Renormalization group, BRST cohomology, and the problem of confinement

Reinhard Oehme
*Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637,**
Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg-Institut für Physik, Munich, Federal Republic of Germany
(Received 2 August 1990)

Provided the number of matter fields is limited, the structure analysis of the gauge-field propagator results in superconvergence relations which provide a link between long- and short-distance properties of the theory. The information contained in these relations is combined with specific consequences of the Becchi-Rouet-Stora-Tyutin algebra in order to argue for the confinement of transverse gauge-field excitations (gluons) if the number of flavors is less than ten for quantum chromodynamics. With a larger number of matter fields (between ten and sixteen flavors in QCD), confinement is not required. Hence there could be a phase transition. The question of quark confinement is considered only briefly.

I. INTRODUCTION

Superconvergence relations of structure functions are obtained on the basis of analyticity and with the help of renormalization-group methods. They provide a link between high-energy and low-energy features of a gauge theory with asymptotic freedom. In combination with results following from the Becchi-Rouet-Stora-Tyutin (BRST) algebra, we use this information in order to give arguments for the confinement of transverse gauge-field excitations in continuum gauge theories with a limited number of matter fields (less than ten flavors in quantum chromodynamics). Confinement is understood in the sense that the corresponding states are not representatives of physical states. With the number of matter fields above a certain value, while retaining asymptotic freedom, our methods provide no restriction. Confined and unconfined phases are possible in principle, but the latter situation appears quite plausible, so that a phase transition is a possibility. Questions of quark confinement will be discussed only briefly and are deferred to a later paper. Within the framework described above, the emergence of an approximately linear quark-antiquark potential has been discussed in another publication.[1]

A preliminary account of our work may be found in Ref. 2. Some of the results have been presented in Ref. 3. In several papers,[4] Nishijima has discussed confinement using the structure analysis of Refs. 5 and 6 and BRST methods, but employing a noncovariant formalism. His conclusions are somewhat more restricted, but in agreement with ours.

As has been pointed out, the tools of our analysis are the BRST algebra and the structure analysis of the gauge-field propagator. Even though the structure functions of propagators are gauge-dependent quantities and do not represent directly physical observables, they nevertheless contain important information about the theory. When we find, working in the Landau gauge, for example, that certain states do not belong to the physical subspace and hence do not contribute to the unitarity relations of the S matrix, we have a gauge-dependent argument for a physical result.

We consider the gauge-field theory in isolation and ignore the fact that it actually should be embedded into a more comprehensive scheme. At least near the Planck mass, it is to be expected that such an embedding becomes important. But as far as confinement is concerned, we assume that it is not the dominant feature.

Non-Abelian gauge theories, as constrained systems, are most easily quantized within the framework of the BRST symmetry.[7] The theory is formulated in a covariant fashion in a state space $\mathcal{V}$ of indefinite metric, which contains many unphysical excitations such as ghosts, antighosts, longitudinal and timelike gauge quanta, and possibly others.[8–10] The BRST and ghost number operators form a graded algebra which can be used to define a physical subspace as a particular cohomology group of the nilpotent BRST operator Q.

We certainly expect that excitations corresponding to ghosts and to longitudinal and timelike gluons do not belong to the physical subspace of quantum chromodynamics (QCD). This should be the case even in the weak-coupling perturbation expansion, which is the formal asymptotic limit of the theory for $g^2 \rightarrow +0$. However, in the nonperturbative theory, we expect that there is a phase where gluons and quarks are confined so that they are not created as free particles in collisions. There are many approaches to the problem of confinement. Here we concentrate on covariant continuum gauge theory. We say that certain excitations are confined if they do not belong to the physical subspace of the general state space $\mathcal{V}$, the subspace being defined by the BRST cohomology. For ghosts and similar unphysical quanta, this confinement is kinematical and persists also in perturbation theory. In contrast, for gluons and quarks, it is of dynamical origin and a nonperturbative effect.

The definition of the physical subspace $\mathcal{H}$ in terms of a BRST cohomology is invariant under equivalence transformations and under Poincaré transformations.[8,11–15] Equivalence transformations are "unitary" mappings in the general state space $\mathcal{V}$ with indefinite metric. They leave invariant matrix elements of BRST-invariant (physical) operators between physical states and therefore do not change any physical result of the theory. For the study of confinement, we need to isolate states of the form $A^{\mu\nu}|0\rangle$ which are representatives of physical states defined by the cohomology $\mathcal{H}$. Here $A^{\mu\nu}$ is the curl of the gauge field operator. In a specific frame with respect to the transformations mentioned above, this is quite possible. The BRST algebra induces a decomposition of the state space $\mathcal{V}$ into orthonormal subspaces $\mathcal{V}_p$ and $\mathcal{V}_u$, where $\mathcal{V}_p$ is isomorphic to $\mathcal{H}$. Further, $\mathcal{V}_p$ is a nondegenerate subspace of $\mathcal{V}$ so that there is a well-defined self-adjoint projection operator $P(\mathcal{V}_p)$. But a direct projection is not invariant under Lorentz and equivalence transformations, and it is necessary to proceed more indirectly. As a first step, we use a projection P_+ which selects states of positive norm.[5,6] We only need to consider the states $P_+\tilde{A}_a^{\mu\nu}(-k)|0\rangle$ or, more conveniently, $\Psi_+=P_+\Psi$, where

$$\Psi(c,k^2)=\int d^4q\,\delta(q^2-k^2)c_{\mu\nu}^a(q)\tilde{A}_a^{\mu\nu}(-q)|0\rangle\ . \qquad (1.1)$$

Here the coefficients $c_{\mu\nu}^a(q)$ are test functions, and $\tilde{A}_a^{\mu\nu}(-q)$ is the Fourier transform of the curl of the gauge-field operator. The norm of Ψ in $\mathcal{V}$ is given directly by the discontinuity $\rho(k^2)$ of the transverse gluon propagator, except for a positive factor involving the test functions. The norm of Ψ_+ is consequently related to $\rho_+(k^2)=\rho(k^2)$ for $\rho>0$ and zero otherwise. States corresponding to possible multipole terms contributing to $\rho(k^2)$ are not included in ρ_+, which is therefore essentially a positive measure. We can divide the positive-norm states Ψ_+ into two classes C_p and C_{u+} which satisfy $Q\Psi_p=0$ or $Q\Psi_{u+}\neq 0$, respectively, where Q is the nilpotent BRST operator. For a given $k^2\geq 0$, we have then either $\rho_+=\rho_p$ or $\rho_+=\rho_{u+}$. Since no new dimensionful parameter is introduced in this decomposition, the discontinuities ρ_p and ρ_{u+} are defined in a renormalization-group-invariant way. In general, they can have support in nonoverlapping regions of the real, positive k^2 axis. The states Ψ_p are representatives of the elements of $\mathcal{H}$, the physical subspace, while the Ψ_{u+} are unphysical states which, by a similarity transformation, can be brought into a form having no component in $\mathcal{V}_p$. Lorentz and similarity transformations leave C_p and C_u invariant, and hence ρ_p and ρ_{u+} are also invariantly defined.

In addition to the BRST structure of the state space, our arguments for confinement rest upon the structure analysis of the gluon propagator.[5,6] The general principles of gauge-field theories imply analytic properties of the structure function, and an extensive use of the renormalization group provides asymptotic expressions which depend upon the number of matter fields (number of flavors N_F for QCD). It is an important result of this analysis that the structure function $D(k^2)$ of the transverse gauge-field propagator vanishes faster than k^{-2} in all directions of the cut k^2 plane provided $\gamma_{00}<0$ and $\beta_0<0$ (corresponding to less than ten flavors in QCD: $N_F<10$). Here γ_{00} and β_0 are the usual lowest-order coefficients of the anomalous dimension of the gauge-field operator in the Landau gauge and of the renormalization-group function. Hence we have the superconvergence relation for $\pi\rho(k^2)=\mathrm{Im}D(k^2+i0)$:

$$\int_{-0}^{\infty}dk^2\rho(k^2)=0\ . \qquad (1.2)$$

For $\gamma_{00}>0$, $\beta_0<0$ ($10\leq N_F\leq 16$ in QCD), this relation is not valid.

With the help of the projection P_+ and the classes C_p and C_u described above, the structure analysis can be extended to D_+ and D_p with the discontinuities ρ_+ and ρ_p, respectively. But in these cases the renormalization-group equations have two different solutions, *a priori*. For $\gamma_{00}<0$, $\beta_0<0$, ($N_F<10$), we find that D_+ is either identically zero, or it diverges for $g^2\to+0$ and fixed k^2 at least like $(g^2)^{-\gamma_{00}/\beta_0}$. The situation is the same for D_p.

In the case of D_p, which via ρ_p has only contributions from physical states, we can argue from unitarity and from other requirements that the divergent solution is excluded for $N_F<10$. Then ρ_p vanishes, except possibly at a set of points of measure zero, which are not of physical interest. This indicates that all positive-norm states Ψ_+ of the form (1.1) must be unphysical states Ψ_{u+}. Transverse gauge quanta are not in the physical subspace $\mathcal{H}$; they must be confined in this sense. On the other hand, with $D_p=0$ we have $D_+=D_{u+}$. Here we can choose only the second and divergent solution. Otherwise the theory becomes inconsistent. There are no difficulties with the divergence of D_+ for $g^2\to+0$. Since all contributing states are unphysical, they do not contribute to unitarity relations involving physical operators and states.

We emphasize again that the superconvergence relations used in our arguments imply a connection between low- and high-energy properties of the gauge theory. They are not simply consequences of the asymptotic behavior, but depend in an important way upon the cut-plane analyticity, which follows from general principles. Because of these features, the superconvergence relations make it possible to obtain results about problems such as confinement, which are basically connected with low-energy features of the theory.

For $\gamma_{00}>0$, $\beta_0<0$ ($10\leq N_F\leq 16$ for QCD), the theory looks quite different from the confined case described above. There is no superconvergence, and consequently our methods do not require confinement. However, it is also not excluded that the confined phase persists for other reasons. Nevertheless, within the framework we have considered, it is not implausible that there is a phase transition as γ_{00} changes sign as a function of the number of matter fields (between $N_F=9$ and $N_F=10$ for QCD). In this case the condition $\gamma_{00}<0$ would be necessary and not only sufficient for confinement. It is to be expected that, with increasing numbers of matter fields (number of flavors N_F), the corresponding vacuum polarization provides the appropriate screening of color charges. It has been suggested by Nishijima[16] to use numerical simulations in order to look for an indication of a phase transition in QCD as a function of N_F. Such calculations may become feasible in the near future.[17]

In this article we discuss only gluon confinement. An analogous approach to quark confinement with the help of the quark propagator is not possible. There are no superconvergence relations for the corresponding relevant structure function. However, the results concerning gluon confinement may be connected with the criterion for general color confinement given by Kugo and Ojima[8] and discussed further by Nishijima.[18] The sufficiency of this condition has been shown rigorously, and there are indications that it may also be necessary.[19] The necessity, together with our results on gluon confinement, would also give an argument for quark confinement provided $\gamma_{00}<0, \beta_0<0$.

On a more phenomenological level, it is possible to make a connection between the superconvergence of the gluon propagator function $D(k^2)$ for $\gamma_{00}<0$, $\beta_0<0$ ($N_F<10$) and an approximately linear potential between static quark color charges.[16,1,3]. Again the superconvergence relation and the negative sign of the discontinuity $\rho(k^2)$ for larger values of k^2 are essential. We have discussed these matters in Ref. 1.

II. BRST COHOMOLOGY

For our discussion of confinement, certain properties of the state space $\mathcal{V}$ and the physical subspace $\mathcal{H}$ will be of importance. In this section we summarize the essential tools[8,11–15] and introduce invariant classes of states.

In a non-Abelian gauge theory, we can construct a nilpotent and self-adjoint BRST-operator Q. As a consequence, the state space $\mathcal{V}$ of the theory has indefinite metric. In addition to an anti-BRST operator, there is also a self-adjoint ghost number operator N_c which provides a grading of $\mathcal{V}$. The operator Q satisfies the commutation relation

$$i[N_c,Q]=Q \tag{2.1}$$

and changes the ghost number by one unit. But we do not need to consider the ghost number explicitly in the following, nor the anti-BRST operator. Of course, as far as the physical subspace is concerned, we are interested only in states with ghost number zero.

The space $\mathcal{H}$ of physical states is defined by the "BRST cohomology"

$$\mathcal{H}\equiv \ker Q/\mathrm{im}Q \ , \tag{2.2}$$

where the kernel is the subspace

$$\ker Q=\{Q\Psi=0, \quad \Psi\in\mathcal{V}\} \ , \tag{2.3}$$

and the image is given by

$$\mathrm{im}Q=\{\Psi=Q\Phi, \ \Phi\in\mathcal{V}\} \ . \tag{2.4}$$

We have $\mathrm{im}Q\subset\ker Q$. Because of

$$Q^2=0 \ , \tag{2.5}$$

it follows that all states $\Psi\subset\mathrm{im}Q$ have zero norm and are orthogonal to the states of $\ker Q$.

In quantum chromodynamics we expect that all zero-norm states of $\ker Q$ belong to $\mathrm{im}Q$. Then the norm of states in $\ker Q$ is of one sign only, and we can choose $\mathcal{H}$ to have a positive-definite metric as desired for a physical subspace.[14]

We denote the general inner product in the indefinite metric space $\mathcal{V}$ by

$$(\Psi,\Phi), \quad \Psi\in\mathcal{V}, \quad \Phi\in\mathcal{V} \ . \tag{2.6}$$

In order to obtain a complete decomposition of $\mathcal{V}$, we introduce a self-adjoint involution $\mathcal{C}$ so that the indefinite metric inner product (2.6) in $\mathcal{V}$ is converted into a definite product, which we denote by[11–15]

$$(\Psi,\Phi)_{\mathcal{C}}\equiv(\Psi,\mathcal{C}\Phi) \ . \tag{2.7}$$

We have

$$\mathcal{C}^{\dagger}=\mathcal{C} \quad \text{and} \quad \mathcal{C}^2=1 \ . \tag{2.8}$$

The BRST-operator Q is self-adjoint with respect to the inner product (2.6) in $\mathcal{V}$:

$$(\Psi,Q\Phi)=(Q\Psi,\Phi) \quad \text{or} \ Q^{\dagger}=Q \ . \tag{2.9}$$

With respect to the $\mathcal{C}$ product (2.7), we have then

$$(\Psi,Q\Phi)_{\mathcal{C}}=(Q^*\Psi,\Phi)_{\mathcal{C}} \ , \tag{2.10}$$

and the definition (2.7) implies

$$Q^*=CQC \ . \tag{2.11}$$

We call Q^* the conjugate BRST operator.[14] It is also nilpotent and self-adjoint in $\mathcal{V}$:

$$Q^{*2}=0 \quad \text{and} \ Q^{*\dagger}=Q^* \ , \tag{2.12}$$

in accordance with Eq. (2.9). For vanishing ghost number, the cohomology

$$\mathcal{H}'=\ker Q^*/\mathrm{im}Q^* \tag{2.13}$$

is then isomorphic to $\mathcal{H}$ as defined in Eq. (2.2): $\mathcal{H}'\simeq\mathcal{H}$. Q and Q^* are adjoint operators with respect to the product (2.10). Hence we find that

$$(\mathrm{im}Q)_{\perp}=\ker Q^*, \quad (\mathrm{im}Q^*)_{\perp}=\ker Q \ , \tag{2.14}$$

and we can write the decompositions

$$\mathcal{V}=\ker Q\oplus\mathrm{im}Q^*=\ker Q^*\oplus\mathrm{im}Q \ , \tag{2.15}$$

which imply, in view of the isomorphism $\mathcal{H}'\simeq\mathcal{H}$,

$$\mathcal{V}=\mathcal{V}_p\oplus\mathrm{im}Q\oplus\mathrm{im}Q^* \ , \tag{2.16}$$

where $\mathcal{V}_p$ is isomorphic to $\mathcal{H}$.[8,11–15] This result is obtained by showing that $\mathcal{H}$ and $\mathcal{H}'$ are isomorphic to $\ker Q\cap\ker Q^*$.

While $\mathcal{V}_p$, $\mathrm{im}Q$, and $\mathrm{im}Q^*$ are orthogonal subspaces of $\mathcal{V}$ with respect to the $\mathcal{C}$ product (2.7), in connection with the indefinite metric product (2.6), the spaces $\mathrm{im}Q$ and $\mathrm{im}Q^*$ have zero norm. They are orthogonal to $\mathcal{V}_p$. But states in $\mathrm{im}Q$ and $\mathrm{im}Q^*$ are paired so that for every $\Psi\in\mathrm{im}Q$ there is a $\Psi'\in\mathrm{im}Q^*$ so that $(\Psi,\Psi')\neq0$.

The operators Q and Q^* carry ghost number $+1$ and -1, respectively. We can define the BRST Laplacian Δ as an operator with ghost number zero by[12–15]

$$\Delta \equiv \{Q, Q^*\} = -(Q - Q^*)^2 . \tag{2.17}$$

The Laplacian Δ commutes with Q, Q^* and $\mathcal{C}$. The kernel of Δ directly gives the cohomology of Q or of Q^*:

$$\mathcal{H} = \ker\Delta . \tag{2.18}$$

For our later discussions, we are interested in mappings U in $\mathcal{V}$ which leave the inner product (2.6) invariant:

$$(U\Psi, U\Phi) = (\Psi, \Phi), \quad \forall \Psi, \Phi \in \mathcal{V} . \tag{2.19}$$

The operators U are "unitary" in this sense, and we write formally $U^\dagger = U^{-1}$. Further, we want to impose the restriction that these mappings, like observable operators, commute with Q and consequently leave $\ker Q$ and $\mathrm{im} Q$ invariant. With respect to the $\mathcal{C}$ inner product (2.1), we can then write U as a matrix corresponding to the decomposition (2.16):

$$U = \begin{bmatrix} U_{11} & 0 & U_{13} \\ U_{21} & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} , \tag{2.20}$$

where the indices refer to the subspaces $\mathcal{V}_p$, $\mathrm{im}Q$, and $\mathrm{im}Q^*$, respectively. The unitarity condition is then given by

$$U^\dagger = \mathcal{C} U^* \mathcal{C} = U^{-1} , \tag{2.21}$$

and implies a set of relations between the operator U_{ij}. The BRST operator can be written, in a representation corresponding to Eq. (2.20), as

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & q \\ 0 & 0 & 0 \end{bmatrix} \tag{2.22}$$

and $[Q, U] = 0$ implies $U_{22} q = q U_{33}$. Note that, in the matrix representation, the states are written in the form

$$\Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} , \tag{2.23}$$

and the inner product in $\mathcal{V}$ is given by

$$(\Psi, \Phi) = (\Psi, \mathcal{C}\Phi)_{\mathcal{C}} = \Psi_1^* \Psi_1 + \Psi_2^* \Psi_3 + \Psi_3^* \Psi_2 , \tag{2.24}$$

with $\mathcal{C}$ having the representation

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} . \tag{2.25}$$

From the relation $\ker\Delta = \mathcal{H}$ in Eq. (2.18), it follows that all zeros of the Laplacian Δ are associated with the cohomology $\mathcal{H}$ of Q, which is isomorphic to $\mathcal{V}_p$. Expressing Δ in terms of the suboperator q appearing in Eq. (2.22), we then infer that q is invertible. A more complete discussion of these matters, including connections to the index theorem, may be found in Refs. 13–15.

Let A be an operator in $\mathcal{V}$ which is BRST invariant and of the form (2.20). If Ψ_p and Φ_p are states in $\ker Q$, satisfying $Q\Psi_p = Q\Phi_p = 0$, then we have

$$(\Psi_p, U^{-1} A U \Phi_p) = \Psi_{p1}^* U_{11}^* A_{11} U_{11} \Phi_{p1} , \tag{2.26}$$

since the states have representations

$$\Psi_p = \begin{bmatrix} \Psi_{p1} \\ \Psi_{p2} \\ 0 \end{bmatrix}, \quad \Phi_p = \begin{bmatrix} \Phi_{p1} \\ \Phi_{p2} \\ 0 \end{bmatrix} . \tag{2.27}$$

We see that only the suboperator U_{11} of U is involved in the transformation of matrix elements of BRST-invariant operators with respect to states in $\ker Q$, which includes physical operators between physical states. Consequently we call all mappings U with $U_{11} = 1$ "equivalence transformations." They do not change any physical quantities.

Other BRST-invariant unitary operators of the type (2.20) are the representations of the invariance groups of the theory. In particular, the Poincaré transformations $U(a, \Lambda)$ are important for us. Also in this case, only the elements $U_{11}(a, \Lambda)$ are of physical relevance, since they transform the physical quantities as indicated in Eq. (2.26).

We now define three exclusive classes of states $\Psi \in \mathcal{V}$:

(1) A state $\Psi = \Psi_p$ belongs to the class C_p if it satisfies $Q\Psi_p = 0$ and has positive norm. Then Ψ_p must be of the form (2.27) with a nonvanishing component in $\mathcal{V}_p$, i.e., $\Psi_{p1} \neq 0$. By an equivalence transformation with $U_{11} = 1$, we get

$$U\Psi_p = \begin{bmatrix} \Psi_{p1} \\ U_{21}\Psi_{p1} + U_{22}\Psi_{p2} \\ 0 \end{bmatrix} , \tag{2.28}$$

with the U_{ij} satisfying Eq. (2.21). We can then find operators U_{21} and U_{22} such that $U\Psi_p$ has only components in $\mathcal{V}_p$. The states in the class C_p are representatives of physical states defined by the cohomology $\mathcal{H}$.

(2) A state $\Psi = \Psi_u$ belongs to the class C_u if $Q\Psi_u \neq 0$. In components, this means that we must have $\Psi_{u3} \neq 0$ since

$$Q\Psi_u = \begin{bmatrix} 0 \\ q\Psi_{u3} \\ 0 \end{bmatrix} . \tag{2.29}$$

In general, an equivalence transformation with $U_{11} = 1$ implies

$$U\Psi_u = \begin{bmatrix} \Psi_{u1} + U_{13}\Psi_{u3} \\ U_{21}\Psi_{u1} + U_{22}\Psi_{u2} + U_{23}\Psi_{u3} \\ U_{33}\Psi_{u3} \end{bmatrix} . \tag{2.30}$$

Either $\Psi_{u1} = 0$, *a priori*, or because $\Psi_{u3} \neq 0$, we can find an U with U_{13} such that $\Psi_{u1} + U_{13}\Psi_{u3} = 0$. This implies that we can transform the state Ψ_u into a form where it has no component in $\mathcal{V}_p$. The states in the class C_u are unphysical. They have positive, negative, or zero norm in the space $\mathcal{V}$. As we have seen, a state in C_u may well have a nonvanishing component in $\mathcal{V}_p$, but it can be transformed away with the help of an equivalence transformation which does not change any of the physical consequences of the theory. The features described above show that a projection operator $P(\mathcal{V}_p)$ from $\mathcal{V}$ into $\mathcal{V}_p$ does not commute with these transformations, nor with Poincaré transformations $U(a,\Lambda)$. Projections into $\ker Q$ have an analogous problem; in addition, the projection operator $P(\ker Q)$ is not self-adjoint since $\ker Q$ is a degenerate subspace. We rather have $P^\dagger(\ker Q) = P(\ker Q^*)$.

(3) There is a third class of states, denoted by C_0, which satisfy $Q\Psi_0=0$ and have zero norm. They have nonvanishing components in $\mathrm{im}Q$ only, and hence are of the form

$$\Psi_0=\begin{bmatrix}0\\ \Psi_{02}\\ 0\end{bmatrix} . \tag{2.31}$$

The classes C_p, C_u, and C_0 are exclusive. In particular, we later need the fact that a state with negative norm must belong to C_u, while a state with positive norm is either in C_p or C_u. Another important advantage is that the classes, and in particular C_u, are invariant under Poincaré transformations and equivalence transformations. This follows from the previous discussion. For C_u it must be noted in addition that

$$U_{33}\Psi_{u3}\neq 0 \quad \text{if } \Psi_{u3}\neq 0 , \tag{2.32}$$

since Eq. (2.21) implies

$$U_{22}U_{33}^\dagger = U_{33}U_{22}^\dagger = 1 . \tag{2.33}$$

We note that C_p and C_0 are linear vector spaces, but C_u is not. A sum of unphysical states, where the components in $\mathrm{im}Q^*$ cancel, gives rise to a state in C_0 or in C_p. In the latter case we would get a representative of a physical state. A related feature of C_u is that sum of a physical state and an unphysical state results in an unphysical state. For instance, let us consider the states $\Psi_{p1}\in\mathcal{V}_p$ and $\Psi_3\in\mathrm{im}Q^*$; in components

$$\Psi_{p1}=\begin{bmatrix}\psi_{p1}\\ 0\\ 0\end{bmatrix},\quad \Psi_3=\begin{bmatrix}0\\ 0\\ \psi_3\end{bmatrix} . \tag{2.34}$$

Then $\Psi_u=\Psi_{p1}+\lambda\Psi_3$ is an unphysical state for $\lambda\neq 0$. We see that $\lambda\Psi_3=\Psi_u-\Psi_{p1}$ is a null vector. Hence there is no relevance to the magnitude of λ as far as the situation described above is concerned. The states Ψ_{p1} and Ψ_u have the same norm. Given $\lambda\Psi_3\neq 0$, we can find an equivalence transformation U which brings Ψ_u into the form

$$\Psi_u'=U\Psi_u=\begin{bmatrix}0\\ \Psi_2'\\ \Psi_3'\end{bmatrix} , \tag{2.35}$$

where the component in $\mathcal{V}_p$ is removed.

We also remark that the subset of states in $\mathcal{V}$ with positive norm is in general not a subspace. Given the existence of a state $\Psi_+\in\mathcal{V}$ with positive norm, we can write any $\Psi\in\mathcal{V}$ as $\Psi=\Phi_\lambda+\lambda\Psi_+$, where $\Phi_\lambda=\Psi-\lambda\Psi_+$ has positive norm for sufficiently large values of $|\lambda|$, even if Ψ has negative norm.[20]

The features of the classes of states described above can already be studied explicitly in very simple examples. For instance, we may consider the Fock space of noninteracting quantum electrodynamics. In the one-particle sector for a given momentum, one has the two transverse photon states as physical states, and the longitudinal and timelike photons as unphysical states. All these states are orthogonal to each other in $\mathcal{V}$, and the latter has negative norm. The sum and the difference of the unphysical states are zero-norm states in the subspaces $\mathrm{im}Q^*$ and $\mathrm{im}Q$, respectively. Of course, the tranverse photon states are in $\mathcal{V}_p$.

Although some of the properties of states in $\mathcal{V}$ as described in this section, may appear to be unconventional, they are typical for indefinite metric spaces.[11–15,20] For our arguments in the following, we only have to decide whether or not positive-norm states of the form $\tilde{A}^{\mu\nu}(-k)|0\rangle$ are representatives of physical states. This is a rather straightforward problem as far as the cohomology is concerned, but because we have to preserve relativistic covariance, we need the tools collected in this section.

III. SUPERCONVERGENCE

The second ingredient of importance for our discussion of confinement is the structure of the gluon propagator. Detailed derivations have been given in previous papers.[5,6] Hence we restrict ourselves to a summary of the assumptions and a formulation of the results as they are needed in the following.

We assume the usual postulates of covariant gauge theories with a state space of indefinite metric. As spectral conditions, we require only non-negative eigenvalues of $P_\mu P^\mu$ and of P^0, where P^μ is the energy-momentum operator. At least for the first few orders of perturbation theory in powers of the gauge coupling parameter g, we assume that the exact Green's functions are connected with the corresponding formal perturbation expansion in the limit $g^2\to +0$. This requirement appears to be natural in view of the asymptotic freedom of QCD and the physical relevance of the weak-coupling limit. The connection with perturbation theory is assumed only for unprojected Green's functions within the framework of the general state space $\mathcal{V}$. Projections may not commute with the limit $g^2\to +0$.

Since later we will also introduce projected propagators, it is convenient to define the transverse gluon propagator in terms of

$$A_a^{\mu\nu}(x)=\partial^\mu A_a^\nu(x)-\partial^\nu A_a^\mu(x) . \tag{3.1}$$

In this way the longitudinal part is eliminated in all cases, and the two point function is completely determined by the transverse structure function $D_c(k^2)$. We write

$$\int d^4x\, e^{+ik\cdot x}\langle 0|TA_a^{\mu\nu}(x)A_b^{\rho\sigma}(0)|0\rangle$$
$$= -i\delta_{ab}D_c(k^2)$$
$$\times(k^\mu k^\rho g^{\nu\sigma}-k^\mu k^\sigma g^{\nu\rho}+k^\nu k^\sigma g^{\mu\rho}-k^\nu k^\rho g^{\mu\sigma}) , \quad (3.2)$$

where use has been made of relativistic covariance, the invariance of the ground state $|0\rangle$, and the unbroken global gauge symmetry. As a consequence of Lorentz invariance and spectral conditions, the structure function $D_c(k^2)$ is the boundary value of an analytic function:

$$D_c(k^2)=D(k^2+i0) \text{ for real } k^2\geq 0 , \quad (3.3)$$

where $D(k^2)$ is analytic in the cut k^2 plane. *A priori,* with the space-time expression for the two-point function being a tempered distribution, the analytic function $D(k^2)$ is bounded by a polynomial for $k^2\to\infty$.

We obtain further information for the asymptotic behavior of $D(k^2)$ from a renormalization-group analysis. As explained in the Introduction, it is sufficient and convenient to use the Landau gauge, and to consider the fields as functions of x, g, and κ^2, where $\kappa^2<0$ is the renormalization point:

$$-k^2D(k^2,\kappa^2,g)\equiv R\left[\frac{k^2}{\kappa^2},g\right]=1 \text{ for } k^2=\kappa^2<0 . \quad (3.4)$$

Possible intrinsic quark masses can be accommodated with the help of a mass-independent renormalization scheme.[21] The renormalization-group equation for R can be written in various forms. At first, we use the relation

$$R\left[\frac{k^2}{\kappa^2},g\right]=R\left[\left|\frac{k^2}{\kappa^2}\right|,g\right]R\left[e^{i\phi},\bar{g}\left[\left|\frac{k^2}{\kappa^2}\right|,g\right]\right] \quad (3.5)$$

with

$$R(u,g)=\exp\left[\int_{g^2}^{\bar{g}^2(u,g)}dx\,\gamma(x)/\beta(x)\right] . \quad (3.6)$$

Here $k^2=-|k^2|e^{i\phi}$, $|\phi|\leq\pi,\bar{g}(u,g)$ is the effective coupling, $\beta(g^2)$ the renormalization-group function and $\gamma(g^2)=\gamma(g^2,\ \alpha=0)$ the anomalous dimension of the gauge field. In the limit $g^2\to+0$, we have $\beta(g^2)=\beta_0g^4+\beta_1g^6+\cdots,\gamma(g^2,\alpha)=(\gamma_{00}+\alpha\gamma_{01})g^2+\cdots$. For QCD, the coefficients are given by

$$\beta_0=-(16\pi^2)^{-1}(11-\tfrac{2}{3}N_F) ,$$

$$\gamma_{00}=-(16\pi^2)^{-1}(\tfrac{13}{2}-\tfrac{2}{3}N_F) ,$$

$$\gamma_{01}=(16\pi^2)^{-1}3/2 ,$$

with N_F being the number of flavors. We always assume $\beta_0<0$ corresponding to asymptotic freedom. With the limits

$$\lim_{u\to\infty}\bar{g}^2(u,g)\simeq(-\beta_0\ln u)^{-1}+\cdots ,$$
$$\lim_{g^2\to+0}R\left[\frac{k^2}{\kappa^2},g\right]\simeq 1+\gamma_{00}g^2\ln\frac{k^2}{\kappa^2}+\cdots , \quad (3.7)$$

we obtain for the asymptotic behavior of the structure function

$$D(k^2,\kappa^2,g)\simeq -C_V(g^2)k^{-2}\left[\ln\left|\frac{k^2}{\kappa^2}\right|\right]^{-\gamma_{00}/\beta_0} \quad (3.8)$$

for $k^2\to\infty$ in *all directions* in the complex k^2 plane. The coefficient $C_V>0$ is given by

$$C_V(g^2)=(g^2|\beta_0|)^{-\gamma_{00}/\beta_0}$$
$$\times\exp\left[\int_0^{g^2}dx\left[\frac{\gamma_{00}}{\beta_0x}-\frac{\gamma(x)}{\beta(x)}\right]\right] . \quad (3.9)$$

The renormalization group is needed here only for g^2 in the neighborhood of $g^2=+0$, where it is controlled by the asymptotic expressions of $\beta(g^2)$ and $\gamma(g^2)$, and where possible higher fixed points are not relevant. Also in $C_V(g^2)$, we consider only values of g^2 below possible higher zeros of $\beta(g^2)$.

For the discontinuity $\pi\rho=\operatorname{Im}D(k^2+i0)$, we find correspondingly

$$\rho(k^2,\kappa^2,g)\simeq -C_V(g^2)\frac{\gamma_{00}}{\beta_0}k^{-2}\left[\ln\frac{k^2}{|\kappa^2|}\right]^{-\gamma_{00}/\beta_0-1} , \quad (3.10)$$

where $k^2\to+\infty$ along the real axis. In view of the analytic properties of $D(k^2)$ and the asymptotic behavior (3.8) in all directions, we obtain the unsubtracted dispersion relation

$$D(k^2)=\int_{-0}^{\infty}dk'^2\frac{\rho(k'^2)}{k'^2-k^2} . \quad (3.11)$$

Because of Eq. (3.10), this formula can be rewritten as a dipole representation

$$D(k^2)=\int_{-0}^{\infty}dk'^2\frac{\sigma(k'^2)}{(k'^2-k^2)^2} \quad (3.12)$$

with

$$\sigma(k^2)\equiv\int_{-0}^{k^2}dq^2\rho(q^2) , \quad (3.13)$$

where for $k^2\to+\infty$

$$\sigma(k^2)\simeq C_V(g^2)\left[\ln\frac{k^2}{|\kappa^2|}\right]^{-\gamma_{00}/\beta_0} . \quad (3.14)$$

These relations are of interest for quark confinement. They have been discussed in Ref. 1 in connection with an approximately linear quark-antiquark potential.

The discontinuity ρ of the structure function is also of importance because it is characterized by the norm of the states $A_a^{\mu\nu}|0\rangle$. Using the Fourier transform of the

Heisenberg operator

$$\tilde{A}_a^{\mu\nu}(k)=\int d^4x\, e^{ik\cdot x} A_a^{\mu\nu}(x)\ , \tag{3.15}$$

we have

$$\langle 0|\tilde{A}_a^{\mu\nu}(k')\tilde{A}_b^{\rho\sigma}(-k)|0\rangle=\delta_{ab}\delta(k'-k)\theta(k^0)\pi\rho(k^2)\times(-2)(2\pi)^4\times(k^\mu k^\rho g^{\nu\sigma}-k^\mu k^\sigma g^{\nu\rho}+k^\nu k^\sigma g^{\mu\rho}-k^\nu k^\rho g^{\mu\sigma})\ . \tag{3.16}$$

With test functions $c^a_{\mu\nu}(k)=-c^a_{\nu\mu}(k)$, the norm of the states

$$\Psi(c)=\int d^4k\, c^a_{\mu\nu}(k)\tilde{A}_a^{\mu\nu}(-k)|0\rangle \tag{3.17}$$

is then given by

$$(\Psi(c),\Psi(c))=\int d^4k\ \theta(k^0)\pi\rho(k^2)C(k)\ , \tag{3.18}$$

with $C(k)>0$, where, for $k^2\geq 0, k^0\geq 0$:

$$C(k)\equiv -8(2\pi)^4 k^\mu \bar{c}^a_{\mu\nu}(k)k^\rho c^a_{\rho\sigma}(k)g^{\nu\sigma}\ . \tag{3.19}$$

Hence the sign of the norm in Eq. (3.18) is determined solely by $\rho(k^2)$. It is convenient to introduce states with a fixed value of k^2 by writing

$$\Psi(c,k^2)=\int d^4q\ \delta(q^2-k^2)c^a_{\mu\nu}(q)\tilde{A}_a^{\mu\nu}(-q)|0\rangle\ . \tag{3.20}$$

They have the norm

$$(\Psi(c,k'^2),\Psi(c,k^2))=\pi\rho(k^2)\delta(k'^2-k^2)\times\int d^4q\ \theta(q^0)\delta(q^2-k^2)C(q)\ , \tag{3.21}$$

which is directly given by $\rho(k^2)$ except for a positive factor. For possible multipole states, the representation (3.20) is less useful. But these states are not particularly important for our discussion of confinement. They have been considered in Refs. 5 and 6.

The formulas (3.8) and (3.10) exhibit a critical dependence of the theory upon the sign of γ_{00}/β_0. We assume asymptotic freedom corresponding to $\beta_0<0$. For $\gamma_{00}/\beta_0>0$ (corresponding to $N_F<10$ in QCD), it then follows that $D(k^2)$ vanishes faster than k^{-2} for $k^2\to\infty$. Hence we have a *superconvergence relation*[5,6]

$$\int_{-0}^{\infty} dk^2\rho(k^2,\kappa^2,g)=0\ . \tag{3.22}$$

Furthermore, as seen from Eq. (3.10), the sign of the discontinuity ρ is opposite to that of γ_{00}/β_0 for sufficiently large values of k^2, and hence it is negative for $\gamma_{00}/\beta_0>0$.

The distribution properties of $\rho(k^2)$ have been discussed in detail in Refs. 5 and 6. Here we mention only that there must be a smallest $K^2(\kappa^2,g)\geq 0$ so that for $\gamma_{00}/\beta_0>0$ the discontinuity ρ is a negative measure for $k^2>K^2$. The position $K^2(\kappa^2,g)$ must be invariant with respect to the renormalization group:

$$K^2(\kappa^2,g)=K^2(\kappa'^2,g') \tag{3.23}$$

with

$$\frac{\kappa'^2}{\kappa^2}=u(g',g)\equiv\exp\left[\int_{g^2}^{g'^2}dx\ \beta^{-1}(x)\right]\ . \tag{3.24}$$

In the weak-coupling limit $g^2\to+0$, we have then

$$K^2(\kappa^2,g)\simeq C_K\kappa^2\exp(1/\beta_0 g^2)(g^2)^{\beta_1/\beta_0^2}+\cdots\ , \tag{3.25}$$

so that K^2 vanishes exponentially.

Let us note here also that the states $\Psi(c,k^2)$ introduced in Eq. (3.20), considered as functions of k^2 and g^2, are related by

$$\Psi(c,k^2,g)=\sqrt{R(u(g',g),g)}\Psi(c,k'^2,g')u^{-1}(g',g) \tag{3.26}$$

for renormalization-group equivalent points (k^2,g) and (k'^2,g'), where either

$$g'=\bar{g}\left[\left|\frac{k^2}{k'^2}\right|,g\right]\quad\text{or}\quad k'^2=k^2u^{-1}(g',g)\ , \tag{3.27}$$

and where κ^2 is fixed. Here $\bar{g}(u,g),\bar{g}(1,g)=g$, is the effective coupling, and $u(g',g)$ has been defined in Eq. (3.24). The factor $\sqrt{R}$ is determined by Eq. (3.6) as

$$\sqrt{R(u(g',g),g)}=\exp\left[\tfrac{1}{2}\int_{g^2}^{g'^2}dx\ \gamma(x)/\beta(x)\right]\ . \tag{3.28}$$

In Eq. (3.26) we have assumed that the test functions in Eq. (3.20) do not depend upon κ^2 and g, and that they scale like $c^{\mu\nu}(k)=y^{-2}c^{\mu\nu}(ky^{-1})$ in accordance with their dimension.

In contrast with the case $\gamma_{00}/\beta_0>0,\beta_0<0$, there is no superconvergence relation if $\gamma_{00}/\beta_0<0,\beta_0<0$ $(10\leq N_F\leq 16$ for QCD), and the discontinuity ρ is positive for k^2 larger than some $K^2(\kappa^2,g)\geq 0$.

As we have mentioned before, the superconvergence relation (3.22) provides a connection between infrared and ultraviolet properties of the theory. It is not simply a consequence of the renormalization group, which has been used in order to obtain the asymptotic behavior in the complex k^2 plane. A major role is played by the analyticity of the structure function in the complex plane cut along the positive real axis, which follows from Lorentz covariance and simple spectral properties. The relation (3.22) is not satisfied in weak-coupling perturbation theory, nor in the so-called improved perturbation theory, where one sums an infinite sequence of subgraphs. In the latter case, the structure function has an asymptotic behavior corresponding to that of the exact expression (3.8), but there are Landau-type singularities on the negative real k^2 axis. These singularities contribute to the superconvergence relations so that the resulting formula is quite different from Eq. (3.22). A more detailed discussion will be given elsewhere.

We close this section with a few remarks about general covariant gauges, although, as has been explained in the Introduction, it is sufficient for our purpose to work in the Landau gauge. The situation for arbitrary values of the gauge parameter α has been considered in Ref. 3. We find, for the leading asymptotic term of the structure function R, taking the case $0<\gamma_{00}/\beta_0<1$ as an example,

$$R\left[\frac{k^2}{\kappa^2},g,\alpha\right]\simeq\frac{\alpha}{\alpha_0}+C_V(g,\alpha)\left[\ln\left[\frac{k^2}{\kappa^2}\right]\right]^{-\gamma_{00}/\beta_0}, \quad (3.29)$$

where $\gamma(g,\alpha)=(\gamma_{00}+\alpha\gamma_{01})g^2+\cdots$ for $g^2\to+0$ and $\alpha_0\equiv-\gamma_{00}/\gamma_{01}$. For $\gamma_{00}/\beta_0>0$, there is a modified superconvergence relation

$$-\frac{\alpha}{\alpha_0}+\int_{-0}^{\infty}dk^2\rho(k^2,\kappa^2,g,\alpha)=0\ . \quad (3.30)$$

The discontinuity has the asymptotic form

$$\rho(k^2,\kappa^2,g,\alpha)\simeq-\frac{\gamma_{00}}{\beta_0}C_V(g,\alpha)\times k^{-2}\left[\ln\frac{k^2}{|\kappa^2|}\right]^{-\gamma_{00}/\beta_0-1}, \quad (3.31)$$

and hence the same functional dependence as in the Landau gauge, where $\alpha=0$.

The use of general covariant gauges in the following discussion of confinement is in principle possible. But it involves considerable complications, and we do not consider it in this paper.

IV. GLUON CONFINEMENT

For the discussion of confinement, it is important to consider projected propagators. These have also been explored in detail in previous publications.[5,6] Ideally, we would like to use a Lorentz-covariant formulation of the projection into the physical subspace as defined in Sec. II by the cohomology of the BRST operator. But this is complicated because the projections into $\mathcal{V}_p$ or into ker$Q$ are not Lorentz invariant. It may be possible in principle to obtain a satisfactory formulation by using operator cohomologies. However, in this paper we employ more conventional methods. The only projection which we use is concerned with states of the form $\tilde{A}_a^{\mu\nu}(-k)|0\rangle$ which have positive norm. We write

$$P_+\tilde{A}_a^{\mu\nu}(-k)|0\rangle\ , \quad (4.1)$$

where the operation P_+ selects only those states for which the norm is positive in the sense that the discontinuity ρ in Eq. (3.16) is a positive distribution, which we denote by ρ_+. *A priori,* the weight function ρ is only known to be a temperate distribution. We can write

$$\rho(k^2)=\tau(k^2)+\sum_j\sum_{n=1}^{n_j}c_{n,j}\delta^{(n)}(k^2-K_j^2(\kappa^2,g))\ , \quad (4.2)$$

where $\tau(k^2)$ is a measure except at isolated singular points. The multipole terms involve finite-order derivatives of δ functions. They correspond to states which are not eigenstates of P^2, where P^μ is the energy-momentum operator. Rather, these states satisfy $(P^2-k_j^2)^n\Phi_j=0$, with appropriate powers $n\geq1$. In defining ρ_+, we omit the multipole terms and consider only the positive part of $\tau(k^2)$. For normalizable states, $\rho_+(k^2)$ is then a positive measure. More details may be found in Ref. 6.

As far as the renormalization-group equations are concerned, they are satisfied separately by the contributions from multipole terms and from $\tau(k^2)$ in Eq. (4.2). The multipoles do not contribute to the superconvergence relation (3.22). As we have discussed in Sec. 3, we know that for $k^2>K^2(\kappa^2,g)$, with some $K^2(\kappa^2,g)\geq0$, the weight function ρ is a negative or positive measure for $\gamma_{00}/\beta_0\gtrless0$, respectively. Consequently there are no multipole terms in this region, and the points K_j^2 in Eq. (4.3) are restricted by $0\leq K_j^2\leq K^2$. More detailed discussions of the distribution aspects of ρ may be found in Refs. 5 and 6. The positive-norm states for fixed k^2 are

$$\Psi_+(c,k^2)=P_+\Psi(c,k^2)\ , \quad (4.3)$$

with $\Psi(c,k^2)$ as given in Eq. (3.20). The norm of Ψ_+ is directly proportional to ρ_+ and hence positive:

$$(\Psi_+(c,k'^2),\Psi_+(c,k^2))=\pi\rho_+(k^2)\delta(k'^2-k^2)\times\int d^4q\ \theta(q^0)\delta(q^2-k^2)C(q)\ , \quad (4.4)$$

with $C(q)>0$ as defined in Eq. (3.19).

We now introduce a projected propagator with the structure function $D_+(k^2)$ by

$$\int d^4x\ e^{ik\cdot x}\langle0|TA_a^{\mu\nu}(x)P_+A_b^{\rho\sigma}(0)|0\rangle=-i\delta_{ab}D_+(k^2+i0)(k^\mu k^\rho g^{\nu\sigma}-k^\mu k^\sigma g^{\mu\rho}+k^\nu k^\sigma g^{\mu\rho}-k^\nu k^\rho g^{\mu\sigma})\ , \quad (4.5)$$

since the projection is defined in a Lorentz-covariant fashion. If we want to see more explicitly how $D_+(k^2+i0)$ is given in terms of the basic projected discontinuity $\rho_+(k^2)$, we can express the time-ordered product in Eq. (4.5) in terms of a commutator times $\epsilon(x^0)$ plus an anticommutator of the Heisenberg operators. Then we find

$$D_+(k^2+i0)=\int d^4x\ e^{ik\cdot x}\int_{-0}^{\infty}d\lambda^2\rho_+(\lambda^2)\Delta_c(x,\lambda^2)+\sum_{n=0}^{N-1}c_n(k^2)^n\ , \quad (4.6)$$

where

$$\Delta_c(x,\lambda^2)=\tfrac{1}{2}[\epsilon(x^0)\Delta(x,\lambda^2)+i\Delta_1(x,\lambda^2)] \quad (4.7)$$

with

$$\Delta(x,\lambda^2)=i\int\frac{d^4q}{(2\pi)^3}e^{iq\cdot x}\epsilon(q^0)\delta(q^2-\lambda^2)\ ,$$
$$\Delta_1(x,\lambda^2)=\int\frac{d^4q}{(2\pi)^3}e^{iq\cdot x}\delta(q^2-\lambda^2)\ . \quad (4.8)$$

In Eq. (4.6), the polynomial with real coefficients has its origin in the possible appearance of products of $\epsilon(x^0)$ and distributions $\delta^{(n)}(x^2)$ in the integrand of Re$D_+(k^2+i0)$, which are not defined at $x=0$. This feature introduces a certain arbitrariness in the Fourier transform which is just expressed by the polynomial with arbitrary coefficients.[22] Only derivatives $\delta^{(n)}(x^2)$ of finite order are possible in the vacuum expectation value of the projected commutator

$$\langle 0|[A_a^{\mu\nu}(x),P_+A_b^{\rho\sigma}(0)]|0\rangle , \tag{4.9}$$

because Lorentz covariance requires that the expression, which is odd in x^0, vanishes in spacelike regions. An analogous polynomial is present, *a priori,* in the corresponding expression of the unprojected propagator.

From the Lorentz covariance of the projection P_+, and the usual spectral condition, it follows that $D_+(k^2+i0)$ is, as already indicated, the boundary value of an analytic function $D_+(k^2)$ which is regular in the cut k^2 plane. Up to possible subtractions and corresponding polynomials, we therefore have the representation

$$D_+(k^2)=\int_{-0}^{\infty}dk'^2\frac{\rho_+(k'^2)}{k'^2-k^2} . \tag{4.10}$$

Further information about the asymptotic behavior of $D_+(k^2)$ can again be obtained from the renormalization group. The projection P_+ does not introduce a new dimensionful parameter, and therefore does not disturb the renormalization-group invariance. The points, which characterize the regions where $\rho_+(k^2)$ has support, are invariant. They are given by functions $K^2(\kappa^2,g)$ which have been discussed in Sec. III, Eqs. (3.23)–(3.25).

Defining the dimensionless structure function

$$R_+\left[\frac{k^2}{\kappa^2},g\right]=-k^2D_+(k^2,\kappa^2,g) , \tag{4.11}$$

we find that it satisfies a renormalization-group equation analogous to Eq. (3.5).[5,6] We obtain

$$R_+\left[\frac{k^2}{\kappa^2},g\right]=R\left[\left|\frac{k^2}{\kappa^2}\right|,g\right] \times R_+\left[e^{i\phi},\bar{g}\left[\left|\frac{k^2}{\kappa^2}\right|,g\right]\right] , \tag{4.12}$$

with $k^2=-|k^2|e^{i\phi}$, $|\phi|\leq\pi$. However, although Eqs. (4.12) and (3.5) are of the same form, there is an essential difference. While $R(k^2/\kappa^2,g)$ is normalized to "one" for $k^2=\kappa^2$, the expression $R_+(1,g)$ in an unknown function of g^2. Given the known asymptotic form (3.8) of $R(u,g)$, we get from Eq. (4.12) a correlation between the behavior of R_+ for $k^2\to\infty$, g^2 fixed and the limit $g^2\to+0$ with k^2 fixed. Somewhat more general, this correlation is seen also by writing the renormalization-group equation in the form[5,6]

$$R_+\left[\frac{k^2}{\kappa^2},g\right]=R(u(g',g),g)R_+\left[\frac{k^2}{\kappa^2}u^{-1}(g',g),g'\right] , \tag{4.13}$$

with $u(g',g)$ as given in Eq. (3.24), and

$$R(u(g',g),g)=\left[\frac{g^2}{g'^2}\right]^{-\gamma_{00}/\beta_0} \times\exp\left[\int_{g^2}^{g'^2}dx\left[\frac{\gamma(x)}{\beta(x)}-\frac{\gamma_{00}}{\beta_0 x}\right]\right] . \tag{4.14}$$

Since the integral in Eq. (4.14) is bounded, R behaves like $(g^2)^{-\gamma_{00}/\beta_0}$ for $g^2\to+0$.

The essential consequences of the renormalization-group equations are the following.

(1) For $\gamma_{00}/\beta_0>0$ ($N_F<10$ for QCD), we see from Eq. (4.12) that R_+ vanishes for $k^2\to\infty$ in all directions in the cut plane and fixed g unless we allow it to diverge for $g^2\to+0$ and fixed k^2, so that the last factor R_+ in Eq. (4.12) compensates the decrease of R in view of the limits in Eq. (3.7). But if R_+ vanishes asymptotically, D_+ vanishes faster than k^{-2} at infinity. Then we have again a superconvergence relation

$$\int_{-0}^{\infty}dk^2\rho_+(k^2)=0 . \tag{4.15}$$

Since ρ_+ is positive, Eq. (4.15) implies that ρ_+ vanishes everywhere for $k^2\geq0$. [An exception are contributions from a possible set of measure zero. Here, and in the following, we do not consider this mathematical possibility. Furthermore, such functions do not contribute to D_+ as given by the dispersion relation (4.10).] The superconvergence for $\gamma_{00}/\beta_0>0$ is avoided only if

$$\left|R_+\left[\frac{k^2}{\kappa^2},g\right]\right|\geq O((g^2)^{-\gamma_{00}/\beta_0}) \tag{4.16}$$

for $g^2\to+0$ and fixed k^2.

(2) For $\gamma_{00}/\beta_0<0$ ($10\leq N_F\leq16$ for QCD), on the other hand, we may assume that $R_+(k^2/\kappa^2,g)$ approaches a constant for $g^2\to+0$ without having superconvergence or other relevant restrictions.

We note that polynomial terms in $D_+(k^2)$, which are possible *a priori* as indicated in Eq. (4.6), do not change the above conclusions. The polynomials, and corresponding subtractions in the dispersion relation D_+, have been discussed in detail in Ref. 6. As a consequence of the renormalization-group equations (4.14), the coefficients of the polynomials would diverge exponentially for $g^2\to+0$, like the appropriate power of

$$u^{-1}(g',g)\simeq\exp(-1/\beta_0 g^2)(g^2)^{-\beta_1/\beta_0^2} , \tag{4.17}$$

multiplied by $(g^2)^{-\gamma_{00}/\beta_0}$.

Let us now return to the classes of states in the space $\mathcal{V}$ which have been introduced in Sec. II in a generic fashion.

(1) First, we are interested in states of the form (4.1) which satisfy

$$QP_+\tilde{A}_a^{\mu\nu}(-k)|0\rangle=0 \tag{4.18}$$

as antimetric Lorentz tensors and SU(3)-octet vectors in color space. As before, we assume that there is no spontaneous breaking of global color symmetry. Since the operators $U(\Lambda)$ representing Lorentz transformations in $\mathcal{V}$ commute with the BRST operator Q, we find that Eq.

(4.18), for a given four-vector k, implies that all states with the same value of $k^2 \geq 0$ also satisfy Eq. (4.18). In terms of the states $\Psi_+(c,k^2)$ introduced in Eqs. (3.2) and (4.3), the validity of Eq. (4.18) and its transforms implies then that

$$Q\Psi_+(c,k^2)=0 \tag{4.19}$$

is valid for all test functions $c_a^{\mu\nu}(k)$ for a given $k^2 \geq 0$. Hence these states belong to ker$Q$, and because of the positive norm, there must be a nonvanishing component in $\mathcal{V}_p$. This implies that the states considered are representatives of physical states as defined by the cohomology $\mathcal{H}$ of Q.

(2) The alternative to Eq. (4.18) is that, for a given four-vector k, the state $P_+\tilde{A}|0\rangle$ satisfies

$$QP_+\tilde{A}_a^{\mu\nu}(-k)|0\rangle \neq 0 \tag{4.20}$$

as Lorentz tensor and color vector. Because of $[Q,U(\Lambda)]=0$, we find again that all corresponding states with the same value of $k^2 \geq 0$ also satisfy Eq. (4.20). For the states $\Psi_+(c,k^2)$, Eq. (4.20) implies that there are test functions $c_a^{\mu\nu}(k)$ for which

$$Q\Psi_+(c,k^2)\neq 0 \ . \tag{4.21}$$

If Eq. (4.20) is valid for a given test function c, then the same is true for all its Lorentz and/or color transforms

$$c_a'^{\mu\nu}(k)=c_b^{\rho\sigma}(\Lambda^{-1}k)\Lambda_\rho^\mu\Lambda_\sigma^\nu R_{ba} \ . \tag{4.22}$$

The states $\Psi_+(c,k^2)$ and $\Psi_+(c',k^2)$ have, of course, the same norm, which is determined by $\rho_+(k^2)$. According to the discussion in Sec. II, they must have a nonvanishing component in imQ^*. By an equivalence transformation, they can be brought into a form which has only components in kerQ^*, and none in $\mathcal{V}_p$. Hence they are unphysical states, even though they have positive norm.

The two possibilities for positive-norm states are exclusive. In the neighborhood of a given point $k^2 \geq 0$, or as a δ-function contribution, the discontinuity ρ_+ is either due to physical states in the class C_p of Sec. II, or unphysical states in C_u:

$$\begin{aligned} &\rho_+(k^2)=\rho_p(k^2) \ \text{ if } \tilde{A}(-k)|0\rangle \in C_p \ , \\ &\rho_+(k^2)=\rho_{u+}(k^2) \ \text{ if } \tilde{A}(-k)|0\rangle \in C_{u+} \ . \end{aligned} \tag{4.23}$$

As we have pointed out in Sec. III, the discontinuity ρ is a negative measure for $k^2 > K^2(\kappa^2,g)$ and $\gamma_{00}/\beta_0 > 0$, with $K^2(\kappa^2,g)$ as defined in Eqs. (3.23)–(3.25). Consequently ρ_+ and ρ_p vanish for $k^2 > K^2$, and the separation discussed above is of importance only in the interval $0 \leq k^2 \leq K^2(\kappa^2,g)$. We may well have either $\rho_+=\rho_p$ or $\rho_+=\rho_{u+}$ in this whole interval, but there is no difficulty in dealing with the more general situation considered above.

So far, we have discussed the separation of ρ_p and ρ_{u+} for fixed values of κ^2 and g, but it is actually invariant under renormalization-group substitutions of (κ^2,g) by (κ'^2,g') where $\kappa'^2=\kappa^2 u(g',g)$ as described in Sec. III. At these renormalization-group equivalent points the states $\tilde{A}^{\mu\nu}(-k,\kappa^2,g)|0\rangle$ are proportional to each other, and so are the BRST operators (see, in this connection, Ref. 8). Since the factors are real, it follows that the classes C_p and C_{u+} are invariant. This implies that possible points of separation between ρ_p and ρ_{u+} must be invariant functions $K_s^2(\kappa^2,g)$ as given in Eqs. (3.23)–(3.25). Similarly, the position $K_i(\kappa^2,g)$ of possible δ-function contributions are invariant.

In Refs. 5 and 6, the renormalization-group equations for the discontinuity ρ as a distribution, and of ρ_+ as a measure, have been studied in detail. Because of the renormalization-group invariance of C_p and the related invariant separation of ρ_p from ρ_{u+}, the weight function ρ_p must satisfy an equation analogous to the one for ρ_+. Defining a dimensionless expression by

$$\chi_p\left[\frac{k^2}{\kappa^2},g\right] \equiv k^2\rho_p(k^2,\kappa^2,g) \ , \tag{4.24}$$

we have the relation

$$\chi_p\left[\frac{k^2}{\kappa^2},g\right] = R(u(g',g),g)\chi_p\left[\frac{k^2}{\kappa^2}u^{-1}(g',g),g\right] \ . \tag{4.25}$$

Here R is given by Eq. (4.14) and $u(g',g)$ by Eq. (3.24). The function $u(g',g)$ vanishes proportional to $\exp(1/\beta_0 g^2)(g^2)^{\beta_1/\beta_0^2}$. Note that, substituting $k^2 = K^2(\kappa^2,g)$ on both sides of Eq. (4.24) and using Eqs. (3.23) and (3.24) with

$$K^2(\kappa^2,g')=K^2(\kappa^2,g)u^{-1}(g',g) \ , \tag{4.26}$$

we obtain

$$\chi_p\left[\frac{K^2(\kappa^2,g)}{\kappa^2}\right] = R(u(g',g),g)\chi_p\left[\frac{K^2(\kappa^2,g')}{\kappa^2},g'\right] \ . \tag{4.27}$$

So, for example, zeros of ρ_p match on both sides of the renormalization-group equation, their location being defined in an invariant fashion.

In this section we discuss only the case $\gamma_{00}/\beta_0 > 0$ ($N_F < 10$ for QCD). As has been mentioned above, there is a minimal point $K^2(\kappa^2,g) \geq 0$ so that

$$\rho_+(k^2,\kappa^2,g)=0 \ \text{ for } k^2 > K^2(\kappa^2,g) \ , \tag{4.28}$$

where K^2 is invariant according to Eq. (3.23) and vanishes exponentially for $g^2 \to +0$. For the projected structure function $D_+(k^2)$, we then write the representation (4.10) in the form

$$D_+(k^2)=\int_{-0}^{K^2} dk'^2 \frac{\rho_+(k'^2)}{k'^2-k^2} \ . \tag{4.29}$$

In principle, and as indicated in Eq. (4.6), there could be a polynomial in k^2 with real coefficients in Eq. (4.29). From the renormalization-group equations, we learn that these coefficients diverge exponentially for $g^2 \to +0$. As indicated before, a possible polynomial does not change our later conclusions, and we omit it in the following.

In view of Eq. (4.23), we can split Eq. (4.29) and write separate relations involving ρ_p and ρ_{u+}, both having support for *different* values of $k^2 \geq 0$. So we have

$$D_p(k^2)=\int_{-0}^{K^2} dk'^2 \frac{\rho_p(k'^2)}{k'^2-k^2} , \tag{4.30}$$

and a corresponding relation for D_{u+}.

The formula (4.30) collects the contributions from all possible regions of support of ρ_p which have renormalization-group-invariant boundary points $K_s(\kappa^2,g)$ or positions $K_i(\kappa^2,g)$ as discussed before. The function D_p is the part of the general structure function D which is due to possible physical states of the form $\tilde{A}^{\mu\nu}(-k)|0\rangle$. Because of the invariance of C_p, which we have discussed, the function D_p must also satisfy a renormalization-group equation such as the one for D_+. Either directly, or via Eqs. (4.25) and (4.30), we obtain

$$R_p\left[\frac{k^2}{\kappa^2},g\right]=R(u(g',g),g)R_p\left[\frac{k^2}{\kappa^2}u^{-1}(g',g),g'\right] \tag{4.31}$$

with $R_p=-k^2D_p$.

Another derivation of a renormalization-group equation corresponding to Eq. (4.31) can be found in Ref. 4, where Nishijima uses an explicit decomposition of the propagator with respect to physical states for a given reference frame in $\mathcal{V}$.

In analogy with the situation for D_+ discussed earlier in this section, there are two possibilities for the solution of Eq. (4.31).

(1) If we want to keep D_p bounded for $g^2 \to +0$, it must vanish asymptotically faster than k^{-2}. But then it is superconvergent and we get

$$\int_{-0}^{K^2} dk'^2 \rho_p(k'^2)=0 \tag{4.32}$$

and hence $\rho_p(k^2)\equiv 0$ and $D_p(k^2)\equiv 0$. We again ignore a set of measure zero.

(2) The superconvergence can be avoided if we allow divergence for $g^2 \to +0$. The requirement is

$$|D_p(k^2,\kappa^2,g)| \geq O((g^2)^{-\gamma_{00}/\beta_0}) . \tag{4.33}$$

The dispersion relation (4.30) can be written in the form

$$D_p(k^2)=-\frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \rho_p(k'^2) + \frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \frac{k'^2\rho_p(k'^2)}{k'^2-k^2} . \tag{4.34}$$

From the renormalization-group equation (4.31), we learn then that the first term diverges like $(g^2)^{-\gamma_{00}/\beta_0}$ for $g^2 \to +0$, unless Eq. (4.32) holds. The second term is proportional to $\exp(1/\beta_0 g^2)$ and vanishes exponentially for $g^2 \to +0$.

We would now like to give some arguments which indicate that the second possibility can be excluded.

If the transverse gluons are physical quanta, we expect that the k^{-2} term appearing in the weak-coupling limit $g^2 \to +0$ of the structure function D is associated with the physical gluon contributions to the exact propagator, so that $D_p \simeq -c^2k^{-2}$ for $g^2 \to +0$, with c^2 being a finite constant. This is excluded for $\gamma_0/\beta_0 > 0$ ($N_F < 10$) in view of Eq. (4.33). On the other hand, for $\gamma_{00}/\beta_0 < 0$ ($10 \leq N_F \leq 16$), there is no problem in satisfying the requirement described above, although D_p may vanish for some other reason.

A more formal argument can be given by studying unitarity relations for Wightman functions involving BRST- and gauge-invariant operators such as, for example, $G=F_{\mu\nu}F^{\mu\nu}$. We can use the methods worked out in collaboration with Zimmermann,[23] but for the particular situation with possible physical gluons as defined by the BRST cohomology. The unitarity relations for the G propagator are resummed so that integrals over ρ_p appear. If $\gamma_{00}/\beta_0 > 0$, these lead to divergencies in the limit $g^2 \to +0$ which are not compatible with unitarity. On the other hand, if gluons are confined, they are not states in the physical subspace and hence do not contribute to the unitarity relations. Then the divergences for $g^2 \to +0$ do not matter.

In addition to the arguments in favor of gluon confinement given above, we recall that for $\gamma_{00}/\beta_0 > 0$ ($N_F < 10$) there are no states of the form $\tilde{A}^{\mu\nu}(-k)|0\rangle$ with positive norm for $k^2 > K^2(\kappa^2,g)$, and hence no corresponding physical states. Here the bound $K^2(\kappa^2,g)$ vanishes exponentially for $g^2 \to +0$. Although, in principle, a theory with a state space of indefinite metric may have unusual features, we would nevertheless expect that the Heisenberg operator $\tilde{A}^{\mu\nu}(-k)$ can generate physical many-particle states for all values of k^2 if there existed physical gluon states of the form $\tilde{A}^{\mu\nu}(-k)|0\rangle$ for $k^2 < K^2$. As a contrast, we note that for $\gamma_{00}/\beta_0 < 0$ ($10 \leq N_F \leq 16$), the discontinuity ρ_+ does not vanish for large values of k^2, as will be discussed in the next section. Hence, there is no obstacle to the existence of such physical states for any value of $k^2 \geq 0$ if $\gamma_{00}/\beta_0 < 0$.

We now turn to the contribution of unphysical states from the class C_u to the dispersion relation (4.29). We can write

$$D_{u+}(k^2)=\int_{-0}^{K^2} dk'^2 \frac{\rho_{u+}(k'^2)}{k'^2-k^2} , \tag{4.35}$$

and we have the same two possibilities as discussed in connection with D_p. However, since the states involved in ρ_{u+} are not physical, the objections mentioned above against the divergent solution do not apply now, and we can allow D_{u+} to diverge at least like $(g^2)^{-\gamma_{00}/\beta_0}$ for $g^2 \to +0$. In fact, this is the only acceptable solution, since $\rho_{u+}(k^2)\equiv 0$, together with $\rho_p \equiv 0$, would result in $\rho_+(k^2)\equiv 0$. With $\rho_+ \equiv 0$, the superconvergence relation (3.22) for the discontinuity ρ of the full propagator would imply that $\rho(k^2)$ vanishes for all k^2, with the possible exception of multipole terms such as

$$\sum_j \sum_{n=1}^{n_j} c_{n,j}\delta^{(n)}(k^2-K_j^2(\kappa^2,g)) . \tag{4.36}$$

As we have pointed out before, these multipole contributions involve derivatives of δ functions of finite order, and they do not contribute to the superconvergence relation (3.22) for ρ. However, the expression (4.36) *alone* is not compatible with the asymptotic form (3.10) for ρ, nor with the weak-coupling limit $g^2 \to +0$, which should reproduce the perturbation theory expression in the case of the full propagator.

We conclude that for $\gamma_{00}/\beta_0 > 0$ ($N_F < 10$ for QCD) there should be no physical states of the form $\tilde{A}^{\mu\nu}(-k)|0\rangle$, and hence no physical gluon states. There must be unphysical states of this type with both positive and negative norm (sign of ρ). The case $\gamma_{00}/\beta_0 < 0$ ($10 \le N_f \le 16$) will be discussed in the following section.

V. MANY FLAVORS

In the previous section, we have found that the gauge-field quanta are confined if $\gamma_{00}/\beta_0 > 0$ ($N_F < 10$ for QCD). We now want to discuss briefly the situation for $\gamma_{00}/\beta_0 < 0$, $\beta_0 < 0$ ($10 \le N_F \le 16$), and to compare the two cases with the help of the dispersion representations of $D_+(k^2)$ and $D_-(k^2)$.

As seen from Eq. (3.8), the structure function $D(k^2)$ for the full propagator decreases less fast than k^{-2} for $k^2 \to \infty$ if $\gamma_{00}/\beta_0 < 0$. Hence we have no superconvergence relation. For the projected propagator functions $D_+(k^2)$, etc. the renormalization-group equations show that nonvanishing expressions, which are bounded for $g^2 \to +0, k^2$ fixed, are allowed in this case. Therefore, for $D_p(k^2)$, we are *not* forced to take the trivial solution. The situation is completely compatible with the states $P_+ \tilde{A}^{\mu\nu}|0\rangle$ of positive norm being in ker$Q$ with nonvanishing component in $\mathcal{V}_p$, and therefore gluons may not be confined. On the other hand, we could *choose* the solution $\rho_p \equiv 0$ corresponding to confinement. We see that, on the basis of our methods, unconfined and confined phases are possible for $\gamma_{00}/\beta_0 < 0$, $\beta_0 < 0$. Nevertheless, it may be quite plausible that the condition $\gamma_{00}/\beta_0 > 0$ is not only sufficient but also necessary for the confinement of gluons and perhaps also of quarks. Then we would have a deconfining phase transition as γ_{00}/β_0 changes sign with an increasing number of matter fields in the theory. In general, it is to be expected that quark vacuum polarization provides the appropriate screening of color charges with increasing values of N_F. As explained in the Introduction, it may eventually be possible to decide the question of a deconfining phase transition around $\gamma_{00} = 0$ with the help of computer simulations.[16,17]

An interesting picture of the different characteristics of the states $\tilde{A}|0\rangle$ for both signs of γ_{00} is provided by a more detailed study of the dispersion relations for $D_+(k^2)$ and $D_-(k^2)$.[6] D_+ has been discussed in the previous section, D_- contains contribution from negative-norm states and possible multipole terms of the type (4.36); we have $D = D_+ + D_-$.

We first consider the case $\gamma_{00}/\beta_0 > 0, \beta_0$ ($N_F < 10$): As has been mentioned before, we can write the representation (4.29) for D_+ in the form

$$D_+(k^2) = -\frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \rho_+(k'^2) + \frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \frac{k'^2 \rho_+(k'^2)}{k'^2 - k^2} . \tag{5.1}$$

From the renormalization-group equations we learn that the first term in Eq. (5.1) diverges like $(g^2)^{-\gamma_{00}/\beta_0}$ for $g^2 \to +0$, while the second term is proportional to $\exp(1/\beta_0 g^2)$, vanishing exponentially in the weak-coupling limit. None of the terms in D_+ contributes to the power series in g^2 which, by assumption, is the formal asymptotic limit for $g^2 \to +0$ of the full propagator function D. These perturbation theoretical terms must then come from D_-, for which we can write the decomposition

$$D_-(k^2) = \frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \rho_-(k'^2) + \frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \frac{k'^2 \rho_-(k'^2)}{k'^2 - k^2} + \int_{K^2}^{\infty} dk'^2 \frac{\rho_-(k'^2)}{k'^2 - k^2} . \tag{5.2}$$

For $g^2 \to +0$, the first two terms behave as described before for the corresponding terms of D_+. In particular, the diverging first terms cancel in $D = D_+ + D_-$ in view of the superconvergence relation (3.22) for $\rho = \rho_+ + \rho_-$. It is the last term in Eq. (5.2) which gives rise, for $g^2 \to +0$, to the perturbative power-series expansion of D. The latter is obtained by inserting the asymptotic expansion of $\rho_-(k^2) = \rho(k^2)$ for $k^2 \to \infty$, and hence is connected only with unphysical states of the general, nonperturbative theory. It should be recalled here that, as shown in Eq. (3.25), the boundary point $K^2 = K^2(\kappa^2, g)$ vanishes exponentially for $g^2 \to +0$.

According to our previous results, we have $D = D_u$ for $\gamma_{00}/\beta_0 > 0$, and we see from Eqs. (5.1) and (5.2) and the related discussion that the representations are in complete accord with this conclusion.

As a comparison, we consider briefly the situation for $\gamma_{00}/\beta_0 < 0$, $\beta_0 < 0$ ($10 \le N_F \le 16$). In this case with many flavors, we have

$$D_+(k^2) = -\frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \rho_+(k'^2) + \frac{1}{k^2}\int_{-0}^{K^2} dk'^2 \frac{k'^2 \rho_+(k'^2)}{k'^2 - k^2} + \int_{K^2}^{\infty} dk'^2 \frac{\rho_+(k'^2)}{k'^2 - k^2} .$$

But now the first term *vanishes* like $(g^2)^{-\gamma_{00}/\beta_0}$ for $g^2 \to +0$, while the second term again decreases exponentially. The perturbation expansion for weak coupling comes from the third term and is associated completely with positive-norm states. As we have pointed out in the beginning of this section, we cannot tell whether these positive-norm states are physical or not in the case $\gamma_{00}/\beta_0<0$ considered here, but it may perhaps be suggestive that they are. The function $D_-(k^2)$ satisfies an equation analogous to Eq. (5.1) in the present case. The whole function vanishes for $g^2\to+0$.

We see that the dispersion representations for D_+ and D_- give a good picture of the very different structure of the theory for positive and negative signs of the anomalous dimension coefficient γ_{00}.

Finally, we remark that it is of interest to study supersymmetric gauge theories and theories with conformal invariance in the light of the features presented in this paper. Some such theories have been discussed in Ref. 24.

ACKNOWLEDGMENTS

Some of this work was completed while I was at the Research Institute for Fundamental Physics, Kyoto University. It is a pleasure to thank Kazuhiko Nishijima, and the members of the Institute, for their very kind hospitality. I would like to thank Wolfhart Zimmermann, and the members of the Theory Group of the Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, for their very kind hospitality in München. For discussions and helpful conversations, I am indebted to P. G. O. Freund, M. Fukugita, H. Miyazawa, N. Nakanishi, Y. Nambu, K. Nishijima, I. Ojima, K. Sibold, and W. Zimmermann. This work has been supported in part by the National Science Foundation Grant No. PHY90-00386.

*Permanent address.

[1] R. Oehme, Phys. Lett. B **232**, 498 (1989).
[2] R. Oehme, Phys. Lett. B **195**, 60 (1987); Fermi Institute Report No. EFI 89-33 (unpublished).
[3] R. Oehme, in *Perspectives on Particle Physics—From Mesons and Resonances to Quarks and Strings,* Festschrift in honor of Prof. H. Miyazawa, edited by S. Matsuda, R. Sasaki, and T. Muta (World Scientific, Singapore, 1988).
[4] K. Nishijima, Prog. Theor. Phys. **75**, 1221 (1986); Prog. Theor. Phys. Suppl. **86**, 253 (1986).
[5] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 475 (1980).
[6] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 1661 (1980).
[7] C. Becchi, A. Rouet, and R. Stora, Ann. Phys. (N.Y.) **98**, 287 (1976); I. V. Tyutin, Lebedev Report No. FIAN 39, 1975 (unpublished).
[8] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979); Phys. Lett. **73B**, 495 (1978).
[9] G. Curci and R. Ferrari, Nuovo Cimento A **35**, 276 (1976); **35**, 474 (1976).
[10] N. Nakanishi, Prog. Theor. Phys. Suppl. **51**, 1 (1972).
[11] K. Nishijima, Nucl. Phys. **B238**, 601 (1984).
[12] K. Nishijima, Prog. Theor. Phys. **80**, 987 (1988); **80**, 905 (1988).
[13] I. B. Frenkel, H. Garland, and G. J. Zuckerman, Proc. Natl. Acad. Sci. USA **83**, 8442 (1986).
[14] M. Spiegelglas, Nucl. Phys. **B283**, 205 (1987).
[15] J. M. Figueroa-O'Farrill and T. Kimura, Stony Brook Report No. ITB-SB-88-34 (unpublished).
[16] K. Nishijima, Prog. Theor. Phys. **77**, 1035 (1987).
[17] T. Fukugita and A. Ukawa, in *Wandering in the Fields,* edited by K. Kawarabayashi and A. Ukawa (World Scientific, Singapore, 1987).
[18] K. Nishijima, Prog. Theor. Phys. **74**, 889 (1985).
[19] K. Nishijima and Y. Okada, Prog. Theor. Phys. **72**, 294 (1984).
[20] J. Bognar, *Indefinite Inner Product Spaces* (Springer, Berlin, 1974).
[21] S. Weinberg, Phys. Rev. D **8**, 3497 (1973); G.'t Hooft, Nucl. Phys. **B61**, 455 (1973); J. C. Collins and A. J. MacFarlane, Phys. Rev. D **10**, 1201 (1974).
[22] H. J. Bremermann, R. Oehme, and J. G. Taylor, Phys. Rev. **109**, 2178 (1958); N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).
[23] R. Oehme and W. Zimmermann, Phys. Lett. **108B**, 416 (1982).
[24] R. Oehme, in *Leite Lopes Festschrift,* edited by N. Fleury, S. Joffily, J. A. M. Simões, and A. Troper (World Scientific, Singapore, 1988).