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Multiplicity moments and nuclear geometry in relativistic heavy-ion collisions

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The energy, target-mass, and rapidity-window independence or approximate independence of the multiplicity moments in high-energy nucleus-nucleus collisions are analyzed. It is pointed out that all of these properties are due to nuclear geometry. It is proved under general conditons that, when the target mass is not extremely light and the rapidity window not very narrow, the normalized moments of multiplicity are approximately equal to that of the number of participating nucleons. The calculated results for both minimum-bias and central events agree with recent experimental data.

I. INTRODUCTION

A lot of data have been obtained from the successful acceleration of oxygen and sulphur ions to 60 and 200A GeV at the CERN Super Proton Synchrotron, among which are the following observations on the charged-particle multiplicity distribution (MD).

(i) The multiplicity distribution in terms of the Kuba-Nielsen-Olesen normalized variable $n/\langle n\rangle$ (KNO MD) scales with respect to the incident energy.[1,2]

(ii) The KNO MD is independent of the target mass.[1]

(iii) The normalized multiplicity moments C_2–C_4 are approximately independent of the rapidity window.[3]

(iv) The ratio of dispersion $D=(\langle n^2\rangle-\langle n\rangle^2)^{1/2}$ to mean multiplicity $\langle n\rangle$, and S, the slope of $C_2\langle n\rangle$ vs $\langle n\rangle$, are constants and independent of the energy and the target mass.[1,3,4]

Many authors[5,6] have considered some of these properties, especially that about the KNO MD, using phenomenological models. According to popular opinion,[7–10] elementary nucleon-nucleon collision and nuclear geometry are two main ingredients of nuclear collision. Obviously, the nuclear geometry does not depend on incident energy, and in the presently available energy region, the KNO MD of the nucleon-nucleon collisions scales with respect to energy also.[11] A combination of both these properties naturally results in the energy scaling of the KNO MD of the nucleus-nucleus collisions. The question is as follows: When the incident energy becomes so high, e.g., at the colliding energy 200A GeV of the planned Brookhaven Relativistic Heavy Ion Collider (RHIC),[12] that the scaling of the KNO MD at the corresponding energy is violated for nucleon-nucleon collisions, does the KNO MD of the nucleus-nucleus collision still keep energy scaled? In addition, why are the KNO MD and the normalized multiplicity moments also independent of the target mass and the rapidity window? All of these problems need further discussions.

As is well known, the multiplicity moments are important characteristics in multiparticle production. The properties of the KNO MD can be fully described by the normalized moments

$$C_q=\frac{\langle n^q\rangle}{\langle n\rangle^q},\quad q=2,3,\ldots . \tag{1}$$

In this paper, we will concentrate our attention to the study of the independence or approximate independence of C_q on energy, the target mass, and rapidity window. We will see that all of these properties are due to the following fact: The nuclear geometry dominates the nuclear collisions provided that the target mass is not too light and the rapidity window not very narrow. As a result, all of the above-mentioned independences do not depend on the concrete behavior of elementary nucleon-nucleon collisions. We will prove this statement in the next section under very general conditions.

II. DOMINANCE OF THE NUCLEAR GEOMETRY IN THE MULTIPLICITY MOMENTS

The current models of heavy-ion collision are all based on the following hypothesis: The experimental data of nucleus-nucleus collisions are the geometrical average of the corresponding data for the colliding processes at a fixed impact parameter, and the straight-line trajectories of the colliding nuclei determine their participating nucleons at a fixed impact parameter. On the other hand, on how these nucleons contribute to the final state of the collision process different models have different assumptions. For simplicity, we will assume that every participating nucleon can be regarded as a secondary particle source. In fact, such kind of assumption has been used in some models such as FRITIOF (Ref. 9) and the multisource model in the central rapidity region.[10]

If the average contribution of each participating nucleon to the MD is $g(n')$, the MD at fixed b is the superposition of the contributions of $N(b)$ participating nucleons:

$$G^{(N)}(n)=\sum_{n_1',n_2',\ldots}\delta\left[n-\sum_{i=1}^{N}n_i'\right]\prod_{i=1}^{N}g(n_i') . \tag{2}$$

The number $N(b)$ is determined by

$$\begin{aligned}N(b)&=N_P(b)+N_T(b) ,\\ N_P(b)&=\int d^3\mathbf{r}\,\rho_P(\mathbf{r})\theta(R_P-(x^2+y^2)^{1/2})\\ &\quad\times\theta(R_T-[(x-b)^2+y^2]^{1/2}) ,\\ N_T(b)&=\int d^3\mathbf{r}\,\rho_T(\mathbf{r})\theta(R_T-(x^2+y^2)^{1/2})\\ &\quad\times\theta(R_P-[(x-b)^2+y^2]^{1/2}) ,\end{aligned} \tag{3}$$

where ρ_P, ρ_T are the nuclear densities of the beam and the target, respectively. A Wood-Saxon distribution (WS)

$$\rho(r)=\frac{K 4\pi r^2}{1+\exp[(r-r_0 A^{1/3})/c_0]} \tag{4}$$

will be used provided $A>8$.[13] In Eq. (4) K is a normalization constant; the values of the parameter c_0 and the radius r_0 of the participating nucleons are taken from Ref. 13. The radii R_P and R_T of the maximal geometrical cross sections for the beam and the target are determined by

$$\int d^3\mathbf{r}\,\rho_a(\mathbf{r})\theta(r_0-[(x-R_a)^2+y^2]^{1/2})=1\ , \qquad a=P,T\ . \tag{5}$$

The meaning of this equation is that there is just one nucleon in the cylinder whose radius is r_0 and whose center is R_a away from the axis passing through the center of the nucleus a $(a=P,T)$ and perpendicular to the cross section being considered.

Putting $N=2$ in Eq. (2), we can determine $g(n')$ from the multiplicity distribution $G(n'')$ of the nucleon-nucleon collisions. (In this paper, the physical variables marked with primes and double primes represent those variables of a single participating nucleon and a nucleon-nucleon collision, respectively.) A possible form of $g(n')$ in the energy range of the CERN Intersecting Storage Rings has been given in Ref. 5. Bu the results in this paper are not concerned with its concrete form.

Taking into account all the processes with different b, the final-state multiplicity distribution of nucleus-nucleus collision is

$$P(n)=\sum_N p(N)G^{(N)}(n)\ . \tag{6}$$

The distribution $p(N)$ of the participating nucleons can be obtained from Eq. (3) as

$$p(N)\sim b(N)\frac{db}{dN}\ . \tag{7}$$

The curves for ^{16}O-Cu, Ag, Au collisions are shown in Fig. 1.

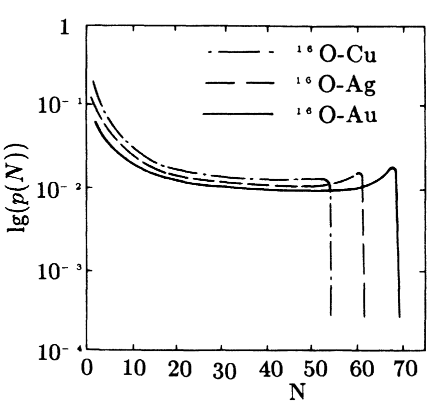

FIG. 1. The normalized distributions of the number of participating nucleons for ^{16}O-A collisions.

Similar to Ref. 14, we introduce generating functions $F_P(\theta)$ and $F_g(\theta)$ for nucleus-nucleus and nucleon-nucleon collisions, respectively. From Eq. (6),

$$\begin{aligned} F_P(\theta)&=\sum_n P(n)\theta^n=\sum_N p(N)[F_g(\theta)]^N\ , \\ F_g(\theta)&=\sum_{n'} g(n')\theta^{n'},\quad -1\le\theta\le 1\ . \end{aligned} \tag{8}$$

Differentiating Eq. (8) with respect to θ and making use of

$$\begin{aligned} &\left.\frac{\partial}{\partial\theta}F_P(\theta)\right|_{\theta=1}=\langle n\rangle,\quad \left.\frac{\partial}{\partial\theta}F_g(\theta)\right|_{\theta=1}=\langle n'\rangle\ , \\ &\left.\frac{\partial^2}{\partial\theta^2}F_P(\theta)\right|_{\theta=1}=\langle n(n-1)\rangle\ , \\ &\left.\frac{\partial^2}{\partial\theta^2}F_g(\theta)\right|_{\theta=1}=\langle n'(n'-1)\rangle,\ \ldots\ , \end{aligned} \tag{9}$$

the average multiplicity $\langle n\rangle$ and the normalized multiplicity moments C_q can be written as

$$\langle n\rangle=\langle N\rangle\langle n'\rangle=\tfrac{1}{2}\langle N\rangle\langle n''\rangle\ , \tag{10}$$

$$\begin{aligned} C_2&=\frac{\langle N^2\rangle}{\langle N\rangle^2}+\frac{d_2'}{\langle N\rangle}\ , \\ C_3&=\frac{\langle N^3\rangle}{\langle N\rangle^3}+\frac{3}{\langle N\rangle^3}(\langle N^2\rangle-\langle N\rangle)d_2'+\frac{1}{\langle N\rangle^2}d_3'\ , \\ C_4&=\frac{\langle N^4\rangle}{\langle N\rangle^4}+\frac{6}{\langle N\rangle^4}\langle N(N-1)^2\rangle d_2' \\ &\quad+\frac{3}{\langle N\rangle^4}\langle N(N-1)\rangle d_2'^2 \\ &\quad+\frac{4}{\langle N\rangle^4}\langle N(N-1)\rangle d_3'+\frac{1}{\langle N\rangle^3}d_4',\ \ldots\ , \end{aligned} \tag{11}$$

where

$$d_i'=\frac{\langle n'^i\rangle}{\langle n'\rangle^i}-1,\quad \langle N^i\rangle=\sum_N p(N)N^i\ . \tag{12}$$

Equation (10) can be directly tested against the existing data. In all the discussions above, the effects of leading particles are not included; therefore, from the conservation of isospin we have

$$\frac{\langle n_-\rangle}{\langle n_-''\rangle}=\frac{\langle n\rangle}{\langle n''\rangle}=\frac{\langle N\rangle}{2}\ . \tag{13}$$

The results for different targets are compared with the data in Fig. 2.

In Eq. (11), the first term of C_q is completely due to nuclear geometry. Now, let us prove that when the colliding nuclei are not too light and the rapidity window of final-state particles not very narrow, the first term dominates C_q. This means

$$C_q\equiv\frac{\langle n^q\rangle}{\langle n\rangle^q}\simeq\frac{\langle N^q\rangle}{\langle N\rangle^q}\ . \tag{14}$$

In order to do this, we first rewrite Eq. (11) as

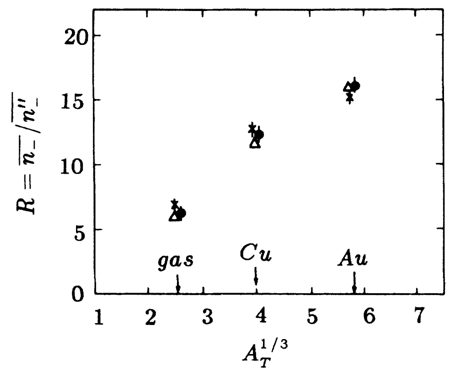

FIG. 2. The ratio R of the average multiplicity of negative particles observed in ^{16}O-gas (80% Ne, 20% He), Cu, Au collisions to that in nucleon-nucleon collisions at the same energy. Our results (△) from Eq. (13) are compared with the data (×, 60A GeV; ●, 200A GeV) from Ref. 1.

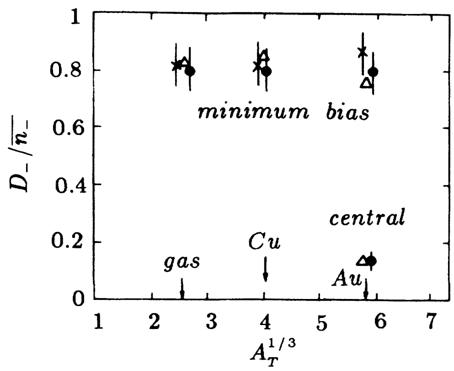

FIG. 3. The ratio of the dispersion to the average multiplicity for negative particles. Our results (△) are compared with the data (×, 60A GeV; ●, 200A GeV) from Ref. 1.

$$
\begin{aligned}
C_2 &= \frac{\langle N^2\rangle}{\langle N\rangle^2}\left[1+\frac{1}{\langle N\rangle}\left[\frac{\langle N\rangle^2}{\langle N^2\rangle}d'_2\right]\right],\\
C_3 &= \frac{\langle N^3\rangle}{\langle N\rangle^3}\left[1+\frac{1}{\langle N\rangle}\left[\frac{3\langle N\rangle\langle N^2\rangle}{\langle N^3\rangle}d'_2\right]\right.\\
&\qquad\left.+\frac{1}{\langle N\rangle^2}\left[\frac{\langle N\rangle^3}{\langle N^3\rangle}\right](d'_3-3d'_2)\right],\ \ldots
\end{aligned}
\tag{15}
$$

The terms in the square brackets of C_q are an expansion in terms of $1/\langle N\rangle$. The factors d'_i $(2\le i\le q)$ and $\prod_j \langle N^{q_j}\rangle/\langle N^q\rangle$ $(q_j\ge 1,\ \sum_j q_j=q)$ are involved in all the coefficients of this expansion except for the first term. It is ready to see that the inequality

$$\prod_j \langle N^{q_j}\rangle/\langle N^q\rangle<1 \quad \left[q_j\ge 1,\ \sum_j q_j=q\right] \tag{16}$$

is always satisfied (cf. Table I), regardless of whether we use the Wood-Saxon distribution or, in the first approximation, use the step function

$$\rho(r)=\rho_0 4\pi r^2\delta(r_0 A^{1/3}-r) \tag{17}$$

as the nuclear density distribution. The reason is that $p(N)$ is a smooth function for both of these two distributions in a wide region of N (cf. Fig. 1), and if we put $p(N)=\text{const}$, the left-hand side of Eq. (17) is equal to $(q+1)/\prod_j(q_j+1)<1$. In addition, d'_i is completely determined by the normalized multiplicity moments C''_q of the nucleon-nucleon collisions:

$$
\begin{aligned}
d'_2 &= 2(C''_2-1),\\
d'_3 &= 2(2C''_3-3C''_2+1),\\
d'_4 &= 4(2C''_4-3C''^2_2-4C''_3+9C''_2-4),\ \ldots
\end{aligned}
\tag{18}
$$

If the rapidity window is not very narrow, $d'_i\sim 1$. Hence, when the colliding nuclei are not too light and the peripheral interactions are not considered alone, so that $\langle N\rangle \gg 1$, the first term of C_q in Eq. (11) [or Eq. (15)] is much more important than the other terms and Eq. (14) is a good approximation.

III. INDEPENDENCE OF NORMALIZED MULTIPLICITY MOMENTS ON ENERGY, TARGET MASS, AND RAPIDITY WINDOW

(i) Equation (11) shows that the contribution of elementary nucleon-nucleon collisions to the nucleus-nucleus ones is sharply suppressed by the nuclear geometry. The normalized moments of the final-state particles for nucleus-nucleus collision are approximately equal to that of the participating nucleons. Hence, the energy scaling of the multiplicity moments remains valid for nucleus-nucleus collision, no matter how the MD's of the elementary nucleon-nucleon collisions vary with the incident energy. The D_-/n_- for different targets are calculated using the formulas given above and compared with the data in Fig. 3. It shows clearly this energy scaling.

(ii) If the pseudorapidity window $\Delta\eta$ is symmetric around the pseudorapidity η_c of the center of mass of two colliding nucleons in the laboratory frame, the final-state particles come from all of the N particle sources. There is no restriction on the position and width of the rapidity window in deriving Eq. (11) in Sec. II. The influence of

TABLE I. $\prod_j\langle N^{q_j}\rangle/\langle N^q\rangle$ of ^{16}O-Cu collisions $(q_j\ge 1,\ \sum_j q_j=q)$.

	$q=2$	$q=3$		$q=4$			
Distribution	$\frac{\langle N\rangle^2}{\langle N^2\rangle}$	$\frac{\langle N\rangle\langle N^2\rangle}{\langle N^3\rangle}$	$\frac{\langle N\rangle^3}{\langle N^3\rangle}$	$\frac{\langle N\rangle\langle N^3\rangle}{\langle N^4\rangle}$	$\frac{\langle N^2\rangle^2}{\langle N^4\rangle}$	$\frac{\langle N\rangle^2\langle N^2\rangle}{\langle N^4\rangle}$	$\frac{\langle N\rangle^4}{\langle N^4\rangle}$
WS	0.56	0.47	0.26	0.44	0.37	0.21	0.12
Step func.	0.67	0.60	0.40	0.57	0.50	0.34	0.23

phase space on C_q is just reflected in d_i' and is determined only by the normalized multiplicity moments of nucleon-nucleon collisions, as shown in Eq. (18). When the window is not very narrow ($\Delta\eta < 1$), $d_i' \sim 1$, Eq. (14) is always satisfied. Only when the window is so narrow that C_i'' increases sharply,[15] is Eq. (14) no longer effective. So, C_q is rapidity window independent provided that the window is not very narrow. Using the pp data[16] as input, according to Eq. (11), Tables II and III give the fluctuation of C_q moments in pseudorapidity windows together with the experimental data for nearly central collision events.

(iii) Similar to other popular models,[7–10] in this paper, for the dynamical aspects of nucleus-nucleus collision, the properties of nucleon-nucleon collisions are used as input. On the basis of this, the nuclear geometry is added.

The influence of geometry is twofold. First, when the impact parameter b is not too large, the number N of the participating nucleons is much larger than 1, $\langle N \rangle \gg 1$, for both minimum-bias and central events and dominates the collision. It is this property that was used in deriving Eq. (14). Therefore, the energy and rapidity-window independences of the normalized multiplicity moments are based on this property.

On the other hand, the number N of the participating nucleons will fluctuate around $\langle N \rangle$. This fluctuation is the second influence of the nuclear geometry. It directly effects the value of C_q.

Let us consider two extreme cases.

First, we discuss the case with no fluctuation, $N = \langle N \rangle$. It corresponds to a collision at fixed b. In this case,

$$\frac{\langle N^q \rangle}{\langle N \rangle^q} = 1 \quad [p(N) = \delta_{N\langle N \rangle}] , \tag{19}$$

the value of C_q is determined by the terms involving d_i' in Eq. (11). Because the contribution of these terms decreases with the increasing of N as mentioned above, we have

TABLE II. The values of $\langle n \rangle / D$, C_2, C_3, C_4, and S in different intervals of pseudorapidity ($\Delta\eta$) for nearly central ^{16}O-emulsion collisions at 200A GeV. The data (in parentheses) are from Ref. 3.

$\Delta\eta$	$\frac{\langle n \rangle}{D}$	C_2	C_3	C_4	S
0.25	1.60	1.39	2.23	3.97	1.32
	(1.58)	(1.40)	(2.40)	(4.86)	(1.33)
0.50	1.69	1.35	2.07	3.46	1.31
	(1.72)	(1.34)	(2.12)	(3.80)	(1.31)
1.00	1.75	1.33	1.98	3.18	1.31
	(1.77)	(1.32)	(2.02)	(3.43)	(1.30)
1.50	1.77	1.32	1.95	3.08	1.30
	(1.82)	(1.30)	(1.96)	(3.26)	(1.29)
2.00	1.79	1.31	1.92	3.02	1.30
	(1.86)	(1.29)	(1.92)	(3.13)	(1.28)
2.50	1.80	1.31	1.91	2.98	1.30
	(1.88)	(1.28)	(1.90)	(3.07)	(1.27)
3.00	1.81	1.31	1.90	2.95	1.30
	(1.91)	(1.27)	(1.87)	(3.02)	(1.26)

TABLE III. The same as Table II for 200A-GeV ^{32}S-emulsion collisions.

$\Delta\eta$	$\frac{\langle n \rangle}{D}$	C_2	C_3	C_4	S
0.25	2.13	1.22	1.62	2.30	1.18
	(1.99)	(1.25)	(1.80)	(2.89)	(1.21)
0.50	2.24	1.20	1.54	2.10	1.18
	(2.22)	(1.20)	(1.62)	(2.39)	(1.18)
1.00	2.31	1.19	1.50	1.99	1.18
	(2.31)	(1.19)	(1.56)	(2.20)	(1.18)
1.50	2.34	1.18	1.48	1.95	1.17
	(2.40)	(1.17)	(1.52)	(2.11)	(1.16)
2.00	2.36	1.18	1.47	1.92	1.17
	(2.46)	(1.16)	(1.49)	(2.04)	(1.15)
2.50	2.37	1.18	1.47	1.91	1.17
	(2.51)	(1.16)	(1.48)	(2.01)	(1.15)
3.00	2.38	1.18	1.46	1.90	1.17
	(2.54)	(1.15)	(1.46)	(1.98)	(1.15)

$$C_q'' = C_q(2) \geq C_q(N) \simeq 1 . \tag{20}$$

For central events, $N \gg 1$, $C_q(N) \simeq 1$, $D_-/n_- = \sqrt{C_2 - 1} \simeq 0$, as shown in Fig. 3.

We now turn to another extreme case: The probability for different participating nucleons is the same: $p(N) = \text{const}$. This means that there is no peak at $N = \langle N \rangle$. From simple deduction, we have

$$\frac{\langle N^q \rangle}{\langle N \rangle^q} = \frac{2^q}{q+1} \quad [p(N) = \text{constant}] . \tag{21}$$

For minimum-bias samples, there is a wide plateau in the middle part of N (cf. Fig. 1). As a rough approximation, $p(N)$ can be treated as a constant, and

$$C_q \simeq \frac{\langle N^q \rangle}{\langle N \rangle^q} = \frac{2^q}{q+1} \tag{22}$$

is independent of the target mass. But in the region where N is small, $p(N)$ increases rapidly with the decreasing of N as shown in Fig. 1. Physically this is because, when b is so large that the collision approaches peripheral interaction, the corresponding geometrical cross section is very large. This increase of $p(N)$, on one hand, violates the strict target mass independence of C_q [cf. Fig. 3, values of $D = \sqrt{C_2 - 1}$ are roughly equal to 0.8 for ^{16}O-gas, Cu, Au collisions], and, on the other hand, makes $\langle N^q \rangle / \langle N \rangle^q$ larger than that in Eq. (21):

$$C_q > \frac{2^q}{q+1} \quad (\text{minimum bias}) . \tag{23}$$

For example, $C_2 > 4/3$, $D_- / \langle n_- \rangle > 1/\sqrt{3}$, as shown in Fig. 3.

For nearly central events, the fluctuating range of N around $\langle N \rangle$ is neither so small as for central events nor so wide for minimum-bias samples. Therefore, the values of C_q in Tables II and III are larger than that in central events and smaller than that in minimum-bias samples.

From the above discussions, we see that when the number of participating nucleons is large enough and the rapidity window is not too narrow, the characteristics of the normalized moments are mainly determined by nuclear geometry. To analyze dynamical fluctuations beyond geometry, Hwa[17] expressed the correlation between C_2 moments and average multiplicity $\langle n \rangle$ as

$$C_2 \langle n \rangle = 1 + S \langle n \rangle \ . \tag{24}$$

Substituting Eq. (10) into Eq. (24), the local slope S can be written as

$$S = C_2 - \frac{2}{\langle N \rangle} \langle n'' \rangle \ . \tag{25}$$

In Tables II and III, the agreement of our results with the data indicates that if there exists collective dynamics beyond that involved in nucleon-nucleon collisions, it is not sensitive to multiplicity distributions.

IV. SUMMARY

As mentioned in Sec. II, in this paper, we have used the following two hypotheses to calculate the multiplicity distribution $G^N(n)$ in the process with fixed participating nucleons N. First, the number of secondary particle sources is just equal to N, calculated from purely geometrical consideration. Second, each source contributes to the process independently. In fact, the latter has been tested by experiments and is popularly used in the models treating high-energy nucleon collisions. For example, in hadron-nucleus collisions, one feature of the data[18] is that $R = \langle n \rangle / \langle n'' \rangle$ may be parametrized as

$$R = \tfrac{1}{2}(1 + \langle \nu \rangle) \ . \tag{26}$$

This means that there are $(1 + \langle \nu \rangle)$ independent particle sources, and each source contributes $\frac{1}{2}\langle n'' \rangle$ particles on the average. In principle, the calculation of the number of these sources is concerned with both nuclear geometry and dynamics. On the basis of the dual parton model,[8] if $q\bar{q}$ chains are neglected, $\langle \nu \rangle$ can be calculated by purely geometry of the target nucleus and interpreted as the average number of hadron-nucleon collisions experienced by the hadron when it passes through the target. This simple geometrical treatment agrees approximately with the data.[7] Obviously, the average number $\langle \nu \rangle$ of hadron-nucleon collisions and the average number $\langle N_T \rangle$ of participating nucleons in the target are of the same for hadron-nucleus collisions

$$R = \tfrac{1}{2}(1 + \langle N_T \rangle) \ . \tag{27}$$

This means that there is no difference whether the elementary hadron-nucleon collisions or the participating nucleons are treated as the particle sources.

For high-energy nucleus-nucleus collisions, the dominant part played by the nuclear geometry in the determination of normalized multiplicity moments C_q is discussed in some detail in this paper. It manifests itself in two respects: The hugeness of the average number of participating nucleons in minimum-bias and central nucleus-nucleus collisions, $\langle N \rangle \gg 1$, results in the energy and rapidity-window independence of C_q. The fluctuation of N around $\langle N \rangle$ determines the value of C_q and its approximate target-mass independence. The agreement of our results with the experimental data shows that the nuclear geometry can really reproduce the global features, such as multiplicity distribution, of high-energy nuclear collisions.

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