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Electron propagator in a strong electromagnetic field in the very-high-energy limit

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The solution of the Klein-Gordon and Dirac equation for the propagator (Green's function) in the very-high-energy limit ($1/E$ approximation) is studied in the context of quantum "beamstrahlung." The results obtained are necessary to consider bremsstrahlung energy loss by electrons and positrons in high-energy e^+e^- linear colliders when multiphoton radiation processes are taken into account. The results can be useful also in other physical processes, e.g., photon radiation in crystals.

In the near future beams of particles of very high energy (1 TeV and more) will be used, in machines which are now being considered and built.[1] In this context, considering the physics in such high-energy limits, it is worthwhile to study the perturbation theory in which the $1/E$ expansion is used. By $1/E$ expansion we mean the expansion in the power series of $1/E$, where E is an energy of the particle. In many physical situations, high-energy particles interact with the strong external field, which practically makes it impossible to apply standard perturbation theory. The $1/E$ expansion may (in some cases) help us solve this problem. Moreover, in many problems some effects can be treated perturbatively in the standard sense (small coupling constant), and some cannot be. The "classical" example of such a situation is quantum radiation processes in the presence of a strong external field. The "beamstrahlung" process, which has been recently studied in detail,[2-6] is of this type. In Refs. 2–5 the beamsstrahlung energy loss by electrons and positrons in very-high-energy e^+e^- linear colliders was calculated for the case of longitudinally uniform and nonuniform bunches. A different approach to the problem is presented in a series of papers.[6]

One of the next steps in considering beamstrahlung is to calculate the multiphoton radiation processes. In the method proposed and developed by Jacob and Wu,[2,3] beamstrahlung is calculated in terms of Feynman diagrams. To consider multiphoton radiation, we should find first of all the formula for the electron propagation function in the "external field" produced by a very dense positron bunch.

In our paper we propose a method which allows us to solve, in a systematic way, the equation for the propagator in the high-energy approximation. The method is presented in Secs. II and III. In Sec. IV we calculate the propagator for the spinless case in the $1/E$ approximation. The Dirac case (electron with spin) is discussed in Sec. V.

II. THE PROBLEM

First we consider the spinless case (we neglect the spin of the electron). We will be looking for a propagation function of an electron in the high-energy regime, where the energy of the electron $E\simeq k$ is going to infinity ($k=|\mathbf{k}|$, where $\mathbf{k}$ is a momentum of the electron). We have to solve the stationary Klein-Gordon equation in the external potential field for the propagator (Green's function) in the $1/k$ approximation. The $1/k$ approximation means that we consider the terms independent of k and the terms proportional to $1/k$, neglecting terms $1/k^2$ and higher powers of $1/k$. Such an approximation is consistent with the high-energy regime where $k\to\infty$ and with earlier papers.[2,3] The external field is a potential generated by the positron bunch inside which the electron is running. We assume that the potential is independent of time, which allows us to consider the stationary Klein-Gordon equation. The simplest way to obtain the desired result is to rewrite the Klein-Gordon equation in the form of a set of the differential equations which can be solved much more easily. Let us present this method first.

We have the following differential Klein-Gordon equation for the electron propagator $G(\mathbf{r},\mathbf{r}')$:

$$[k^2+\nabla^2-2kU(\mathbf{r})+U^2(\mathbf{r})]G(\mathbf{r},\mathbf{r}')=-\delta(\mathbf{r}-\mathbf{r}') , \qquad (2.1)$$

where $k=|\mathbf{k}|\simeq E$ is the energy of the electron ($E^2=k^2+m^2$), $k^2\gg m^2$, and $U(\mathbf{r})$ is a potential generated by the positron bunch. Let $\mathbf{R}=\mathbf{r}-\mathbf{r}'$, $R=|\mathbf{R}|$, and $\hat{\mathbf{n}}=\mathbf{R}/R$. We assume that $G(\mathbf{r},\mathbf{r}')$ has the form

$$G_0(\mathbf{r},\mathbf{r}')=G_0(\mathbf{R})S_0(\mathbf{r},\mathbf{r}') ,$$

and (2.2)

$$S(\mathbf{r},\mathbf{r}')=S_0(\mathbf{r},\mathbf{r}')\left[1+\frac{1}{k}\Psi(\mathbf{r},\mathbf{r}')\right] .$$

$G_0(R)=(1/4\pi R)e^{ikR}$ is a "free" propagator for the Klein-Gordon equation,

$$(k^2+\nabla^2)G_0(R)=-\delta(\mathbf{R}) . \qquad (2.3)$$

S_0 and Ψ are unknown functions.

The propagator should be symmetric: $G(\mathbf{r},\mathbf{r}')=G(\mathbf{r}',\mathbf{r})$, which implies that $S(\mathbf{r},\mathbf{r}')=S(\mathbf{r}',\mathbf{r})$. We assume also that $S_0(\mathbf{r},\mathbf{r})=S_0(\mathbf{R}=0)=1$. Now we can put the assumed form of $G(\mathbf{r},\mathbf{r}')$ into the Klein-Gordon equation (2.1) and [using (2.3)] obtain the result

$$\begin{aligned}\nabla^2 S_0 &+ \frac{1}{k}\Psi\nabla^2 S_0 + \frac{2}{k}\overline{\nabla}S_0\cdot\overline{\nabla}\Psi + \frac{1}{k}S_0\nabla^2\Psi + 2ik\hat{\mathbf{n}}\cdot\overline{\nabla}S_0 \\ &+ 2i\Psi\hat{\mathbf{n}}\cdot\overline{\nabla}S_0 + 2iS_0\hat{\mathbf{n}}\cdot\overline{\nabla}\Psi + U^2 S_0 \\ &+ \frac{1}{k}U^2\Psi S_0 - 2U\Psi S_0 \\ &- 2kUS_0 - \frac{2}{R}\hat{\mathbf{n}}\cdot\overline{\nabla}S_0\left[1+\frac{1}{k}\Psi\right] - \frac{2}{Rk}S_0\hat{\mathbf{n}}\cdot\overline{\nabla}\Psi = 0 , \\ \hat{\mathbf{n}}\cdot\overline{\nabla} &= \frac{\partial}{\partial R} . \end{aligned} \tag{2.4}$$

Consistent with our approximation, we limit ourselves to considering only terms proportional to k and terms independent of k ($1/k$ approximation in G). Neglecting the terms in (2.4) proportional to $1/k$ and $1/k^2$, we find that

$$\begin{aligned}\nabla^2 S_0 &+ 2k(i\hat{\mathbf{n}}\cdot\overline{\nabla} - U)S_0 + U^2 S_0 + 2iS_0\hat{\mathbf{n}}\cdot\overline{\nabla}\Psi \\ &- 2US_0\Psi + 2\left[i\Psi - \frac{1}{R}\right]\hat{\mathbf{n}}\cdot\overline{\nabla}S_0 = 0 . \end{aligned} \tag{2.5}$$

Equation (2.5) can be rewritten as a set of two equations:

$$\begin{aligned} &i\hat{\mathbf{n}}\cdot\overline{\nabla}S_0 = US_0 , \\ &\nabla^2 S_0 + U^2 S_0 - \frac{2}{R}\hat{\mathbf{n}}\cdot\overline{\nabla}S_0 + 2iS_0\hat{\mathbf{n}}\cdot\overline{\nabla}\Psi = 0 . \end{aligned} \tag{2.6}$$

Now it is easy to solve (2.6) under the following conditions: $S_0(\mathbf{r},\mathbf{r}')=S_0(\mathbf{r}',\mathbf{r})$, $\Psi(\mathbf{r},\mathbf{r}')=\Psi(\mathbf{r}',\mathbf{r})$, and $\Psi(\mathbf{r},\mathbf{r}) = \Psi(R=0)=0$. The solutions are

$$\begin{aligned} S_0(\mathbf{r},\mathbf{r}') &= \exp\left[-i\int_0^R U(\hat{\mathbf{n}}s+\mathbf{r}')ds\right] , \\ \Psi(\mathbf{r},\mathbf{r}') &= \tfrac{1}{2}[U(\mathbf{r})-U(\mathbf{r}')] + \tfrac{1}{2}\int_0^R S_0^{-1}\nabla_t^2 S_0(\hat{\mathbf{n}}s+\mathbf{r}')ds , \end{aligned} \tag{2.7}$$

where $\nabla^2 = (\hat{\mathbf{n}}\cdot\overline{\nabla})^2 + (2/R)\hat{\mathbf{n}}\cdot\overline{\nabla} + \nabla_t^2$.

The more elegant and systematic way is to solve the integral equation instead of the set of differential ones. However more complicated, it may appear more efficient, especially in the case when we cannot limit ourselves to the $1/k$ approximation and must take into account terms of order $1/k^2$ and more.

Now we will solve the integral equation in the $1/k$ approximation. The method used here is the method of the stationary phase, applied and developed in the 1950s by Wu, Schiff, Saxon, and others[7] in the context of the high-energy potential scattering, where it was used to calculate wave functions and the S-matrix elements.

From Eq. (2.1) we can write the following integral equation for $G(\mathbf{r},\mathbf{r}')$:

$$\begin{aligned} G(\mathbf{r},\mathbf{r}') = G_0(R) - 2k\int G_0(\mathbf{r}-\mathbf{r}'')U(\mathbf{r}'')\left[1-\frac{U(\mathbf{r}'')}{2k}\right] \\ \times G(\mathbf{r}'',\mathbf{r}')d^3r'' . \end{aligned} \tag{2.8}$$

After simple modifications we have

$$\begin{aligned} S(\mathbf{r},\mathbf{r}') = 1 - 2k\int G_0(\mathbf{r}-\mathbf{r}'')G_0(\mathbf{r}'-\mathbf{r}'')G_0^{-1}(\mathbf{R})U(\mathbf{r}'') \\ \times\left[1-\frac{U(\mathbf{r}'')}{2k}\right]S(\mathbf{r}'',\mathbf{r}')d^3r'' . \end{aligned} \tag{2.9}$$

Now we define a new function $J(\mathbf{r},\mathbf{r}') \equiv U(\mathbf{r})S(\mathbf{r},\mathbf{r}')$ which obeys the following integral equation:

$$J(\mathbf{r},\mathbf{r}') = U(\mathbf{r}) - 2kU(\mathbf{r})I , \tag{2.10}$$

where

$$I = \int G_0(\mathbf{r}-\mathbf{r}'')G_0(\mathbf{r}'-\mathbf{r}'')G_0^{-1}(\mathbf{R})J(\mathbf{r}'',\mathbf{r}') \times\left[1-\frac{U(\mathbf{r}'')}{2k}\right]d^3r'' .$$

In the next sections we will solve this equation in the high-energy approximation.

III. INTEGRAL EQUATION IN THE HIGH-ENERGY LIMIT $k\to\infty$

Let us consider integral I from Eq. (2.10):

$$I = \frac{R}{4\pi}\int e^{ik(|\mathbf{r}-\mathbf{r}''|+|\mathbf{r}'-\mathbf{r}''|-R)}\left[1-\frac{U(\mathbf{r}'')}{2k}\right] \times\frac{J(\mathbf{r}'',\mathbf{r}')}{|\mathbf{r}'-\mathbf{r}''||\mathbf{r}-\mathbf{r}''|}d^3\mathbf{r}'' . \tag{3.1}$$

To calculate I let us choose the coordinate system defined as

$$\mathbf{r}=(0,0,z),\quad \mathbf{r}'=(0,0,-z),\quad z>0 ,$$

$$R=2z ,$$

$$|\mathbf{r}'-\mathbf{r}''| = (\rho^2+z_+^2)^{1/2},\quad |\mathbf{r}-\mathbf{r}''| = (\rho^2+z_-^2)^{1/2} ,$$

where $\rho^2 = x''^2+y''^2$, $z_- = z''-z$, and $z_+ = z''+z$. The integral I will be calculated using the method of stationary phase. Because $k\to\infty$, the dominant contribution proceeds from such configurations where the expression $|\mathbf{r}-\mathbf{r}''|+|\mathbf{r}'-\mathbf{r}''|-R \simeq 0$. This means that $\rho^2\simeq 0$, and so we can express integrand in (3.1) in terms of a Taylor series near the configuration where $\rho^2=0$ ($x''=y''=0$). We have

$$|\mathbf{r}-\mathbf{r}''| = |z_-|(1+\tfrac{1}{2}\rho_-^2 - \tfrac{1}{8}\rho_-^4 + \cdots) , \quad \text{where } \rho_-^2 = \left[\frac{\rho}{z_-}\right]^2 ,$$

$$|\mathbf{r}'-\mathbf{r}''| = |z_+|(1+\tfrac{1}{2}\rho_+^2 - \tfrac{1}{8}\rho_+^4 + \cdots) , \quad \text{where } \rho_+^2 = \left[\frac{\rho}{z_+}\right]^2 ,$$

$$\frac{1}{|\mathbf{r}-\mathbf{r}''|} = \frac{1}{|z_-|}\phi(\rho_-^2),\quad \frac{1}{|\mathbf{r}'-\mathbf{r}''|} = \frac{1}{|z_+|}\phi(\rho_+^2) ,$$

$$\phi(x^2) = 1 - \tfrac{1}{2}x^2 + \tfrac{3}{8}x^4 - \cdots ,$$

$$\begin{aligned} J(\mathbf{r}'',\mathbf{r}') = J_0 + J_x x'' + J_y y'' + \tfrac{1}{2}J_{xx}x''^2 \\ + J_{xy}x''y'' + \tfrac{1}{2}J_{yy}y''^2 + \cdots , \end{aligned}$$

where

$$J_0=J(0,0,z'',\mathbf{r}')\ ,$$

$$J_x=\frac{\partial J}{\partial x''}(0,0,z'',\mathbf{r}')\ ,$$

$$J_{xx}=\frac{\partial^2 J}{\partial x''^2}(0,0,z'',\mathbf{r}')\ ,$$

and the same for J_y, J_{xy}, J_{yy}:

$$U(\mathbf{r}'')=U(0,0,z'')+\cdots\ . \tag{3.2}$$

The Taylor series for J is correct only if J is a slowly varying function of x'', y'', and z''. This assumption will be checked after the calculations. Using above series (3.2)–(3.1), we get

$$I=\frac{z}{2\pi}\int\frac{1}{|z_+z_-|}e^{ik(|z_+|+|z_-|-2z)}\times\int\left[1-\frac{1}{2k}U(0,0,z'')+\cdots\right]\left[J_0+\tfrac{1}{2}J_{xx}x''^2+\frac{1}{2}J_{yy}y''^2+\cdots\right]\phi(\rho_-^2)\phi(\rho_+^2)e^{ikb^2\rho^2}\chi(\rho^2)dx''\,dy''\,dz''\ , \tag{3.3}$$

where

$$\chi(\rho^2)=e^{-ika^2\rho^4+\cdots}\ ,\quad a^2=\left[\frac{1}{2|z_-|}\right]^3+\left[\frac{1}{2|z_+|}\right]^3,\quad b^2=\frac{1}{2|z_-|}+\frac{1}{2|z_+|}\ .$$

The terms proportional to J_x, J_y, J_{xy} give no contribution to I.

Now let us change the variables:

$$x''=\frac{1}{\sqrt{k}}\xi_1,\quad y''=\frac{1}{\sqrt{k}}\xi_2,\quad \rho^2=\frac{1}{k}(\xi_1^2+\xi_2^2)=\frac{1}{k}\alpha^2\ ,$$

where $\alpha^2=\xi_1^2+\xi_2^2$. Then

$$\chi(\rho^2)=\exp\left[-i\frac{1}{k}a^2\alpha^4+\cdots\right]\ .$$

In the high-energy limit, χ can be expressed by the $1/k$ power series

$$\chi=1-\frac{i}{k}a^2\alpha^4+\cdots\ .$$

In this way a systematic procedure is created which allows us to express integral I as a power series in $1/k$. Now it is easy to calculate integrals over ξ_1 and ξ_2 (they are of the Gaussian type). We limit ourselves to $(1/k)^2$ terms in I (we calculate the propagator in the $1/k$ approximation). It is consistent with the results obtained by Jacob and Wu for the wave functions.[2,3]

We have

$$I=\frac{z}{2\pi}\int\frac{1}{k|z_+z_-|}e^{ik(|z_+|+|z_-|-2z)}\times\int\left\{J_0-\frac{1}{k}J_0\left[\frac{1}{2}U(0,0,z'')+i\alpha^4a^2+\frac{1}{2}\left[\frac{1}{z_-}\right]^2\alpha^2+\frac{1}{2}\left[\frac{1}{z_+}\right]^2\alpha^2\right]+\frac{1}{2k}(J_{xx}\xi_1^2+J_{yy}\xi_2^2)\right\}e^{ib^2\alpha^2}d\xi_1 d\xi_2 dz''\ , \tag{3.4}$$

$$\alpha^2=\xi_1^2+\xi_2^2\ .$$

To calculate integrals in (3.4), we have to consider three integration ranges for variable z'': $(-\infty,-z)$, $(-z,z)$, and (z,∞). The result is

$$I=-\frac{1}{2(ik)}\int_{-z}^{z}J(0,0,z'',\mathbf{r}')dz''+\left[\frac{1}{2(ik)}\right]^2\int_{-z}^{z}iU(0,0,z'')J(0,0,z'',\mathbf{r}')dz''-\left[\frac{1}{2(ik)}\right]^2\frac{1}{z}\int_{-z}^{z}J(0,0,z'',\mathbf{r}')dz''$$
$$+\left[\frac{1}{2(ik)}\right]^2\frac{1}{2z}\int_{-z}^{z}(z^2-z''^2)\nabla_T^2 J(0,0,z'',\mathbf{r}')dz''+\left[\frac{1}{2(ik)}\right]^2[J(0,0,-z,\mathbf{r}')+J(0,0,z,\mathbf{r}')]\ , \tag{3.5}$$

where $\nabla_T^2=J_{xx}+J_{yy}$, $\mathbf{r}'=(0,0,-z)$.

Now we must come back to the previous coordinate system from our particularly defined one. In the arbitrary frame the result is

$$I=-\left[\frac{1}{2(ik)}\right]\int_0^R J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds+\left[\frac{1}{2(ik)}\right]^2\left[J(\mathbf{r},\mathbf{r}')+J(\mathbf{r}',\mathbf{r}')-\frac{2}{R}\int_0^R J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds+i\int_0^R U(\mathbf{r}'+\hat{\mathbf{n}}s)J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds\right.$$
$$\left.+\frac{1}{R}\int_0^R s(R-s)\nabla_T^2 J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds\right], \tag{3.6}$$

where

$$\nabla_T^2=\nabla^2-(\hat{\mathbf{n}}\cdot\overline{\nabla})^2,\quad \nabla^2=(\hat{\mathbf{n}}\cdot\overline{\nabla})^2+\frac{2}{R}\hat{\mathbf{n}}\cdot\overline{\nabla}+\nabla_t^2,\quad J(\mathbf{r}',\mathbf{r}')=U(\mathbf{r}')\ .$$

IV. SOLUTION OF THE INTEGRAL EQUATION FOR THE PROPAGATOR

To calculate the propagator $[S(\mathbf{r},\mathbf{r}')]$, we need to solve Eq. (2.10) with integral I given by formula (3.6).

First, we find the solution to zeroth order in the $1/k$ approximation (only terms independent on k). In such a case we have, instead of (2.10),

$$J(\mathbf{r},\mathbf{r}')=U(\mathbf{r})-iU(\mathbf{r})\int_0^R J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds\ , \tag{4.1}$$

with the condition $J(\mathbf{r},\mathbf{r})=U(\mathbf{r})$. The solution is simple:

$$J(\mathbf{r},\mathbf{r}')=U(\mathbf{r})\exp\left[-i\int_0^R U(\mathbf{r}'+\hat{\mathbf{n}}s)ds\right]. \tag{4.2}$$

Now it is easy to find the first approximation for $S(\mathbf{r},\mathbf{r}')$:

$$S_0(\mathbf{r},\mathbf{r}')=\exp\left[-i\int_0^R U(\mathbf{r}'+\hat{\mathbf{n}}s)ds\right], \tag{4.3}$$

$$S_0(\mathbf{r},\mathbf{r})=S_0(R=0)=1\ ,$$
$$S_0(\mathbf{r},\mathbf{r}')=S_0(\mathbf{r}',\mathbf{r})$$

(see Ref. 7), which is exactly the result of Saxon and Schiff[7] and our result (2.7).

The first approximation of S can now be substituted in (2.10) in order to obtain a second approximation to J and, as a consquence, to $S(\mathbf{r},\mathbf{r}')$. Let $S_0(\mathbf{r},\mathbf{r}')=e^{K(\mathbf{r},\mathbf{r}')}$, where

$$K(\mathbf{r},\mathbf{r}')=-i\int_0^R U(\mathbf{r}'+\hat{\mathbf{n}}s)ds\ .$$

Let us introduce a new function $f(\mathbf{r},\mathbf{r}')$:

$$f(\mathbf{r},\mathbf{r}')=1-i\int_0^R J(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds\ . \tag{4.4}$$

We have $f'\equiv\partial f/\partial R=-iJ(\mathbf{r},\mathbf{r}')$ and the first iteration for f (f_1) is equal to $f_1=S_0$.

Now we can rewrite Eq. (2.10) as an equation for f' in the $1/k$ approximation:

$$f'+iU(\mathbf{r})f+\frac{1}{k}U(\mathbf{r})\left[\frac{1}{R}(f_1-1)+i\tfrac{1}{2}U(\mathbf{r}')-\tfrac{1}{2}\int_0^R U^2(\mathbf{r}'+\hat{\mathbf{n}}s)f_1(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds+\frac{i}{2}U(\mathbf{r})f_1\right.$$
$$\left.-\frac{1}{2R}\int_0^R s(R-s)\nabla_T^2 f_1'(\mathbf{r}'+\hat{\mathbf{n}}s,\mathbf{r}')ds\right]=0\ . \tag{4.5}$$

Because of the fact that

$$\nabla_T^2 f_1'=(\nabla_T^2 S_0)'-\frac{2}{R^2}S_0'+\frac{2}{R}\nabla_T^2 S_0\ ,$$

where the prime means $\hat{\mathbf{n}}\cdot\overline{\nabla}$, in the last terms in the above formula (with ∇_T^2), the integration by parts can be done. The solution of (4.5) is easy, and because of the connections

$$if'=J(\mathbf{r},\mathbf{r}'),\quad \text{and}\quad J(\mathbf{r},\mathbf{r}')=U(\mathbf{r})S(\mathbf{r},\mathbf{r}')\ ,$$

it is easy to find the function $S(\mathbf{r},\mathbf{r}')$ in the $1/k$ approximation.

Our final result is

$$S(\mathbf{r},\mathbf{r}')=S_0(\mathbf{r},\mathbf{r}')\left[1+\frac{1}{k}\Psi(\mathbf{r},\mathbf{r}')\right], \tag{4.6}$$

where

$$\Psi(\mathbf{r},\mathbf{r}')=\frac{1}{2}U(\mathbf{r})-\frac{1}{2}U(\mathbf{r}')+\frac{i}{2}\int_0^R S_0^{-1}\nabla_t^2 S_0(\mathbf{r}'+\hat{\mathbf{n}}s)ds\ .$$

This is exactly the result (2.7).

From (4.6) we see that our assumption about J (J should be a slowly varying function) is justified: J does not contain any exponential factor involving k (has no oscillation terms). The propagator S is a symmetric function in $\mathbf{r}$ and $\mathbf{r}'$, as should be.

As an example, we consider the simple potential $U(\mathbf{r})=\alpha(x^2+y^2)=\alpha\rho^2$ (α means constant) discussed in Refs. 2 and 3. For $K(\mathbf{r},\mathbf{r}')$ and $\Psi(\mathbf{r},\mathbf{r}')$, we find

$$K=-\tfrac{1}{3}i\alpha R(\rho^2+\rho'^2+\rho\rho'\cos\phi)\ ,$$

$$\Psi=\tfrac{1}{3}\alpha R\left[R-\frac{i}{2}\alpha[(\rho\rho')^2+\rho\rho'(R^2-\rho^2-\rho'^2+\rho\rho'\cos\phi)\cos\phi+\tfrac{4}{15}(z-z')^2(\rho^2+\rho'^2-2\rho\rho'\cos\phi)]\right]\ , \qquad (4.7)$$

where $\rho\rho'\cos\phi=xx'+yy'$ and $S_0=\exp(K)$.

V. THE DIRAC CASE

The Dirac equation for propagator (propagation function where the spin of electron is taken into account) is given by

$$[E-U(\mathbf{r})+i\hat{\boldsymbol{\alpha}}\cdot\overline{\nabla}-m\hat{\beta}]G_D(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\ , \qquad (5.1)$$

where m is the mass of electron, $E\approx k$ is energy, $\hat{\boldsymbol{\alpha}},\hat{\beta}$ are Dirac matrices, and $U(r)$ is an "effective potential" inside the positron bunch. It is not difficult to guess that the solution of (5.1) is

$$G_D(\mathbf{r},\mathbf{r}')=-[E-U(\mathbf{r})+m\hat{\beta}-i\hat{\boldsymbol{\alpha}}\cdot\overline{\nabla}]G(\mathbf{r},\mathbf{r}')\ , \qquad (5.2)$$

where $G(\mathbf{r},\mathbf{r}')$ is the propagator for the Klein-Gordon equation discussed in Secs. II–IV. Because

$$G(\mathbf{r},\mathbf{r}')=G_0(R)S_0(\mathbf{r},\mathbf{r}')\left[1+\frac{1}{k}\Psi(\mathbf{r},\mathbf{r}')\right]$$

[see Eq. (4.6)], we can conclude, consistent with our considered approximation, that G_D has the form

$$G_D=-G_0(R)S_0(\mathbf{r},\mathbf{r}')\Big\{(1+\hat{\boldsymbol{\alpha}}\hat{\mathbf{n}})[k-U(\mathbf{r})+\Psi(\mathbf{r},\mathbf{r}')] +m\hat{\beta}+i\frac{1}{R}\hat{\boldsymbol{\alpha}}\cdot\hat{\mathbf{n}}-\hat{\boldsymbol{\alpha}}\cdot\overline{\nabla}_tK(\mathbf{r},\mathbf{r}')\Big\}\ , \qquad (5.3)$$

where $\overline{\nabla}=\hat{\mathbf{n}}(\hat{\mathbf{n}}\cdot\overline{\nabla})+\overline{\nabla}_t$.

VI. CONCLUDING REMARKS

This paper can be considered as the next of the series of papers in which the "beamstrahlung" is discussed. However, we hope that the results obtained by us will be useful not only in the context of beamstrahlung, but also in other contexts, for example, in solid-state physics, when we consider the processes with very energetic electrons (from a high-energy beam) running inside the crystals.

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