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Extracting V_{bu}/V_{bc} from semileptonic B decays

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Recent CLEO and ARGUS results for charmless semileptonic decays of B mesons are compared to several models for exclusive channels under the assumption that the π, ρ, D, and D^* saturate the region of high electron momentum of interest in determining V_{bu}/V_{bc}. The effect of current-algebra-inspired $\pi\pi$ backgrounds and threshold constraints in the ρ channel, as well as the error made by the narrow-width approximation, are examined with a view to estimating the model dependence inherent in extracting V_{bu}/V_{bc} from the data. Total semileptonic rates to π and ρ differ widely, model to model, while rates to D and D^* final states have much less model dependence. Model dependence is somewhat smaller when only high lepton energies are concerned. V_{bc} calculated from exclusive semileptonic decay to D and D^* final states vary about 25% and 10%, respectively, model to model. The model dependence of V_{bu}/V_{bc} is also considerable. The narrow-width approximation in the ρ channel can be a 20% effect in the determination of this ratio because of the sensitivity of the high-energy region to the lowest mass of the final hadronic system. If current-algebra-inspired backgrounds are present, V_{bu}/V_{bc} may be considerably smaller than suggested by many models. If the D^{**} component of b to c is as large as recent CLEO data suggest, V_{bu}/V_{bc} may be 16% larger than expected by models that ignore this contribution. Electron spectra for various models are presented, illustrating the high variation of shape features in the end-point region.

I. INTRODUCTION

The recent discovery by CLEO[1] and ARGUS[2] of noncharm final states in semileptonic B decay stimulated a renewed surge of theoretical interest in semileptonic decays of B mesons, particularly in connection with a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{bu}, one of the fundamental parameters of the standard model. The experimental data include inclusive electron spectra above the end point for charmed decays and branching ratios to selected exclusive states. To extract this parameter from the experimental data one needs theoretical input in the form of weak matrix elements between the initial B and the final hadron states, which may be exclusive single-meson or -resonance states, many-particle backgrounds, or an inclusive mixture of all of these.

Since the processes of heavy-meson decay are clearly nonperturbative, they cannot be reliably calculated from the QCD Lagrangian, and workers have been forced to rely on phenomenological models. The free-quark model[3] with and without QCD corrections[4] was for some time the favored tool for analyzing these decays. However, as has been emphasized many times, and in much detail recently by Isgur, Scora, Grinstein, and Wise,[5] the free-quark model is not able to predict the shape or normalization of the lepton spectrum in the interesting end-point region of low recoil masses where the spectrum is controlled by a set of discrete exclusive states: D and D^*, etc., for $b \rightarrow c$ transitions and π, ρ, etc., for $b \rightarrow u$ transitions.

There is a long history of interest in these exclusive channels. Rough estimates were already given in Ref. 3. Later work employed current algebra,[6] flavor-SU(4) symmetry, and various quark-model approaches to calculate form factors in transitions to pseudoscalar, vector, and higher-spin states.[7] In Ref. 8 effective chiral Lagangians were used, including a Wess-Zumino term, to predict weak matrix elements, especially for final hadronic states with two pseudoscalars. The chiral Lagrangian yields constraints on the weak matrix elements at low energy which are fully determined by the pion-decay constant. This approach is particularly suited for calculating K_{e4} decays, which occur almost at threshold, and less suitable for semileptonic decays, such as B and D, which are characterized by a large energy release. For these cases, in Ref. 9 (called CPK in the following) we constructed amplitudes that had the right low-energy behavior as given by the chiral Lagrangian but also had the correct resonant structure at higher energy.

The low-energy terms, which we call contact terms, arise from tree diagrams of an effective Lagrangian; however, because both B and D are heavier than the chiral-symmetry-breaking scale, there are significant and incalculable corrections from higher-derivative interactions, loop diagrams, and higer-order symmetry-breaking terms. We take the current-algebra symmetry-breaking terms, therefore, as educated guesses for nonresonant backgrounds to the resonant terms when the final hadron state consists of two pseudoscalars. As we demonstrated in CPK, these terms are more important the larger the width of the produced resonance. For $D \to K^* + e + \nu$ the contact terms contribute a background that increased the decay rate by about 10%, consistent with experiment E691 (Ref. 10) at Fermilab.

The purpose of this paper is to study the model dependence of estimates for V_{bu}/V_{bc}, in particular by using the current-algebra-inspired backgrounds as a calibration of "theoretical" uncertainty. In addition we will calculate the effect of resonance widths. To this end we will present rates for the channels $B \to \rho + e + \nu$ (including the π-π background as modeled by the current-algebra contact terms) for a variety of models for the vector and axial-vector form factors, in narrow-width approximation as well as with full two-body kinematics for ρ-meson final states. We calculate $B \to \pi + e + \nu$ also, and will later make the assumption, for the most part, that π and ρ saturate $b \to u + e + \nu$.

We also present rates for $B \to D$ and $B \to D^*$ because these are the rates most important for populating the $b \to c$ transitions. In particular, they are needed if the ratio V_{bu}/V_{bc} is to be extracted from the data. It is also interesting to inquire whether much of the model dependence cancels out if this ratio is extracted from quotients of data above and below the $b \to c$ production threshold. To this end we will present electron-spectra results, derived from a variety of form-factor models, in terms of the CKM matrix elements, and integrate them over the kinematical regions of interest. Above the $b \to c$ region ($E > 2.3$ GeV) and in the $b \to c$ region ($2.0 < E < 2.3$ GeV) as presented in the ARGUS[2] data and in the regions $2.2 < E < 2.4$ GeV and $2.4 < E < 2.6$ GeV as presented in the CLEO[1] data. We will then determine V_{bu}/V_{bc}. Again, we make a saturation assumption that D and D^* account for all of the $b \to c + e + \nu$ rate in the pertinent energy regions. The effect of additional states (e.g., D^{**}) will be roughly estimated in a worst-case calculation.

Many authors, as we have said, have worked on models of these transitions. We will present results using the form factors of Koerner and Schuler[11] (KS), Isgur, Scora, Grinstein, and Wise[12] (ISGW), Hagiwara, Martin, and Wade[13] (HMW), and a variant of our own (KP). In addition we will present the $B \to \pi$ modifications suggested by Isgur and Wise.[14] KS used form factors similar to Wirbel, Stech, and Bauer[15] (WSB), except that KS use dipole behavior for certain terms. For this reason we also will present some modified KS results, without dipoles, which is a model close to that of WSB. The HMW form factors are similar to those written down earlier by Suzuki.[16] We use the pole form of their form factors but rather than regarding the pole position as an adjustable parameter, we use the same vector-meson masses in the current channel as in the KS and KP models. Our aim here is to give an indication of model dependence, not an exhaustive survey of all form-factor models.[7]

II. FORM FACTORS

Kinematics, normalizations, and current-algebra terms are given in CPK. We use p_1 to denote the B momentum and p_i to denote the final hadron momentum, where i may be π, ρ, D, or D^*. Masses which control the q^2 dependence in vector and axial-vector channels are denoted by m_V^i or m_A^i. $q_{\max}$ is the maximum momentum transfer $m_B - m_i$.

When lepton masses may be neglected, the current matrix element for a B meson of momentum p_1 to decay semileptonically into a pseudoscalar meson of momentum p_i is given by

$$\langle p_i | j_\mu | B(p_1) \rangle = f_+(q^2)(p_1 + p_i)_\mu ,$$

where

$$p_1 = p_i + q$$

and q is the momentum transfer to the leptons. The form factor $f_+(q^2)$ is given as follows:

KS:

$$f_+^i(q^2) = I_i \frac{m_V^{i2}}{m_V^{i2} - q^2} ,$$

$$I_\pi = 0.33, \quad m_V^\pi = 5.33 \text{ GeV} ,$$

$$I_D = 0.7, \quad m_V^D = 6.34 \text{ GeV} ;$$

ISGW:

$$f_+^\pi(q^2) = 1.905 \exp[-0.1165(q_{\max}^2 - q^2)] ,$$

$$f_+^D(q^2) = 1.184 \exp[-0.002\,957(q_{\max}^2 - q^2)] ,$$

$$q_{\max} = m_B - m_i ;$$

IW (additional term):

$$f_+^\pi(q^2) = \frac{32(f_{B^*})}{m_V^\pi - q^2} \exp[-0.3249(q_{\max}^2 - q^2)] ,$$

$$f_{B^*} = 0.7 \text{ GeV} ;$$

HMW:

$$f_+^i(q^2) = \frac{m_V^{i2} - q_{\max}^2}{m_V^{i2} - q^2} \frac{m_B + m_i}{[(m_B + m_i)^2 - q^2]^{1/2}} ,$$

m_V^i as in KS above ;

KP:

$$f_+^i(q^2) = \frac{m_V^{i2} - q_{\max}^2}{m_V^{i2} - q^2} .$$

The additional IW term, Isgur and Wise[14] argue, is necessary to account for a direct coupling to B^* in the lepton current.

Again, when lepton masses may be neglected, the current matrix element for a B meson decaying semileptonically to a vector meson of momentum p_i is given by

$$\langle V_i(p) | j_\mu | B(p_1) \rangle = \epsilon^{*\nu} F_1^A(q^2) g_{\mu\nu} + F_2^A(q^2) p_{1\mu} p_{1\nu} + iF^V(q^2) \epsilon_{\mu\nu\rho\sigma} p_1^\rho p^\sigma .$$

The generalization and form-factor correspondence for a two-body final state is given in CPK. The various form-factor assumptions are as follows:

KS:

$$F_1^A=(m_1+m_i)I_i\frac{m_A^{i2}}{m_A^{i2}-q^2}\ ,$$

$$F_2^A=\frac{-2}{m_1+m_i}I_i\left[\frac{m_A^{i2}}{m_A^{i2}-q^2}\right]^2,$$

$$F^V=\frac{-2}{m_1+m_i}I_i\left[\frac{m_V^{i2}}{m_V^{i2}-q^2}\right]^2,$$

$$I_\rho=0.33,\quad I_{D^*}=0.7\ ,$$

$$m_V^\rho=5.33,\quad m_V^{D^*}=6.34\ ,$$

$$m_A^\rho=5.33,\quad m_A^{D^*}=6.34\ ;$$

ISGW($B\to\rho+e+\nu$):

$$F_1^A=3.579\exp[-0.1168(q_{\max}^2-q^2)]\ ,$$

$$F_2^A=-0.5021\exp[-0.1168(q_{\max}^2-q^2]\ ,$$

$$F^V=-0.699\exp[-0.1168(q_{\max}^2-q^2)]\ ;$$

ISGW ($B\to D^*+e+\nu$) :

$$F_1^A=6.833\exp[-0.029\,57(q_{\max}^2-q^2)]\ ,$$

$$F_2^A=-0.291\,51\exp[-0.029\,57(q_{\max}^2-q^2)]\ ,$$

$$F^V=-0.3068\exp[-0.029\,57(q_{\max}^2-q^2)]\ ;$$

HMW :

$$B\to V_i,\quad i=\rho \text{ or } D^*\ ,$$

$$F_1^A=\sqrt{(m_1+m_i)^2-q^2}\,\frac{m_A^{i2}-q_{\max}^2}{m_A^{i2}-q^2}\ ,$$

$$F_2^A=\frac{-2}{\sqrt{(m_1+m_i)^2-q^2}}\,\frac{m_A^{i2}-q_{\max}^2}{m_A^{i2}-q^2}\ ,$$

$$F^V=\frac{-2}{\sqrt{(m_1+m_i)^2-q^2}}\,\frac{m_V^{i2}-q_{\max}^2}{m_V^{i2}-q^2}\ ,$$

$m_{V,A}^i$ as in KS ;

KP:

$$F_1^A=(m_1+m_i)\frac{m_A^{i2}-q_{\max}^2}{m_A^{i2}-q^2}\ ,$$

$$F_2^A=\frac{-2}{m_1+m_i}\,\frac{m_A^{i2}-q_{\max}^2}{m_A^{i2}-q^2}\ ,$$

$$F^V=\frac{-2}{m_1+m_i}\,\frac{m_V^{i2}-q_{\max}^2}{m_V^{i2}-q^2}\ .$$

The contact term can be treated either as a literal constant, with no q^2 dependence, or a low-energy term which also can be modified by further development of the chiral perturbation series. They are written down by CPK. For the KP illustration we use the contact term damping factors

$$\frac{m_{V,A}^{i2}-(m_1-m_2-m_3)^2}{m_{V,A}^i-q^2}$$

which is unity at the current-algebra point $q^2=(m_1-m_2-m_3)^2$. Compared to our original work (CPK) we propose here a new extrapolation of the form factor for the contact term if we depart from the chiral limit $m_1=m_2=m_3=\sqrt{q^2}=0$. This new form factor reduces the contact term appreciably. In the old, naive approach, the form factor had no explicit dependence on the pseudoscalar masses and was equal to unity for $q^2=0$. Now it is equal to one when the recoil momentum of the final-state meson system vanishes ($q^2=q_{\max}^2$).

III. RESULTS

$B^-\to\pi^0+e^-+\nu$

In Table I and Fig. 1 we present the partial decay rates for selected energy intervals for the decay of B^- to a neutral pion (half the rate for B^0 to decay to a charged pion) for the KS, ISGW, HMW, and KP models.

For the ISGW model we also present these rates with the additional IW term included coherently for positive and negative interference. All rates are reduced rates $\tilde{\Gamma}$ in the units $|V_{bu}|^2\times10^{-11}$ GeV, so that the true rate is given by $\Gamma=\tilde{\Gamma}|V_{bu}|^2\times10^{-11}$ GeV. The model dependence of this rate, a simple process depending on only one form factor, is quite large, the smallest and largest estimates varying by an order of magnitude. Since large momentum transfers q^2 are kinematically allowed, it is not surprising that the models differ so drastically.

We defer discussion of these results and comparison with experimental data until $B^-\to\rho^0+e^-+\nu$ and $B^-\to D^0(D^{0*})+e^-+\nu$ are presented.

$B^-\to D^0+e^-+\nu$

In Table II and Fig. 2 we present the partial decay rate for the semileptonic decay of B^- to a neutral D meson for the same models considered in $B\to\pi+e+\nu$ decay. In contrast with the latter channel, $B^-\to D^0+e^-+\nu$ decay has much less model dependence, because of the

TABLE I. Partial decay rates $\tilde{\Gamma}$ for $B^-\to\pi^0+e^-+\nu$ in units $|V_{bu}|^2\times10^{-11}$ GeV, in the indicated lepton energy regions, for the form-factor models described in the text. The IW term describes an additional direct coupling of the lepton current to B^* and has been added coherently to the ISGW model, with positive and negative relative phase.

	All E	$E>2.3$	$2.4<E<2.6$	$2.2<E<2.4$
KS	0.239	0.036 2	0.018 4	0.031 3
ISGW	0.069 4	0.015 8	0.008 10	0.013 2
IW addition	0.008 50	0.007 01	0.005 36	0.002 38
ISGW+IW	0.101	0.035 0	0.022 1	0.022 9
ISGW−IW	0.056 1	0.010 3	0.005 58	0.008 32
HMW	0.018 1	0.004 04	0.002 36	0.003 15
KP	0.011 0	0.001 57	0.000 843	0.001 44

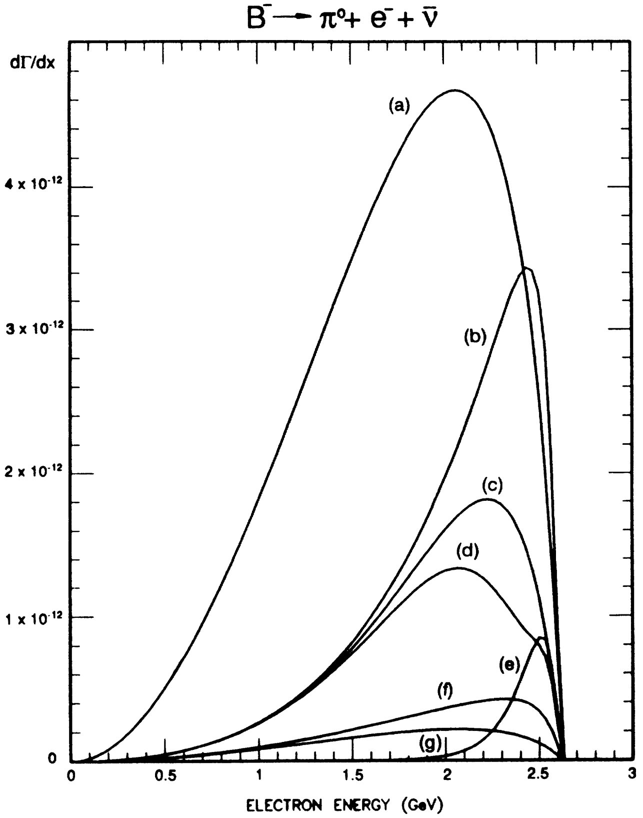

FIG. 1. Electron spectrum $d\Gamma/dx$ in GeV units for the semilepton decay $B^- \to \pi^0 + e^- + \nu$ for the models described in the text. $x = 2E_e/m_B$. (a) KS, (b) ISGW+IW, (c) ISGW, (d) ISGW− IW, (e) IW, (f) HMW, (g) KP.

low-momentum transfers that are allowed. To compare Table II to data, we need the exclusive branching ratio and the B^- lifetime. Using the world-average B lifetime[17] τ_B (we assume $\tau_{B^0} = \tau_{B^-}$),

$$\tau_B = 1.18 \pm 0.14 \text{ psec} ,$$

we find

$$\Gamma(B) = (5.58 \pm 0.70) \times 10^{-13} \text{ GeV} .$$

CLEO[18] reports the branching ratios

$$B(B^- \to D^0 + e^- + \nu) = (1.6 \pm 0.6^{+0.9}_{-0.6})\% ,$$

$$B(B^0 \to D^+ + e^- + \nu) = (2.0 \pm 0.4 \pm 0.2)\% ,$$

and ARGUS reports[19]

$$B(B^0 \to D^- + e^+ + \nu) = (1.8 \pm 0.6 \pm 0.5)\% .$$

Using the nominal average $B(B \to De\nu) = (1.8 \pm 0.6)\%$ and the results of Table II, we can calculate $|V_{bc}|$:

Model	$\lvert V_{bc} \rvert$
KS	0.043±0.007
ISGW	0.035±0.007
HMW	0.041±0.007
KP	0.042±0.007

TABLE II. Partial decay rates $\tilde{\Gamma}$ for $B^- \to D^0 + e^- + \nu$ in units $|V_{bc}|^2 \times 10^{-11}$ GeV, in the indicated energy regions, for the form-factor models described in the text.

	All E	$2.0 < E < 2.3$ GeV
KS	0.544	0.0246
ISGW	0.801	0.0365
HMW	0.6059	0.0270
KP	0.563	0.0255

$B^- \to \rho^0 + e^- + \nu$

In Table III and Fig. 3 we present the partial decay rates of the semileptonic decay of B^- to a neutral ρ meson (half the rate for B^0 decay to a charged ρ meson) for the various models already described. Each model has been calculated for full two-body kinematics, without contact terms (2BK), in the narrow-width approximation (NWA) and in full two-body kinematics with contact terms (CON). In the KS case we do not present the very-large-contact effect given in CPK (increases rate by factor of 50–100) because our (KP) less naive extrapolation to higher q^2 essentially replaces it.

The narrow-width approximation, conventionally used

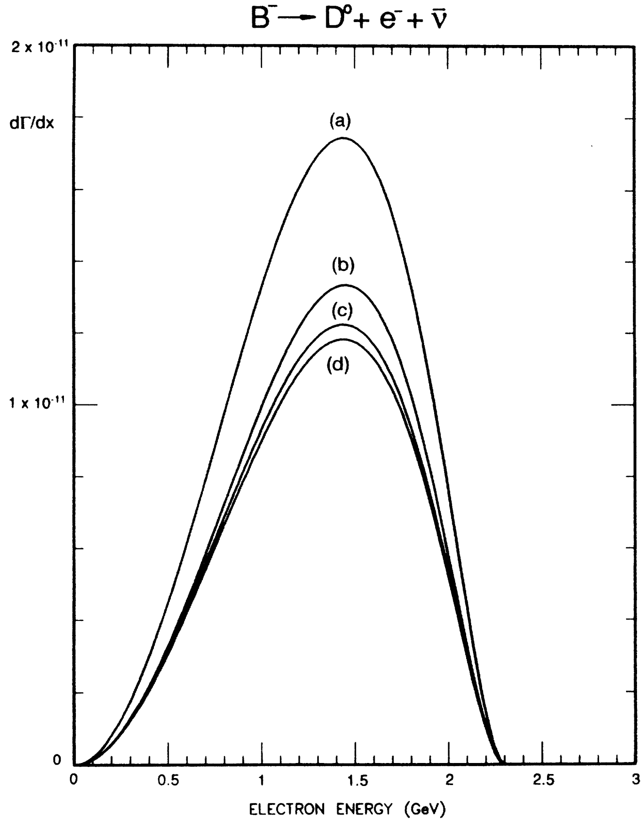

FIG. 2. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B \to D + e + \nu$ for the models described in the text. $x = 2E_e/m_B$. (a) ISGW, (b) HMW, (c) KP, (d) KS.

TABLE III. Partial decay rates $\tilde{\Gamma}$ for $B^- \rightarrow \rho^0 + e^- + \nu$ in units $|V_{bu}|^2 \times 10^{-11}$ GeV, in the indicated energy regions, for the form-factor models described in the text. 2BK=full two-body kinematics for $\pi\pi(\rho)$ final hadron state; NWA=narrow-width approximation for ρ; CON=current-algebra-inspired-contact-term background. For KS, which have dipole behavior for F^V and F_2^A, results are also presented for monopole behavior of these form factors.

	All E	$E > 2.3$ GeV	$2.2 < E < 2.4$	$2.4 < E < 2.6$
KS(2BK)	1.01	0.253	0.295	0.123
KS(NWA)	1.09	0.342	0.327	0.183
KS(2BK) monopole	1.29	0.206	0.275	0.0952
KS(NWA) monopole	1.41	0.280	0.312	0.129
ISGW(2BK)	0.245	0.0776	0.0813	0.0353
ISGW(NWA)	0.272	0.0988	0.0924	0.0492
ISGW(CON)	1.09	0.106	0.107	0.0527
HMW(2BK)	0.611	0.119	0.134	0.0538
HMW(NWA)	0.573	0.152	0.151	0.0761
HMW(CON)	6.08	0.216	0.265	0.101
KP(2BK)	0.951	0.152	0.210	0.0864
KP(NWA)	1.05	0.208	0.300	0.0951
KP(CON)	2.35	0.145	0.214	0.0823

by almost all authors, consistently overestimates the ρ rates by about 8%. As data improves, the ρ width, therefore, must be taken into account.

The KS results have also been run for the case where the KS dipole behavior for the F^V and F_2^A has been modified to monopole behavior. The KS form factors yield the largest semileptonic ρ rates, by factors of 4–5 over the smallest, those of ISGW.

The effect of the current-algebra contact terms are dramatic, boosting rates by factors of 2–10. The differences are there because the contact terms as we have calculated them here are modified by the different q^2 dependence of the form factors. However, the contact terms have a much smaller influence on the end-point region, even in the extreme case of CPK where only 2% of the contact rate is in the $2.4 < E < 2.6$ region. It is important to note that contact-term effect in the high-energy-tail region is very model dependent—negligible in the KP model but significant for HMW and ISGW.

In CPK it was pointed out that there was experimental support in the E691 and Mark III (Ref 10) results for a nonresonant $D \rightarrow K^* + e + \nu$ background of about the same size as or larger than that suggested by the current-algebra contact term. If this is also the case in $B^- \rightarrow \rho^0 + e + \nu$, then we see that rates may be boosted by at least a factor of 2 over those given by simple resonance production, with significant influence on determinations of V_{bu}/V_{bc}, as shown below.

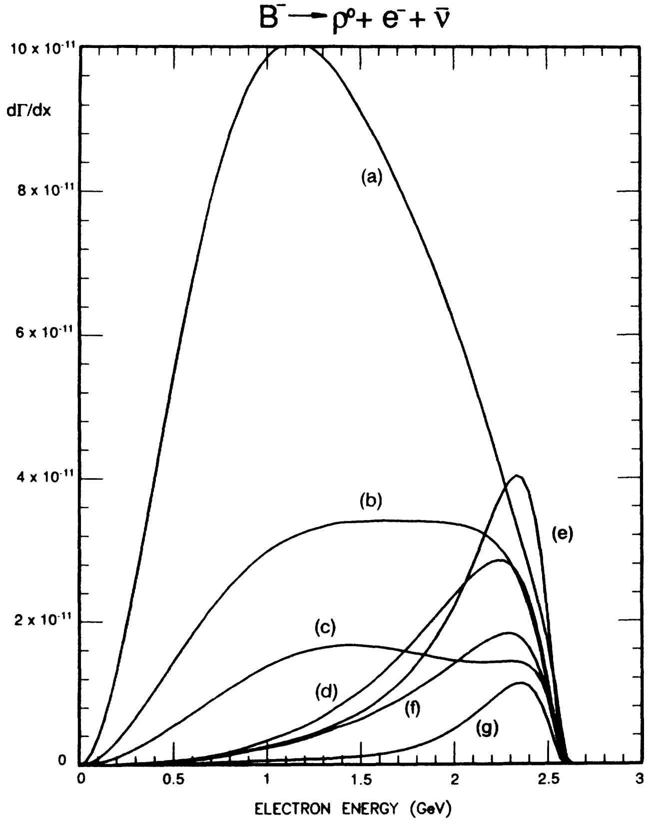

FIG. 3. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B^- \rightarrow \rho^0 + e^- + \nu$ for the models described in the text. $x = 2E_e/m_B$. (a) HMW+CON, (b) KP+CON, (c) ISGW+CON, (d) KP, (e) KS, (f) HMW, (g) ISGW.

$B^- \rightarrow D^{*0} + e^- + \nu$

In Table IV and Fig. 4 we present partial semileptonic decay rates for B^- to a neutral D^* meson. As was the

TABLE IV. Partial decay rates for $B^- \to D^{*0} + e^- + \nu$ in units $|V_{bc}|^2 \times 10^{-11}$ GeV, in the indicated energy regions, for the models discussed in the text. For KS, which have dipole behavior for F^V and F_2^A, results are also presented for monopole behavior of these form factors.

	All E	$2.0 < E < 2.3$ GeV
KS	1.72	0.135
KS monopole	1.77	0.135
ISGW	1.66	0.135
HMW	1.74	0.136
KP	1.95	0.143

case for $B \to D + e + \nu$ decay, the models are in good agreement because of the limited range of momentum transfers that are kinematically possible.

Using the lifetime quoted in the $B^- \to D^0 + e^- + \nu$ discussion and the CLEO branching ratio[18]

$$B(B^- \to D^{*0} e^- \nu) = (4.4 \pm 0.6 \pm 1.2)\% \ ,$$

$$\Gamma(B^- \to D^{*0} e^- \nu) = (2.2 \pm 0.7) \times 10^{-14}\ \text{GeV} \ ,$$

we find from Table IV that

$$|V_{bc}| = \begin{cases} 0.038 \pm 0.005, & \text{KS} \ , \\ 0.039 \pm 0.005, & \text{ISGW} \ , \\ 0.039 \pm 0.005, & \text{HMW} \ , \\ 0.035 \pm 0.005, & \text{KP} \ . \end{cases}$$

The general trend is for a smaller $|V_{bc}|$ from $B^- \to D^* e^- \nu$ than $B^- \to D^0 e^- \nu$ but the effect is not statistically significant. The reported branching ratio for $\bar{B}^0 \to D^{*+} e^- \nu$ is $(4.6 \pm 0.5 \pm 0.7)\%$ (CLEO[18]) and $(6.0 \pm 1.0 \pm 1.4)\%$ (ARGUS[19]) corresponding to

	CLEO $\|V_{bc}\|$	ARGUS $\|V_{bc}\|$
KS	0.039 ± 0.004	0.045 ± 0.006
ISGW	0.040 ± 0.004	0.046 ± 0.006
HMW	0.040 ± 0.004	0.046 ± 0.006
KP	0.036 ± 0.004	0.041 ± 0.006

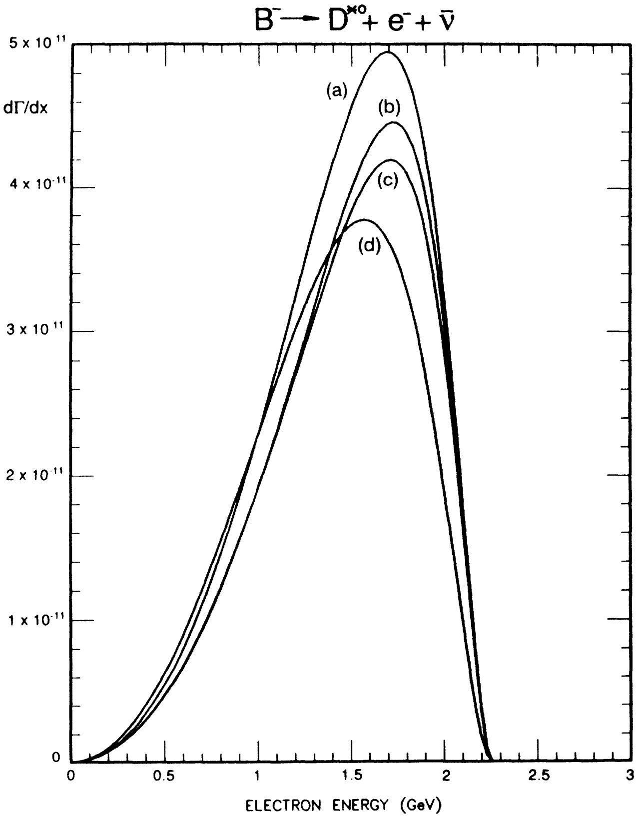

FIG. 4. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B \to D^* + e + \nu$ for the models described in the text. $x = 2E_e/m_B$. (a) KP, (b) KS, (c) HMW, (d) ISGW.

Comparison with ARGUS results

Table V reports the ratio $|V_{bu}/V_{bc}|$ as determined from the branching ratios in the energy region reported by ARGUS:[2]

$$\Delta_1:\ 2.0\ \text{GeV} < E < 2.3\ \text{GeV}$$

and

$$\Delta_2:\ 2.3 < E < 2.6\ \text{GeV} \ .$$

TABLE V. $|V_{bu}/V_{bc}|$ as extracted from the ARGUS and CLEO partial branching-ratio data, for the models discussed in the text. CLEO presented results in two energy intervals, which give independent determinations. Also presented are KS results for monopole form factors in F^V and F_2^A.

		CLEO	
		Energy interval (GeV)	
	ARGUS	2.2–2.4	2.4–2.6
Model	$\|V_{bu}/V_{bc}\|$	$\|V_{bu}/V_{bc}\|$	$\|V_{bu}/V_{bc}\|$
KS(2BK)	0.131	0.0826	0.137
KS(NWA)	0.114	0.0787	0.115
KS(2BK) monopole	0.143	0.0861	0.155
KS(NWA) monopole	0.126	0.0812	0.135
ISGW(2BK)	0.240	0.157	0.255
ISGW(NWA)	0.214	0.149	0.221
ISGW(CON)	0.210	0.139	0.215
HMW(2BK)	0.204	0.127	0.217
HMW(NWA)	0.180	0.120	0.184
HMW(CON)	0.152	0.0908	0.160
KP(2BK)	0.185	0.108	0.184
KP(NWA)	0.158	0.103	0.175
KP(CON)	0.189	0.107	0.188

TABLE VI. V_{bu} and V_{bc}, as determined from the CLEO data, using the B lifetime to calculate $|V_{bu}|$ and Table V results to obtain $|V_{bc}|$.

Model	2.2<E<2.4 GeV $\|V_{bu}\|$	2.2<E<2.4 GeV $\|V_{bc}\|$	2.4<E<2.6 GeV $\|V_{bu}\|$	2.4<E<2.6 GeV $\|V_{bc}\|$
KS(2BK)	0.0041	0.050	0.0069	0.050
KS(NWA)	0.0039	0.050	0.0058	0.050
KS(2BK) monopole	0.0043	0.050	0.0077	0.050
KS(NWA) monopole	0.0040	0.049	0.0067	0.050
ISGW(2BK)	0.0077	0.049	0.012	0.048
ISGW(NWA)	0.0073	0.046	0.011	0.049
ISGW(CON)	0.0068	0.049	0.010	0.049
HMW(2BK)	0.0064	0.050	0.0109	0.050
HMW(NWA)	0.0060	0.050	0.0108	0.050
HMW(CON)	0.0046	0.051	0.0081	0.050
KP(2BK)	0.0051	0.047	0.0088	0.047
KP(NWA)	0.0049	0.047	0.0083	0.048
KP(CON)	0.0051	0.047	0.0090	0.048

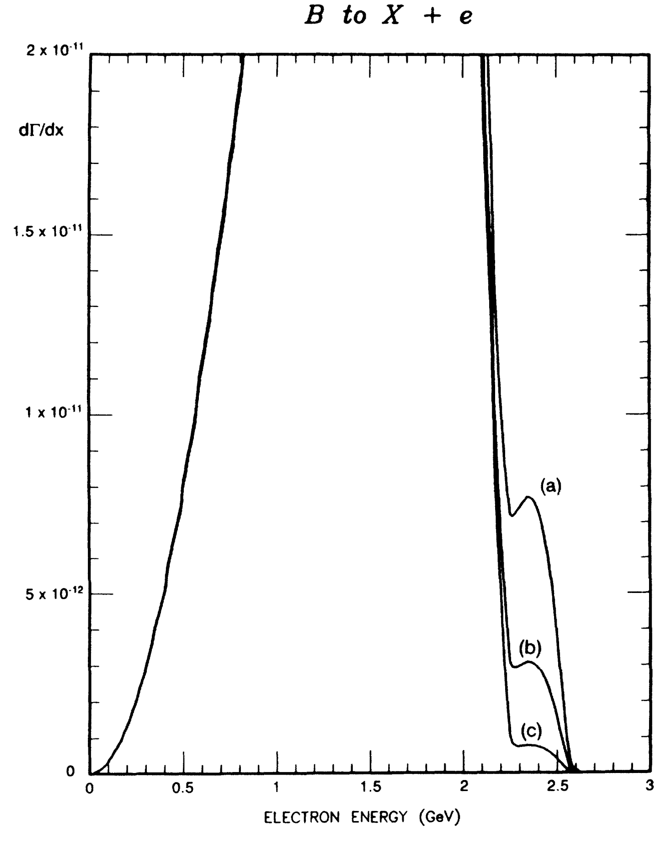

FIG. 5. Electron spectrum $d\Gamma/dx$ in GeV units for $B^- \rightarrow X + e^- + \nu$, where $X = \pi + \rho + D + D^*$ in the KS model without contact terms. $x = E_e/m_B$. (a) $|V_{bu}/V_{bc}| = 0.32$, (b) $|V_{bu}/V_{bc}| = 0.20$, (c) $|V_{bu}/V_{bc}| = 0.10$.

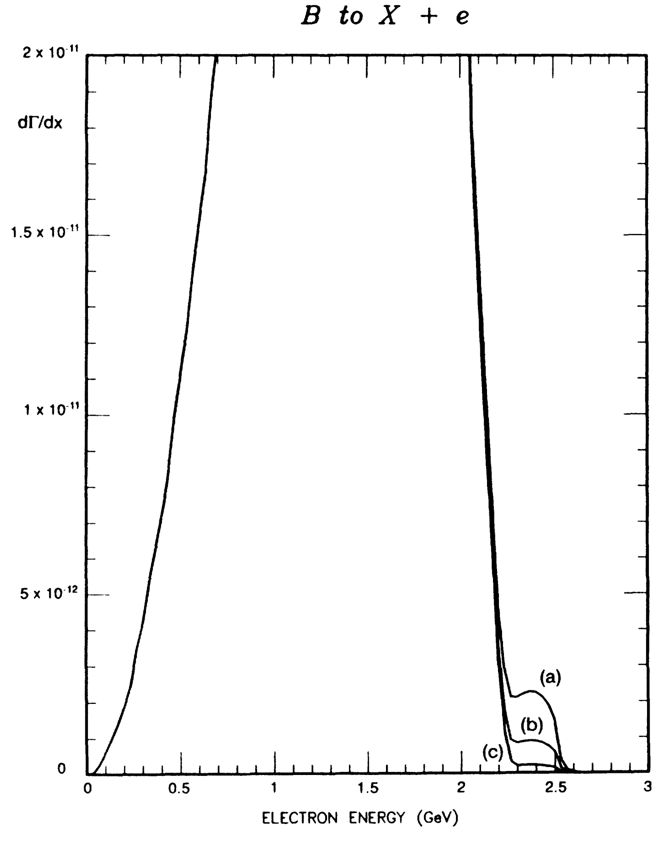

FIG. 6. Electron spectrum $d\Gamma/dx$ in GeV units for $B^- \rightarrow X + e^- + \nu$, where $X = \pi + \rho + D + D^*$ in the ISGW model without contact terms. $x = 2E_e/m_B$. (a) $|V_{bu}/V_{bc}| = 0.32$, (b) $|V_{bu}/V_{bc} = 0.20$, (c) $|V_{bu}/V_{bc}| = 0.10$.

If it is assumed that the beam mixture is 50% charged and 50% neutral B mesons, that the rates are saturated in these regions by π, ρ, D, and D^*, and that the exclusive rates add incoherently, then

$$B(\Delta_2)/B(\Delta_1)=\frac{(\Gamma_{\pi^0}+\Gamma_{\pi^+}+\Gamma_{\rho^0}+\Gamma_{\rho^+})_{\Delta_2}}{(\Gamma_{D^0}+\Gamma_{D^+}+\Gamma_{D^{*0}}+\Gamma_{D^{*+}})_{\Delta_1}} .$$

In terms of the reduced rates $\tilde{\Gamma}$ quoted in Tables I–IV, we have

$$B(\Delta_2)/B(\Delta_1)=\tfrac{3}{2}|V_{bu}/V_{bc}|^2[\tilde{\Gamma}(\pi^0)+\tilde{\Gamma}(\rho^0)]_{\Delta_2}/[\tilde{\Gamma}(D^0)+\tilde{\Gamma}(D^{*0})]_{\Delta_1} .$$

The results for $|V_{bu}/V_{bc}|$ given in Table V are derived from the ARGUS[2] result for semileptonic branching ratios

$$\frac{B_{\mathrm{SL}}(2.3-2.6)}{B_{\mathrm{SL}}(2.0-2.3)}=4.7\pm1.2\% .$$

The variation is roughly $0.11<|V_{bu}/V_{bc}|<0.19$ in the absence of contact terms and $0.14<|V_{bu}/V_{bc}|<0.18$ when they are included. In the ISGW case we have not included their 1s and 1p terms; we estimate that including these would reduce the V_{bu} result by 20%. There is also experimental indication that D and D^* do not saturate the $b\rightarrow c$ semileptonic rate. CLEO,[18] for example, reports

$$B(D+D^*+D^{**})=(8.0\pm1.4)\%$$

with D^{**} accounted for 25% of the total. Since D^{**} is heavier than D and D^*, it should have a softer momentum spectrum. In the worst case that its momentum spectrum is similar to the D and D^* average, adding a 25% D^* component would increase our $|V_{bu}/V_{bc}|$ estimate by 10%.

Comparison with CLEO results

CLEO reports the branching ratios $B(2.4-2.6)$ and $B(2.2-2.4)$ for semileptonic decay in the indicated energy intervals (GeV) and the total semileptonic branching

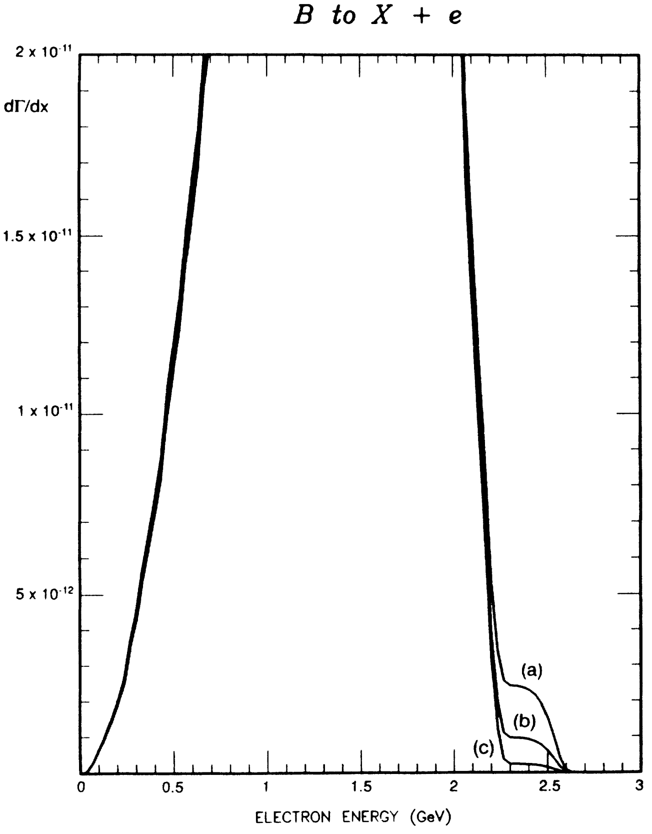

FIG. 7. Electron spectrum $d\Gamma/dx$ in GeV units for $B^-\rightarrow X+e^-+\nu$, where $X=\pi+\rho+D+D^*$ in the ISGW model with contact terms. $x=2E_e/M_B$. (a) $|V_{bu}/V_{bc}|=0.32$, (b) $|V_{bu}/V_{bc}|=0.20$, (c) $|V_{bu}/V_{bc}|=0.10$.

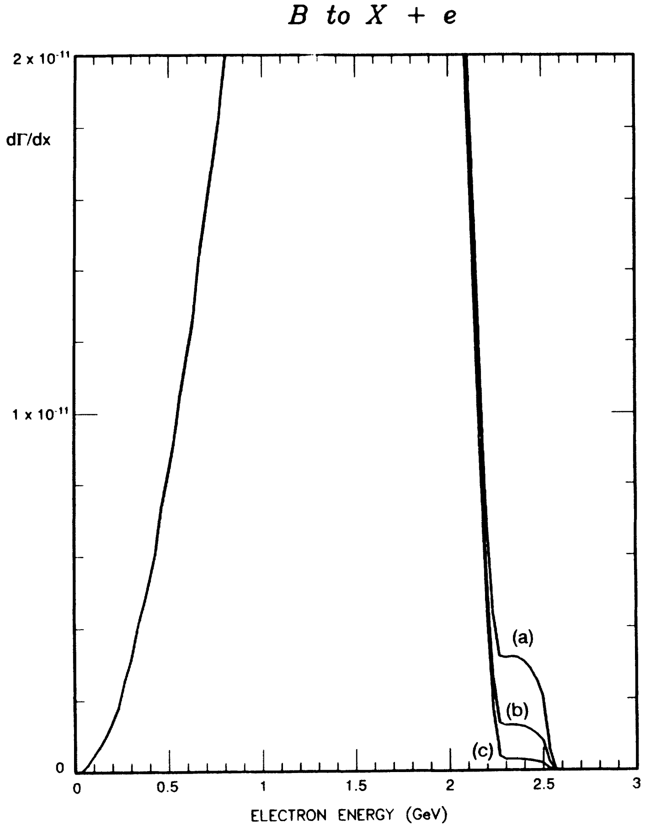

FIG. 8. Electron spectrum $d\Gamma/dx$ in GeV units for $B^-\rightarrow X+e^-+\nu$, where $X=\pi+\rho+D+D^*$ in the HMW model without contact terms. $x=2E_e/M_B$. (a) $|V_{bu}/V_{bc}|=0.32$, (b) $|V_{bu}/V_{bc}|=0.20$, (c) $|V_{bu}/V_{bc}|=0.10$.

$B_{\rm SL}$ ratio:

$$B(2.4\text{–}2.6)\approx B_{bu}(2.4\text{–}2.6)$$

$$=(1.8\pm0.4\pm0.3)\times10^{-4}\ ,$$

$$B_{bu}(2.2\text{–}2.4)=(1.5\pm0.7\pm0.7)\times10^{-4}\ ,$$

$$B_{\rm SL}\approx B_{bc}=0.102\pm0.002\pm0.007\ .$$

Again we assume a 50%/50% mixture of B^0 and B^- mesons in the CLEO data set. For each energy interval Δ we calculate

$$\left|\frac{V_{bu}}{V_{bc}}\right|^2=\frac{B_{bu}(\Delta)\tilde{\Gamma}_D(\text{all})+\tilde{\Gamma}_{D^*}(\text{all})}{B_{bc}(\text{all})\frac{3}{2}[\tilde{\Gamma}_{\rho^0}(\Delta)+\Gamma_{\pi^0}(\Delta)]}$$

using the results of Tables I–IV. The outcome is shown in Table V. Again the estimate could be increased by as much as 16% if a D^{**} component is included in $b\to c$.

The upper-interval CLEO results are quite consistent, model to model, with the ARGUS results for $|V_{bu}/V_{bc}|$. The CLEO results for $|V_{bu}/V_{bc}|$ tend to be systematically lower when the lower-energy interval is used for the estimate, model by model. Perhaps the models are all wrong in the same way in their description of the lepton energy dependence or the effect is somehow related to the assumptions which the CLEO group needed to make to subtract the $b\to c+e+\nu$ tail from the branching ratio reported as that of $b\to u$ in the interval $2.2<E<2.4$ GeV.

We see from Table V that the results for $|V_{bu}/V_{bc}|$ for the various models differ only very little. This is not so astonishing since, apart from the contact term model, all of these models are really quite similar. On the other hand, we have to keep in mind that the conventional assumptions on form factors might be all wrong as it seems to be the case for the semileptonic decay of $D\to K^*$.

The clear importance of two-body kinematics as opposed to narrow-resonance approximation on the determination of $|V_{bu}/V_{bc}|$ is evident from Table V; all models using the narrow-width approximation under estimate $|V_{bu}/V_{bc}|$ by about 10%.

The comparison of theoretical form-factor models with CLEO data in Table V is based on a model's ability to predict the *ratio* of $b\to c$ to $b\to u$ rates but is independent of the absolute magnitudes of the rates. Another, more stringent test, which does require prediction of the absolute magnitude of $b\to u$, is possible if the B lifetime is known. Using the world-average B lifetime data

$$\Gamma_B=5.58\times10^{-13}\ \text{GeV}\ ,$$

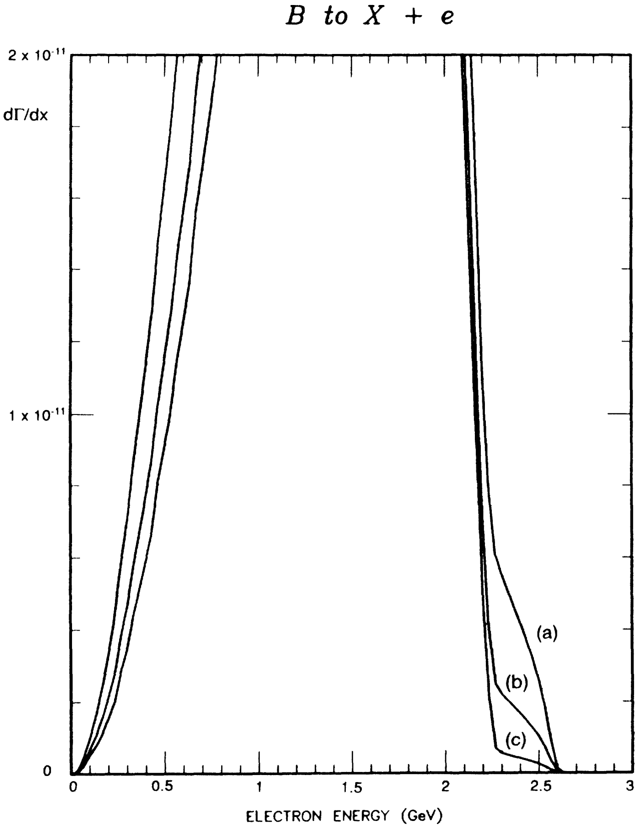

FIG. 9. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B^-\to X+e^-+\nu$, where $X=\pi+\rho+D+D^*$ in the HMW model with contact terms. $x=2E_e/M_B$. (a) $|V_{bu}/V_{bc}|=0.32$, (b) $|V_{bu}/V_{bc}|=0.20$, (c) $|V_{bu}/V_{bc}|=0.10$.

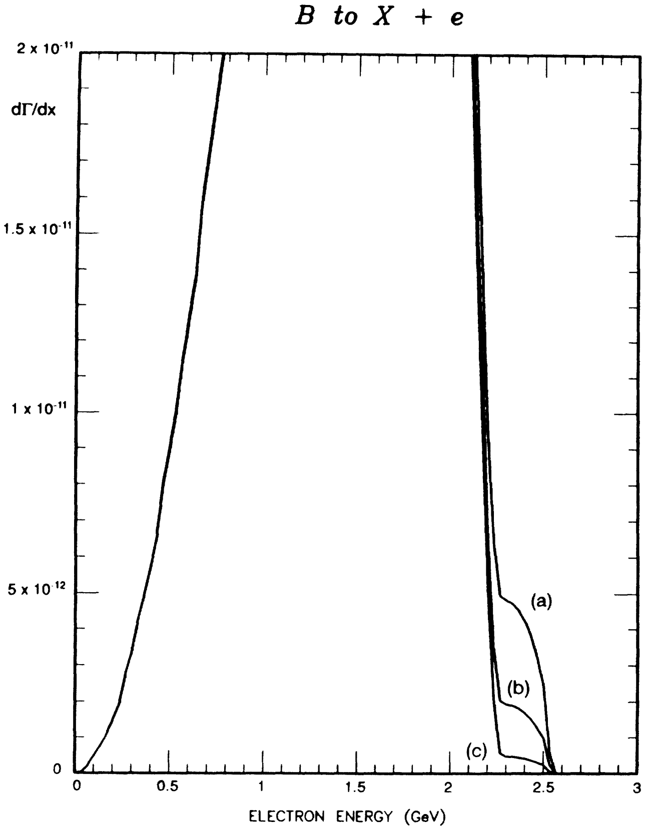

FIG. 10. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B^-\to X+e^-+\nu$, where $X=\pi+\rho+D+D^*$ in the KP model without contact terms. $x=2E_e/M_B$. (a) $|V_{bu}/V_{bc}|=0.32$, (b) $|V_{bu}/V_{bc}|=0.20$, (c) $|V_{bu}/V_{bc}|=0.10$.

we can calculate

$$|V_{bu}|^2=\frac{\Gamma_B B_{ub}(\Delta)}{\frac{3}{2}[\tilde{\Gamma}_{\pi^0}(\Delta)+\tilde{\Gamma}_{\rho^0}(\Delta)]}\ .$$

where Δ are the two CLEO intervals 2.2–2.4 GeV and 2.4–2.6 GeV. From this determination of V_{bu}, model by model, we may compare this result to those of Table V, yielding V_{bu} and V_{bc} as given in Table VI.

While there is little variation in $|V_{bc}|$—this determination is independent of V_{ub} and depends only on the models ability to correctly predict $\Gamma_{D^+}+\Gamma_{D^*}$— the variation in V_{bu} is considerable, consistent with variations of the ratio in Table V. It is notable, however, that $|V_{bc}|$, quite stable in Table VI ($V_{bc}\approx 0.049$) is systematically higher here than in the determination from D and D^* semileptonic decays. This would be the case if our assumption were wrong that D and D^* saturated the $b\rightarrow c$ semileptonic rate in the pertinent energy intervals.

Electron spectra for the various models are given in Fig. 5 (KS), Figs. 6 and 7 (ISGW), Figs. 8 and 9 (HMW), and Figs. 10 and 11 (KP). The spectra have been run for $|V_{bu}/V_{bc}|^2=0.01, 0.04, 0.1$, $|V_{bu}/V_{bc}|=0.10, 0.20, 0.32$. The inclusive spectra are based only on π, ρ, D, and D^* production. The KS and ISGW are distinctly "humped" in the $b\rightarrow u$ region, HMW and KP somewhat less so. The hump is visible only for large $|V_{bu}/V_{bc}|$. The effect of the contact term is to smooth out this feature.

IV. SUMMARY AND CONCLUSIONS

We have studied the model dependence of four theories for the $b\rightarrow u$ and $b\rightarrow c$ exclusive semileptonic channels $B\rightarrow\pi$, ρ, D, and D^*. For each theory we have discussed the effect of a current-algebra contact term as a model for a background in $B\rightarrow\rho$ as well as the effect of the narrow-width approximation for the same channel. We have also estimated the effects of other states that may contribute to $b\rightarrow u$ (1s and 1p quark-model states) and to $b\rightarrow c$ (D^{**} final state.) Under the assumption that the exclusive states π, ρ, D, and D^* saturate the rates in the high lepton energy intervals used to extract the CKM parameters, we calculate V_{bu} and V_{bc} to determine the effect of model dependence on these parameters.

Total semileptonic rates for $B\rightarrow\pi$ and $B\rightarrow\rho$ differ widely, model to model. Rates for $B\rightarrow D$ and $B\rightarrow D^*$ have much less model dependence, as expected, because the models generally agree for small momentum transfers. The model dependence of $B\rightarrow\pi$ and $B\rightarrow\rho$ are somewhat smaller when only higher lepton energy rates are considered. The contact background term in $B\rightarrow\rho$ is particularly suppressed in these regions.

V_{bc} calculated from exclusive $B\rightarrow D$ decay varies about 25% model to model whereas a 10% model-to-model variation is obtained if $B\rightarrow D^*$ decays are used. This model variation is about the magnitude of the errors in the experimental rates.

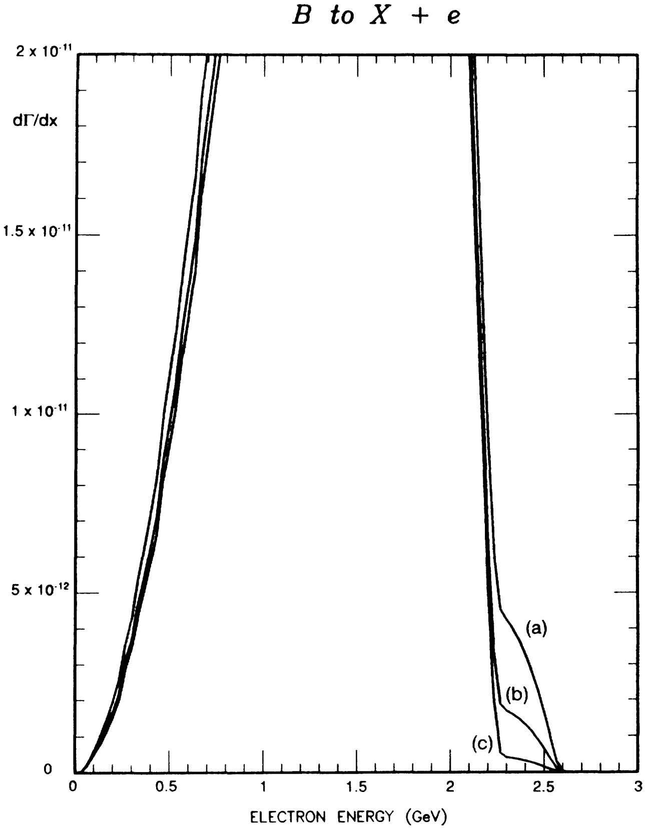

FIG. 11. Electron spectrum $d\Gamma/dx$ in GeV units for the semileptonic decay $B^-\rightarrow X+e^-+\nu$, where $X=\pi+\rho+D+D^*$ in the KP model with contact terms. $x=2E_e/M_B$. (a) $|V_{bu}/V_{bc}|=0.32$, (b) $|V_{bu}/V_{bc}|=0.20$, (c) $|V_{bu}/V_{bc}|=0.10$.

V_{bu}/V_{bc} as extracted from the high-lepton-energy rates varies considerably model to model (as well as energy region to energy region for the CLEO determination). The narrow-width approximation gives ratios incorrect by as much as 20%. If the contact backgrounds are present, V_{bu}/V_{bc} may be considerably smaller than suggested by models that ignore them. If the D^{**} component of $b\rightarrow c$ is as large as recent CLEO data suggests, V_{bu}/V_{bc} may be 16% larger than suggested by models that ignore this contribution.

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