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Rapid Communications

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Possible evidence for Abelian dominance in quark confinement

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Performing numerical simulations and Abelian projection in SU(2) QCD, we find that Wilson loops composed of a residual U(1) gauge field alone show a strong enhancement when the Abelian projection is done in a U(1)-covariant gauge. Remarkably, the Creutz ratios determined from the Abelian Wilson loops approach the scaling curve with $\sqrt{\sigma}\sim 58\Lambda_L$. Nonrenormalizable unitary gauges do not show such interesting types of behavior.

It is crucial in QCD to understand the mechanism of quark confinement. 't Hooft[1] and Mandelstam[2] have conjectured that the dual Meissner effect due to color-magnetic-monopole condensation is responsible for color-charge confinement. Especially interesting is 't Hooft's idea of Abelian projection.[3] When one fixes the gauge degrees of freedom in such a way that the maximal torus group remains unbroken, QCD can be regarded as an Abelian theory with color charges and color magnetic monopoles. If the monopoles make Bose condensation, color charges and then quarks are confined.[4] In this scheme, however, there seems a difficult problem concerning how to choose the Abelian group, i.e., a gauge choice problem.

Using numerical simulations, Kronfeld *et al.*[5] have tested the picture in SU(2) gauge theory in comparison with compact U(1) lattice gauge theory. The latter is known to realize confinement due to monopole condensation.[6,7] Evaluating Abelian monopole currents directly, they have shown that the monopole condensation also occurs in SU(2) gauge theory and that the deconfinement mechanism may be understood in terms of color magnetic monopoles. However the effect of the monopole condensation seems to depend on the gauge choice. A U(1)-covariant gauge (specified later) looks more favorable than nonrenormalizable unitary gauges.

In this Rapid Communication we report another interesting Monte Carlo test of the Abelian confinement mechanism, evaluating various Wilson loops in four-dimensional SU(2) gauge theory. If the Abelian degrees of freedom would play the dominant role in color confinement after the Abelian projection as suggested in Ref. 3, one could expect some significant effects in Wilson loops which are composed of a residual Abelian gauge field. We find possible evidence for Abelian dominance in the quark confinement mechanism.

We adopt the usual Wilson action expressed in terms of the product of link gauge variables $U(s,\hat{\mu})$ around a plaquette. We consider two types of gauge fixings. The U(1)-covariant gauge in the lattice theory is given in the manner of Kronfeld *et al.*[5] by performing a local gauge transformation $V(s)$ such that

$$R=\sum_{s,\hat{\mu}}\mathrm{Tr}[\sigma_3\tilde{U}(s,\hat{\mu})\sigma_3\tilde{U}^\dagger(s,\hat{\mu})] \tag{1}$$

is maximized. Here $\tilde{U}(s,\hat{\mu})=V(s)U(s,\hat{\mu})V^{-1}(s+\hat{\mu})$. In this gauge,

$$X_1(s)=\sum_{\mu}[U(s,\hat{\mu})\sigma_3U^\dagger(s,\hat{\mu}) + U^\dagger(s-\hat{\mu},\hat{\mu})\sigma_3U(s-\hat{\mu},\hat{\mu})] \tag{2}$$

is diagonalized. This corresponds to $D_\mu A^{+\mu}=(\partial_\mu+iga_\mu)\times A^{+\mu}=0$, where a_μ ($A^{\pm\mu}$) are (off) diagonal gluons.

As nonrenormalizable unitary gauges, we choose two (composite) adjoint fields $X_i(s)$ $(i=2,3)$ which are made diagonal. They are

$$X_2(s)=U(s,\hat{1})U(s+\hat{1},\hat{2})U^\dagger(s+\hat{2},\hat{1})U^\dagger(s,\hat{2}) \tag{3}$$

and

$$X_3(s)=\prod_{t=0}^{L_4-1}U(s+t\hat{4},\hat{4})\,, \tag{4}$$

where L_4 is the extent of the lattice in the fourth direction.

Since $X_i(s)$ is a functional of $U(s,\hat{\mu})$, a gauge function $V(s)$ which diagonalizes $X_i(s)$ is also a functional of $U(s,\hat{\mu})$. It is important to see the transformation property of $V(s)$ under any SU(2) transformation $W(s)$. Let us fix the U(1) ambiguity of $V(s)$ in some way. Then we see

$$V(s)\rightarrow V^W(s)=d(s)V(s)W^{-1}(s)\,, \tag{5}$$

where $d(s)$ [$\in$ U(1)] is determined uniquely by $V(s)$ and $W(s)$.

Using the definition $\tilde{U}(s,\hat{\mu})=V(s)U(s,\hat{\mu})V^\dagger(s+\hat{\mu})$ and (5), we get

$$[\tilde{U}(s,\hat{\mu})]^W=d(s)\tilde{U}(s,\hat{\mu})d^\dagger(s+\hat{\mu}) \tag{6}$$

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for any SU(2) $W(s)$. Hence all U(1)-invariant quantities composed of $\tilde{U}(s,\hat{\mu})$ are automatically SU(2) invariant after the Abelian projection.

It is possible to separate the diagonal U(1) gauge field uniquely from $\tilde{U}(s,\mu)$:

$$\tilde{U}(s,\hat{\mu})=A(s,\hat{\mu})u(s,\hat{\mu})\,. \tag{7}$$

These transform as $A^{W}(s,\hat{\mu})=d(s)A(s,\hat{\mu})d^{\dagger}(s)$ and $u^{W}(s,\hat{\mu})=d(s)u(s,\hat{\mu})d^{\dagger}(s+\hat{\mu})$, that is, $u(s,\hat{\mu})$ transforms as a U(1) gauge field, whereas $A(s,\hat{\mu})$ as a matter field.

The Abelian Wilson loop $W_3(c)$ is defined as a path-ordered product of $u(s,\hat{\mu})$ around some loop c. This is U(1) invariant and then also SU(2) invariant. Define $B(s,\hat{\mu})=V^{\dagger}(s)A^{\dagger}(s,\hat{\mu})V(s)$ and $\hat{U}(s,\hat{\mu})=B(s,\hat{\mu})\times U(s,\hat{\mu})$. One finds from (7) that $W_3(c)$ can be expressed also as a path-ordered product of $\hat{U}(s,\hat{\mu})$ around the same loop. Since $B(s,\hat{\mu})$ transforms as the adjoint representation, $\hat{U}(s,\hat{\mu})$ transforms like $U(s,\hat{\mu})$. This also shows the SU(2) invariance of $W_3(c)$. It is to be emphasized that the Abelian Wilson loops defined in different gauges correspond to different SU(2)-invariant quantities. Using the Stokes theorem, we find $W_3(c)$ measures a flux expressed in terms of an SU(2) gauge-invariant Abelian field strength first introduced by 't Hooft[8,9] in connection with a classical monopole solution.

We perform Monte Carlo simulations on 8^4, 10^4, and 12^4 lattices at various values of $\beta(=4/g^2)$ and on a 16^4 lattice from $\beta=2.4$ to $\beta=3.0$ using HITAC S820/80 at the National Laboratory for High-Energy Physics (KEK) and at Tokyo University and FACOM VP400 at Kyoto University. We apply a vectorized heat-bath algorithm to generate gauge configurations, and then, in the covariant gauge, choose appropriate ones by maximizing R in (1). The covariant gauge fixing is done such that quantity corresponding to $\langle|D_\mu A^{+\mu}|^2\rangle$ in the continuum becomes less than 10^{-4}, which needs an additional several hundred iterations for the gauge fixing. Some data are calculated under a more severe condition ($\langle|D_\mu A^{+\mu}|^2\rangle<10^{-5}$) in order to see whether or not gauge fixing is satisfactory. In the unitary gauges, gauge functions are fixed analytically. All measurements are done typically every ten sweeps after a thermalization of 1000 sweeps. We use the so called "jackknife method"[10] which is well suited to estimating statistical errors reliably. We obtain the Creutz ratios whose statistical errors are always less than 10%, which needs about 3000 Monte Carlo sweeps on the 16^4 lattice. The Monte Carlo link update of a 16^4 lattice can be done in 0.27 sec and the gauge-fixing time is typically 10.5 sec on an HITAC S820/80 computer.

Let us show our results of various Wilson loops. An Abelian Wilson loop $W_3(c)$ composed of the Abelian link variable $u(s,\hat{\mu})$ shows interesting behavior after the Abelian projection using the U(1)-covariant gauge. Before the gauge fixing, the 1×1 Wilson loop behaves like $8\beta/81$ as derived by the strong-coupling expansion. After the covariant gauge fixing, however, the Wilson loop enhances considerably as shown in Fig. 1. It is totally unexpected that 1×1 $W_3(c)$ obeys $\beta/4$ in the strong-coupling region which is given by the strong-coupling expansion of the full non-Abelian 1×1 Wilson loop. Also

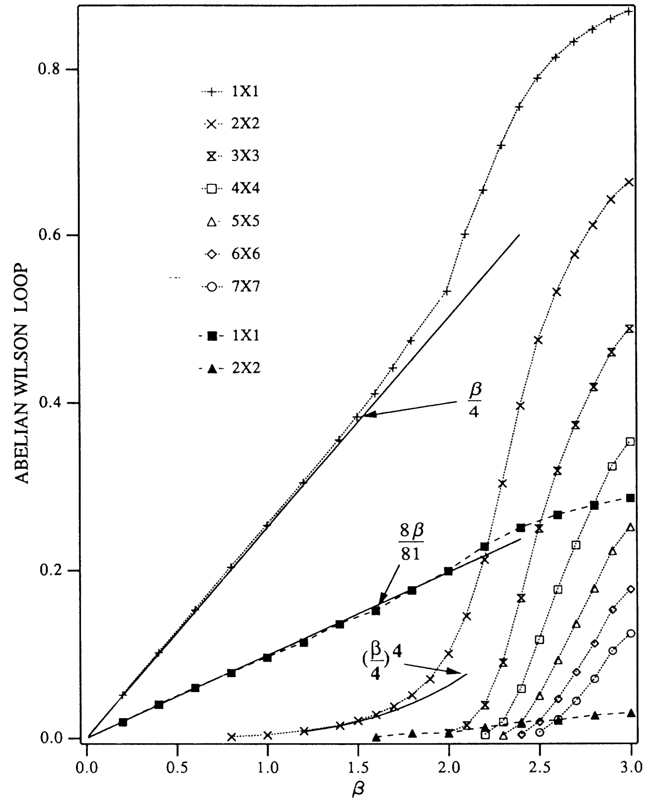

FIG. 1. Abelian Wilson loops before (solid squares and solid triangles) and after (other symbols) Abelian projection in the U(1)-covariant gauge. The upper and the lower solid lines denote the curves in the strong-coupling expansion of the 1×1 and the 2×2 full Wilson loops. The middle solid line denotes the curve $8\beta/81$ in the strong-coupling expansion before the Abelian projection.

2×2 $W_3(c)$ lies on the curve $(\beta/4)^4$ for small β. We check that the full Wilson loops are really similar to $W_3(c)$ for small β up to around $\beta\sim1.5$. It should be noticed that the Abelian and the full Wilson loops are different SU(2)-invariant quantities. If the system would reduce simply to the compact U(1) theory after the Abelian projection, 1×1 $W_3(c)$ should behave like $\beta/2$ for small β. Our results deny such a simple scenario.

To study gauge dependence, we evaluate the same quantity in the unitary gauges using $X_2(s)$ and $X_3(s)$. Figure 2 is the case with $X_2(s)$. Since the gauge condition breaks rotational invariance, the Wilson loops depend on the plane they lie on. In Fig. 2 is the data when the Wilson loop is on the 3-4 plane. Other types of Wilson loops are smaller in the scaling region, although they are about the same for small β. Similar results are obtained for $X_3(s)$. No large enhancement is observed. It is much the same as that without the gauge fixing especially in the strong-coupling region. Such a clear distinction between two types of gauges is also unexpected.

Data of the Creutz ratios are more surprising. They are presented in Fig. 3 for the Abelian Wilson loop in the covariant gauge. We also measure the quantity for the full SU(2) Wilson loops. Comparison of the two data indicates that they agree with each other in the strong-

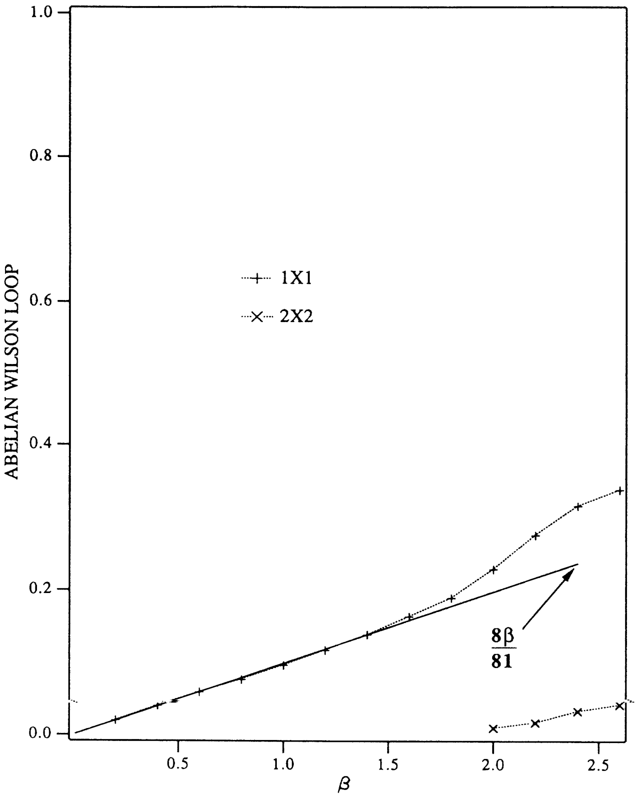

FIG. 2. Abelian Wilson loops after Abelian projection in the unitary gauge. The solid line shows the curve in the strong-coupling expansion before the Abelian projection.

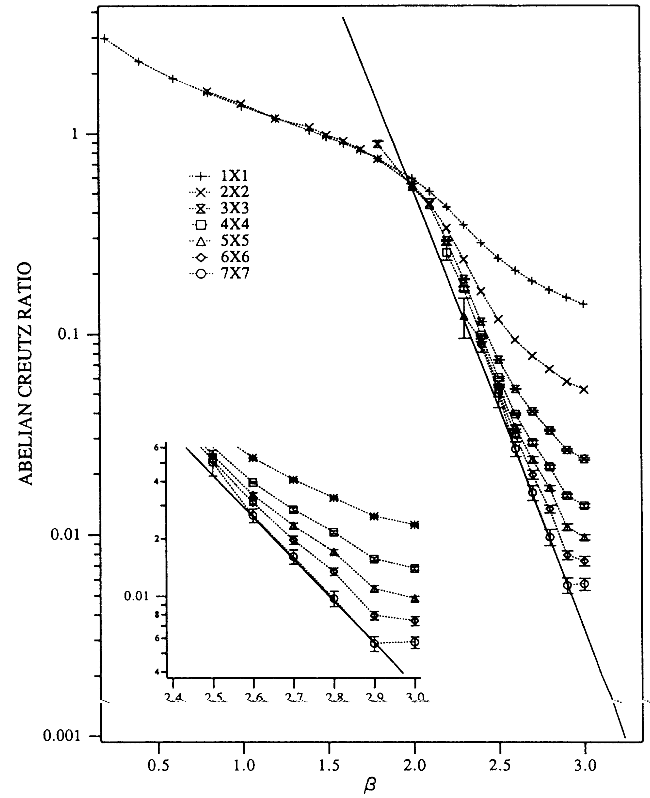

FIG. 3. Creutz ratios from the Abelian Wilson loops. The solid line shows the scaling curves with $\sqrt{\sigma}=58\Lambda_L$ for reference.

coupling region. In the weak-coupling region, the Abelian Creutz ratios approach the scaling curve with $\sqrt{\sigma}\sim 58\Lambda_L$ in the scaling region. The S/N ratio is better in the $W_3(c)$ case than in the full case when compared in the same CPU time. We have obtained the clear data of Abelian $\chi_3(7,7)$, whereas full $\chi(7,7)$ cannot have been determined due to large noises. See the recent data gathered by Campostrini *et al.*[11] On the 16^4 lattice they perform a total of 15500 sweeps. Nevertheless, they could not get sensible data of $\chi(7,7)$ before cooling. After several cooling steps, the noises are drastically reduced and they obtain the string tension $\sqrt{\sigma}\sim 58\Lambda_L$. Also we note that the finite-size effects are smaller in the Abelian case.

We have also tried to evaluate the Creutz ratios in the unitary gauges. Even more sweeps than those in the covariant case makes meaningful only the small-size ratios which are, however, much contaminated with large finite-size effects. To derive the scaling limit of the string tension seems impossible in the unitary gauges.

In conclusion, we have carried out Monte Carlo simulations and the Abelian projection. We have observed very interesting results of the Abelian Wilson loops when we adopt the U(1)-covariant gauge. Then the Abelian Wilson loops show large enhancement after the Abelian projection. The strong-coupling types of behavior coincide with those of the full SU(2) Wilson loops. QCD is not reduced simply to the compact U(1) theory after the Abelian projection. Such behaviors critically depend on the gauge choice. The Abelian Wilson loop corresponding to nonrenormalizable unitary gauges do not give rise to such an enhancement. The Creutz ratios derived from the Abelian Wilson loops in the covariant gauge approach the scaling curve in agreement with that of the full Wilson loop which is obtained after several cooling steps.

It is interesting to study, in the strong-coupling expansion, why the Abelian Wilson loops corresponding to different gauge choice show such a clear distinction. The property of the quantity $B(s,\hat{\mu})$ defined above is important. It is possible to show that the same area law as that of the full Wilson loop is obtained when the Abelian Wilson loop with large enough size is considered in a local gauge like one with $X_2(s)$. In such a gauge, $V(s)$ and $B(s,\hat{\mu})$ are determined only by near-link variables around $U(s,\hat{\mu})$. However, the U(1)-covariant gauge is very nonlocal. We need informations of all link variables to fix $V(s)$. Hence it is impossible analytically to prove such area-law behavior in the covariant gauge in strong-coupling expansion. Nevertheless, the Monte Carlo data show the area law from small β to large β in the nonlocal U(1)-covariant gauge.

Smit and van der Sijs[12] have recently proposed that a classical solution (the so-called dyon solution) may be responsible for quark confinement. It is interesting to study whether the monopole found in Ref. 5 is a lattice artifact as in compact QED or is an object corresponding to the classical solution.

The details and other data will be published elsewhere.[13]

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[1] G. 't Hooft, in *High Energy Physics*, edited by A. Zichichi (Editrice Compositori, Bologna, 1976).

[2] S. Mandelstam, Phys. Rep. **23C**, 245 (1976).

[3] G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).

[4] T. Suzuki, Prog. Theor. Phys. **80**, 929 (1988); S. Maedan and T. Suzuki, *ibid.* **81**, 229 (1989); T. Suzuki, *ibid.* **81**, 752 (1989).

[5] A. S. Kronfeld *et al.*, Phys. Lett. B **198**, 516 (1987); A. S. Kronfeld, G. Schierholz, and U.-J. Wiese, Nucl. Phys. **B293**, 461 (1987).

[6] T. Banks, R. Myerson, and J. Kogut, Nucl. Phys. **B129**, 493 (1977); J. L. Cardy, *ibid.* **B170**, 369 (1980).

[7] T. DeGrand and D. Toussaint, Phys. Rev. D **22**, 2478 (1980).

[8] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974).

[9] J. Arafune, P. G. O. Freund, and C. J. Goebel, J. Math. Phys. **16**, 433 (1975).

[10] B. Efron, SIAM Rev. **21**, 460 (1979).

[11] M. Campostrini, A. Di Giacomo, M. Maggiore, H. Panagopoulos, and E. Vicari, Phys. Lett. B **225**, 403 (1989).

[12] J. Smit and A. van der Sijs, Amsterdam Report No. ITFA-89-17, 1989 (unpublished).

[13] I. Yotsuyanagi, S. Kiura, T. Suzuki, Y. Matsubara, and K. Shiokawa (unpublished).