Why does electromagnetism conserve parity?

K. S. Babu and Rabindra N. Mohapatra
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 4 May 1990)

The assumption of electrical neutrality of the neutrino in the context of the standard model is shown to explain why electromagnetism conserves parity. We then construct an extension of the standard model where the neutrino has a nonzero but tiny charge. Such theories necessarily imply a parity-violating component in QED and nonconservation of electric charge ($\Delta Q \neq 0$). The strengths of the parity-violating component of QED as well as $\Delta Q \neq 0$ interactions are connected to the nonvanishing neutrino charge Q_ν which is shown to be bounded by $Q_\nu \lesssim 10^{-28}e$ in the context of these models.

I. INTRODUCTION

Two of the fundamental mysteries of electromagnetism are the following: (i) Why is electric charge quantized? (ii) Why is electromagnetism vectorlike (or parity conserving)? In the context of the standard model and its extensions the first question has recently been addressed by a number of authors.[1-3] The basic strategy employed in these papers is to analyze the constraints[1-4] implied by the Adler-Bell-Jackiw anomaly and impose the additional constraint of vectorlike nature of electromagnetism. The first assumption is required for renormalizability of the theory and the second may be reasonable in view of several suggestions that QED of massless fermions is inconsistent[5] and in order to give fermions their mass, QED has to be necessarily vectorlike.

In this paper we seek an understanding of why QED is vectorlike in the context of the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model.[6] We find that if we demand the neutrino to be electrically neutral, the anomaly constraints imply relations between weak hypercharges that lead to vectorlike QED as well as quantization of electric charges. Note that *a priori*, because of the U(1) factor, hypercharge quantum numbers of the various fermion multiplets can take a continuous set of values, and therefore neither the quantization of electric charge nor the vectorlike nature of QED follows from the choice of the gauge group.

We find this result interesting because, as is well known, the masslessness of the neutrino led to the *V-A* theory of weak interaction of Marshak, Sudarshan, Feynman, and Gell-Mann, elucidating the nature of one of the four fundamental forces. The result of this paper would be that another property of the neutrino, its electrical neutrality, provides an understanding of why electromagnetism conserves parity as well as why the electric charges of known fermions and bosons are quantized.

We then discuss the situation in left-right-symmetric theories and show how again the vanishing of the neutrino charge (both of the left- and the right-handed neutrinos) in conjunction with anomaly constraints leads to vectorlike QED as well as electric charge quantization. An extension of the standard model is then presented where the neutrino has a nonzero charge which leads to a parity-violating component in QED. In such theories, nonvanishing fermion masses imply violation of electric charge conservation. From these considerations we obtain a very stringent limit on the electric charge of the neutrino ($Q_\nu \lesssim 10^{-28}e$).

II. ELECTRICAL NEUTRALITY OF NEUTRINO AND VECTORLIKE QED IN THE STANDARD MODEL

The standard model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The quarks and leptons are assigned to the gauge group as

$$\begin{aligned} Q_L &\equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} :(3,2,Y_q)\ , \\ \psi_L &\equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} :(1,2,Y_l)\ , \\ u_R &:(3,1,Y_u)\ , \\ d_R &:(3,1,Y_d)\ , \\ e_R &:(1,1,Y_e)\ . \end{aligned} \tag{1}$$

The assignments under $SU(3)_C$ and $SU(2)_L$ are chosen to match the observed properties of strong and weak interactions (e.g., β decay and muon decay). The hypercharge quantum numbers are left free. It is these hypercharge quantum numbers which decide (a) whether QED is vectorlike and (b) whether the electric charge is quantized. An understanding of the above two questions requires precise values for the Y_i and our purpose is to explore the nature of theoretical constraints that can uniquely fix the hypercharge quantum numbers.

The first set of constraints is obtained by demanding the theory to be anomaly-free.[1-4] There are four such constraints related to the vanishing of $[U(1)_Y]^3$, $U(1)_Y[SU(3)_C]^2$, $U(1)_Y[SU(2)_L]^2$, and $U(1)_Y$ (gravity)^2 anomalies and are given by

$$6Y_q^3+2Y_l^3-3Y_u^3-3Y_d^3-Y_e^3=0\ , \tag{2a}$$

$$2Y_q-Y_u-Y_d=0\ , \tag{2b}$$

$$3Y_q+Y_l=0\ , \tag{2c}$$

$$6Y_q+2Y_l-3Y_u-3Y_d-Y_e=0\ . \tag{2d}$$

These four equations involve five hypercharge parameters leaving one of the parameters arbitrary. Therefore, we cannot conclude from anomaly constraints alone whether QED is vectorlike or if the electric charges are quantized.

To proceed further we note that in the standard model by rescaling the hypercharge of the Higgs field, we can write the electric charge as

$$Q=I_{3L}+\frac{Y}{2}\ . \tag{3}$$

Now, if we demand that neutrino is electrically neutral, it implies

$$Q_\nu=0 \text{ or } Y_l=-1\ . \tag{4}$$

Substituting Eq. (4) into (2a)–(2d) we obtain $Y_u=\frac{4}{3}$, $Y_d=-\frac{2}{3}$, $Y_q=\frac{1}{3}$, and $Y_e=-2$. Clearly, this results in quantization of electric charge. Furthermore, since both the left and the right helicities of quarks and leptons have the same charge, QED turns out to be vectorlike.

It is worth pointing out that if one adds the right-handed neutrino to the standard-model spectrum, while a vanishing neutrino charge implies vector QED, it is not required. Thus the tight relation between the neutrality of neutrino and vector QED is a property only of the minimal standard model.

III. LEFT-RIGHT-SYMMETRIC MODEL AND VECTOR ELECTRODYNAMICS

The anomaly constraints in the left-right-symmetric model have been discussed in several papers[2,4] where it has been shown that, for the standard assignment of quarks and leptons,

$$\begin{aligned}
Q_L &\equiv \begin{bmatrix} u_L \\ d_L \end{bmatrix} \quad (3,2,1,Y_{q_L})\ ,\\
Q_R &\equiv \begin{bmatrix} u_R \\ d_R \end{bmatrix} \quad (3,1,2,Y_{q_R})\ ,\\
\psi_L &\equiv \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \quad (1,2,1,Y_{l_L})\ ,\\
\psi_R &\equiv \begin{bmatrix} \nu_R \\ e_R \end{bmatrix} \quad (1,1,2,Y_{l_R})\ ,
\end{aligned} \tag{5}$$

mixed-gravitational and mixed-color anomalies imply $Y_{q_L}=Y_{q_R}\equiv Y_q$ and $Y_{l_L}=Y_{l_R}\equiv Y_l$ whereas $U(1)[SU(2)]^2$ anomalies lead to

$$3Y_q+Y_l=0\ . \tag{6}$$

Again, we see that one U(1) parameter is arbitrary and therefore neither quantization of electric charge nor vectorlike QED follows from the anomaly constraints alone. One might think that left-right symmetry of the hypercharge assignment may lead to vector QED; but this is not true since the electric charge is determined by the unbroken U(1) generator left over after symmetry breaking and, in general, we have

$$Q=I_{3L}+bI_{3R}+cY\ . \tag{7}$$

The constants b and c depend on the quantum numbers of the Higgs bosons responsible for symmetry breaking. For instance, if $SU(2)_R$ symmetry is broken by a Higgs boson $\Delta_L\equiv(a,0,Y_\Delta)\oplus\Delta_R\equiv(0,a,Y_\Delta)$ with hypercharge Y_Δ and the remaining $SU(2)_L\times U(1)$ symmetry is broken by $\phi(a,b,Y_\phi)+(a\to b)$, then

$$\begin{aligned}
b&=-\frac{I_{3L}(\phi)}{I_{3R}(\phi)-\dfrac{Y_\phi}{Y_\Delta}I_{3R}(\Delta)}\ ,\\
c&=\frac{I_{3L}(\phi)I_{3R}(\Delta)}{I_{3R}(\phi)Y_\Delta-Y_\phi I_{3R}(\Delta)}\ .
\end{aligned} \tag{8}$$

Note that vectorlike QED requires $b=1$ and Eq. (8) tells us that b is in general different from 1. If we now require that both the neutrino helicities have the same charge, i.e., $Q(\nu_L)=Q(\nu_R)$, then

$$Q=I_{3L}+I_{3R}+cY\ . \tag{9}$$

This implies vectorlike QED, but not electric charge quantization. If we further demand that $Q(\nu_L)=Q(\nu_R)=0$, it fixes $c=-1/2Y_l$ which implies electric charge quantization.

At this point, it is worth emphasizing that the economy in the leptonic sector as well as the low rank of the gauge groups have played a crucial role in drawing our conclusions. As soon as we go to higher-rank gauge groups ($r\geq 6$) or more fermions, we would lose both these properties of QED.

IV. LIMITS ON PARITY VIOLATION IN QUANTUM ELECTRODYNAMICS

We saw in Sec. II that a zero electric charge of the neutrino implies parity conservation by QED. We could turn this argument around to conclude that, if the neutrino had a tiny electric charge in the standard model, via anomaly equations, this would imply parity violation in the interactions of a photon with charged particles such as the electron and the proton. In fact, the anomaly equations (2a)–(2d) imply that, if $Q_\nu=\epsilon/2$, then, from the electric charge formula $Q=I_{3L}+Y/2$,

$$\begin{aligned}
&Y_l=-1+\epsilon,\quad Y_q=\tfrac{1}{3}(1-\epsilon)\ ,\\
&Y_e=-2(1-\epsilon)\ ,\\
&Y_u=\tfrac{4}{3}(1-\epsilon),\quad Y_d=-\tfrac{2}{3}(1-\epsilon)\ ,
\end{aligned} \tag{10}$$

leading to $Q_{u_L}-Q_{u_R}=\epsilon/2$, $Q_{d_L}-Q_{d_R}=-\epsilon/2$, $Q_{e_L}-Q_{e_R}=-\epsilon/2$. This results in a parity-violating component of QED given by

$$\mathcal{L}_{\rm QED}^{(-)}=\frac{ie\epsilon}{2}(-\bar{e}\gamma_\mu\gamma_5 e+\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d)A^\mu\ . \tag{11}$$

The most stringent bound on the electric charge of the neutrino is obtained from SN 1987A observations[7] to be $Q_\nu \le 10^{-19}e$ which implies $\epsilon \le 10^{-19}$. This means that the strength of the parity-violating QED interactions is $\le e \times 10^{-19}$. In the nonrelativistic limit, Eq. (11) leads to spin-dependent interactions of the form $\psi^\dagger \sigma \psi \cdot \mathbf{A}$ as well as $\psi^\dagger \sigma \cdot \nabla \psi \phi$, where $A_\mu = (\phi, \mathbf{A})$. We will discuss the detailed phenomenological implications of this in a subsequent paper. Here, we simply note that in this model the atoms will not be neutral; in such a case, the stability of galaxies requires $\epsilon \le (G_N m_p^2/10)^{1/2} = 10^{-20}$. Furthermore, in our model $Q(e)+Q(p)=0$ whereas $Q_n = -Q_\nu = -\epsilon/2$. (This applies both to the vector and axial-vector components.) The most stringent experimental bound on the charge of neutron is from a Grenoble experiment by Mampe *et al.*[8]: $Q_n \le 10^{-21}e$. When $Q_p + Q_e = 0$, a more stringent bound can be obtained from the following considerations. The Earth will be electrically charged due to its neutron content and will have a radial electric field $E \approx \epsilon \times 10^{30}$ V/m. If we assume that E should not exceed 100 V/m, this implies $\epsilon \le 10^{-28}e$ which is the most stringent observational bound on the neutrino charge in the context of the $SU(2)_L \times U(1)_Y$ model.

V. FERMION MASSES AND BREAKDOWN OF ELECTRIC CHARGE CONSERVATION

In order to study further implications of a nonvanishing neutrino charge, we note that once the fermions acquire mass, there will be breakdown of electric charge conservation since a Dirac mass connects the left- and the right-handed chirality components. To show this explicitly we first realize that the Higgs doublet which defined the electric charge by causing the breakdown of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ had its hypercharge Y_φ scaled to $Y_\varphi = 1$; it therefore has no gauge-invariant Yukawa couplings. Let us therefore introduce a second Higgs doublet φ_2 with $Y_2 = 1-\epsilon$. Its components have electric charges $Q(\varphi_2^{+1/2}) = 1-\epsilon/2$ and $Q(\varphi_2^{-1/2}) = -\epsilon/2$. It has gauge-invariant Yukawa couplings to the fermions. To get fermion masses we have to give a nonzero vacuum expectation value (VEV) to $\langle \varphi_2^{-1/2} \rangle = V_2$. This not only breaks electric charge conservation[9] but also gives the photon a mass:

$$m_\gamma = \frac{e}{\sqrt{2}} \epsilon \frac{V_1 V_2}{\sqrt{V_1^2 + V_2^2}} \ . \tag{12}$$

Because of the Z-γ mixing in the mass matrix [the mixing angle is given by $\theta = (M_\gamma / M_Z)(V_2/V_1)$], additional couplings (both parity violating and parity conserving) of the photon to the fermions proportional to ϵ are induced. These new couplings also obey $Q_e + Q_p = 0$ and $Q_n + Q_\nu = 0$, so that the bounds on ϵ discussed in Sec. IV are still valid. Since φ_2 must give mass to the t quark, if we demand its Yukawa couplings to be less than order 1, we expect $V_2 \simeq 100$ GeV. The present upper limit[10] on $m_\gamma \le 10^{-25}$ GeV implies $\epsilon \le 10^{-27}$.

We wish to point out that, in contrast with previous speculations on electric charge breakdown which were put in an *ad hoc* manner to explore new phenomena, we are led to electric charge violation from considerations of a nonvanishing neutrino charge. We can further show that in the context of the standard model the breakdown of the electric charge also implies that the neutrino has a nonzero charge. To see this, let us consider the standard model with the usual hypercharge assignment and add to it a second Higgs doublet φ_2 with $Y_{\varphi_2} = 1+\epsilon$, if we give the $I_3 = -\frac{1}{2}$ component of φ_2 a VEV, it will break electric charge and at the same time will lead to Z-γ mixing in the mass matrix. Diagonalization of this matrix will cause the neutrino to acquire an electric charge proportional to ϵ. Since Z coupling violates parity, the photon coupling will now acquire a parity-violating piece. Similar results are obtained if we introduce an isosinglet scalar with hypercharge ϵ into the standard model and give it a VEV. Thus, in the context of the $SU(2)_L \times U(1)_Y$ model, neutrino charge, parity violation in QED, and breakdown of electric charge conservation are related to each other in the sense that assuming one leads to the other two.

ACKNOWLEDGMENTS

We would like to thank J. Sucher for useful discussions. This work was supported by a grant from National Science Foundation.

[1] N. G. Deshpande, Oregon Report No. OITS-107, 1979 (unpublished); R. Foot, H. Lew, G. Joshi, and R. Volkas, Mod. Phys. Lett. A **5**, 95 (1990).

[2] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **63**, 938 (1989); Phys. Rev. D **41**, 271 (1990).

[3] R. Foot, University of Wisconsin report, 1989 (unpublished); X-G. He, G. C. Joshi, H. Lew, B. McKeller, and R. Volkas, Phys. Rev. D **40**, 3140 (1989); C. Geng, *ibid.* **41**, 1292 (1990); S. Rudaz, *ibid.* **41**, 2619 (1990); E. Golowich and P. B. Pal, *ibid.* **41**, 3537 (1990).

[4] C. Q. Geng and R. E. Marshak, Phys. Rev. D **39**, 693 (1989); J. A. Minahan, P. Ramond, and R. C. Warner, *ibid.* **41**, 715 (1990).

[5] See, for instance, V. N. Gribov, Nucl. Phys. **B206**, 103 (1982); A. S. Gorsky, B. L. Ioffe, and A. Yu Khodjamirian, Phys. Lett. B **227**, 474 (1989).

[6] For an earlier discussion of this question, see A. Zee, in *Weak Interaction as Probes of Unification—1980,* proceedings of the workshop, edited by G. B. Collins, L. N. Cheng, and J. R. Ficenec (AIP Conf. Proc. No. 72) (AIP, New York, 1981), p. 475.

[7] G. Barbiellini and G. Cocconi, Nature (London) **329**, 21 (1987).

[8] J. Bauman, R. Gahler, A. I. Ioffe, J. Klaus, and W. Mampe, Nucl. Intrum. Methods **A284**, 130 (1989).

[9] For theoretical discussions of electric charge nonconservation, see L. B. Okun and Ya B. Zeldovich, Phys. Lett. **78B**, 597

(1978); A. Yu. Ignatiev, V. Kuzmin, and M. E. Shaposnikov, *ibid.* **84B** 315 (1978); R. N. Mohapatra, Phys. Rev. Lett. **59**, 1510 (1987); S. Nussinov, *ibid.* **59**, 2401 (1987); M. Suzuki, Phys. Rev. D **38**, 1544 (1988); M. M. Tsypin, Yad. Fiz. **50**, 431 (1989) [Sov. J. Nucl. Phys. **50**, 269 (1989)].

[10]A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. **43**, 277 (1971); L. Davis, A. S. Goldhaber, and M. M. Nieto, Phys. Rev. Lett. **35**, 1402 (1975).