_{Hn}^2} , \tag{13}$$

$$\tilde{A}_0(q^2) = \sum_n \frac{\sqrt{2}G_F \tilde{Z}_{0n}}{q^2 + m_{Hn}^2} , \tag{14}$$

$$A_1(q^2) = \sum_n \frac{\sqrt{2}G_F \tilde{Z}_{1n}}{q^2 + m_{Hn}^2} , \tag{15}$$

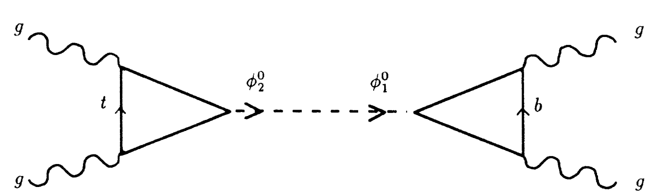

FIG. 3. Contribution of neutral-Higgs-boson exchange to the CP-violating four-gluon operator $GGG\tilde{G}$, $\text{Im}\,A_0$.

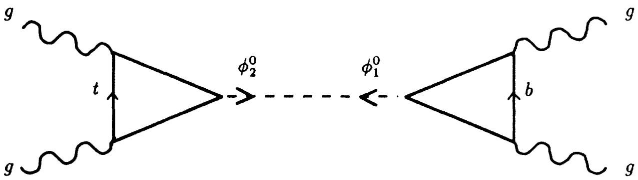

FIG. 4. Contribution to $GGG\tilde{G}$ involving $\mathrm{Im}\,\tilde{A}_0$.

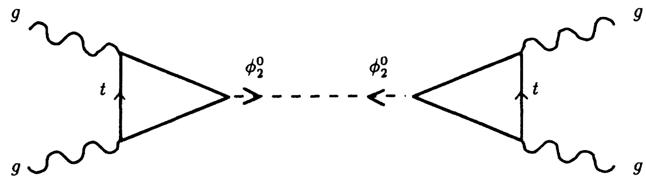

FIG. 6. Contribution to $GGG\tilde{G}$ involving $\mathrm{Im}\,A_2$.

$$A_2(q^2)=\sum_n \frac{\sqrt{2}G_F\tilde{Z}_{2n}}{q^2+m_{Hn}^2} , \tag{16}$$

where $m_{H'n}$ and m_{Hn} are, respectively, the nth charged- and neutral-scalar mass eigenvalues, and the factor $\sqrt{2}G_F$ has been inserted to make the coefficients Z_n, etc., dimensionless. The contribution of scalar exchange to any observable will then be given by a corresponding sum. For instance, the exchange of a neutral scalar makes a contribution to the coefficient C in Eq. (1) of the form[1]

$$C=(4\pi)^{-1}\sqrt{2}G_F\zeta\sum_n h(m_t/m_{Hn})\mathrm{Im}Z_{2n} , \tag{17}$$

where h is the function[2]

$$h(\sigma)=\frac{\sigma^4}{2}\int_0^1 dx\int_0^1 du\frac{u^3x^3(1-x)}{[\sigma^2x(1-ux)+(1-u)(1-x)]^2} \tag{18}$$

and $\zeta\simeq 10^{-4}$ is a QCD factor.[3,8] (For charged-Higgs-boson exchange h is somewhat larger[2] and ζ is an order of magnitude larger.[10])

The Z coefficients in (12)–(16) satisfy important sum rules. Invariance under SU(2)×U(1), together with whatever discrete symmetry enforces Eq. (2), tell us that the kinematic part of the scalar Lagrangian is a linear combination of $\partial_\mu\phi_1^\dagger\partial^\mu\phi_1$ and $\partial_\mu\phi_2^\dagger\partial^\mu\phi_2$, plus terms depending only on any other scalars. It follows that the equal-time commutators $[\phi_2^0,\dot\phi_1^0]$, $[\phi_1^0,\dot\phi_1^0]$, and $[\phi_2^0,\dot\phi_2^0]$ all vanish, so that

$$\sum_n \tilde{Z}_{0n}=\sum_n Z_{1n}=\sum_n Z_{2n}=0 . \tag{19}$$

Also, if ϕ_1 and ϕ_2 are distinct then the equal-time commutators $[\phi_2^+,\dot\phi_1^{+*}]$ and $[\phi_2^0,\dot\phi_1^{0*}]$ also vanish, so the same applies to Z_n and Z_{0n}:

$$\sum_n Z_n=\sum_n Z_{0n}=0 . \tag{20}$$

Equation (20) is not true if ϕ_1 and ϕ_2 are proportional,

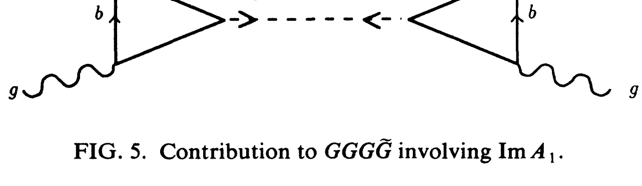

FIG. 5. Contribution to $GGG\tilde{G}$ involving $\mathrm{Im}\,A_1$.

but in this case Eq. (10) tells us that there is no *CP* violation in A or A_0 anyway.

From these sum rules, we see that if all charged or neutral scalars have equal mass, then there is no *CP* violation in charged- or neutral-scalar exchange. There is no reason to expect such a degeneracy. Instead, we will later consider the simplifying assumption used in earlier work,[1,2] that one mass eigenstate (presumably the lightest) dominates the effects of Higgs-boson exchange. In this approximation, we can drop the sums and the indices n, with $m_{H'}$ and m_H then understood to be the masses of the dominant-charged and neutral-mass eigenstates, and Z, Z_0, $\tilde{Z}_0$, Z_1, and Z_2 to be the coefficients of the dominant terms in (12)–(16).

III. TWO-DOUBLET THEORIES

If the electroweak SU(2)×U(1) symmetry were broken by the vacuum expectation value of a single SU(2)×U(1) doublet, then in unitarity gauge there would be just a single real scalar coupled to the quarks, and *CP* would be automatically conserved in Higgs-boson exchange. We, therefore, turn immediately to the next-simplest case, in which SU(2)×U(1) is broken by the vacuum expectation values of just two independent scalar doublets. In addition to these two doublets, the theory may involve any other scalars whose vacuum expectation values do not participate in breaking SU(2)×U(1), either because they are gauge singlets, or because for one reason or another they have vanishing vacuum expectation values.

We must distinguish between two subcases, in which the doublets ϕ_1 and ϕ_2 that give mass to the charge $-\frac{1}{3}$ and charge $\frac{2}{3}$ quarks are or are not independent. We shall concentrate on the case where ϕ_1 and ϕ_2 are independent, and then briefly describe the results found for the other case.

If ϕ_1 and ϕ_2 are independent, then these are also the only two scalar doublets whose vacuum expectation values break SU(2)×U(1). We will normalize then so that the kinetic Lagrangian reads

$$\mathcal{L}_{\mathrm{kin}}=-\partial_\mu\phi_1^\dagger\partial^\mu\phi_1-\partial_\mu\phi_2^\dagger\partial^\mu\phi_2+\cdots , \tag{21}$$

where the ellipsis refers to fields other than ϕ_1 and ϕ_2. [Recall that whatever discrete symmetry enforces Eq. (2) will also rule out any off-diagonal terms $\partial_\mu\phi_2^\dagger\partial^\mu\phi_1$ or $\partial_\mu\phi_1^\dagger\partial^\mu\phi_2$, or any kinematic terms connecting ϕ_1 or ϕ_2 to other scalar doublets. These kinematic terms are all "hard," so the form of (21) would be unaffected if the discrete symmetry are allowed to be softly broken, by terms of dimensionality ≤ 3.] For fields normalized in this way, the Fermi coupling constant is given by

$$\sqrt{2}G_F = \frac{1}{|\lambda_1|^2+|\lambda_2|^2} . \tag{22}$$

The unitarity gauge condition here reads[14]

$$\mathrm{Im} \sum_{n=1}^{2} \left[\begin{pmatrix} 0 \\ \lambda_n \end{pmatrix}^{\dagger} t_\alpha \begin{pmatrix} \phi_n^+ \\ \phi_n^0 \end{pmatrix} \right] = 0 \tag{23}$$

for all SU(2)$\times$U(1) gauge generators t_α. In detail, this is

$$\lambda_1^*\phi_1^+ + \lambda_2^*\phi_2^+ = 0 , \tag{24}$$

$$\mathrm{Im}(\lambda_1^*\phi_1^0 + \lambda_2^*\phi_2^0) = 0 . \tag{25}$$

We see immediately that in this sort of theory the charged-Higgs-boson amplitude (4) is *real,*

$$A(q^2) = \frac{-1}{|\lambda_1|^2} \langle \phi_2^+ \phi_2^{+*} \rangle_q , \tag{26}$$

and so there is no *CP* violation in the exchange of a single charged Higgs boson between quarks. On the other hand, Eq. (25) provides us with just one constraint on two complex neutral fields, so it leaves us with three degrees of freedom, and plenty of opportunity for *CP* violation. This condition allows us to write the complex neutral scalars in terms of three real scalars Φ_n:

$$\phi_1^0 = \frac{\lambda_2}{\sqrt{2}|\lambda_1|} \left[\Phi_1 - \frac{i|\lambda_2|}{\sqrt{|\lambda_1|^2+|\lambda_2|^2}} \Phi_3 \right] , \tag{27}$$

$$\phi_2^0 = \frac{\lambda_2}{\sqrt{2}|\lambda_2|} \left[\Phi_2 + \frac{i|\lambda_1|}{\sqrt{|\lambda_1|^2+|\lambda_2|^2}} \Phi_3 \right] . \tag{28}$$

These new fields are canonically normalized, in the sense that the kinematic Lagrangian (21) in unitarity gauge is just

$$\mathcal{L}_{\mathrm{kin}} = -\tfrac{1}{2} \sum_{n=1}^{3} (\partial_\mu \Phi_n)(\partial^\mu \Phi_n) + \cdots \tag{29}$$

as usual for real scalars. Inserting (27) and (28) in Eqs. (5)–(8) gives the *CP*-nonconserving parts of the neutral-Higgs-boson-exchange amplitudes as

$$\mathrm{Im}\, A_0(q^2) = \frac{|\lambda_1|\langle \Phi_1\Phi_3\rangle_q + |\lambda_2|\langle \Phi_2\Phi_3\rangle_q}{2|\lambda_1\lambda_2|\sqrt{|\lambda_1|^2+|\lambda_2|^2}} , \tag{30}$$

$$\mathrm{Im}\, \tilde{A}_0(q^2) = \frac{|\lambda_1|\langle \Phi_1\Phi_3\rangle_q - |\lambda_2|\langle \Phi_2\Phi_3\rangle_q}{2|\lambda_1\lambda_2|\sqrt{|\lambda_1|^2+|\lambda_2|^2}} , \tag{31}$$

$$\mathrm{Im}\, A_1(q^2) = \frac{-|\lambda_2|\langle \Phi_1\Phi_3\rangle_q}{|\lambda_1|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}} , \tag{32}$$

$$\mathrm{Im}\, A_2(q^2) = \frac{|\lambda_1|\langle \Phi_2\Phi_3\rangle_q}{|\lambda_2|^2\sqrt{|\lambda_1|^2+|\lambda_2|^2}} . \tag{33}$$

(We are using the fact that $\langle \Phi_a\Phi_b\rangle_q$ is real and symmetric in a and b, which follows from the reality of the Φ, Lorentz invariance, and translation invariance. We ignore the presence of possible cuts in the q^2 plane, along which these propagators would be complex, because we will eventually be evaluating the propagators in the tree approximation.) Evidently the four *CP*-violating amplitudes are subject to two linear relations, which may be written as

$$\mathrm{Im}\, A_1(q^2) = -\left|\frac{\lambda_2}{\lambda_1}\right|^2 [\mathrm{Im}\, A_0(q^2) + \mathrm{Im}\, \tilde{A}_0(q^2)] , \tag{34}$$

$$\mathrm{Im}\, A_2(q^2) = \left|\frac{\lambda_1}{\lambda_2}\right|^2 [\mathrm{Im}\, A_0(q^2) - \mathrm{Im}\, \tilde{A}_0(q^2)] . \tag{35}$$

We will now apply the approximation discussed in the previous section of taking the effect of scalar exchange to be dominated by a single neutral-scalar particle of mass m_H. In (30)–(33), we then replace

$$\langle \Phi_a \Phi_b \rangle_q \simeq \frac{u_a u_b}{q^2+m_H^2} \tag{36}$$

with u_a real. This gives the A amplitudes in the form assumed earlier:

$$A_0(q^2) \simeq \frac{\sqrt{2}G_F Z_0}{q^2+m_H^2} , \tag{37}$$

$$\tilde{A}_0(q^2) \simeq \frac{\sqrt{2}G_F \tilde{Z}_0}{q^2+m_H^2} , \tag{38}$$

$$A_1(q^2) \simeq \frac{\sqrt{2}G_F Z_1}{q^2+m_H^2} , \tag{39}$$

$$A_2(q^2) \simeq \frac{\sqrt{2}G_F Z_2}{q^2+m_H^2} . \tag{40}$$

Using (36) in (30)–(33), and comparing with (13)–(16) and (22), we see that the *CP*-violating amplitudes in (13)–(16) are

$$\mathrm{Im} Z_0 = \frac{1}{2}\left[1+\left|\frac{\lambda_1}{\lambda_2}\right|^2\right]^{1/2} u_1 u_3 + \frac{1}{2}\left[1+\left|\frac{\lambda_2}{\lambda_1}\right|^2\right]^{1/2} u_2 u_3 . \tag{41}$$

$$\mathrm{Im} \tilde{Z}_0 = \frac{1}{2}\left[1+\left|\frac{\lambda_1}{\lambda_2}\right|^2\right]^{1/2} u_1 u_3 - \frac{1}{2}\left[1+\left|\frac{\lambda_2}{\lambda_1}\right|^2\right]^{1/2} u_2 u_3 , \tag{42}$$

$$\mathrm{Im} Z_1 = -\left[\left|\frac{\lambda_2}{\lambda_1}\right|^2 + \left|\frac{\lambda_2}{\lambda_1}\right|^4\right]^{1/2} u_1 u_3 , \tag{43}$$

$$\mathrm{Im} Z_2 = \left[\left|\frac{\lambda_1}{\lambda_2}\right|^2 + \left|\frac{\lambda_1}{\lambda_2}\right|^4\right]^{1/2} u_1 u_3 . \tag{44}$$

Also, because the fields are chosen to give the kinematic Lagrangian the form (29), the u's are subject to the inequality

$$(u_1)^2+(u_2)^2+(u_3)^2\le 1 \ . \qquad (45)$$

(This is an inequality, because in general other neutral-mass eigenstates can contribute to the Φ commutators.) Using this, it is elementary to show that the quantities (41)–(44) are subject to the upper bounds

$$|\mathrm{Im}Z_0|\le\frac{1}{4}\left[\left|\frac{\lambda_1}{\lambda_2}\right|+\left|\frac{\lambda_2}{\lambda_1}\right|\right] , \qquad (46)$$

$$|\mathrm{Im}\tilde{Z}_0|\le\frac{1}{4}\left[\left|\frac{\lambda_1}{\lambda_2}\right|+\left|\frac{\lambda_2}{\lambda_1}\right|\right] , \qquad (47)$$

$$|\mathrm{Im}Z_1|\le\frac{1}{2}\left[\left|\frac{\lambda_1}{\lambda_2}\right|^2+\left|\frac{\lambda_2}{\lambda_1}\right|^4\right]^{1/2} , \qquad (48)$$

$$|\mathrm{Im}Z_2|\le\frac{1}{2}\left[\left|\frac{\lambda_1}{\lambda_2}\right|^2+\left|\frac{\lambda_1}{\lambda_2}\right|^4\right]^{1/2} . \qquad (49)$$

Note that if $|\lambda_1|\gg|\lambda_2|$ or $|\lambda_2|\gg|\lambda_1|$, then, respectively, $|\mathrm{Im}Z_1|$ or $|\mathrm{Im}Z_2|$ must be small. This no surprise, because in these limits $\mathrm{SU}(2)\times\mathrm{U}(1)$ is effectively broken by only one scalar doublet, respectively, ϕ_1 or ϕ_2, and the only neutral field left from the Higgs mechanism in this doublet is a single real scalar.

Now let us briefly consider the other case mentioned earlier, where the doublets ϕ_1 and ϕ_2 in the Yukawa interaction are proportional, and there is another doublet ϕ_3 that does not couple to quarks but whose vacuum expectation value also participates in breaking SU(2)–U(1). As already remarked in Sec. II, with ϕ_1 and ϕ_2 proportional, there is only one independent *CP*-violating amplitude, $\tilde{A}_0=A_1=A_2$. We can use the unitarity gauge condition here to express the two complex neutral scalars ϕ_2^0 and ϕ_3^0 in terms of three real scalars, just as in Eqs. (27) and (28). Following the same procedure as before, and assuming the dominance of a single-mass eigenstate, we now find

$$\tilde{A}_0(q^2)=A_1(q^2)=A_2(q^2)\simeq\frac{Z_2\sqrt{2}G_F}{q^2+m_H^2} , \qquad (50)$$

$$\mathrm{Im}Z_2=\left[\left|\frac{\lambda_3}{\lambda_2}\right|^2+\left|\frac{\lambda_3}{\lambda_2}\right|^4\right]^{1/2}u_2u_3 \ . \qquad (51)$$

From (45), we then find

$$|\mathrm{Im}Z_2|\le\tfrac{1}{2}(|\lambda_3/\lambda_2|^2+|\lambda_3/\lambda_2|^4)^{1/2} \ . \qquad (52)$$

Here the *CP* violation in neutral-scalar exchange is small if $|\lambda_3|\ll|\lambda_2|$, again because in this limit the model approaches the minimal standard model.

IV. REACHING THE BOUND

We will now show that the bounds (46)–(49) are in general the best that can be derived, by showing that there are physically allowed models that realize these bounds. Specifically, we will consider a model in which the *only* scalar fields are the two doublets, ϕ_1 and ϕ_2. Any such model with an exact discrete symmetry that distinguishes ϕ_1 and ϕ_2 and enforces Eq. (2) (such as $\phi_1\to-\phi_1$, $D_R\to-D_R$, $\phi_2\to+\phi_2$, $U_R\to U_R$, $Q_L\to Q_L$) will rule out *CP* violation in renormalizable Lagrangians, but *CP* nonconservation can be introduced by letting this discrete symmetry be broken by soft terms in the Lagrangian.[15] We, therefore, take the Lagrangian for the ϕ fields in the form

$$\mathcal{L}_\phi=-\partial_\mu\phi_1^\dagger\partial^\mu\phi_1-\partial_\mu\phi_2^\dagger\partial^\mu\phi_2-V(\phi_1,\phi_2) , \qquad (53)$$

$$\begin{aligned}V=&\ m_1^2\phi_1^\dagger\phi_1+m_2^2\phi_2^\dagger\phi_2+\eta\phi_1^\dagger\phi_2+\eta^*\phi_2^\dagger\phi_1\\&+\tfrac{1}{2}g_1(\phi_1^\dagger\phi_1)^2+\tfrac{1}{2}g_2(\phi_2^\dagger\phi_2)^2+g(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\\&+g'|\phi_1^\dagger\phi_2|^2+\tfrac{1}{2}h(\phi_1^\dagger\phi_2)^2+\tfrac{1}{2}h^*(\phi_2^\dagger\phi_1)^2 ,\end{aligned} \qquad (54)$$

where m_1^2, m_2^2, g_1, g_2, g, and g' are real, but η and h may be complex. Aside from the soft $\phi_1^\dagger\phi_2$ and $\phi_2^\dagger\phi_1$ terms, this respects the symmetry $\phi_1\to-\phi_1$, $\phi_2\to\phi_2$, which can be used to enforce the assumed form (2) of the hard Yukawa interactions. This Lagrangian conserves *CP* if and only if h/η^2 is real; we will allow it to be complex.

With arbitrary values of the hard couplings g_1,g_2,g,g', and h, we can always adjust the soft couplings m_1^2,m_2^2, and η to make the vacuum expectation values λ_1 and λ_2 anything we like. To see this, note that (54) may be rewritten

$$\begin{aligned}V=&\ \tfrac{1}{2}g_1(\phi_1^\dagger\phi_1-|\lambda_1|^2)^2+\tfrac{1}{2}g_2(\phi_2^\dagger\phi_2-|\lambda_2|^2)^2\\&+g(\phi_1^\dagger\phi_1-|\lambda_1|^2)(\phi_2^\dagger\phi_2-|\lambda_2|^2)\\&+g'|\phi_1^\dagger\phi_2-\lambda_1^*\lambda_2|^2+\mathrm{Re}[h(\phi_1^\dagger\phi_2-\lambda_1^*\lambda_2)^2]\\&+\xi\left[\frac{\phi_1}{\lambda_1}-\frac{\phi_2}{\lambda_2}\right]^\dagger\left[\frac{\phi_1}{\lambda_1}-\frac{\phi_2}{\lambda_2}\right] ,\end{aligned} \qquad (55)$$

where ξ is an arbitrary real quantity, and λ_1 and λ_2 satisfy the conditions

$$m_1^2=-g_1|\lambda_1|^2-g|\lambda_2|^2+\xi/|\lambda_1|^2 , \qquad (56)$$

$$m_2^2=-g_2|\lambda_2|^2-g|\lambda_1|^2+\xi/|\lambda_2|^2 , \qquad (57)$$

$$\eta=-g'\lambda_2^*\lambda_1-h\lambda_1^*\lambda_2-\xi/\lambda_1^*\lambda_2 \ . \qquad (58)$$

It is obvious from the form of (55) that V is stationary when $\phi_1^0=\lambda_1$, $\phi_2^0=\lambda_2$, and $\phi_1^+=\phi_2^+=0$. Instead of taking the independent parameters of the theory to be all the couplings in (54), we will work with Eq. (55), and take the independent parameters to be the hard couplings g_1, g_2, g, g', and h, plus the soft coupling ξ and the vacuum expectation values λ_1 and λ_2. (That is, we replace the four real parameters m_1^2, m_2^2, $\mathrm{Re}\eta$, $\mathrm{Im}\eta$ with the set ξ, $|\lambda_1|^2$, $|\lambda_2|^2$, and $\mathrm{Arg}\lambda_1^*\lambda_2$.) The mark of *CP* violation (intrinsic *or* spontaneous) will now be an imaginary term in $h\lambda_1^{*2}\lambda_2^2$.

We will now work in unitarity gauge, using Eqs. (27) and (28) to express the neutral components of ϕ_1 and ϕ_2 in terms of the three real canonically normalized fields Φ_a. (The charged components of ϕ_1 and ϕ_2 are set equal to zero here, since our concern is with the propagator of the neutral scalars.) Shifting the Φ_a by their expectation values and expanding (55) to second order in the shifted fields, we find the Φ mass matrix:

$$M_{11}^2 = g_1|\lambda_1|^2 + g'|\lambda_2|^2 + \frac{\mathrm{Re}(h\lambda_1^{*2}\lambda_2^2)+\xi}{|\lambda_1|^2} , \quad (59)$$

$$M_{22}^2 = g_2|\lambda_2|^2 + g'|\lambda_1|^2 + \frac{\mathrm{Re}(h\lambda_1^{*2}\lambda_2^2)+\xi}{|\lambda_1|^2} , \quad (60)$$

$$M_{33}^2 = (|\lambda_1|^2+|\lambda_2|^2)\left[g' + \frac{\xi - \mathrm{Re}(h\lambda_1^{*2}\lambda_2^2}{|\lambda_1\lambda_2|^2}\right] , \quad (61)$$

$$M_{12}^2 = |\lambda_1\lambda_2|(g+g') + \frac{\mathrm{Re}(h\lambda_1^{*2}\lambda_2^2)-\xi}{|\lambda_1\lambda_2|} , \quad (62)$$

$$M_{13}^2 = -\frac{\sqrt{|\lambda_1|^2+|\lambda_2|^2}}{|\lambda_1^2\lambda_2|}\mathrm{Im}(h\lambda_1^{*2}\lambda_2^2) , \quad (63)$$

$$M_{23}^2 = -\frac{\sqrt{|\lambda_1|^2+|\lambda_2|^2}}{|\lambda_1\lambda_2^2|}\mathrm{Im}(h\lambda_1^{*2}\lambda_2^2) . \quad (64)$$

In order to show that the bound (49) on $|\mathrm{Im}Z_2|$ can be realized for any vacuum expectation values λ_1,λ_2, we need to show that we can adjust the parameters in the Lagrangian (with fixed λ_1 and λ_2) to make $|u_2u_3| = \frac{1}{2}$, where u_a is the normalized eigenvector of M^2 with the smallest eigenvalue. One convenient choice of parameters that obviously satisfies all positivity conditions is

$$g_1 \to +\infty, \quad g_2 = g', \quad \xi = 0, \quad \mathrm{Re}(h\lambda_1^{*2}\lambda_2^2) = 0 . \quad (65)$$

[This is not unique; we only need to impose one linear relation among g_2, g', ξ, and $\mathrm{Re}(h\lambda_1^{*2}\lambda_2^2)$.] Taking $g_1 \to \infty$ makes the largest eigenvalue of M^2 go to infinity and gives it an eigenvector in the one-direction. The remaining eigenvectors and eigenvalues can be found by diagonalizing the remaining 2×2 matrix, which here has elements

$$M_{22}^2 = M_{33}^2 = (|\lambda_1|^2+|\lambda_2|^2)g_2 , \quad (66)$$

$$M_{23}^2 = -\frac{\sqrt{|\lambda_1|^2+|\lambda_2|^2}}{|\lambda_1\lambda_2^2|}\mathrm{Im}(h\lambda_1^{*2}\lambda_2^2) . \quad (67)$$

The normalized eigenvectors of any such 2×2 matrix with equal diagonal elements have components with $u_2 = \pm u_3 = 1/\sqrt{2}$, thus reaching the bound (49). Note that this is accomplished without imposing any sort of degeneracy on the scalar masses; the eigenvalues of M^2 here are infinity and $M_{22}^2 \pm M_{23}^2$, which can be as different as we like.

It is easy to show in the same way that the bound on $|\mathrm{Im}Z_1|$ can also be reached; simply reverse the roles of the 1 and 2 axes in the above argument. The bounds on $|\mathrm{Im}Z_0|$ and $|\mathrm{Im}\tilde{Z}_0|$ can be examined in the same way after first changing the basis for the Φ fields, but we shall not bother with this here.

V. CONCLUSIONS

We have found a bound on the CP-nonconservation parameter $\mathrm{Im}Z_2$ of Ref. 1 in models where $SU(2)\times U(1)$ is broken by the expectation values of two scalar doublets, and we have shown that this bound can actually be reached for physically reasonable values of the parameters of the models. Of course, there is no reason to expect that these parameters will be fine tuned so as to maximize $|\mathrm{Im}Z_2|$. The point here is that, unless new physical constraints on the models are discovered,[16] it would require an unnatural fine tuning to make the value of $|\mathrm{Im}Z_2|$ very much less than its upper bound (49). Therefore, this bound provides a plausible rough estimate of the value of $|\mathrm{Im}Z_2|$ to be expected in these nonminimal models.

The bound (49) is of order unity if the two scalar vacuum expectation values λ_1,λ_2 are of the same order of magnitude. On the other hand, it is possible that $|\lambda_1| \ll |\lambda_2|$, in which case (49) can be considerably less than unity. Indeed, the fact that the charge $+\frac{2}{3}$ quarks of the second and third generations are both much heavier than their charge $-\frac{1}{3}$ counterparts suggests that this may actually be the case.

Of course, if $SU(2)\times U(1)$ were broken by the vacuum expectation values of three or more doublets, then the bound (49) would not apply. Still, in the absence of new physical constraints, the most plausible value for $|\mathrm{Im}Z_2|$ would be of order unity.

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[15]The possibility of *spontaneous CP* violation in two-doublet models with a softly broken discrete symmetry was pointed out by G. C. Branco and M. N. Rebelo, Phys. Lett. **160B**, 117 (1985); J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987). (I thank H. Haber and P. Roy for these two references.) Here we are allowing the *CP* violation to be either intrinsic or spontaneous.

[16]For instance, a "custodial" SO(4) symmetry, under which $\mathrm{Im}\phi_n^+$, $\mathrm{Re}\phi_n^+$, $\mathrm{Im}\phi_n^0$, and $\mathrm{Re}\phi_n^0$ rotate as a four-vector, would automatically rule out *CP* violation in the scalar potential for arbitrary numbers of scalar doublets. Such a symmetry is not respected by quark-scalar interactions, so it seems unnatural to impose it in the scalar potential, just as for *CP* itself.