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Low-energy technicolor Lagrangian with vector mesons

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We apply techniques used in QCD and the implied chiral symmetry to motivate a low-energy Lagrangian for technicolor theories. This allows one to cleanly introduce the gluon and the weak gauge bosons at the technifermion level. The Higgs mechanism and techni-vector-meson dominance are displayed. The effect of vector dominance on the W and Z masses, the techni-ω and techni-ρ couplings, and the sizable enhancement by the latter on W and Z production are demonstrated. Differences between our results and some published results are discussed. The incorporation of extended technicolor interactions is also presented.

I. INTRODUCTION

In recognition of its nonperturbative nature, it is generally accepted that the low-energy region of QCD is represented by a nonlinear σ-type model.[1] The effective Lagrangian of the model consists of pseudoscalar fields which are the pseudo-Goldstone bosons resulting from a global chiral-symmetry breakdown. Interactions with the low-lying vector particles can be introduced via external flavor currents coupled to the pseudoscalars. Furthermore, the effective Lagrangian can be divided into normal-parity and abnormal-parity parts. The normal-parity part[2-5] contains the kinetic energy terms, and the abnormal-parity part contains the Wess-Zumino term.[3-13]

The general nature of the derivation of the low-energy phenomenological Lagrangian of QCD suggests that the approach may be applied to other strongly interacting theories based on non-Abelian gauge groups, in particular, to technicolor theory.[14,15] The technicolor theory, which is designed to provide a dynamical mechanism to break the electroweak gauge interactions, shares some common features with confining theories such as QCD but is confined at a much higher energy and has a much richer structure.[16,17] Predictions of technicolor at the Superconducting Super Collider (SSC) energy regime obtained by drawing analogies with QCD can be found in the literature.[18-23]

In this article we propose a low-energy effective Lagrangian for a class of technicolor theories motivated by the chiral symmetry of the technicolor force and the observation of vector-meson dominance[24] in QCD. The general form of the normal and anomalous chiral Lagrangians of the technicolor theory is expected to be a valid phenomenological framework for new strong-interaction physics between 1 TeV and around the electroweak energy scale. This effective Lagrangian allows us to extend a previous treatment[20] of technicolor phenomenology which described interactions among low-lying technicolor bosons and the light gauge bosons, γ, Z^0, $W^{\pm}$ and gluons.

Since we concern ourselves only with the low-energy effective Lagrangian for the strongly interacting, composite sector of the technicolor theory, we do not address some of the important open questions of the theory, such as the problem of the flavor-changing neutral current[16,25] and its proposed solutions.[19,26,27] However, we shall examine the low-energy effect of the extended technicolor theory relevant to the present purpose. The baryonic sector, which is above the range of interest in this work, will not be considered.

In Sec. II, we present and examine a low-energy effective Lagrangian containing pseudo-Goldstone-boson condensates and low-lying vector excitations that are composites of the fundamental fermion fields. A line of argument that can possibly arrive at such an effective Lagrangian and which has been widely reported in the literature is briefly recapitulated in the Appendix. This argument may offer some insight into the effective Lagrangian used but the subsequent phenomenological development is independent of the specific argument. We also demonstrate the Higgs mechanism and vector-meson dominance. Section III contains a discussion of the phenomenology involving the technivectors and a comparison with some of the earlier work.[21,23] In Sec. IV, we discuss the effect of the extended technicolor interaction which introduces direct coupling between the technicolor bosons and the light fermions. Such couplings could have significant phenomenological implications. Section V is the summary section.

II. EFFECTIVE LAGRANGIAN FOR TECHNICOLOR MODELS

There is at present no derivation from first principles of the low-energy phenomenology of a strongly interacting field theory. However, for the case of QCD, much is known about such a phenomenological Lagrangian both empirically and from general principles such as chiral symmetry. The argument that leads to the low-energy QCD effective Lagrangian is extracted from phenomenology below 1 GeV; however, we envisage that it applies in general to non-Abelian theories below their confinement scales. The essential ingredients of such a low-energy Lagrangian are the pseudo-Goldstone bosons (PGB's) in the nonlinear realization and the low-lying massive spin-1 excitations which couple to the PGB's in flavor-symmetric form mimicking a gauge interaction. In order to include the standard-model gauge bosons, e.g., the gluon, W, Z, and γ, we take the vector-meson-dominance (VMD) approach. In this approach the light gauge bosons appear only in the mass terms of some of the vector- and axial-vector-meson collective fields as shifts to the latter. This form of effective Lagrangian with VMD for QCD has been obtained in various approaches in the literature such as the bosonization argument,[3,4] the Legendre transform,[28] the hidden-symmetry approach,[29] and the minimal gauging method.[8] In order to explore in systematic detail the physical assumptions involved and to attempt to make some reasonable first approximation for some extended technicolor parameters to be discussed in Sec. IV, we offer a heuristic argument along the lines of Refs. 3 and 4 in the Appendix. The subsequent development of this section and Sec. III is of course independent of this particular argument.

Adopting the general result of QCD as outlined above we can write down the following effective action, based on which we can carry out phenomenological analyses:

$$S_{\rm eff}=\int d^4x\,{\rm tr}\left[\frac{F^2}{4}(\mathcal{D}U)^\dagger\mathcal{D}U-\frac{1}{2}(F^VF^V+F^AF^A)+m_V^2[(V-\mathcal{V})^2+(A-\mathcal{A})^2]\right]+\Gamma_A(U,V,A)\,, \tag{2.1}$$

where

$$\begin{aligned}
U&=\exp\left[\frac{2i\phi}{F}\right],\\
D_\mu&=\partial_\mu-ig[V_\mu,\ ]+ig\{A_\mu,\ \}\,,\\
F^V_{\mu\nu}&=\partial_\mu V_\nu-\partial_\nu V_\mu+ig[V_\mu,V_\nu]+ig[A_\mu,A_\nu]\,,\\
F^A_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu+ig[A_\mu,V_\nu]+ig[V_\mu,A_\nu]\,.
\end{aligned} \tag{2.2}$$

The quantity F is the "unrenormalized" technipseudoscalar decay constant; ϕ is the $N_f\times N_f$ matrices of Goldstone bosons (all but three will become massive pseudo-Goldstone bosons); V and A are the $N_f\times N_f$ matrices for the vector and axial-vector technimesons; and $\mathcal{V}$ and $\mathcal{A}$ represent the standard-model gauge bosons. All of these will be further explained below.

Although the technifermion degree of freedom is lost below the technicolor condensate scale, the symmetry of the effective Lagrangian reflects the flavor symmetry of the fundamental fermions and hence the latter needs to be specified. We consider Farhi-Susskind-type models which contain N_f right-handed singlet technifermions and $N_d=N_f/2$ left-handed doublets. The flavor symmetry of this model is therefore SU(N_f) and the generators can be written in a convenient form:

$$\begin{aligned}
t^0&=(1_2\times 1_{N_d})/2\sqrt{N_d}\,,\\
\Sigma^i&=(\sigma_i\times 1_{N_d})/2\sqrt{N_d},\quad i=1,2,3\,,\\
\Lambda_a&=(1_2\times\lambda_a)/2\sqrt{2},\quad a=1,\ldots,N_d^2-1\,,\\
\Omega_a^i&=(\sigma_i\times\lambda_a)/2\sqrt{2}\,,
\end{aligned} \tag{2.3}$$

where the σ_i represent the SU(2)-isospin generators and the λ_a are the generators for SU(N_d).[30] When SU(N_d) contains the ordinary color SU(3), we take the first eight λ_a to coincide with the color-SU(3) generators. The gauge vector and axial-vector currents can thus be written as

$$\begin{aligned}
\mathcal{V}&=\frac{\sqrt{2}g_c}{g}G^a\Lambda_a^{(3)}+\frac{\sqrt{N_d}g_L}{2g}W_i\Sigma^i+\frac{g_Y}{2g}B(Y_L+\sqrt{N_d}\Sigma^3)\,,\\
\mathcal{A}&=-\frac{\sqrt{N_d}g_L}{2g}W_i\Sigma^i+\frac{g_y}{2g}B\sqrt{N_d}\Sigma^3\,.
\end{aligned} \tag{2.4}$$

Before we proceed further, let us eliminate the true Goldstone bosons by utilizing the Higgs mechanism and going into the unitary gauge. Since Σ^j, $j=1,2,3$, in (2.3) form a subalgebra of the flavor group, we can make a right coset decomposition of U,

$$U=\exp\left[\frac{i\pi^j\Sigma^j}{F}\right]\exp\left[\frac{i\phi_\alpha X_\alpha}{F}\right],$$

where the set $\{X_\alpha\}$ contains the generators in (2.3) excluding the Σ^j. Then the field π^j can be eliminated by an SU$_L$(2) gauge transformation with the transformation matrix, $\mathcal{U}_L=\exp(-i\pi^j\Sigma^j/F)$. The Goldstone bosons are now eliminated from U and only pseudo-Goldstone bosons remain. We shall work only in the unitary gauge in the following discussions, and we use the same symbols to denote V, A, and U in this gauge.

One can now diagonalize the vector states and calculate the W and Z masses. The constraint from anomaly cancellation[31] in the electroweak gauge sector requires the left-handed hypercharge matrix Y_L to be traceless. Thus we can write Y_L as a linear combination of the diagonal Λ_a:

$$Y_L\equiv\sqrt{N_d}\,\overline{Y}\,\overline{\Lambda} \tag{2.5}$$

with

$$\begin{aligned}
\overline{Y}^2&=\frac{2}{N_d}{\rm Tr}\,Y_L^2\,,\\
\overline{\Lambda}&=b_a\Lambda_a,\quad \Sigma_a b_a^2=1\,.
\end{aligned} \tag{2.6}$$

To obtain the mass eigenstates, we note that the relevant quadratic terms in (2.1) which determine the mixing are

$$\mathcal{L}_2=\mathrm{Tr}(\partial_\mu\phi\partial^\mu\phi+F^2g^2A_\mu A^\mu+2FgA_\mu\partial^\mu\phi$$
$$+m_V^2A_\mu A^\mu+m_V^2V_\mu V^\mu-2m_V^2V_\mu\mathcal{V}^\mu$$
$$+m_V^2\mathcal{V}_\mu\mathcal{V}^\mu-2m_V^2A_\mu\mathcal{A}^\mu+m_V^2\mathcal{A}^\mu\mathcal{A}_\mu)\ . \tag{2.7}$$

We denote our states by

$$\begin{aligned}\phi&=P_0't^0+P_0\overline{\Lambda}+P_a\Lambda_a'+P_a^i\Omega_a^i\ ,\\ V&=\omega't^0+\omega\overline{\Lambda}+\rho_i\Sigma_i+\omega_a\Lambda_a'+\rho_a^i\Omega_a^i\ ,\\ A&=\alpha't^0+\alpha\overline{\Lambda}+\beta_i\Sigma_i+\alpha_a\Lambda_a'+\beta_a^i\Omega_a^i\ ,\end{aligned}\tag{2.8}$$

where the original set of orthogonal generators Λ^a in (2.3) has been replaced by an equivalent orthogonal set $\overline{\Lambda}$ and Λ'^a. In the exotic sector involving only technivectors whose quantum numbers are different from the gauge particles, there are no contributions from $\mathrm{Tr}V\mathcal{V}$ and $\mathrm{Tr}A\mathcal{A}$. The physical parameters and mass eigenstates are

$$\begin{aligned}A_p&=A+\frac{gF_p}{m_V^2}\partial\phi_p\ ,\\ V_p&=V\ ,\\ \phi_p&=R\phi\ ,\\ F_p&=RF\ ,\\ m_A^2&=R^{-2}m_V^2\ ,\\ R&=\left[1-\frac{g^2F_p^2}{m_V^2}\right]^{1/2}.\end{aligned}\tag{2.9}$$

Note that the mass degeneracy of the vector and axial vector is lifted and that the physical value of the decay constant has been shifted. If some of the technifermions are in a fundamental representation of SU(3) color, then there will be mixing of gluons with the corresponding technivectors denoted as ω_8^a, $a=1-8$, through the term contained in (2.7):

$$-2m_V^2\mathrm{Tr}V\mathcal{V}=-m_V^2\frac{\sqrt{2}g_3}{g}\omega_8^aG^a+\cdots\ . \tag{2.10}$$

The mass eigenstates are

$$\begin{aligned}\begin{bmatrix}\omega_8\\ G\end{bmatrix}_{\mathrm{phys}}&=\begin{bmatrix}\cos\theta_8 & -\sin\theta_8\\ \sin\theta_8 & \cos\theta_8\end{bmatrix}\begin{bmatrix}\omega_8\\ G\end{bmatrix},\\ \cos\theta_8&=\left[1+2\left[\frac{g_3}{g}\right]^2\right]^{-1/2}\end{aligned}\tag{2.11}$$

and have masses $m_{\omega_8}\simeq m_V$ and $m_G=0$. Here we have vector-meson dominance of QCD by a color octet of technivector mesons.

The mixing is the electroweak sector involving the generators Σ_i and $\overline{\Lambda}$ is more complicated and more interesting. First we note that in this sector $\mathrm{Tr}A\partial\phi$ contains only $\frac{1}{2}\alpha\partial P_0$ while $\mathrm{Tr}A\mathcal{A}$ contains no α terms. Thus the isosinglet axial vector α and pseudoscalar P_0 decouple from the rest and mix only with each other as indicated in (2.9).

For the remaining fields, we must diagonalize:

$$\begin{aligned}\mathcal{L}_L^{\mathrm{EW}}=&\left[\frac{F^2g^2}{2}+\frac{m_V^2}{2}\right]\beta^2+\frac{m_V^2}{2}(\rho^2+\omega^2)\\ &-m_V^2x\rho\cdot\mathbf{W}-m_V^2y\rho^3B-m_V^2y\overline{Y}\omega B\\ &+m_V^2x\beta\cdot\mathbf{W}-m_V^2x\beta^3B+m_V^2x^2\mathbf{W}^2\\ &+m_V^2y^2B^2\left[1+\frac{\overline{Y}^2}{2}\right],\end{aligned}\tag{2.12}$$

where $x=\sqrt{N_d}(g_L/2g)$ and $y=\sqrt{N_d}(g_Y/2g)$. We will do this in the approximation that the gauge couplings are much smaller than the technimeson effective coupling g. For the charge sector we find the approximate eigenstates

$$\begin{aligned}\beta_{\mathrm{phys}}^{\pm}&\simeq\beta^{\pm}+xR^2W^{\pm}\ ,\\ \rho_{\mathrm{phys}}^{\pm}&\simeq\rho^{\pm}-xW^{\pm}\ ,\\ W_{\mathrm{phys}}^{\pm}&\simeq W^{\pm}+x\rho^{\pm}-xR^2\beta^{\pm}\ .\end{aligned}\tag{2.13}$$

The axial-vector and vector masses m_A and m_V are given in (2.9) and the W-boson mass is

$$M_W^2\simeq\frac{N_d}{4}g_L^2F_p^2[1-(1+R^4)x^2]\ . \tag{2.14}$$

We give the next-order correction of M_W for later use. One can check that the technipseudoscalar decay constant is given to leading order by the standard result $F_p\simeq 250\ \mathrm{GeV}/\sqrt{N_d}$. In (2.13) we have neglected higher-order terms which involve, for example, a tiny mixing of ρ and β. This mixing will lead to a small parity-violating coupling, such as the $\beta\phi\phi$ coupling.

To diagonalize the neutral bosons we use the usual redefinitions for the Z and the photon A_γ in terms of the Weinberg angle θ; $Z=-\cos\theta\, W^3+\sin\theta\, B$, $A_\gamma=\sin\theta\, W^3+\cos\theta\, B$ and we find, for the mass eigenstates,

$$\begin{aligned}\beta_{\mathrm{phys}}^3&\simeq\beta^3-\frac{x}{\cos\theta}R^2Z\ ,\\ \rho_{\mathrm{phys}}^3&\simeq\rho^3-\frac{e}{g}\sqrt{N_d}A_\gamma+x\frac{\cos2\theta}{\cos\theta}Z\ ,\\ \omega_{\mathrm{phys}}&\simeq\omega-x\sin\theta\overline{Y}(A_\gamma+\tan\theta Z)\ ,\\ Z_{\mathrm{phys}}&\simeq Z+x\sin\theta\,\overline{Y}\tan\theta\,\omega-x\frac{\cos2\theta}{\cos\theta}\rho^3+\frac{x}{\cos\theta}R^2\beta^3\ ,\\ A_{\gamma\,\mathrm{phys}}&\simeq A_\gamma+\frac{e}{2g}\sqrt{N_d}\overline{Y}\omega+\frac{e}{g}\sqrt{N_d}\rho^3\ ,\end{aligned}\tag{2.15}$$

with a Z mass

$$M_Z^2\simeq\frac{N_d}{4}\frac{g_L^2F_p^2}{\cos^2\theta}\left[1-\frac{x^2}{\cos^2\theta}(\overline{Y}^2\sin^4\theta+\cos^22\theta+R^4)\right], \tag{2.16}$$

where we have again given the next-order corrections to M_Z^2. It is clear from (2.14) and (2.16) that $M_W^2/M_Z^2\cos^2\theta\simeq 1$ to lowest order but will deviate from unity in the next-leading-order x^2. This correction must be consistent with experimental bounds from actual measurements of the W and Z masses and electroweak coupling strengths. A detailed discussion of this constraint including radiative corrections will be presented elsewhere.

III. TECHNI-VECTOR-MESON DOMINANCE AND PHENOMENOLOGY

We begin this section by reviewing the predictions for the various parameters in the model (2.1). In addition to the known $SU(3)\times SU(2)\times U(1)$ gauge couplings there is the strong coupling g associated with the technimeson sector. Using scaling arguments[32] for large N_T one can estimate g from our knowledge of $g^2_{\rho\pi\pi}/4\pi \simeq 2.7$ in QCD:

$$g \simeq g_{\rho\pi\pi}\left[\frac{3}{N_T}\right]^{1/2} \simeq \frac{10}{\sqrt{N_T}} . \tag{3.1}$$

Furthermore, the technidecay constant is determined by the electroweak breaking scale and the number of technidoublets in the model, N_d:

$$F_p \simeq \frac{250\ \text{GeV}}{\sqrt{N_d}} . \tag{3.2}$$

Finally, the mass of the technivectors can be estimated using the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation:[33]

$$\frac{g^2F_p^2}{m_V^2} \simeq \frac{1}{2} , \tag{3.3}$$

which gives

$$m_V \simeq 885\left[\frac{4}{N_T}\right]^{1/2}\left[\frac{4}{N_d}\right]^{1/2} \text{GeV} ,$$

agreeing with the result obtained from scaling arguments using m_ρ.[18,21,34]

In the pseudoscalar sector the weak gauge interactions and the extended technicolor interactions combine to contribute masses[34] to all of these states. As an illustration of the types of pseudoscalars that arise in these models, one can consider the one family technicolor model of Farhi and Susskind.[35] In this model there are six techniquarks and two technileptons. The techniquarks form three left-handed weak isodoublets, each with a different QCD color, and six right-handed weak isosinglets. The color-singlet technileptons form one left-handed weak isodoublet and two right-handed weak isosinglets:

$$\begin{bmatrix} U_a \\ D_a \end{bmatrix}_L , U_{aR}, D_{aR}, \begin{bmatrix} N \\ E \end{bmatrix}_L , N_R, E_R, \quad a=1,2,3 .$$

The indices of the technicolor group $SU(N_T)$, $N_T \ge 4$ are omitted. The flavor group of this model is SU(8) and hence it has an $SU(8)\times SU(8)\times U(1)$ global chiral symmetry at energies above the condensate scale. The pseudoscalars can occur in color singlets, triplets, or octets. For color singlets, one finds[22]

$$8\ \text{GeV} < M(P^{\pm}) < 40\ \text{GeV} ,$$

$$2\ \text{GeV} < M(P^0, P^{0\prime}) < 40\ \text{GeV} ,$$

and for the color triplet P_3 and color octet P_8 one has (see, for example, the third article of Ref. 22)

$$M(P_3) \simeq 160\ \text{GeV}\left[\frac{4}{N_T}\right]^{1/2} ,$$

$$M(P_8) \simeq 240\ \text{GeV}\left[\frac{4}{N_T}\right]^{1/2} .$$

It should be noted that the above estimates of pseudoscalar masses may have a sizable uncertainty. The upper limit of the color singlet could exceed 100 GeV and the masses of the color triplet and octet could be increased by 50%. These uncertainties come from two sources, both related to the extended technicolor contribution. One uncertainty is the Λ_{ETC} scale which is related to the scale of ordinary fermion masses and can be much larger than that used in the above estimates in view of the apparent higher quark mass limit.[36] For a discussion we refer to Ref. 22. Another uncertainty is caused by the unknown value of the technifermion condensate scale, which can be much larger in an extended technicolor theory with a slowly running coupling constant. We will be omitting the axial vectors from the following discussions since, analogous to QCD, they are expected to be heavier and thus less accessible to experiment. It is clear from (2.12) that (in addition to the extended technicolor interactions to be discussed in the next section) the primary coupling of the technimesons to ordinary fermions is through the mixing of the technivectors with the gauge particles. Therefore, it is of interest to clarify the form of the interactions of the techni-ρ, techni-ω and, in some models, the color octet ω_8.

Let us first consider the couplings of the technivectors to the pseudoscalars in (2.3). The primary vertex is

$$\mathcal{L}_{V\phi\phi} \simeq 2ig\, \text{Tr} V[\partial\phi, \phi] , \tag{3.4}$$

where all the fields are the physical fields while the small mixing due to the (2.12) can be neglected here. The decay width of the techni-ρ into pseudoscalars is

$$\Gamma(\rho^i \to P_a^j P_a^k) \simeq \frac{g^2}{4\pi}\frac{m_V}{12N_d} , \tag{3.5}$$

where we have neglected the pseudoscalar mass with respect to the vector mass. The index a is not summed over and corresponds to the generators of the $SU(N_d)$ group and hence there are N_d^2-1 channels for decay. Thus for $N_d/N_T > \frac{3}{2}$ the techni-ρ total width would exceeds its mass and, for practical purposes, would not appear as a resonance. The decays of the ω and ω_8 are more model dependent but can be given in terms of structure constants f_{abc} of the particular $SU(N_d)$ generators of the model,

$$\Gamma(\omega^a \to P^b P^c) = \Gamma(\omega^a \to P_b^i P_c^i) = \frac{g^2}{4\pi}\frac{(f_{abc})^2 m_V}{24} , \tag{3.6}$$

and summing over final states gives a total width into pseudoscalars of

$$\Gamma_{\mathrm{tot}}(\omega\to\phi\phi)=\frac{g^2}{4\pi}\frac{m_V}{6}N_d \ ,$$

which is twice the width for techni-ρ decay. At this point one might ask what has happened to the analog of the QCD decay $\rho\to\pi\pi$. This channel would have appeared in (3.4) had we not been working in the unitary gauge. Now, of course, it must show up as the decay into longitudinally polarized gauge bosons. To illustrate this and other features of the model we consider the ρWW vertex.

The interaction of the technivectors and gauge bosons arises when one replaces the original interaction states with the physical mass eigenstates. Equivalently[37] one could work in a perturbation expansion with insertions in, for example, the W propagator of the ρ and β mixing terms given in (2.12). The ρWW vertex arises from two sources: the strongly coupling $\rho\rho\rho$ vertex and the WWW gauge vertex,

$$\begin{aligned}\mathcal{L}_{\rho\rho\rho}&=-\frac{g\epsilon_{ijk}}{\sqrt{N_d}}\partial_\mu\rho^i_\nu\rho^j_\mu\rho^k_\nu \ ,\\ \mathcal{L}_{WWW}&=-g_L\epsilon_{ijk}\partial_\mu W^i_\nu W^j_\mu W^k_\nu \ ,\end{aligned} \tag{3.7}$$

where we have to rewrite ρ and W in terms of the physical states given in (2.13) and (2.15). Combining the two contributions from (3.7) and working to first order in (g_L/g) in mixing, we find

$$\mathcal{L}_{\rho WW}\simeq\frac{-g}{\sqrt{N_d}}\left[\frac{g_L\sqrt{N_d}}{g}\right]^2\epsilon_{ijk}\partial_\mu\rho^i_\nu W^j_\mu W^k_\nu+\cdots \ , \tag{3.8}$$

which has a suppression of $(g_L/g)^2$, compared to the strong coupling of ρ to pseudoscalars. This suppression is exactly compensated for in the decay rate by a kinematical enhancement due to the longitudinal polarization of the W's. The decay width is given by

$$\Gamma(\rho^0\to W^+W^-)\simeq\frac{1}{16\pi m_V}|\mathcal{M}|^2 \ ,$$

where

$$\mathcal{M}=\frac{ig_L^2}{2g}(P_+-P_-)^\alpha\epsilon^\mu(P_+)\epsilon^\mu(P_-)\epsilon^\alpha(P_++P_-) \tag{3.9}$$

is the amplitude derived from $\mathcal{L}_{\rho WW}$. In computing the sum over polarization of the W's the dominant contributions come from the $k_\mu k_{\mu'}/M_W^2$ terms. Keeping only these leading terms and using (2.15) we find

$$\Gamma(\rho\to WW)\simeq\frac{g^2}{4\pi}\frac{m_V}{12N_d}\left[\frac{1}{2}\left[\frac{m_V^2}{g^2F_p^2}\right]\right]^2 . \tag{3.10}$$

A comparison with (3.5) and (3.3) shows that (3.10) agrees with the equivalence theorem[38] and a scaled-up version of QCD. However, this is not necessarily the case for the techni-ω decays.

It has been proposed[23] that since the ordinary ω decay modes in QCD contain information on the constituent quantum numbers, one might be able to use the techni-ω to obtain similar information about the underlying technicolor theory. For example, $\omega\to\pi^0\gamma$ contains information on the number of colors in QCD. As (2.15) shows, the coupling of the techni-ω does depend on the charges of the techniquarks through $\overline{Y}$ [see (2.5)]. But because the flavor-symmetry group for technicolor includes the ordinary low-energy gauge group there are restrictions from the requirement of anomaly[31] on the possible ω couplings. From the general construction of the effective Lagrangian in Sec. II it can be shown that there is no analog to the $\rho\omega\pi$ coupling of QCD. The anomalous $VV\phi$ coupling can be taken from Refs. 7 and 8:

$$\begin{aligned}\mathcal{L}_{VV\phi}&=g_{VV\phi}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\{\partial^\mu V^\nu,\partial^\alpha V^\beta\}\phi) \ ,\\ g_{VV\phi}&=-\frac{2g^2N_T}{(4\pi)^2F_p} \ .\end{aligned} \tag{3.11}$$

Since the group generator corresponding to the techni-ω is $\overline{\Lambda}$, from anomaly constraints, one sees that there is no $\rho_T\omega_T\pi_T$ coupling in $\mathcal{L}_{VV\phi}$ of (3.11). This is a general feature of these models and is not the result of a specific substructure. The techni-ω will not be produced by gauge-boson fusion in pp colliders but could still be produced via a Drell-Yan process through its mixing with the Z^0 and γ. The cross section for ω_T production can be taken from Table II of Ref. 23 with their factor $(2A-1)^2$ being replaced by

$$\left[\frac{N_d}{4}\overline{Y}^2\right]\left[\frac{2\ \mathrm{TeV}}{m_V}\right]^4$$

to generalize it to our model. The authors of Ref. 23 also computed the Drell-Yan cross section for techni-ρ production. However, in their approximations they did not include the mixing of the ρ_0 and the B gauge boson. Including this mixing would give, for inclusive production,

$$\sigma(pp\to\rho^0)\simeq0.7\sigma(pp\to\rho^+)+0.4\sigma(pp\to\rho^-) \tag{3.12}$$

instead of their Eq. (4.37).

In view of the strong decay of the techni-ρ into W's and Z's it has been proposed[21] that a possible signature of technicolor might be a resonant enhancement of $\bar{q}_iq_j\to WZ$. The cross section for this process can be conveniently written as[39]

$$\begin{aligned}\frac{d\sigma}{dt}(\bar{u}d\to W^-Z)=\frac{\pi\alpha^2|U_{ud}|^2}{6s^2X_W}\Bigg\{&\frac{e_s^2}{(s-M_W^2)^2}\left[\left[2+\frac{M_Z^2}{4M_W^2}\right]f_1+\left[\frac{M_Z^2}{2M_W^2}-2\right]f_2\right]+\frac{2e_s}{s-M_W^2}\left[\frac{e_t}{t}-\frac{e_u}{u}\right](f_1-f_2)\\ &+\left[\left[\frac{e_u}{u}\right]^2+\left[\frac{e_t}{t}\right]^2\right]f_1+2\left[\frac{e_u}{u}\right]\left[\frac{e_t}{t}\right]f_2\\ &+\left[\frac{f_1+2f_2}{4M_Z^2M_W^2}\right]\left[(e_s+e_u-e_t)^2-2e_s(e_s+e_u-e_t)\frac{M_Z^2}{s-M_W^2}\right]\Bigg\} \ ,\end{aligned} \tag{3.13}$$

where

$$X_W=\sin^2\theta \ ,$$
$$f_1=ut-M_Z^2M_W^2 \ , \qquad (3.14)$$
$$f_2=s(M_Z^2+M_W^2) \ ,$$

and quark masses are taken to be zero. The e_s, e_u, and e_t parameters are the relative s-, u-, and t-channel couplings; i.e., in the standard model,

$$e_s=\cot\theta \ ,$$
$$e_u=\frac{-1+\frac{2}{3}X_W}{\sin2\theta} \ ,$$
$$e_t=\frac{1-\frac{4}{3}X_W}{\sin2\theta} \ .$$

Hence in the standard model the last term of (3.13) is zero and one does not see an enhancement at large ut. In a theory with a techni-ρ one can appeal to the equivalence theorem[38] to replace the longitudinal gauge bosons with the Goldstone bosons and then couple the Goldstone bosons to the transverse gauge bosons through vector-meson dominance with the techni-ρ. This amounts to replacing, in the s channel, the Higgs-gauge-boson vertex of the standard model in diagram (a) of Fig. 1 by the techni-ρ exchange in diagram (b).

The couplings for diagram (b) of Fig. 1 can be obtained from (2.12) and (3.4). Computing the contributions to the cross section, we find the modification to standard cross section to be the additional term

$$\Delta\frac{d\sigma}{dt}=\frac{\pi\alpha^2}{6s^2X_W^2}\times\left[\frac{1}{(s-M_W^2)^2}\frac{ut}{4}\left[\frac{m_V^4}{(s-M_V^2)^2}-1\right]\right] . \qquad (3.15)$$

It is clear that this modification is small for s small but is resonantly enhanced for s around the techni-ρ mass m_V. Let us now see how this effect comes about directly from (3.13), using the physical mass eigenstates. We are interested in the last term of (3.13) which shows an enhancement for large ut. Thus we must compute e_s, e_u, and e_t in the physical basis. The s-channel coupling e_s now involves two diagrams for W and ρ exchange. The u- and t-channel couplings e_u and e_t change due to the mixing angles of the W and Z and the technivectors. We will work to order $(g_L/g)^2\sim M_W^2/m_V^2$ in the mixings which is equivalent to the perturbative mixing approach taken earlier.

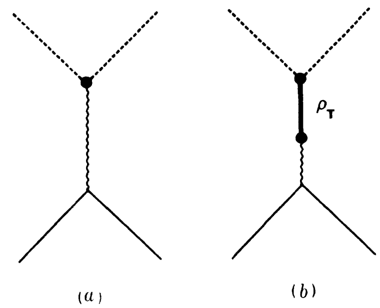

FIG. 1. Vector-meson dominance in WZ production. In diagram (a) we show the conventional coupling of the intermediate W to a longitudinal W and Z. In diagram (b) we show this same coupling in the vector-meson dominance picture of a technicolor theory where the coupling to the Higgs technipions is through an intermediate techni-ρ,ρ_T.

Using (2.13) and (2.15) it is straightforward to show

$$(e_s+e_u-e_t)\simeq\cot\theta\left[\frac{M_Z^2}{s-m_V^2+i\Gamma m_V}\right] , \qquad (3.16)$$

where Γ is a width for the techni-ρ. Since we are interested in the energy region around $s=m_V^2$, we take Γ to be the decay width of the techni-ρ. However, a more complete treatment[40] of the composite nature of the techni-ρ propagator, such as the threshold effect, would be required at energies far from the resonance region. See, for example, Ref. 21 for their treatment of such effects in the pair production of pseudoscalar technimesons by gluon fusion. From (3.13) and (3.16), we obtain an additional term to the standard cross section without the techni-ρ:

$$\Delta\frac{d\sigma}{dt}\simeq\frac{\pi\alpha^2|U_{ud}|^2}{6s^2X_W}\left[\left[\frac{f_1+2f_2}{4M_Z^2M_W^2}\right]\cot^2\theta\left[\frac{M_Z^4}{(s-m_V^2)^2+\Gamma^2m_V^2}\right]\left[1-2\frac{s-m_V^2}{s-M_W^2}\right]\right]$$
$$\simeq\frac{\pi\alpha^2|U_{ud}|^2}{6s^2X_W^2}\left[\frac{ut}{4(s-M_W^2)^2}\left[\frac{m_V^4+m_V^2\Gamma^2}{(s-m_V^2)^2+m_V^2\Gamma^2}-1\right]\right] , \qquad (3.17)$$

which is in agreement with the perturbative VMD result (3.15). The enhancement for $pp\to W+Z+X$ is computed in Ref. 21 for two technicolor models: the "minimal" model with $N_d=1$ and the Farhi-Susskind model with $N_d=4$. The difference in the models is basically the width of the techni-ρ which increases relative to its mass as N_d increases: $\Gamma\sim3m_V/4(N_d/N_T)$. Even for the case $N_d=N_T=4$ their calculations show that it is quite possible that such an enhancement may be seen at future pp colliders if technicolor is relevant to electroweak symmetry breaking. We should also remark that our expression for the enhancement differs slightly from that of Ref. 21 in the dependence on techni-ρ width. This expression for the modification can be written in the form

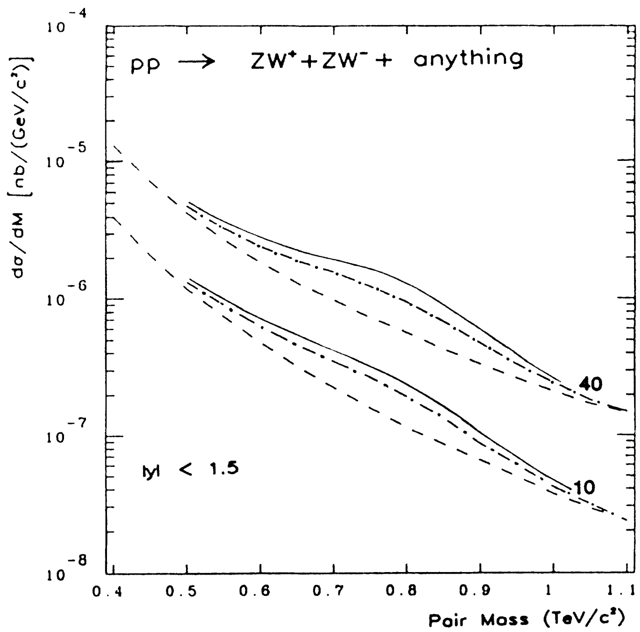

FIG. 2. W and Z production in pp colliders at 10 and 40 TeV. The y cut has to be satisfied by both the Z^0 and $W^{\pm}$. The curves are without the techni-ρ (dashed curves), with the ρ_T enhancement given by the first article of Ref. 21 (dashed-dotted curves), and with the ρ_T enhancement according to (3.13) and (3.17) (solid curves). The dashed and dashed-dotted curves are part of Fig. 180, first article of Ref. 21.

$$\Delta\frac{d\sigma}{dt}\simeq\frac{\pi\alpha^2|U_{ud}|^2}{6s^2X_W^2}\frac{ut}{4(s-M_W^2)^2}\times\left[\frac{m_V^4}{(s-m_V^2)^2+\Gamma^2m_V^2}-1\right]. \tag{3.18}$$

One notes that the factor in large parentheses is not suppressed for s well below the resonance as one would expect. This is in contrast with our corresponding factor in (3.17), which vanished as $s\to 0$. For the Farhi-Susskind model when Γ is close to m_V our expression (3.17) would yield roughly 50% more enhancement than (3.18). We illustrate this additional enhancement, which should make it easier to detect the techni-ρ effect over the QCD background, by comparing in Fig. 2 our enhancement with that of Ref. 21. In the minimal technicolor model, the width of the techni-ρ is about $\frac{3}{16}$ of its mass and the difference between the enhancement in (3.17) and that of (3.18) is negligible. However, in this case, the resonant effect of the techni-ρ is apparent (see Fig. 179 of the first article of Ref. 21) and the detection of the presence of the techni-ρ is unambiguous.

IV. EXTENDED TECHNICOLOR CONSIDERATIONS

The technicolor model described in Sec. II is incomplete in that it provides no mechanism of mass generation for the quarks and leptons. This can be amended by introducing extended technicolor (ETC) interactions[16] coupling technifermions to ordinary fermions. The technicolor group $G_{\rm TC}$ is embedded into a larger group $G_{\rm ETC}$ which is assumed to break down to $G_{\rm TC}$ at a scale $\Lambda_{\rm ETC}>\Lambda$ where Λ is the technicolor scale. There is as yet no completely satisfactory extended technicolor model, however, progress[19,26] has been made to remedy the problem of possible flavor-changing neutral currents (FCNC's). In this section we will consider briefly some consequences of the ETC interactions for our effective Lagrangian.

Let us denote[19] the currents of the ETC vector bosons coupling to ordinary fermions and technifermions as

$$J_\mu^A=\bar f^{\,n}_{Li}g^{An}_{Lia}\gamma_\mu\psi_{La}+\bar f^{\,n}_{Ri}g^{An}_{Ria}\gamma_\mu\psi_{Ra}+\text{H.c.}\,, \tag{4.1}$$

where ψ_{La} is a left-handed technifermion of species a, f^n_{Li} is an ordinary left-handed fermion of species i and generation n, and g^A is the appropriate coupling to the ETC vector boson A. In general, the interactions in (4.1) will lead to flavor-changing neutral currents which must be suppressed in any realistic model. One mechanism, referred to as monophagy,[19] does this by requiring each fermion to be coupled only to one type of technifermion. In the Farhi-Susskind model, for example, this can be done by assuming the extended technicolor gauge bosons which couple to the ordinary fermions carry none of the electroweak quantum numbers so that each fermion is coupled to the technifermion of the same type, i.e., $(u,c,t)\leftrightarrow U$, $(d,s,b)\leftrightarrow D, (e,\mu,\tau)\leftrightarrow E$, and $(\nu_e\nu_\mu\nu_\tau)\leftrightarrow N$. We define the following coupling matrices in techniflavor space:

$$\begin{aligned}
[G^{nm}_{ij}]_{ab}&=\frac{2N_T\Lambda^2}{g_0}\sum_A g^{An}_{Lia}g^{Am*}_{Rjb}/M_A^2\,,\\
[G^{nm}_{Lij}]_{ab}&=\frac{2N_T\Lambda^2}{g_0}\sum_A g^{An}_{Lia}g^{Am*}_{Ljb}/M_A^2\,,\\
[G^{nm}_{Rij}]_{ab}&=\frac{2N_T\Lambda^2}{g_0}\sum_A g^{An}_{Ria}g^{Am*}_{Rjb}/M_A^2\,,
\end{aligned} \tag{4.2}$$

where M_A is the effective ETC gauge-boson mass. Then the direct coupling terms of the technipseudoscalars technivectors to the ordinary fermions can be written as a trace over techniflavor indices:

$$\begin{aligned}
\mathcal{L}_{\rm ETC}=&\frac{2i\mu}{F_p}\bar f^{\,n}_i\mathrm{Tr}\left\{t^\alpha\left[G^{nm}_{ij}\left[\frac{1+\gamma_5}{2}\right]-G^{mn*}_{ji}\left[\frac{1-\gamma_5}{2}\right]\right]\right\}f^m_j\phi^\alpha\\
&-\frac{g}{2}\left[\frac{g_0}{g_1}\right]\bar f^{\,n}_i\gamma^\mu\mathrm{Tr}\left\{t^\alpha\left[G^{nm}_{Rij}\left[\frac{1+\gamma_5}{2}\right]+G^{nm}_{Lij}\left[\frac{1-\gamma_5}{2}\right]\right]\right\}f^m_jV^\alpha_\mu\,.
\end{aligned} \tag{4.3}$$

The parameters g_0 and g_1 are model dependent and their ratio is expected[3] to be of order one [see (A18)]. The parameter μ is related to the condensate scale and is estimated[3] to be $\mu=m_V/\sqrt{6}$, assuming a KSRF-type relation [see (2.9), (3.3), and (A12)]. We have omitted axial-vector couplings and contact four-fermion interactions. The axial vectors are dropped as before because we anticipate a heavy mass for them. The main pseudoscalar interactions in (4.3) can be simplified somewhat in a monophageous-type model. In Ref. 19 these interactions are written for the Farhi-Susskind model and it is explicitly shown that there are no FCNC's. For definiteness we will now specialize our discussion of the technivector interactions to this model. We remark that even with these simplifications the vector and also the leptoquark pseudoscalar interactions are not amenable to definite predictions due to the complexity of (4.2) for the couplings. To get some feeling for the strength of these ETC couplings we make the further assumption that generation mixing can be neglected. In particular, the ETC couplings are taken to be diagonal:

$$\frac{\Lambda}{M_A}\left[\frac{2N_T\mu}{g_0}\right]^{1/2} g_{Lia}^{An}=\alpha_d^n\sqrt{\overline{m}_d^n}\,\delta_{An}\delta_{ia}\ , \tag{4.4}$$

$$\frac{\Lambda}{M_A}\left[\frac{2N_T\mu}{g_0}\right]^{1/2} g_{Ria}^{An}=\frac{m_i}{\alpha_d^n\sqrt{\overline{m}_d^n}}\delta_{An}\delta_{ia}\ , \tag{4.5}$$

where the subscript d stands for the particular isodoublet the ith fermion belongs to, $\overline{m}_d^n$ is the average mass of that doublet for the nth generation, m_i is the mass of the ith fermion, and α_d^n is a free parameter. First we note that (4.4) and (4.5) will give the correct diagonal mass matrices. Next we note that since g_L should be invariant under SU(2) the natural strength should be some average measure of both up and down species; hence, we expect α_d^n to be of order 1. In this simple toy model the left-handed interactions for the techni-ρ are given by

$$\mathcal{L}_{\rho ff}=\sum_{d,n}\frac{g}{2}\left[\frac{g_0}{g_1}\right]\frac{(\alpha_d^n)^2}{4}\frac{\overline{m}_d^n}{\mu}\bar{f}_d\frac{\sigma^k}{2} \times\gamma^\mu\left[\frac{1-\gamma_5}{2}\right]f_d\rho^k\ , \tag{4.6}$$

while the right-handed currents in (4.3) are assumed to be small enough not to be observed in the low-energy electroweak experiments. Recently it has been suggested[41] that perhaps the ETC interactions for the techni-ρ in (4.6) might further enhance the resonant cross section $\sigma(pp\to WZX)$ discussed in Sec. III. Following the treatment of that section we compute the addition to the s-channel coupling e_s from the ETC coupling in (4.6) and compute

$$(e_s+e_u-e_t)=\cot\theta\left[\frac{M_Z^2(1+\epsilon s)}{s-m_V^2+i\Gamma m_V}\right]\ , \tag{4.7}$$

where

$$\epsilon=\frac{g_0}{g_1}\frac{\alpha^2}{32}\left[\frac{m_u+m_d}{\mu}\right]\frac{1}{M_W^2}\ . \tag{4.8}$$

The addition to the differential cross section for $\bar{u}d\to W^-Z$ will then be

$$\Delta\frac{d\sigma}{dt}=\frac{\pi\alpha^2}{6s^2X_W^2}\frac{ut}{4s^2}\left[\frac{(m_V^2+s^2\epsilon)^2+\Gamma^2m_V^2}{(s-m_V^2)^2+\Gamma^2m_V^2}-1\right]\ . \tag{4.9}$$

Our expression differs slightly from that of Ref. 41 in the s behavior of the ETC contribution, but this difference is slight around the region $s\sim m_V^2$ where the formulas are supposed to be valid. What is more to the point is the estimate of ϵ in (4.8):

$$\epsilon s\simeq\frac{1}{10}\frac{m_d}{m_V}\frac{1}{m_W^2}m_V^2\sim10^{-5}\frac{m_V}{m_W}\ , \tag{4.10}$$

and for techni-ρ masses in the TeV range this still leaves a 10^{-4} suppression relative to the VMD contribution. It seems unlikely that in a more complicated model with mixing one could overcome this suppression although with a heavy top-quark mass, $m_t/m_d\sim10^4$, it is not ruled out *a priori*.

V. SUMMARY

We have presented an effective Lagrangian to apply to technicolor theories for energies below the condensate scale of 1 TeV. This general framework includes the technivector mesons as well as the pseudo-Goldstone bosons and the weak gauge interactions via vector-meson dominance. It is hoped that this may serve as an explicit setting for analysis of physics above the electroweak energy scale and below 1 TeV.

We have demonstrated the Higgs mechanism for a general class of models and clarified the mixing of the technivectors with the gauge sector. Because of the constraints of the Higgs mechanism and the cancellation of anomalies, the mixing of the technivectors and gauge vectors depends on only one parameter $\overline{Y}^2$, which is proportional to the sum of the squares of the hypercharge of the left-handed technidoublets in this model. The resulting mass matrices for the vectors lead to an interesting deviation for the W and Z masses at the tree level. It is possible to use experimental limits to then place some constraints on technicolor model parameters.

Of particular note are the interactions of the techni-ω. This techni-ω is not simply a scaled up version of the the QCD ω. For the case of QCD, it is well known that the gauged Wess-Zumino term gives rise to anomalous ω decays. Replacing the QCD mesons with technimesons and using the equivalence theorem to replace the technipions by the longitudinal components of the electroweak gauge bosons would lead one to speculate on $\omega_T\to WWZ$. We have explicitly demonstrated that this mode does not occur in models consisting of technifermions in left-handed weak doublets and right-handed weak singlets only. That is, when identifying the techni-ω that mixes with the gauge bosons, one finds no analog of the $\rho\omega\pi$ vertex in the anomalous part of the Lagrangian. Thus some recent phenomenology suggested for the techni-ω is not consistent with this class of models.

We have also illustrated the use of the effective Lagrangian in developing the phenomenology of the techni-ρ. Our treatment involves ordinary perturbation theory for mass eigenstates rather than the use of approximate vector-meson dominance and the approximate equivalence theorem employed in earlier treatments. In the appropriate limit our results reduce to those in earlier works as we have shown, for example, in (3.17) for the techni-ρ enhancement of W,Z production from quark annihilation. However, we point out that our formalism lends itself to better control over the approximations used and can in principle be used to calculate corrections to the leading-order approximation. For example, we find a significant further enhancement to the W,Z production in pp colliders in the Farhi-Susskind model, which makes the technicolor signature more pronounced.

ACKNOWLEDGMENTS

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APPENDIX: HEURISTIC APPROACH TO AN EFFECTIVE LAGRANGIAN WITH VECTOR-MESON DOMINANCE

Consider an $\mathrm{SU}(N_T)$ theory with N_f flavors of massless fermions. Below an energy Λ_c, fermion-antifermion condensates break the $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)\times\mathrm{U}(1)$-flavor chiral symmetry to a diagonal $\mathrm{SU}(N_f)$ and Goldstone bosons (some become pseudo-Goldstone bosons) which are associated with the broken generators appear. The low-energy world is describable by an effective Lagrangian containing phenomenological fields of pseudoscalars and possibly other low-lying bound states. In the following we illustrate how such a picture may emerge in a certain approximation.

We begin by writing the generating functional of an $\mathrm{SU}(N_T)$ theory, denoting the fundamental fermion fields by ψ and the strong gauge-boson fields by C_μ:

$$Z=\mathcal{N}\int D\psi D\bar{\psi} DC \exp\left[i\int_z L(\bar{\psi},\psi,C)\right] , \tag{A1}$$

where $\int_x$ indicates the integration over space-time, $\mathcal{N}$ is a normalization factor, and the integration over ghost fields is not exhibited. We write L as a free-fermion part L_0, plus an interaction part $L_I=\bar{\psi}\not{C}\psi$, and a gauge piece L_C. In general, one can include the standard-model gauge interactions in $L_0(\psi)$:

$$\begin{aligned}L_0(\psi)&=\bar{\psi}\left[i\not{\partial}+g_c\not{G}+\left[g_L\not{W}+\frac{g_Y}{2}\not{B}Y_L\right]\left[\frac{1-\gamma_5}{2}\right]\right.\\&\qquad\left.+\frac{g_Y}{2}\not{B}Y_R\left[\frac{1+\gamma_5}{2}\right]\right]\psi\\&\equiv\bar{\psi}(i\not{\partial}+\mathcal{V}+\mathcal{A})\psi .\end{aligned} \tag{A2}$$

Note that L_0 could also contain other terms such as effects from the extended technicolor sector discussed in Sec. IV or a bare-quark-mass term for an application to QCD. We can formally integrate out C_μ to obtain[42]

$$\begin{aligned}Z=\mathcal{N}'\int D\psi D\bar{\psi}\exp\left[i\int_x L_0(x)\right]\\ \times\exp\left[\sum_{n=2}^{\infty}\frac{i^n}{n!}\int_n G_c^{[n]}F_1F_2\cdots F_n\right] ,\end{aligned} \tag{A3}$$

where $G_c^{[n]}$ are the pure Yang-Mills connected n-point Green's functions and $\int_n$ indicates integration over the n space-time coordinates; $F_j\equiv\bar{\psi}(x_j)\gamma_{\mu_j}T^{a_j}\psi(x_j)$, and the T^a, $a=1,\ldots,N_c^2-1$ are the generators of $\mathrm{SU}(N_T)$. We note that the leading N_T behavior of $G_c^{[n]}$ is $(1/N_T)^{n/2}$ and simple dimensional analysis would also suggest it scales like $\Lambda^4(1/\Lambda)^{3n}$ where Λ is the confining scale. Thus the expansion in fermion bilinear operators is effectively an expansion in powers of $(1/N_T^{1/2}\Lambda^3)^n$.

In the limit of large N_T one can approximate[32] by truncating the series at $n=2$. As an illustration, we drastically approximate the two-point function keeping only a contact piece:

$$G_{c\mu_1\mu_2}^{[2]a_1a_2}(x_1x_2)\simeq i\frac{\tilde{g}}{N_T}\frac{\delta(x_1-x_2)}{\Lambda^2}\delta_{\mu_1\mu_2}\delta_{a_1a_2} , \tag{A4}$$

where we show the leading N_T behavior. It should be noted that (A4) does not follow from a short-distance approximation of the strong gauge-boson propagation function but instead from the expectation that the strong-interaction gauge bosons are frozen and localized below Λ and the localization region should be smaller than the typical size of a technihadron. This approximation amounts to neglecting all derivative-type terms in (A4).

To obtain technicolor-singlet states, one can perform a Fierz rearrangement and write the effective Lagrangian, using (A3) and (A4),

$$\begin{aligned}L(\psi)=L_0(\psi)&+\frac{g_0}{2N_T\Lambda^2}[(\bar{\psi}t^a\psi)^2-(\bar{\psi}t^a\gamma_5\psi)^2]\\&-\frac{g_1}{2N_T\Lambda^2}[(\bar{\psi}t^a\gamma_\mu\psi)^2+(\bar{\psi}t^a\gamma_\mu\gamma_5\psi)^2]\\&+\frac{N_f}{N_T}\frac{\tilde{g}}{2N_T\Lambda^2}(\bar{\psi}t^0\gamma_\mu\psi)^2 ,\end{aligned} \tag{A5}$$

where the t^a are the $\mathrm{U}(N_f)$-flavor generators and $g_0=2g_1=\tilde{g}$.

Although this exercise is by no means a derivation, it does illustrate a connection between the exact functional (A3) and a Nambu–Jona-Lasinio–type[43] model (A5). Note that the $\tilde{g}$ term in (A5) is suppressed by $1/N_T$ and should be dropped in a large-N_T expansion.[32]

We can recast[44] the expression for the effective action (A5) by introducing auxiliary fields representing the mesonic degrees of freedom:

$$Z=\mathcal{N}'\int d\psi\, d\bar{\psi}\, dS\, dP\, dV\, dA \times\exp\left[i\int_x L(\psi,S,P,V,A,\mathcal{V},\mathcal{A})\right],$$
$$L=L_0(\psi)+\bar{\psi}U_p\psi-\frac{N_T\Lambda^2}{g_0}\mathrm{Tr}(S^2+P^2)+\frac{N_T\Lambda^2}{g_1}\mathrm{Tr}(V_\mu^2+A_\mu^2)\,, \tag{A6}$$

where g_0 and g_1 are now treated as independent and $U_p=U_p^a t^a$ is

$$U_p^a=S^a+iP^a\gamma_5+V_\mu^a\gamma^\mu+A_\mu^a\gamma^\mu\gamma^5\,. \tag{A7}$$

We shift the vector and axial-vector fields, $V\to V-\mathcal{V}$, $\mathcal{A}\to\mathcal{A}-\mathcal{A}$, and then integrate over the fermionic degrees of freedom:

$$S_{\rm eff}=-iN_T\ln\det D-\int\left[\frac{N_T\Lambda^2}{g_0}\mathrm{Tr}(S^2+P^2)-\frac{N_T\Lambda^2}{g_1}\mathrm{Tr}[(V-\mathcal{V})^2+(A-\mathcal{A})^2]\right]d^4x\,, \tag{A8}$$

where $D=i\partial\!\!\!/+U_p$ contains only the collective fields S, P, V, and A. Note that the mixing of the weak gauge fields appears only in the mass terms.

The evaluation[2–5,44,45] of $\ln\det D$ involves both a normal-parity piece Γ and an anomalous piece Γ_A. Using the Schwinger proper-time regularization[46] for the normal-parity piece,

$$\Gamma=-i\,\mathrm{Re}(\ln\det D)=\frac{i}{2}\int_{1/\Lambda^2}^{\infty}\frac{d\tau}{\tau}\mathrm{Tr}e^{-\tau D^\dagger D}\,, \tag{A9}$$

we can write,

$$\Gamma=\frac{2\mu^2}{(4\pi)^2}N_T\Gamma(-1,r)\int d^4x\,\mathrm{tr}(M^\dagger M-\mu^2)+\frac{\Gamma(0,r)}{(4\pi)^2}N_T\int d^4x\,\mathrm{tr}[(\mathcal{D}_\mu M)^\dagger(\mathcal{D}^\mu M)-(M^\dagger M-\mu^2)^2-\tfrac{1}{3}(F_{\mu\nu}^V F_V^{\mu\nu}+F_{\mu\nu}^A F_A^{\mu\nu})]\,, \tag{A10}$$

where we have introduced a new scale, $\mu, r=\mu^2/\Lambda^2$, and $M=S+iP$, $\mathcal{D}_\mu=\partial_\mu-i[V_\mu,\ ]+i\{A_\mu,\ \}$. The trace is only over flavor space now and $\Gamma(m,r)$ is the incomplete gamma function. We have omitted all terms finite as $\Lambda\to\infty$. Note that chiral symmetry will be spontaneously broken and μ^2 will be the vacuum value of $M^\dagger M$ provided g_0 is greater than the critical value:[4] $g_0>g_c=8\pi^2$.

We will work in the nonlinear limit $M=\mu U=\mu e^{2i\phi/F}$ to eliminate the heavy scalars from the theory, justified empirically in QCD. Our effective action becomes

$$S_{\rm eff}=\Gamma_A+\Gamma_{\rm finite}+\int d^4x\,\mathrm{tr}\left[\frac{F^2}{4}(\mathcal{D}_\mu U)^\dagger(\mathcal{D}^\mu U)-\frac{1}{2g^2}(F^VF^V+F^AF^A)+\frac{m_V^2}{g^2}(V^2+A^2)\right], \tag{A11}$$

where

$$F^2=6\mu^2/g^2\,,\quad \frac{1}{g^2}=\frac{2}{3}\frac{N_T}{(4\pi)^2}\Gamma(0,r)\,,\quad m_V^2=\frac{N_Tg^2}{g_1}\Lambda^2\,. \tag{A12}$$

Checking the consistency of the mass scales in the QCD case, we find the quite satisfactory values $r=\mu^2/\Lambda^2\simeq0.06$, $\mu=314$ MeV, $\Lambda=1.3$ GeV, and $m_V^2/\Lambda^2=6r=0.36$. We note that the original parameters of the Nambu–Jona-Lasinio model, in particular, the ratio (g_1/g_0), have disappeared in favor of the renormalized parameters in (A12). One sees in Sec. IV, however, that when one includes extended technicolor, (g_1/g_0) reappears. It is therefore useful to get an estimate of this ratio from QCD. This can be done by including a bare mass term for the quarks, $m_0=(m_u+m_d)/2$ in (A2), to explicitly break the chiral symmetry and give a mass to the pion. This gives a new constraint

$$-\frac{N\Lambda^2m_0\mu}{g_0}=\frac{m_\pi^2F_\pi^2}{4} \tag{A13}$$

which together with (A12) yields

$$\left[\frac{g_1}{g_0}\right]=\left[\frac{g^2F_\pi^2}{m_V^2}\right]\left[\frac{m_\pi^2}{m_0m_V}\right]\frac{\sqrt{6(m_V^2/g^2F_\pi^2-1)}}{4}\simeq\left[\frac{7\text{ MeV}}{m_0}\right]\simeq1\,. \tag{A14}$$

Since g_1/g_0 is independent of N_T, we can expect $g_1/g_0\simeq1$ for a technicolor theory as well.

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