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Phenomenology and cosmology of second- and third-family Higgs bosons

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We study the possibility that the Higgs sector of the minimal supersymmetric model consists of three families in the same manner that the fermion sector does. Including only extra doublets and assuming that tree-level flavor-changing neutural currents are naturally eliminated by some symmetry, we find that the lightest of the new scalars is absolutely stable, and that the second lightest is only about 1 GeV more massive. The coupling of these scalars to the Z is independent of any new parameters and if light enough they could be easily discovered (or ruled out) if a search is made for the somewhat unusual signature. The natural stability of the lightest scalar makes it an excellent dark-matter candidate, although the near mass degeneracy of the lightest two of these scalars makes the standard calculational method inadequate.

I. INTRODUCTION

In spite of the many recent phenomenological successes of the standard model, our understanding of the electroweak interactions is far from complete. Two of the most important unsolved questions concern the underlying reasons for the existence of the second and third families of quarks and leptons and the nature of the scalar sector. An electroweak model must contain one or more doublets of Higgs bosons to give mass to the known fermions, and it is quite possible that the Higgs sector is replicated in the same manner as the fermion sector. This is especially possible in supersymmetric models where bosons and fermions are treated in the same framework, and where the Higgs bosons and fermions may even be placed in the same representation. For example, in the popular "superstring-inspired" E_6 models,[1] the Higgs bosons are placed in the same representation as the fermions and the minimal Higgs sector is replicated twice.

In this paper we will study the properties of a possible second and third family of Higgs bosons in the context of a general minimal supersymmetric model. We find, by making the rather mild assumption that flavor-changing neutral currents vanish naturally (i.e., they are removed by some symmetry), that the properties of the additional families of Higgs bosons are rather remarkable. We find that the lightest of these particles is absolutely stable and that the next lightest is probably only a GeV or so more massive. If under 45 GeV in mass, both of these particles are detectable at CERN LEP or the SLAC Linear Collider (SLC), if a search is made for the somewhat unusual signature. Since the lightest is stable, it is a candidate for the dark matter known to exist in galactic halos. While particle dark-matter candidates abound, most rely on *ad hoc* assumptions for their stability. Because the stability of the lightest of these additional Higgs bosons derives from the requirement that there naturally be no flavor-changing neutral currents, these particles join the lightest supersymmetric particle, the axion, and the very light neutrino in being "natural" dark-matter candidates.

The Higgs structure of the minimal supersymmetric model[2] consists of two doublets of opposite hypercharge. If there are several pairs of doublets, we will show that, under our assumptions, a basis can be chosen in which only one pair of doublets couples to quarks and leptons, and that this basis is the same basis in which only that pair acquires a vacuum expectation value (VEV). The other pairs do not acquire VEV's, and so do not, strictly speaking, participate in spontaneous symmetry breaking via the Higgs mechanism. For this reason we will refer to these particles as "pseudo-Higgs" bosons. For previous discussions of these particles see Ref. 3.

The plan of the paper is as follows. In Sec. II we describe the models under consideration. We show that with extra Higgs doublets, the requirement that there *naturally* be no flavor-changing neutral currents (FCNC's) implies a symmetry which distinguishes the Higgs bosons from the pseudo-Higgs bosons, and which results in no quadratic mixing terms between the Higgs bosons and pseudo-Higgs bosons. We also show that only the Higgs bosons get VEVs, that the pseudo-Higgs bosons decouple from all fermions, and that the lightest pseudo-Higgs boson is stable. We then calculate the mass spectrum of the pseudo-Higgs bosons and find a near degeneracy between the lightest and second lightest, although the precise value of the mass splitting depends upon mixing parameters. We also discuss the pseudo-Higgsinos, the supersymmetric partners of the pseudo-Higgs bosons and find their mass spectrum.

In Sec. III we discuss the phenomenology of the pseudo-Higgs bosons. We derive the Feynman rules and discuss detection strategies at particle accelerators. We show that their coupling to the Z is independent of any mixing angles, and if light enough, that they would be copiously produced in Z decays. Because of the near mass degeneracy between the lightest and the second lightest pseudo-Higgs boson, the signature will be quite unusual. We also discuss the limits recent LEP results already place upon the pseudo-Higgs-boson masses. We discuss the possibility of detection at hadron colliders, and finally, we consider briefly the phenomonology of the charged pseudo-Higgs bosons, and the pseudo-Higgsinos.

In Sec. IV we turn to the possibility that the pseudo-Higgs bosons make up the dark matter. We calculate the relic abundance in the limit of complete mass degeneracy and in the limit of a large mass splitting, and find widely different results. For exact degeneracy, the relic abundance calculation, together with the LEP measurements of the Z width, imply an abundance which is too low to make up the bulk of the dark matter, although the pseudo-Higgs bosons may still exist and comprise a minority component. However, with a reasonably large mass splitting, relic abundances in the range to provide the dark matter arise naturally for almost all values of the pseudo-Higgs-boson mass, and we conclude, for this case, that they make a natural dark-matter candidate. We also discuss briefly the possibility of pseudo-Higgsino dark matter and the possibility of detecting pseudo-Higgs-boson dark matter directly in the laboratory. We find the pseudo-Higgs bosons are more difficult to detect than Dirac neutrinos, but may be easier than photinos or neutralinos.

Section V sums up the paper and the appendixes contain the Feynman rules and annihilation cross sections.

II. PSEUDO-HIGGS PARTICLES

The minimal supersymmetric model has two doublets H and $\overline{H}$ of opposite hypercharge. We will extend this model to include several families of such doublets H_i and $\overline{H}_i$. Inclusion of singlets causes more complications and was discussed in Ref. 4. The most general gauge-invariant superpotential is given by

$$W=\mu_{ij}^2 H_i\overline{H}_j+f_{ijk}Q_iU_jH_k+g_{ijk}Q_iD_j\overline{H}_k+h_{ijk}L_iE_j\overline{H}_k \ , \tag{1}$$

where our phase convention will be chosen so that $H_i\overline{H}_j\equiv\epsilon_{ab}(H_i)_a(\overline{H}_j)_b$. As usual, Q, U, D, L, and E are the quark and lepton superfields.

A potential problem with extending the Higgs sector of the standard model is the existence of FCNC's. As shown by Glashow and Weinberg[5] and by Paschos[6] the only way to naturally eliminate such FCNC's is to couple all quarks of a given charge to a single Higgs multiplet. To see this, consider the $f_{ijk}Q_iU_jH_k$ term

$$f_{ij1}Q_iU_jH_1+f_{ij2}Q_iU_jH_2+f_{ij3}Q_iU_jH_3 \ . \tag{2}$$

The mass matrix for the quarks is then given by

$$M_{ij}=f_{ij1}\langle H_1\rangle+f_{ij2}\langle H_2\rangle+f_{ij3}\langle H_3\rangle \ . \tag{3}$$

Clearly, if H_2 and H_3 did not exist, then diagonalization of the mass matrix would be equivalent to diagonalization of f_{ij1}, thus the Yukawa interactions would be flavor diagonal. When they do exist, however, this is not, in general, true. However if f_{ij1}, f_{ij2}, and f_{ij3} are all proportional, then the Yukawa couplings are all flavor diagonal. In this case, one can choose a basis in which only one doublet couples to quarks.

We will explicitly assume that there are no tree-level FCNC's (in either the quark or lepton sector), and that the vanishing of FCNC's occurs due to some symmetry (discrete, continuous, local, or global). A basis in which only H_3 and $\overline{H}_3$ couple to quarks and leptons can then be chosen.[7] In this case, H_3 and $\overline{H}_3$ must have different quantum numbers under the symmetry than the other H_i and $\overline{H}_i$, in order for f_{ij1} and f_{ij2} to vanish while f_{ij3} do not. The detailed nature of that symmetry will not be relevant.[8] Note that we are now following the conventional notation in which H_3 and $\overline{H}_3$ are the ordinary Higgs doublets and H_1, H_2, etc., are the extra families.

The most general soft supersymmetry-breaking terms are mass terms for the scalar fields plus terms proportional to superpotential terms. Ignoring scalar quarks and leptons, these terms are

$$m_{Hij}^2 H_i^\dagger H_j+m_{\overline{H}ij}^2\overline{H}{}_i^\dagger\overline{H}_j-B\mu_{ij}(H_i\overline{H}_j+\text{H.c.}) \ . \tag{4}$$

Since H_3 ($\overline{H}_3$) has different quantum numbers under the symmetry which eliminates tree-level FCNC's than the other H_i ($\overline{H}_i$), $m_{Hi3}^2=m_{H3i}^2=m_{\overline{H}i3}^2=m_{\overline{H}3i}^2=0$ for $i\neq 3$; i.e., the mass terms will not mix the third family of Higgs fields with the other two. What about the μ_{i3} and μ_{3i} terms (for $i\neq 3$)? Because of the above symmetry, if a μ_{33} term exists then the μ_{i3} and μ_{3i} terms do not, and vice versa. It is not hard to see that if the μ_{33} term is absent, then (whether or not the μ_{i3} and μ_{3i} terms are present) the resulting potential will have a global U(1) symmetry under which the third family transforms nontrivially. Since the third-family fields must acquire VEV's to give fermions mass, this global symmetry will be spontaneously broken, resulting in an unacceptable axion (this axion cannot be made invisible, since we have no singlets in the model). As a result, the μ_{33} term must be present, i.e., $\overline{H}_3$ and H_3 have opposite quantum numbers under the symmetry which eliminates tree-level FCNC's. [The dangerous global symmetry is then simply the local U(1) hypercharge symmetry.] Thus, $\mu_{13}=\mu_{23}=\mu_{31}=\mu_{32}=0$, and thus none of the terms in Eq. (4) will mix the third-family fields with the others.

This last fact is a crucial feature of this analysis. It implies that there are no quadratic terms mixing the third family of Higgs fields with the other two. As a result, when H_3 and $\overline{H}_3$ acquire VEV's (as they must to give mass to fermions), there will be no terms which are linear in the other families. Thus, these other families will not be forced to acquire VEV's. We now argue that they will, in general, not acquire VEV's at all.

In the minimal supersymmetric model derived from minimal supergravity, the mass-squared parameters are equal at the unification scale; as they evolve to lower energies the mass-squared parameters of the third-family fields will decrease more rapidly than the others (since only the third-family fields couple to fermions). When one of these parameters (or, actually, a combination of the parameters) goes negative, the electroweak symmetry is broken and the third-family fields will acquire VEV's. Since there is no mixing with the other two families, they will generally not acquire VEV's. To be more specific, consider the potential with only the first family of Higgs fields. In models which come from minimal supergravity, all scalar mass-squared parameters are equal at some grand-unified-theory (GUT) scale. Here, since neither H_1 nor $\bar{H}_1$ couples to fermions, the beta functions for their mass-squared parameters are equal, and thus the equivalence of these parameters at the GUT scale forces the equivalence at all scales. Thus, $m_{H11}^2 = m_{\bar{H}11}^2$. Now examine the potential in the direction $H_1 = \bar{H}_1$, with all other fields zero. In this direction, the requirement that the potential be bounded gives $m_{H11}^2 > B\mu_{11}$. However, this condition also forces all eigenvalues of the curvature matrix at the origin to be positive, thus there is a minimum at the origin and the first-family fields do not acquire VEV's. This result is not changed by including the second family. Thus, the first- and second-family Higgs fields do not acquire VEV's.[9] We refer to them as "pseudo-Higgs bosons." Although these fields come from an extension of the Higgs sector, they have nothing to do with symmetry breaking (since they do not get VEV's), thus they are "like Higgs bosons, but not the same," i.e., pseudo-Higgs bosons.

We thus see that the basis in which the third-family fields decouple from the other two is the same as the basis in which only one family couples to fermions, which in turn is the same as the basis in which only one family acquires VEV's. The only assumptions are that we have no singlets (which could give cubic terms in the potential) and that we have a natural elimination of tree-level FCNC's. The resulting potential is given by

$$\begin{aligned} V = {} & m_1^2|H_1|^2 + m_2^2|H_2|^2 + \bar{m}_1^2|\bar{H}_1|^2 + \bar{m}_2^2|\bar{H}_2|^2 + m_3^2|H_3|^2 + \bar{m}_3^2|\bar{H}_3|^2 \\ & + (\mu'_{33}H_3\bar{H}_3 + \mu'_{11}H_1\bar{H}_1 + \mu'_{22}H_2\bar{H}_2 + \mu'_{12}H_1\bar{H}_2 + \mu'_{21}H_2\bar{H}_1 + \mu'' H_1^\dagger H_2 + \bar{\mu}''\bar{H}_1^\dagger \bar{H}_2 + \text{H.c.}) \\ & + \frac{g^2}{8}\sum_a \left|\sum_i (H_i^* \tau_a H_i + \bar{H}_i^* \tau_a \bar{H}_i)\right|^2 + \frac{g'^2}{8}\left|\sum_i (|H_i|^2 - |\bar{H}_i|^2)\right|^2 , \end{aligned} \tag{5}$$

where $\mu'_{ij} \equiv \mu_{ij}B$ (here, B is the arbitrary soft supersymmetry-breaking parameter), and where

$$\begin{aligned} & m_1^2 = m_{H11}^2 + \mu_{11}^2 + \mu_{12}^2, \quad m_2^2 = m_{H22}^2 + \mu_{22}^2 + \mu_{21}^2 , \\ & \bar{m}_1^2 = m_{\bar{H}11}^2 + \mu_{11}^2 + \mu_{21}^2, \quad \bar{m}_2^2 = m_{\bar{H}22}^2 + \mu_{22}^2 + \mu_{12}^2 , \\ & m_3^2 = m_{H33}^2 + \mu_{33}^2, \quad \bar{m}_3^2 = m_{\bar{H}33}^2 + \mu_{33}^2 , \\ & \mu'' = \mu_{22}\mu_{12} + \mu_{11}\mu_{21} + m_{H12}^2 , \\ & \bar{\mu}'' = \mu_{11}\mu_{12} + \mu_{22}\mu_{21} + m_{\bar{H}12}^2 . \end{aligned} \tag{6}$$

From this potential, it is easy to see that the Lagrangian has a symmetry $H_1, \bar{H}_1, H_2, \bar{H}_2 \to -H_1, -\bar{H}_1, -H_2, -\bar{H}_2$. As a result, the lightest pseudo-Higgs boson must be absolutely stable, since this symmetry is also not spontaneously broken. [Actually, the symmetry is a global U(1) symmetry.] We will refer to this symmetry as PH parity; the pseudo-Higgs bosons have negative PH parity, while all other particles have positive PH parity. We emphasize that the existence of this quantum number is due entirely to our assumption that some symmetry eliminates all tree-level FCNC's.

With four pseudo-Higgs doublets, we have four charged fields and eight neutral fields. The neutral fields can be divided into four "scalars" and four "pseudoscalars," where the term corresponds to the nature of the coupling that they would have to fermions, if such coupling existed (the "pseudoscalars" come from the imaginary part of the neutral fields). The mass matrices can be calculated from the above potential. The mass matrix for the third-family fields completely decouples from that of the pseudo-Higgs fields and is given by the standard matrices in minimal supersymmetry (see Ref. 2). We define $\tan\beta \equiv \langle H_3\rangle / \langle \bar{H}_3\rangle$. The neutral pseudo-Higgs-boson mass matrix divides into two 4×4 matrices. The 4×4 matrix corresponding to the "scalars" is given by

$$\begin{pmatrix} m_1^2 - \frac{1}{2}M_Z^2\cos2\beta & -\mu'_{11} & \mu'' & -\mu'_{12} \\ -\mu'_{11} & \bar{m}_1^2 + \frac{1}{2}M_Z^2\cos2\beta & -\mu'_{21} & \bar{\mu}'' \\ \mu'' & -\mu'_{21} & m_2^2 - \frac{1}{2}M_Z^2\cos2\beta & -\mu'_{22} \\ -\mu'_{12} & \bar{\mu}'' & -\mu'_{22} & \bar{m}_2^2 + \frac{1}{2}M_Z^2\cos2\beta \end{pmatrix} . \tag{7}$$

The 4×4 matrix corresponding to the "pseudoscalars" is given by

$$\begin{pmatrix} m_1^2-\frac{1}{2}M_Z^2\cos2\beta & \mu'_{11} & \mu'' & \mu'_{12} \\ \mu'_{11} & \overline{m}\,_1^2+\frac{1}{2}M_Z^2\cos2\beta & \mu'_{21} & \overline{\mu}'' \\ \mu'' & \mu'_{21} & m_2^2-\frac{1}{2}M_Z^2\cos2\beta & \mu'_{22} \\ \mu'_{12} & \overline{\mu}'' & \mu'_{22} & \overline{m}\,_2^2+\frac{1}{2}M_Z^2\cos2\beta \end{pmatrix} . \tag{8}$$

Note that these mass matrices are *identical* except for the sign of the even-odd elements. This is not surprising, since the terms in the potential which depend on $|H|^2$ will give identical contributions to the real and imaginary parts of the neutral field in H, as long as H does not get a VEV, whereas the $\epsilon_{ij}H_i\overline{H}_j+\text{H.c.}$ term will give opposite contributions. It is easy to see that the matrix in Eq. (8) can be converted into the matrix in Eq. (7) by changing the sign of every even-numbered row and column. Since this does not change the determinant, the secular equation is unchanged and thus the eigenvalues are *identical.* Because of the sign difference, one expects this degeneracy to be broken by radiative corrections. We thus see that the lightest pseudo-Higgs boson (which must be neutral due to the stringent bounds on stable charged particles) is only very slightly lighter than the second lightest.

We will estimate the mass splitting shortly. First, the charged pseudo-Higgs-boson mass matrix must be considered. This matrix is given by

$$\begin{pmatrix} m_1^2+\frac{1}{2}M_Z^2\cos2\overline{\beta} & \mu'_{11} & \mu'' & \mu'_{12} \\ \mu'_{11} & \overline{m}\,_1^2-\frac{1}{2}M_Z^2\cos2\overline{\beta} & \mu'_{21} & \overline{\mu}'' \\ \mu'' & \mu'_{21} & m_2^2+\frac{1}{2}M_Z^2\cos2\overline{\beta} & \mu'_{22} \\ \mu'_{12} & \overline{\mu}'' & \mu'_{22} & \overline{m}\,_2^2-\frac{1}{2}M_Z^2\cos2\overline{\beta} \end{pmatrix} , \tag{9}$$

where $\cos2\overline{\beta}\equiv\cos2\beta\cos2\theta_W$. This matrix is identical to Eq. (8) with $M_Z^2\to-M_Z^2\cos2\theta_W$. It is essential that the smallest eigenvalue of this matrix (the lightest charged pseudo-Higgs boson) be larger than the smallest eigenvalue of Eq. (8) or Eq. (7), so that the stable pseudo-Higgs boson is neutral. To show that there is a large region of parameter space in which this is true, consider the case in which the off-diagonal terms are zero and where $m_i^2=\overline{m}\,_i^2$ (the latter will be approximately true in most models). In this limit, the lightest neutral pseudo-Higgs boson has a mass $m_1^2+\frac{1}{2}M_Z^2\cos2\beta$ and the lightest charged pseudo-Higgs boson has a mass $m_1^2+\frac{1}{2}M_Z^2\cos2\overline{\beta}$. Since $\cos2\beta$ is negative in most models, the lightest charged pseudo-Higgs boson is heavier. Since mixing will lower both eigenvalues, this is generally the case even with mixing included. The only way to have the lightest charged pseudo-Higgs boson be lighter than the neutral pseudo-Higgs boson is to have $\overline{m}\,_1^2\ll m_1^2$. While possible, this occurs only in a small region of parameter space. Note, however, that the matrices are identical in the limit of $M_Z^2\to0$; thus the lightest charged pseudo-Higgs boson is not too much heavier than the lightest neutral pseudo-Higgs boson; we find that it is seldom more than 30 GeV heavier, and never more than 60 GeV heavier. It will thus only be able to decay into a neutral pseudo-Higgs boson and a virtual W.

The spectrum of the lightest pseudo-Higgs bosons thus consists of a pair of neutral pseudo-Higgs bosons with masses degenerate at the tree level, which we call ϕ_S and ϕ_P, and a charged pseudo-Higgs boson ϕ^+ which is on the order of 30 GeV heavier. Since the heavier of ϕ_S and ϕ_P can only decay into the lighter (plus a virtual Z), it is important to estimate the mass splitting, in order to estimate the lifetime. Also, in considering the pseudo-Higgs boson as a dark-matter candidate, the mass splitting will play a crucial role. It is easy to see the origin of the tree-level mass degeneracy. If the sign of the imaginary parts of the $\overline{H}_i$ fields are reversed, then the entire Higgs potential is invariant under the exchange of the real and imaginary parts of the neutral components of the pseudo-Higgs fields. Thus the mass matrices must be identical. This symmetry is clearly not satisfied by gauge interactions, and thus diagrams with gauge bosons and gauge fermions will split the degeneracy. Unfortunately, there are many pseudo-Higgs fields, with many different mixing angles and masses, and thus a meaningful calculation of the mass splitting is not possible. We can, however, estimate the mass splitting. One expects the splitting to be given by

$$O\left[\frac{g^2}{16\pi^2}M\right] ,$$

where M is a typical mass of either a gauge boson or a pseudo-Higgs boson. For $M\sim200$ GeV, this gives a splitting of a 1 GeV. Mixing angles could suppress this, of course. We will take the range of mass splitting to be between 200 MeV and 4 GeV (although one should keep in mind that it could be a bit larger if some of the pseudo-Higgs bosons are quite heavy). This splitting will give a typical lifetime for the second lightest neutral pseudo-Higgs boson of between 10^{-13} and 10^{-7} sec. This result will have interesting phenomenological implications, as will be discussed in the next section.

Finally, we comment on the masses of the supersymmetric partners of the pseudo-Higgs bosons, the pseudo-Higgsinos. There are two charged pseudo-Higgsinos and two neutral pseudo-Higgsinos. Since the pseudo-Higgs bosons do not get VEV's, there will be no mixing in the mass matrix with the gauginos, as happens to Higgsinos in the standard minimal model. For one pseudo-Higgs family, there is only one parameter in the superpotential, and thus the mass matrices for the pseudo-Higgsinos are very simple:

$$\begin{bmatrix} 0 & \mu_{11} \\ \mu_{11} & 0 \end{bmatrix} . \tag{10}$$

The matrix is the same for the charged and for the neutral fields. Since the absolute value of the eigenvalues of this matrix are identical, we find that (in the case of a single family of pseudo-Higgs fields) the pseudo-Higgsinos are all degenerate in mass at the tree level. As above, we expect this degeneracy to be split by of order 1 GeV by radiative corrections. In the more realistic case in which there are two families of pseudo-Higgs bosons, the degeneracy remains: the four pseudo-Higgsinos of each family (two neutral and one charged pair) are all degenerate in mass at the tree level, and the masses are split by radiative corrections.

The pseudo-Higgsinos also have negative PH parity. There are two ways in which the lightest pseudo-Higgsino could be stable since it carries two conserved quantum numbers. It could be the lightest supersymmetric particle (LSP), in which case it is stable due to R parity, or its mass could be less than the sum of the ϕ_S and LSP masses. It is also possible that both the lightest pseudo-Higgsino and pseudo-Higgs boson are stable. For simplicity, we will concentrate on the phenomenology of pseudo-Higgs bosons, assuming that the pseudo-Higgsino can decay. However, the possibility of a light pseudo-Higgsino should be kept in mind; we will also discuss this possibility when relevant.

We now turn to consideration of the phenomenological aspects of pseudo-Higgs bosons, and discuss the possibility of detecting them in the very near future.

III. PHENOMENOLOGY OF PSEUDO-HIGGS PARTICLES

We have seen that the spectrum of the lightest pseudo-Higgs bosons consists of two neutral scalars whose mass splitting is of order 1 GeV, and a charged scalar ϕ^+ with a mass of order 30 GeV higher. In this section, the phenomenological implications of these particles are discussed. For the moment, we will ignore the supersymmetric partners of the pseudo-Higgs bosons. The lighter of the neutral scalars will be denoted ϕ_S and the heavier ϕ_P. In general, mixing with the other, heavier pseudo-Higgs bosons cannot be ignored.

Before discussing how the pseudo-Higgs bosons interact, it is important to emphasize how they do not interact. They do not have any Yukawa couplings to quarks and leptons. They also have no vacuum expectation values, and thus no scalar-vector-vector couplings; and as a result could not be bremsstrahlung scattered off a Z (which is the simplest method of detecting a regular Higgs boson). They have only four-point couplings to vector bosons and other scalars (which will generally be phenomenologically irrelevant), a three-point coupling to a regular Higgs boson, and, most importantly, a three-point pseudo-Higgs–pseudo-Higgs–vector-boson coupling. This latter coupling offers the best hope of detection. Appendix A contains the Feynman rules for a set of relevant interactions.

Since the lightest pseudo-Higgs boson is stable and weakly interacting, ϕ_S will always disappear from a detector. The relevant three-point couplings are the Z-ϕ_S-ϕ_P and the W-ϕ_S-ϕ^+ and W-ϕ_P-ϕ^+ couplings. One must produce a W or Z, which then decays into two pseudo-Higgs bosons. The ϕ_S will disappear, giving missing transverse momentum; the ϕ_P and ϕ^+ will decay into the ϕ_S (which disappears) and a virtual W or Z, which can be detected.

The simplest and most dramatic signal would occur if the ϕ_S has a mass below 40 GeV. In this case, the Z can decay into a ϕ_S and a ϕ_P (which will also have a mass below 40 GeV). The ϕ_S disappears, while the ϕ_P will decay into a ϕ_S and a virtual Z. Since the mass splitting between the ϕ_S and ϕ_P is so small, the decay products of the virtual Z will have very little energy. A typical event would thus have a low-energy muon pair and missing energy. More remarkable would be the fact that, because the lifetime of the ϕ_P is fairly long, the muon pair might not point back to the vertex. If the size of the signal were small, it might be difficult to extract, but as we will now show, the size of the signal is enormous—as much as 3% of all Z decays.

When new particles are introduced, as in minimal supersymmetry, their couplings to Z bosons generally involve various mixing angles—experiments can generally thus constrain the mixing angles, but cannot rule out the existence of the particles (in the appropriate mass range). A remarkable fact about the pseudo-Higgs bosons is that the coupling between the Z, ϕ_S, and ϕ_P is completely independent of any mixing angles. To see this, consider the coupling between the neutral scalars and the Z:

$$\frac{g}{2\cos\theta_W} Z_\mu \left[H_1^* \overleftrightarrow{\partial}^\mu H_1 - \overline{H}_1^* \overleftrightarrow{\partial}^\mu \overline{H}_1 + H_2^* \overleftrightarrow{\partial}^\mu H_2 - \overline{H}_2^* \overleftrightarrow{\partial}^\mu \overline{H}_2 \cdots \right] , \tag{11}$$

where the H fields are the neutral parts of the doublets. In terms of the neutral fields, if we call S_i (P_i) the real (imaginary) part of H_i, then the coupling is

$$\frac{g}{2\cos\theta_W} Z_\mu \left[S_1 \overleftrightarrow{\partial}^\mu P_1 - \overline{S}_1 \overleftrightarrow{\partial}^\mu \overline{P}_1 + S_2 \overleftrightarrow{\partial}^\mu P_2 - \overline{S}_2 \overleftrightarrow{\partial}^\mu \overline{P}_2 \right] . \tag{12}$$

Now, let U^S be the unitary matrix which diagonalizes the S mass matrix, and let U^P be the matrix which diagonalizes the P mass matrix. In terms of ϕ_S and ϕ_P, we have

$$\begin{aligned} S_1 &= U_{11}^S \phi_S + \cdots , \quad \overline{S}_1 = U_{21}^S \phi_S + \cdots , \\ S_2 &= U_{31}^S \phi_S + \cdots , \quad \overline{S}_2 = U_{41}^S \phi_S + \cdots \end{aligned} \tag{13}$$

with an identical expression for ϕ_P. Putting these together, we find the ϕ_S-ϕ_P-Z coupling

$$\frac{g}{2\cos\theta_W} Z_\mu \phi_S \overleftrightarrow{\partial}^\mu \phi_P (U_{11}^{*S} U_{11}^P - U_{21}^{*S} U_{21}^P + U_{31}^{*S} U_{31}^P - U_{41}^{*S} U_{41}^P) . \quad (14)$$

Given that the S and P mass matrices are so similar, it is not surprising that the unitary matrices which diagonalize them are related. Note that one can write $D_i = U_{ij}^\dagger M_{jk} U_{ki}$ (no sum on i) for each mass matrix, where D_i is the ith eigenvalue. Since the M_{jk} are identical if $j+k$ is even, and differ by a sign if $j+k$ is odd, it is easy to see that (since the D_i are the same for each matrix) the unitary matrix U^P can be obtained from U^S by changing the sign of U_{ij}^S when $i+j$ is odd. Plugging that into Eq. (14), we see that the expression in parentheses simply becomes the sum of the squares of the elements in the first column, which is unity. The resulting coupling is thus $(g/2\cos\theta_W) Z_\mu \phi_S \overleftrightarrow{\partial}^\mu \phi_P$, *independent* of any mixing angles.

This result can be understood by considering the single family case. If one examines the general coupling to the Z in, say, minimal supersymmetry, one finds that the vertex is typically multiplied by $\cos(\alpha+\beta)$, where α and β are angles which rotate each field coupling to the Z into their mass eigenstate. Since the one difference in the matrices in the one family case is in the sign of the off-diagonal term, it is obvious that the angles will be equal and opposite, and will thus cancel in the vertex.

The branching ratio can now be calculated. The only dependence on masses will be in the phase-space factor. The branching ratio is

$$\frac{\Gamma(Z \to \phi_S + \phi_P)}{\Gamma(Z \to \nu\bar{\nu})} = \frac{1}{2}\left[1 - \frac{(m_S+m_P)^2}{M_Z^2}\right]^{3/2} \times \left[1 - \frac{(m_S-m_P)^2}{M_Z^2}\right]^{3/2} \approx \tfrac{1}{2}(1-4m_S^2/M_Z^2)^{3/2} , \quad (15)$$

where in the last step we approximated $m_S \approx m_P$. If ϕ_S is fairly light, this decay would occur in roughly 1 in 30 Z decays. The signature is missing transverse momentum and a low-energy fermion pair. The energy of the fermion pair in the ϕ_P rest frame is the mass splitting, i.e., 0.2 to 4 GeV, which could be boosted as high as 20 GeV in the laboratory frame. Since the lifetime of the ϕ_P is between 10^{-13} and 10^{-7} sec (corresponding to roughly 0.1 mm to 10 m), the fermion pair might not point back to the original beam-beam interaction point. The backgrounds to such a low-energy fermion pair (muons would be the easiest to see) might be large, but the signal is also quite large.

Of course, it is not necessary to be on resonance in order to detect $Z \to \phi_S + \phi_P$. In general, the cross section is given by[10]

$$\sigma = \frac{g^4}{192\pi}\left[\frac{8\sin^4\theta_W - 4\sin^2\theta_W + 1}{\cos^4\theta_W}\right] \times \frac{\kappa^3}{\sqrt{s}\,[(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2]} , \quad (16)$$

where $\kappa \equiv \lambda^{1/2}(s, m_S^2)/(2\sqrt{s})$ and $\lambda(a,b) = a^2 - 4ab$. At KEK TRISTAN, where $\sqrt{s} \sim 60$ GeV, this cross section can exceed 0.01 units of R (where R corresponds to the cross section for μ-pair production through a virtual photon), provided the phase-space suppression is not great.

Very recently a bound on possible missing decays of the Z has been announced. A model-independent fit gives the missing width of the Z, in units of neutrino generations, as[11] $N_\nu = 3.02 \pm 0.12$. Taking two standard deviations for an approximate 95% C.L., the contribution to the width from pseudo-Higgs bosons is constrained to be less than 0.26 neutrino species. Using Eq. (15), one finds that $m_S \gtrsim 27$ GeV at a rough 95% C.L. To explore the mass region between this limit and $M_Z/2$, a search for the above signature should be made. (It is likely that searches for very light Higgs bosons or for neutralinos would be sensitive to this signature.)

If ϕ_S is heavier than 45 GeV, it cannot be found at TRISTAN, SLC, or LEP I. It can still be found at higher energy e^+e^- machines, such as LEP II. The cross section, given in Eq. (16), is typically one-tenth of a unit of R. In short, the pseudo-Higgs boson can probably be found in an e^+e^- collider, up to the kinematic limit of the machine, if a search is made for the signature discussed above.

It will be much more difficult to detect the neutral pseudo-Higgs bosons at a hadron collider. The conventional mechanisms of gluon fusion or W fusion do not apply; the lack of fermion couplings eliminates gluon fusion, the lack of a VEV eliminates W fusion. One could still produce pseudo-Higgs bosons via $q\bar{q} \to Z^* \to \phi_S\phi_P$, $W^+W^- \to Z^* \to \phi_S\phi_P$, or through the four-point W-W-ϕ_P-ϕ_P coupling. However, the signature for the ϕ_P (a low-energy μ pair and missing transverse momentum) would probably be impossible to pick out, since the amount of missing energy is very small. It thus appears that electron-positron colliders offer the best hope of detection of the neutral pseudo-Higgs bosons.

In fact, the only way in which hadron colliders could detect the neutral pseudo-Higgs boson would probably be through the decay of a charged pseudo-Higgs boson. One could pair-produce charged pseudo-Higgs bosons; the rate through a virtual photon is smaller than that of a heavy lepton by a factor of 4, the rate through a virtual Z has a cross section given by half the cross section for production of a neutrino. Since each is heavier than the neutral pseudo-Higgs fields, but never more than about 50 GeV heavier, they will decay exclusively into a neutral pseudo-Higgs field (which then disappears, possibly emitting a very-low-energy fermion pair, depending on which neutral field it is) plus a virtual W. The signature is similar to the signature of a heavy lepton (one or two jets, or a single lepton, and missing energy), where the lepton has an associated neutrino which is heavy (about 30 GeV lighter than the lepton). The signature could be distinguished from that of a heavy lepton by considering the angular distribution of the W's, which would be a $\sin^2\theta$ distribution in the pseudo-Higgs-boson case. If the neutral pseudo-Higgs boson is a ϕ_P, it will appear as if the "neutrino" emits a very-low-energy fermion pair, as well. In addition to pair production of the charged pseudo-

Higgs boson, one could produce, through a virtual W, a ϕ^+ and a ϕ_S or a ϕ_P (this corresponds to production of the lepton and its neutrino through a virtual W). Should the pseudo-Higgs boson have a mass above about 80 GeV, then one of these processes would probably be the only method of detection, short of building a higher-energy electron-positron collider. There is little reason to be optimistic about this signature, however. The conventional difficulty with detecting heavy leptons at the Superconducting Super Collider (SSC) is caused by the fact that the leptons decay into real W's, and thus the W^+W^- background is formidable. In this case, the W's must be virtual. It is not clear whether one could distinguish real and virtual W's. A full analysis of the discovery possibilities at the SSC for a heavy lepton which cannot decay into a real W has not yet (to our knowledge) been done; such an analysis would also give the discovery possibilities of the pseudo-Higgs bosons at the SSC. Note that one could also produce pseudo-Higgs bosons through the four-point $VV\phi\phi$ couplings, where $V=(W,Z)$, although the cross section is probably quite small, and the signature may be difficult to detect.

Pseudo-Higgs bosons could also have dramatic implications in the decay of Higgs bosons, which can decay into two (identical) pseudo-Higgs bosons. The rate for decay of a Higgs boson into two pseudo-Higgs bosons, relative to the decay into $\bar{b}b$, is

$$\frac{\Gamma(h_2\to\phi_S\phi_S)}{\Gamma(h_2\to\bar{b}b)}=\left[1-\frac{4m_S^2}{m_{h_2}^2}\right]^{1/2}\times\frac{\cos^2 2\theta_S\sin^2(\alpha+\beta)M_Z^2M_W^2\cos^2\beta}{2m_{h_2}^2m_b^2\sin^2\alpha}\ . \tag{17}$$

Here θ_S is the mixing angle which diagonalizes the neutral pseudo-Higgs-boson mass matrix and α is the angle which diagonalizes the neutral-Higgs-boson mass matrix. The lightest and heaviest of the neutral Higgs bosons which appear in minimal supersymmetry are denoted h_2 and h_1 respectively. The decay into $\phi_P\phi_P$ is identical, and the decay into $\phi^+\phi^-$ is obtained by replacing θ_S with θ_+. The decay of h_1 (assuming h_1 is too light to decay into $t\bar{t}$) can be obtained by replacing $\sin^2(\alpha+\beta)/\sin^2\alpha$ with $\cos^2(\alpha+\beta)/\cos^2\alpha$. Unless there are very small mixing angles, this ratio is very large, indicating that most Higgs boson decays will be into two pseudo-Higgs bosons, if kinematically allowed. Since the lightest pseudo-Higgs bosons are virtually undetectable, this means that most Higgs boson decays could be invisible.

How does this affect Higgs phenomenology? Within a few months, measurements at LEP should place a lower bound of around $M_Z/2$ on the pseudo-Higgs-boson mass (or discover it). This means that h_2 must have a mass near its maximum allowed value (m_Z) for the decay to be kinematically allowed. In this case, the h_2 may not be discovered at LEP, and might have to await the SSC or LHC. By then LEP II will have probed pseudo-Higgs-boson masses up beyond 50 GeV, and thus if h_2 decays primarily into pseudo-Higgs bosons, the pseudo-Higgs bosons will be discovered before the Higgs bosons.

What about the h_1? In minimal supersymmetry the decay $h_1\to W^+W^-$ is always suppressed by a factor of $m_{h_1}^2/m_{h_2}^2$, and thus the invisible decays will dominate (unless h_1 is heavier than twice the top-quark mass). If it is within reach of LEP II, one could look for $Z+h_1$ production, with the Z decaying into a charged fermion pair—the Higgs boson would appear as missing energy. If it is not within reach of LEP II, the discovery will have to be made at a hadron collider. In this case, if the Higgs boson is produced in association with a gauge boson, the invisible signature would be a better signature than the usual hadronic Higgs-boson decay, since QCD backgrounds would be absent.

Finally, the charged Higgs boson would decay into $\phi^+\phi_S$, which would look as if the charged Higgs boson has decayed into a fourth-family lepton and an associated heavy neutrino. Since this possibility exists in the four-family standard model, it will presumably be looked for.

We now turn to the possibility that the supersymmetric partners of the pseudo-Higgs bosons are lighter. In this case, each pseudo-Higgsino family consists of a charged-fermion pair and two neutral Majorana fermions. As discussed above, the lightest family is completely degenerate in mass at the tree level; this degeneracy will be split by radiative corrections. Since new stable charged particles are cosmologically unacceptable, the lightest must be one of the neutral pseudo-Higgsinos. Let us call the lightest $\tilde{\phi}_S$, the other neutral fermion $\tilde{\phi}_P$, and the charged fermion $\tilde{\phi}^+$.

The couplings of the pseudo-Higgsinos are easily found. They do not couple to matter fields; they also do not couple to regular Higgs bosons. They only have gauge couplings to vector bosons (and their partners). The couplings to the W are given by

$$i\frac{g}{2}\overline{\tilde{\phi}}_S\gamma^\mu\tilde{\phi}^+W_\mu^- \quad\text{and}\quad i\frac{g}{2}\overline{\tilde{\phi}}_P\gamma^\mu\gamma_5\tilde{\phi}^+W_\mu^-\ ,$$

and the coupling of the neutral fermions to the Z is given by $i(g/2\cos\theta_W)\overline{\tilde{\phi}}_S\gamma^\mu\gamma_5\tilde{\phi}_PZ_\mu$. We still have the same signal as in the pseudo-Higgs-boson case in Z decays, except that the cross section is larger, roughly twice that of neutrino production and four times that of pseudo-Higgs-boson production. Current data thus excludes pseudo-Higgsinos below about 40 GeV. One can also produce charged $\tilde{\phi}^+\tilde{\phi}^-$ pairs. The main difference between this case and the pseudo-Higgs-boson case is that the charged fields are also only slightly heavier than the neutral fields, and thus will decay into them (via a virtual W) with a long lifetime ($\sim 10^{-10\pm 2}$ sec). The lifetime might even be long enough to leave a visible track in the detectors.

The pseudo-Higgsinos thus look very similar to heavy leptons, in which the charged lepton is only slightly heavier than its neutrino. Detection in a electron-positron collider is straightforward (even Z width measurements can suffice to rule them out) up to the kinematic limit of the machine. In a hadron collider, detection is more difficult; most analyses of heavy lepton production at hadron colliders assume that the neutrino is massless. If it is close enough in mass to the charged lepton to give a long lifetime for the lepton, the phenomenology will be quite different and has yet to be investigated. If discovered, the pseudo-Higgsinos can be distinguished from a standard heavy lepton by the angular distribution in the decay (which is either all V or all A).

We now turn to a discussion of the cosmology of the lightest pseudo-Higgs boson.

IV. PSEUDO-HIGGS PARTICLES AS DARK MATTER

It is widely recognized that somewhere between 90% and 99% of the mass of the Universe is in the form of "dark matter" (DM), material which has not been detected in either emission or absorption of electromagnetic radiation. From studies of the local solar neighborhood and the light emitted from nearby galaxies it has been estimated that the total density of luminous matter (stars, dust, gas, etc.) is $\Omega_{\rm lum} \approx 0.005-0.01$, where Ω is the ratio of matter density to the critical density. Studies of the rotation curves of spiral galaxies, as well as the gravitational dynamics of groups and clusters of galaxies, among other evidence, imply a matter density of at least $\Omega_{\rm DM} \approx .05-.2$. Theoretical predilection has $\Omega_{\rm tot}=1$, but so far there is no strong observational evidence for a critical Universe. The observed age of the Universe places an upper limit on $\Omega_{\rm tot}$ of $\Omega_{\rm tot} h^2 \leq 1$, where, $0.5 \leq h \leq 1$, parametrizes our ignorance of the Hubble parameter.

The nature of the dark matter is still unknown, but some evidence suggests that a new, as yet undiscovered, elementary particle comprises the bulk of it. As a result, dozens of elementary-particle dark-matter candidates have been and are still being proposed. The requirements for a candidate particle are that it is stable (lifetime greater than the age of the Universe), that it interacts at most weakly with ordinary matter, and that it has a relic abundance in the range discussed above $(0.01 \leq \Omega_{\rm DM} h^2 \leq 1)$. In one sense it is easy to invent particles which meet these requirements. Requiring that the particle not be charged or colored ensures at most weak-interaction strength. The candidate particle can be made stable by removing any couplings which allow it to decay into lighter particles, either by hand or by imposing some symmetry. Additional couplings and its mass can be carefully adjusted to give a relic density in the proper range. However, this *ad hoc* procedure is not very satisfying or enlightening from either a theoretical or experimental point of view. One would like to have a particle physics reason for stability and one would like the appropriate relic abundance to appear without fine-tuning. For this reason, certain particle candidates, such as the lightest supersymmetric particle (LSP), the axion, and a very light neutrino have become "best bets," while other candidates have, for the most part, been ignored. We will now argue that in some limits the pseudo-Higgs boson is among the "best bet" candidates.

As we showed in Sec. II, the lightest pseudo-Higgs particle is stable because of the requirement that there be no flavor-changing neutral currents and the general nature of superpotentials involving extra Higgs doublets. It is also at most weakly interacting. Thus, if its relic abundance is in the proper range, the pseudo-Higgs boson satisfies the three criteria above and joins the ranks of "best bet" particle dark-matter candidates.

We previously showed,[4] using standard techniques, that pseudo-Higgs bosons do indeed naturally fall in the proper range of relic abundances and claimed that they therefore belonged in the most favored class of dark-matter candidates. However, the calculation and interpretation of pseudo-Higgs-boson relic abundance has been complicated by two recent developments, both of which have occurred since the completion of Ref. 4. On the experimental side, LEP data on the Z width restricts the mass of the pseudo-Higgs boson, and the LEP Higgs-boson searches constrain the masses of the Higgs bosons which figure prominently in the relevant cross sections. On the theoretical side, it has been shown[12] that the calculation of relic abundances can be drastically affected when additional particles exist near in mass to the DM candidate. As shown in Sec. II, the second lightest pseudo-Higgs boson, ϕ_P may be nearly degenerate in mass with ϕ_S. In this section we calculate the relic abundance both in the degenerate limit where the new methods must be used and in the limit of "moderate" mass splitting, where the results of Ref. 4 are valid. We also show the areas of parameter space which have been eliminated by the LEP experimental results. In the degenerate limit we find that the effective annihilation cross section is very nearly that of a Majorana neutrino (independent of the mixing angle θ_S) and so in analogy with the Majorana neutrino (see Ref. 13), the LEP results make it unlikely that a pseudo-Higgs boson contributes the bulk of the DM—though it still may exist as a minority component. However, for a moderate mass splitting $\Delta=(m_P-m_S)/m_S$ of 10% or more, the effective cross section changes drastically and an appropriate pseudo-Higgs-boson relic density arises naturally for a wide range of masses and mixing angles. Thus, in one of the two limits we consider, pseudo-Higgs bosons are a "best bet" DM candidate, and in the other they are probably ruled out as DM. We also consider the possibility of pseudo-Higgsino DM and find a result very similar to the pseudo-Higgs-boson case.

In discussing the pseudo-Higgs-boson model in Sec. II, several free parameters were found. These included the pseudo-Higgs-boson mass m_S, the pseudo-Higgs-boson mixing angle θ_S and the mass splitting $\Delta=(m_P-m_S)/m_S$. In addition, the Higgs sector of the minimal supersymmetric model has two free parameters which can be taken as the ratio of Higgs vacuum expectation values $\tan\beta=\langle H_3\rangle/\langle \overline{H}_3\rangle$ and the mass of the lightest neutral scalar m_{h2}. From these parameters the couplings and other masses can be found.[2] For example, the masses of the other two neutral Higgs bosons are given by

$$\begin{aligned} m_{h_1}^2 &= (m_{h_2}^2 - m_Z^2)/(m_{h_2}^2 \sec^2 2\beta / m_Z^2 - 1) , \\ m_{h_3}^2 &= m_{h_2}^2 + m_{h_1}^2 - m_Z^2 , \end{aligned} \tag{18}$$

Likewise, the Higgs mixing angle α can be found from formulas in Ref. 2. Since $m_{h_2} \leq m_Z|\cos 2\beta|$, we see that $m_{h_2} \leq m_Z$, $m_{h_1} \geq m_Z$, and $m_{h_2} \leq m_{h_3} \leq m_{h_1}$.

These three neutral Higgs bosons are important because they are involved in the pseudo-Higgs-boson annihilation reactions which govern the pseudo-Higgs-boson relic abundance. Recent results from LEP restrict the masses of these Higgs bosons and so must be taken into consideration. The ALEPH Collaboration[14] reports that for $\tan\beta=1$, $m_{h2}>15$ GeV at 95% C.L. For values of $\tan\beta$ of 1.2, 2, 5, and 10 they find $m_{h2}>14$, 21, 35, and 38 GeV, respectively (reading from their Fig. 1). These results will also be important when we discuss direct detection of pseudo-Higgs-boson dark matter at the end of this section.

The relic abundance of an elementary particle which was once in thermal equilibrium in the early Universe can be found by solving the Boltzmann equation

$$\frac{dn}{dt}=-3Hn-\langle\sigma v\rangle_{\rm eff}(n^2-n_{\rm eq}^2)\ , \tag{19}$$

where n is the actual number density, $n_{\rm eq}$ is the equilibrium number density, H is the Hubble parameter, and $\langle\sigma v\rangle_{\rm eff}$ is the thermally averaged effective annihilation cross section. This equation can be solved in an approximate, but accurate way using a method developed by many workers in the field.[15] For the most part, we use the method described in Ref. 16, but for some estimates we use the method of Ref. 15. In these approximations, the number density is set equal to the equilibrium number density when the temperature T is above the particle

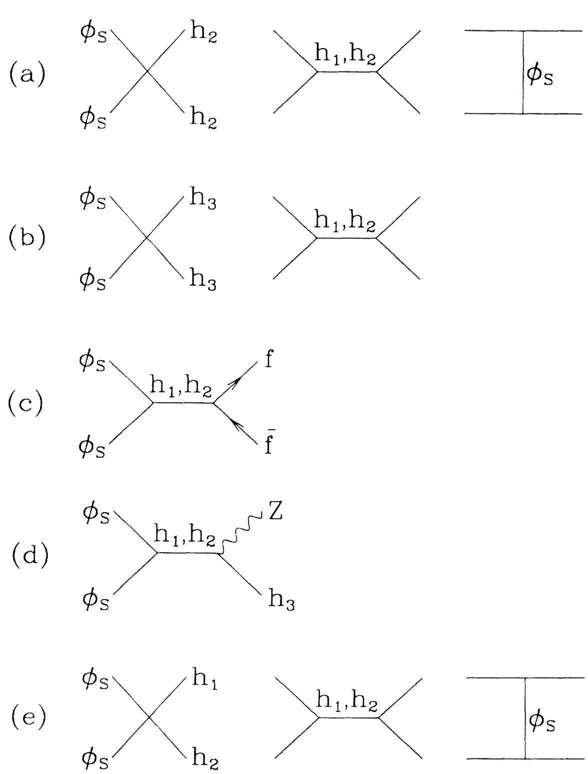

FIG. 1. Feynman diagrams for $\sigma(\phi_S\phi_S\rightarrow{\rm all})$. (a) shows the h_2h_2 final state, (b) the h_3h_3 final state, (c) the $f\bar{f}$ final state, where f is a quark or lepton, (d) shows the Zh_3 final state, and (e) shows the h_1h_2 final state.

mass m. As the Universe cools below the mass, the number density drops exponentially due to the Boltzmann factor $n_{\rm eq}\propto\exp(-m/T)$ until it is so low that the interactions which maintain equilibrium freeze-out. Below this temperature (denoted T_f), $n_{\rm eq}\approx 0$, and the resulting equation can be easily integrated. For the typical case, $\langle\sigma v\rangle_{\rm eff}$ is just the thermal average of the total annihilation cross section $\sigma(\phi_S\phi_S\rightarrow{\rm all})v$, where v is the relative velocity. However, when other particles (such as ϕ_P) exist which are near in mass to the candidate ϕ_S, and when this additional particle shares a conserved quantum number with the candidate (such as PH parity), the effective cross section is modified:[12]

$$\sigma_{\rm eff}=g_{\rm eff}^{-2}[g_S^2\sigma_{SS}+2g_Sg_P\sigma_{SP}(1+\Delta)^{3/2}e^{-x\Delta}+g_P^2\sigma_{PP}(1+\Delta)^3e^{-2x\Delta}]\ , \tag{20}$$

where $g_{\rm eff}=g_S+g_P(1+\Delta)^{3/2}\exp(-x\Delta)$, $\Delta=(m_P-m_S)/m_S$ is the mass splitting, $x=m_S/T$ is the scaled inverse temperature, $g_S=g_P=1$ are the internal degrees of freedom of the ϕ_S and ϕ_P, σ_{SS} is the total $\phi_S+\phi_S$ annihilation cross section, σ_{PP} is the total $\phi_P+\phi_P$ annihilation cross section, and σ_{SP} is the total "coannihilation" cross section $\sigma(\phi_S+\phi_P\rightarrow{\rm all})$. In the limit of degenerate ϕ_S and ϕ_P ($\Delta=0$) the formulas simplify: $\sigma_{\rm eff}({\rm deg})=g_{\rm eff}^{-2}(\sigma_{SS}+2\sigma_{SP}+\sigma_{PP})$, with $g_{\rm eff}=g_S+g_P=2$. Using Eq. (20), the total abundance of negative PH parity particles $n=n_S+n_P$ can be found by solving Eq. (19), where n has been tacitly redefined to mean n_S+n_P. As discussed in Ref. 12, this formula was found by adding the equations for n_S and n_P, and takes into account reactions which create or destroy ϕ_S and ϕ_P particles, or which convert one into the other.

Physically, the effective cross section is a weighted average of the $\phi_S\phi_S$, $\phi_S\phi_P$, and $\phi_P\phi_P$ annihilation cross sections. The reason ϕ_P annihilation must be considered is that any ϕ_P particles left over from the early Universe eventually decay into the (slightly) lighted ϕ_S particles due to conservation of the charge mentioned in Sec. II. [Thus the present-day density n_S is given by n as calculated from Eq. (19).] Since the rate for transforming a ϕ_S into a ϕ_P is proportional to $n_Sn_X\propto T^{3/2}\exp(-m_S/T)$, where n_X is the number density of some relativistic standard-model particle, it is much larger than any annihilation cross section [which is proportional to $n_S^2\sim\exp(-2m_S/T)$]. This means that ϕ_S and ϕ_P stay in "relative equilibrium" during and even after freeze-out. So, as shown in Ref. 12, the freeze-out temperature and final relic abundance are determined by both ϕ_S and ϕ_P annihilation. If, however, the mass difference between ϕ_S and ϕ_P is fairly large, the exponential weighting factors in Eq. (20) remove the contributions of $\phi_S\phi_P$ and $\phi_P\phi_P$ annihilation.

The freeze-out temperature is given by[12]

$$x_f\approx\ln\frac{0.038g_{\rm eff}m_{\rm Pl}m_S\langle\sigma_{\rm eff}v\rangle}{g_*^{1/2}x_f^{1/2}}\ , \tag{21}$$

where $m_{\rm Pl}=1.22\times 10^{19}$ GeV and g_* is the total relativistic degrees of freedom. Because of the logarithm, $x_f\approx 25$

almost independent of the cross section. (Variations in x_f of 10% to 20% are possible for large variations in the cross section.) Using the approximations of Ref. 15 the present-day relic abundance is given by[12]

$$\Omega h^2 = \frac{1.07\times 10^9 x_f}{g_*^{1/2} m_{\rm Pl}({\rm GeV})(I_a + 3I_b/x_f)} , \tag{22}$$

where

$$I_a = x_f \int_{x_f}^{\infty} x^{-2} a_{\rm eff}\, dx, \quad I_b = 2x_f^2 \int_{x_f}^{\infty} x^{-3} b_{\rm eff}\, dx , \tag{23}$$

and where we have Taylor expanded the effective cross section $\sigma_{\rm eff} = a_{\rm eff} + b_{\rm eff} v^2$.

Since in the model of Sec. II, the mass splitting is not precisely known, we must explore both the degenerate and nondegenerate limits. So we must calculate the cross sections σ_{SS}, σ_{SP}, and σ_{PP}. Some Feynman diagrams contributing to σ_{SS} (and σ_{PP}) are shown in Fig. 1. For $m_S < m_Z/2$ the possible final states include $h_2 h_2$ [Fig. 1(a)], $h_3 h_3$ [Fig. 1(b)], and $f\bar{f}$ [Fig. 1(c)], where h_2 is the lightest Higgs scalar, h_3 is the neutral Higgs pseudoscalar, and f is any quark or lepton with $m_f < m_S$. The cross sections for the processes of Fig. 1 can be derived using the Feynman rules given in Appendix A and these are displayed in Appendix B. For simplicity we have taken only the lowest-order term in v^2. For rough estimates this is valid since we do not have any s-wave suppression and one expects the next-order term to be smaller by a factor of $v^2/4 \approx .05$. For $m_S > m_Z/2$, the channels $\phi_S\phi_S \to Z h_3$ and $\phi_S\phi_S \to h_1 h_2$ can open via the processes shown in Figs. 1(d) and 1(e). These cross sections are also given in Appendix B. Note that while the formulas are complicated, nearly all terms are proportional to $\cos^2 2\theta_S$, where θ_S was defined in Sec. II.

For σ_{PP}, we examine the Lagrangian and find that the Feynman rules and therefore the cross sections are the same as for the ϕ_S case. Since $m_S \approx m_P$, we set $\sigma_{PP} = \sigma_{SS}$, ignoring the terms which depend upon the small mass difference. The coannihilation cross section σ_{SP} occurs only through the diagrams of Fig. 2 (when the pseudo-Higgs boson is lighter than the W). This formula is also given in Appendix B. In contrast with the σ_{SS} and σ_{PP} formulas, this formula is much simpler, does *not* involve $\cos 2\theta_S$, and *is* proportional to v^2 (s-wave suppressed).

We are now in a position to find the relic abundances. Using the method of Ref. 16 we first consider the extreme nondegenerate limit where the effects of σ_{PP} and σ_{SP} can be ignored. The pseudo-Higgs-boson relic abundance $\Omega_S h^2$ is shown as a function of m_S in Fig. 3(a) for $\tan\beta = 2$, $\cos 2\theta_S = 0.25$, and several values of m_{h2}. As discussed previously, when $10^{-2} \le \Omega_S h^2 \le 10^0$, pseudo-Higgs bosons can make up the bulk of the dark matter. The structure which is evident in the figure is a result of kinematic thresholds as well as poles in the cross section at $m_S \approx m_{h2}/2$ and $m_S \approx m_{h1}/2$. Recall that $\Omega h^2 \sim 1/\langle \sigma v \rangle$, so increases in cross section result in decreases in relic abundance. Since σ_{SS} depends almost linearly on $\cos^2 2\theta_S$, and this angle is unknown, values of $\Omega_S h^2$ greater or less than those shown in Fig. 3 are allowed. Figure 3(b) is the same as Fig. 4 for $\tan\beta = 10$. It is clear from these plots that abundances in the range

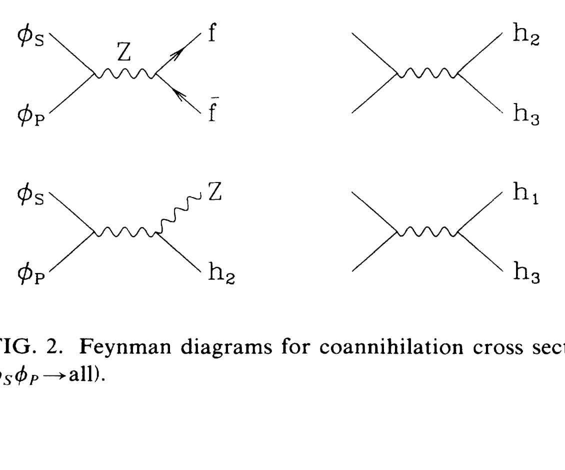

FIG. 2. Feynman diagrams for coannihilation cross section, $\sigma(\phi_S\phi_P \to \text{all})$.

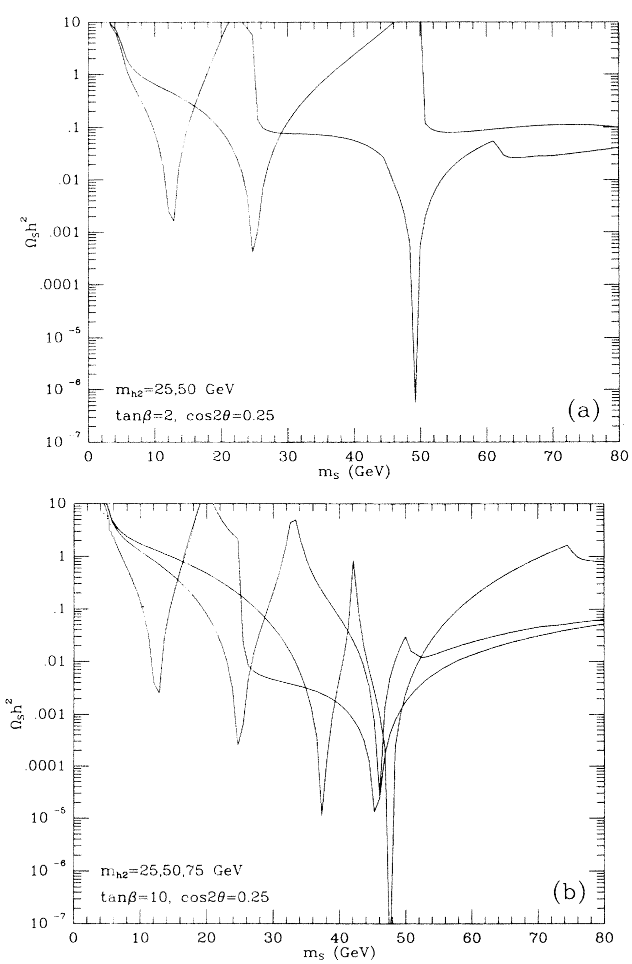

FIG. 3. Relic abundance ($\Omega_S h^2$) of pseudo-Higgs bosons as a function of mass for the nondegenerate limit (ignoring contributions from ϕ_P annihilation). Parameter values $\tan\beta = 2$, $\cos 2\theta_S = 0.25$ are used in (a) for the indicated values of m_{h2}. Parameter values $\tan\beta = 10$, $\cos 2\theta_S = 0.25$ for several values of m_{h2} are used in (b). To provide the bulk of the dark matter requires (roughly) $0.01 \le \Omega_S h^2 \le 1$ (see text). The pole at $m_S \approx m_{h2}/2$ can be used to match the values of m_{h2} to the curves.

necessary to supply the bulk of dark matter occur naturally for most values of m_S and m_{h_2} (in the nondegenerate limit).

As a way of showing this, we give in Fig. 4 a contour plot of $\Omega_S h^2$ in the m_{h_2}, m_S plane for $\tan\beta=2$ and $\cos 2\theta_S=0.25$. Contours of $\Omega_S h^2=0.01$, 0.1, and 1 are shown. Note that parameter values for which $\Omega_S h^2>1$ are ruled out as inconsistent with the observed age of the Universe, and those for which $\Omega_S h^2<0.01$ are inconsistent with pseudo-Higgs bosons contributing the bulk of the dark matter. Of course, changing $\cos 2\theta_S$ would shift these contours considerably. We see that pseudo-Higgs bosons make very good dark matter candidates in the nondegenerate limit.

We now turn to the opposite limit of complete degeneracy of the two lightest pseudo-Higgs-boson masses. Using the effective cross section defined in Eq. (20), the relic abundance for $\Delta=0$ is shown in Fig. 5(a), for the same parameter values as in Fig. 3(a). Note that many of the same pole and threshold features appear, but that the entire relic abundance is shifted downward, especially near the Z pole. This is due to $\phi_S\phi_P$ annihilation via s-channel Z exchange (see Appendix B). For this case, a relic abundance consistent with galactic DM occurs only at a few values of the parameters and only at low values of $\Omega_S h^2$. Including the LEP experimental results which constrain $m_S>27$ GeV, we see that most of the interesting regions are eliminated. However, since $\cos 2\theta_S$ is unknown, we can reduce the cross section by lowering $\cos 2\theta_S$. This however, only reduces σ_{SS} and σ_{PP}, and leaves σ_{SP} dominating the effective cross section. To show this effect we take $\cos 2\theta_S=0$ in Fig. 5(b), where we again plot $\Omega_S h^2$ vs m_S. The relic abundance shown in Fig. 5(b) is an *upper* limit to the abundance in the $\Delta=0$ case. This is because the $\phi_S\phi_P$ cross section is dominated by $f\bar{f}$ final state [Eq. (B9)] which is independent of both $\tan\beta$ and m_{h2}. Again we see many values of m_S where the abundance is too small to make up the bulk of the DM, though ϕ_S could make up an important minority component. Note that for $\Delta=0$ and $\cos 2\theta_S=0$, $\sigma_{\rm eff}$ is very close to being one-half the total annihilation cross section for a massive Majorana neutrino. As shown in Ref. 13, a Majorana neutrino in the GeV mass range cannot be the major constituent of the dark matter, when the constraint coming from the measurement of the Z width is imposed. The constraint here is not quite as strong since a pseudo-Higgs boson contributes only one-half a "neutrino" species to the Z width. More precise measurements of the Z width, may in the near future rule out the pseudo-Higgs boson as the major component of the DM (for $\Delta=0$).

What about the case of intermediate degeneracy where Δ is small but not zero? As Δ increases we move exponentially toward the nondegenerate limit discussed earlier (Figs. 3 and 4). To quantify this movement, we show

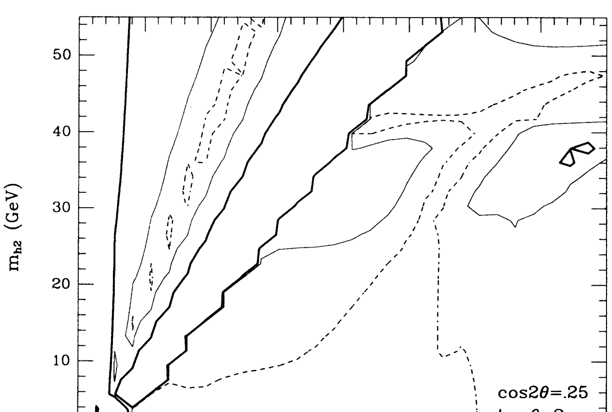

FIG. 4. Contours of pseudo-Higgs-boson relic abundance in the m_S, m_{h_2} plane for the nondegenerate limit. Parameter values $\tan\beta=2$ and $\cos 2\theta_S=0.25$ were chosen. The heavy solid lines indicate $\Omega_S h^2=1$, the light solid lines indicate $\Omega_S h^2=0.1$, and the dashed lines indicate $\Omega_S h^2=0.01$.

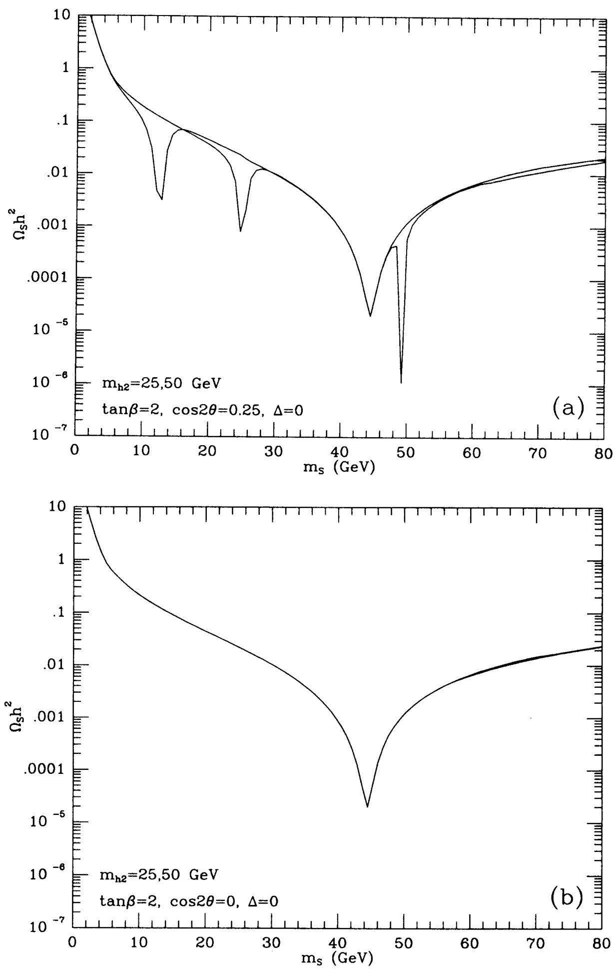

FIG. 5. Relic abundance ($\Omega_S h^2$) of pseudo-Higgs bosons in the degenerate ($m_S=m_P$) limit. Parameter values $\Delta=0$, $\tan\beta=2$, and $\cos 2\theta_S=1$ were chosen in (a) for the two values of m_{h_2} indicated. The pole at $m_S\approx m_{h_2}/2$ can be used to match the values of m_{h_2} to the curves. In (b) $\cos 2\theta_S$ was set to zero, making the result almost independent of $\tan\beta$ and m_{h_2}. This makes (b) a rough upper limit to the pseudo-Higgs-boson relic abundance in the $\Delta=0$ limit.

in Fig. 6 the increase in $\Omega_S h^2$ one finds a function of Δ. Freeze-out temperatures of $x_f=20$, 25, and 30 are shown. Depending on parameters, this spans the range of usual freeze-out temperatures. The increases of Fig. 6 were calculated by finding $(\Omega/\Omega_0)\approx(x_f^2/x_{f0}^2)/(2I_b)$ where I_b was defined in Eq. (23), $x_f=x_{f0}+\ln I_b$, and the zero subscript refers to the $\Delta=0$ limit. Referring to Fig. 5(b), we see that an increase in $\Omega_S h^2$ of between 5 and 100 puts most of the curve into the range relevant for pseudo-Higgs-boson dark matter. From Fig. 6 we see that this corresponds to $0.1\le\Delta\le0.2$. From the estimates of Sec. II, we see that this is reasonable for a light ϕ_S. For more massive ϕ_S, this is a rather large mass splitting, and so pseudo-Higgs-boson dark matter is not as natural as we had hoped. However, the splitting is quite uncertain, and if near the upper end of its range, may be sufficient. For example, if the splitting is somewhat greater than 4 GeV, then for $m_S=40$ GeV, the value of Δ is greater than 0.1. We thus see that pseudo-Higgs-boson dark matter in the more massive range may naturally occur with the appropriate relic abundance. Note that the result of the search at LEP for Higgs bosons[14] makes this the relevant range.

We should also comment on the possibility of pseudo-Higgs-boson dark matter when $m_S>m_W$. In this case several new annihilation channels open, including W^+W^-, ZZ, and final states involving charged Higgs bosons. We have not performed these calculations, but remark that from experience with similar cases involving neutrinos[17] and neutralinos,[18] we expect a large annihilation into the W^+W^- final state which will increase the cross section and decrease the relic abundance. For the $\Delta=0$ limit, this means even lower relic abundances and little possibility for pseudo-Higgs-boson dark matter. For the nondegenerate cases where $\cos2\theta_S$ enters, we expect there will still be many possibilities for pseudo-Higgs-boson DM as long as $\cos2\theta_S$ is small.

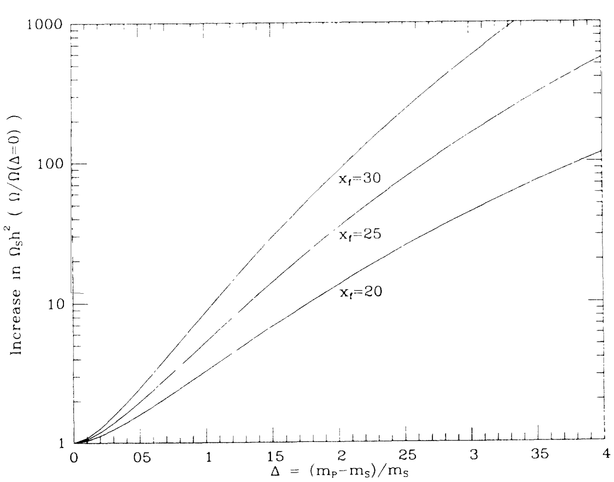

FIG. 6. Increase in $\Omega_S h^2$ as a function of the mass splitting Δ; that is, $\Omega/\Omega(\Delta=0)$. Scaled freeze-out temperatures of $x_f=20$, 25, and 30 are shown. These values span the relevant range.

Next we turn briefly to the possibility that the pseudo-Higgsino is lighter than the pseudo-Higgs boson. Given the assumptions of Sec. II, we would now have three nearly degenerate particles: the lightest pseudo-Higgsino $\tilde\phi_S$, an additional neutral fermion $\tilde\phi_P$, and the charged fermion $\tilde\phi^+$. As discussed in Sec. II, most of the couplings which exist for the pseudo-Higgs boson do *not* exist for the $\tilde\phi_S$, but there is a $\tilde\phi_S\tilde\phi_P Z$ coupling. This means that $\tilde\phi_S+\tilde\phi_S$ and $\tilde\phi_P+\tilde\phi_P$ annihilation will not take place at the tree level $(\sigma_{SS}=\sigma_{PP}\approx0)$, but the coannihilation $\tilde\phi_S+\tilde\phi_P\to f\bar f$ by s-channel Z exchange will occur when the mass splitting is small. In fact, for exact degeneracy, calculation of the effective cross section using the Feynman rules of Appendix A and Eq. (20), shows that it is precisely the same as the total annihilation cross section of a massive Majorana neutrino. In addition, the $Z\to\tilde\phi_S+\tilde\phi_P$ contribution to the Z width is just twice the standard-model contribution of a Majorana neutrino. This makes the LEP data on the Z width more restrictive than for the pseudo-Higgs-boson case ($\tilde m_S>39$ GeV for $N_\nu<3.26$). As shown in Ref. 13, a mass restriction this strong on a Majorana neutrino results in a relic abundance too small to allow it to make up the bulk of the dark matter. Inclusion of $\tilde\phi^+\tilde\phi^-$ annihilation would increase the effective cross section further, thereby resulting in an even smaller relic abundance. We conclude that in the degenerate limit, the pseudo-Higgsino cannot provide the bulk of the DM, although it could exist and provide a minority component. For significant mass splitting $(\Delta\ge0.1)$, the effective cross section will decrease substantially and the pseudo-Higgsino then makes a reasonable dark-matter candidate.

Lastly we consider the possibility of directly detecting pseudo-Higgs-boson dark matter in the laboratory. Two experimental groups[19] have reported results which limit the mass and cross section of elementary-particle dark matter. Both use modified double-β-decay detectors and monitor kilogram size germanium crystals for ionization signals resulting from the elastic scattering of dark-matter particles off germanium nuclei. The observed signals are consistent with background and therefore inconsistent with dark-matter particles having masses and cross sections above the dashed line in Fig. 7. Since these experiments rule out Dirac neutrinos above around 10 GeV as the primary constituent of the dark matter, it is of interest to see whether pseudo-Higgs-boson dark matter could have been detected. In addition, many groups are planning new experiments using improved techniques which should have higher sensitivity and lower backgrounds.

A pseudo-Higgs-boson dark matter particle can elastically scatter off a nucleus by the t-channel exchange of ordinary neutral Higgs bosons (both h_1 and h_2 contribute). Since the Higgs couplings to the quarks in the nucleus are proportional to their very small masses, one might expect the cross section to be very small. However, as shown by Shifman, Vainshtein, and Zakharov[20] and used in this context by Raby and West,[21] the coupling is actually proportional to the mass of the nucleus. The resulting cross section is (in the appropriate $v=0$ limit)

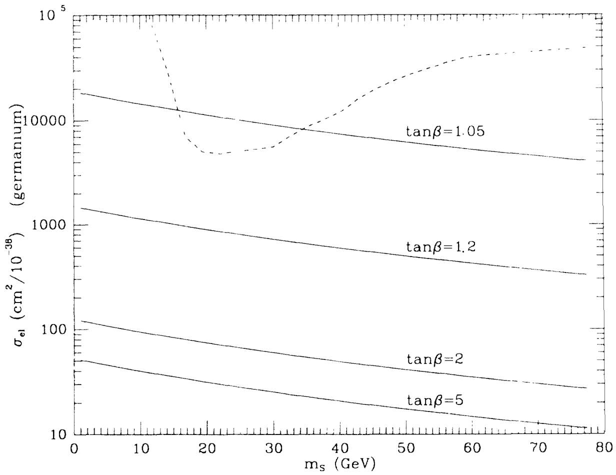

FIG. 7. Direct detection of pseudo-Higgs-boson dark matter. The area above the dashed line is ruled out by experiment (Ref. 19). The solid lines show the theoretical elastic cross section of pseudo-Higgs bosons off germanium nuclei as a function of pseudo-Higgs-boson mass. Several values of $\tan\beta$ are displayed for $\cos2\theta_S=1$. The results are independent of m_{h_2}. The curve for $\tan\beta=1.05$ is ruled out by LEP data (Ref. 14) (see text).

$$\sigma_{\rm el}(\phi_S N\to\phi_S N)\approx\frac{\pi\alpha^2 m_N^4\cos^2 2\theta_S}{324\cos^2 2\beta\sin^4\theta_W m_W^4(m_N+m_S)^2}\ , \tag{24}$$

where m_W is the W mass and m_N is the nucleus mass. This formula is remarkable in that it is independent of the Higgs-boson masses and mixing angle α. Relationships such as Eq. (18) and others from Ref. 2, remove this dependence when both h_1 and h_2 exchange are included. Note that the cross section does depend strongly on $\tan\beta$ and $\cos2\theta_S$.

The cross section, Eq. (24), is plotted (solid lines) in Fig. 7 for $\cos2\theta_S=1$ and several values of $\tan\beta$. For values of $\tan\beta$ near unity one sees that substantial event rates are possible and the cross-section curve can cross into the experimentally excluded region. However, when we take into consideration the LEP Higgs-boson search results we find that values of $\tan\beta$ near unity are disallowed. This is because of the relation[2] $m_{h_2}\ge m_Z|\cos2\beta|$. So for example, $\tan\beta=1.05$ implies $m_{h_2}\le 4.5$ GeV, clearly ruled out by the LEP data.[14] Taking the LEP limit $m_{h_2}\ge 15$ GeV for $\tan\beta$ near unity,[14] we find $\tan\beta\ge 1.2$. For $\tan\beta\ge 2$, we have $m_{h_2}\le 55$ GeV from the relation above and the published LEP data place no restrictions as long as $m_{h_2}\ge 40$. For $\tan\beta=2$ (used previously), the elastic cross section is roughly two orders of magnitude below the present experimental limits. Note that for the small values of $\cos2\theta_S$ which the relic abundance calculation favor, the cross section would be even smaller.

From Fig. 7 and the LEP results we see that direct detection of pseudo-Higgs bosons is more difficult than massive Dirac neutrinos, but may (depending on $\cos2\theta_S$) be easier than the popular photino or neutralino dark-matter candidates, which can have cross sections on germanium two to four orders of magnitude below the present limits.

V. CONCLUSIONS

In this paper we considered the possibility that the Higgs sector of the minimal supersymmetric model consists of three families, in the same way that the fermion sector does. We assumed that the observed smallness of flavor-changing neutral currents is due to the fact that a symmetry of the Lagrangian eliminates them entirely and showed, as a result, that the two extra families of Higgs bosons (denoted pseudo-Higgs bosons) decouple from fermions, do not acquire vacuum expectation values and do not mix with ordinary Higgs bosons. We also showed that the lightest of these states is absolutely stable and that the second lightest of these states is close in mass to the lightest. We derived the interactions of these particles with ordinary particles and showed that they are easily detectable at e^+e^- machines, if a search is made for the somewhat unusual signature. We also discussed detection of the charged pseudo-Higgs boson as well as the supersymmetric partners of the pseudo-Higgs bosons.

The natural stability of the lightest pseudo-Higgs boson makes it a good dark-matter candidate, so we then calculated the pseudo-Higgs-boson relic abundance. We found the relic abundance is probably too small to make up the entirety of the dark matter unless a mixing parameter ($\cos2\theta_S$) is fairly small, and the mass splitting is moderate. There is nothing wrong with a small value of $\cos2\theta_S$ since this parameter is arbitrary, but it does make the direct detection of pseudo-Higgs-boson dark matter more difficult (since the detection rate is proportional to $\cos^2 2\theta_S$). For small values of $\cos2\theta_S$, the pseudo-Higgs boson is an excellent dark-matter candidate, providing an appropriate relic abundance over a very wide range of the other parameters. It is interesting that production of pseudo-Higgs bosons at an e^+e^- machine is independent of $\cos2\theta_S$, so this may be the best way to search for these particles.

ACKNOWLEDGMENTS

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APPENDIX A: PSEUDO-HIGGS-BOSON INTERACTIONS

In this appendix, we list some of the Feynman rules involving pseudo-Higgs bosons and pseudo-Higgsinos. The pseudo-Higgs bosons do not couple to fermions and there are no three-point vector–vector-pseudo-Higgs-boson couplings.

First consider the *vector–pseudo-Higgs-boson–pseudo-Higgs-boson* interactions. We define θ_+ as the angle between the ϕ^+ mass eigenstate and the weak eigenstate. In the single-family case, when $m_1^2=\overline{m}_1^2$, one has $\tan2\theta_S=-\tan2\theta_+\cos2\theta_W$. For all values of θ_S, it is easy to see that $\cos(\theta_S+\theta_+)$ will be within 1% of unity, and $\cos(\theta_S-\theta_+)$ will be within 1% of $\cos 2\theta_S$. The fields and the corresponding vertices (coupling constants) are given by [all vertices should be multiplied by $(g/2)(p+p')^\mu$, where p is the second field's incoming four-momentum, and p' is the third field's outgoing four-momentum]

$$\begin{aligned} &Z_\mu\phi_S\phi_P \to \sec\theta_W, \quad W_\mu^+\phi_S\phi^+ \to -i\cos(\theta_S-\theta_+)\ , \\ &W_\mu^+\phi_P\phi^+ \to \cos(\theta_S+\theta_+)\ , \\ &Z_\mu\phi^+\phi^- \to -i\sec\theta_W\cos 2\theta_W, \quad \gamma_\mu\phi^+\phi^- \to -2i\sin\theta_W\ . \end{aligned} \tag{A1}$$

Next consider the *vector–vector–pseudo-Higgs-boson–pseudo-Higgs-boson* interactions. These couplings will only be of phenomenological relevance if one produces pseudo-Higgs-boson pairs through vector-boson fusion at hadron colliders and will be of cosmological relevance only if the pseudo-Higgs bosons are quite heavy. The fields and corresponding coupling constants are given by [all vertices should be multiplied by $(ig/2)g^{\mu\nu}$]

$$\begin{aligned} &W^+W^-\phi_S\phi_S \to 1, \quad W^+W^-\phi_P\phi_P \to 1\ , \\ &W^+W^-\phi^+\phi^- \to 1\ , \\ &ZZ\phi_S\phi_S \to \sec^2\theta_W, \quad ZZ\phi_P\phi_P \to \sec^2\theta_W\ , \\ &ZZ\phi_+\phi_- \to \sec^2\theta_W\cos^2 2\theta_W\ , \\ &W^+Z\phi^+\phi_S \to -\sin^2\theta_W\sec\theta_W\cos(\theta_S-\theta_+)\ , \\ &W^+Z\phi^+\phi_P \to -i\sin^2\theta_W\sec\theta_W\cos(\theta_S+\theta_+)\ , \\ &\gamma\gamma\phi^+\phi^- \to 4\sin^2\theta_W, \quad Z\gamma\phi^+\phi^- \to 2\cos 2\theta_W\tan\theta_W\ , \\ &W^+\gamma\phi^+\phi_S \to \sin\theta_W\cos(\theta_S-\theta_+)\ , \\ &W^+\gamma\phi^+\phi_P \to i\sin\theta_W\cos(\theta_S+\theta_+)\ . \end{aligned} \tag{A2}$$

Next consider the *Higgs-boson–pseudo-Higgs-boson–pseudo-Higgs-boson* interactions. These are relevant for Higgs-boson decays into pseudo-Higgs bosons, and for cosmological abundance calculations. There are no three-point interactions involving h_3. The fields and corresponding couplings are [all vertices should be multiplied by $(i/2)gm_Z/\sin\theta_W$]

$$\begin{aligned} &h_2\phi_S\phi_S \to \cos 2\theta_S\sin(\alpha+\beta)\ , \\ &h_2\phi_P\phi_P \to \cos 2\theta_S\sin(\alpha+\beta)\ , \\ &h_1\phi_S\phi_S \to -\cos 2\theta_S\cos(\alpha+\beta)\ , \\ &h_1\phi_P\phi_P \to -\cos 2\theta_S\cos(\alpha+\beta)\ , \\ &h_2\phi^+\phi^- \to \cos 2\theta_+\sin(\alpha+\beta)\ , \\ &h_1\phi^+\phi^- \to \cos 2\theta_+\cos(\alpha+\beta)\ , \end{aligned} \tag{A3}$$

where α and β were defined in the text.

Next consider *pseudo-Higgs-boson–pseudo-Higgs-boson–Higgs-boson–Higgs-boson* interactions. These interactions are phenomenologically irrelevant, but do affect the cosmological abundances. Thus we only list interactions involving ϕ_S and ϕ_P. The interactions involving ϕ_P are identical to those involving ϕ_S which are [all vertices should be multiplied by $(i/4)g^2\sec^2\theta_W\cos 2\theta_S$]

$$\begin{aligned} &\phi_S\phi_S h_2h_2 \to \cos 2\alpha, \quad \phi_S\phi_S h_2h_1 \to \sin 2\alpha\ , \\ &\phi_S\phi_S h_1h_1 \to -\cos 2\alpha, \quad \phi_S\phi_S h_3h_3 \to \cos 2\beta\ . \end{aligned} \tag{A4}$$

Finally we consider the *pseudo-Higgsino* interactions. There are no couplings of Higgs bosons to pseudo-Higgsinos, but only gauge interactions. This is because the superpotential has no cubic terms. The couplings are

$$\frac{ig}{2}\bar{\tilde{\phi}}_S\gamma^\mu\tilde{\phi}^+W_\mu^- + \frac{ig}{2}\bar{\tilde{\phi}}_P\gamma^\mu\gamma_5\tilde{\phi}^+W_\mu^- + \frac{ig}{2\cos\theta_W}\bar{\tilde{\phi}}_S\gamma^\mu\gamma_5\tilde{\phi}_PZ_\mu\ . \tag{A5}$$

APPENDIX B: PSEUDO-HIGGS-BOSON ANNIHILATION CROSS SECTIONS

In this appendix we list the annihilation cross sections relevant for calculation of pseudo-Higgs-boson relic abundance. The Feynman rules used are given in Appendix A. When $m_S > m_W$, new channels open and the cross sections given here are not sufficient. All cross sections have been Taylor expanded in the relative velocity v^2 and only the lowest order kept (see Sec. IV). Three cross sections are relevant:

$$\begin{aligned} &\sigma_{SS} = \sigma(\phi_S\phi_S \to \text{all}), \quad \sigma_{PP} = \sigma(\phi_P\phi_P \to \text{all})\ , \\ &\sigma_{SP} = \sigma(\phi_S\phi_P \to \text{all})\ . \end{aligned} \tag{B1}$$

Note that $\sigma_{SS} = \sigma_{PP}$, when the small mass difference between ϕ_S and ϕ_P is ignored.

We first define

$$\begin{aligned} &C = \frac{\pi\alpha^2\cos^2 2\theta_S\beta_f}{64\sin^4\theta_W\cos^4\theta_W}\ , \\ &P_Z = [(4m_S^2-m_Z^2)^2+m_Z^2\Gamma_Z^2]^{-1/2}\ , \\ &P_{h_i} = [(4m_S^2-m_{h_i}^2)^2+m_{h_i}^2\Gamma_{h_i}^2]^{-1/2}\ , \end{aligned} \tag{B2}$$

where $\beta_f = (1-m_f^2/m_S^2)^{1/2}$, m_f is the mass of the final-state particle (for example, m_{h_2}, m_{quark}), Γ_Z is the width of the Z boson, and Γ_{h_i} is the width of h_i $(i=1,2,3)$.

Feynman diagrams contributing to the h_2h_2 final state are shown in Fig. 1(a). The resulting cross section is

$$\sigma(\phi_S\phi_S\to h_2h_2)v\approx\frac{C\cos^2 2\alpha}{m_S^2}\{1+3m_Z^2\sin^2(\alpha+\beta)P_{h_2}+2m_Z^2\cos 2\theta_S\sin^2(\alpha+\beta)/(2m_S^2-m_{h_2}^2)-m_Z^2\cos(\alpha+\beta)[2\tan 2\alpha\sin(\alpha+\beta)-\cos(\alpha+\beta)]P_{h_1}\}^2 . \tag{B3}$$

The diagrams for the h_3h_3 final state are shown in Fig. 1(b) and result in

$$\sigma(\phi_S\phi_S\to h_3h_3)v\approx\frac{C\cos^2 2\beta}{m_S^2}[1+m_Z^2\sin^2(\alpha+\beta)P_{h_2}+m_Z^2\cos^2(\alpha+\beta)P_{h_1}]^2 . \tag{B4}$$

The fermion final-state diagrams are shown in Fig. 1(c) and result in

$$\sigma(\phi_S\phi_S\to f\bar{f})v\approx\sum_f 16c_fC\beta_f^2m_f^2[\sin(\alpha+\beta)P_{h_2}F_{q2}-\cos(\alpha+\beta)P_{h_1}F_{q1}]^2 , \tag{B5}$$

where the sum is over all quarks and leptons with $m_f<m_S$, c_f is the color factor, F_{q1} is $\sin\alpha/\sin\beta$ for up-type quarks and leptons and $\cos\alpha/\cos\beta$ for down types, and F_{q2} is $\cos\alpha/\sin\beta$ for up types and $-\sin\alpha/\cos\beta$ for down types.

In Fig. 1(d) we show the graphs contributing to the Zh_3 final state which can only be open when $m_S>(m_Z+m_{h3})/2$:

$$\sigma(\phi_S\phi_S\to Zh_3)v\approx\frac{2(C/\beta_f)m_Z^2}{m_S^2}[\cos(\alpha-\beta)\sin(\alpha+\beta)P_{h_2}-\sin(\alpha-\beta)\cos(\alpha+\beta)P_{h_1}]^2\times[m_Z^2-2m_{h_3}^2-8m_S^2+(4m_S^2+m_{h_3}^2)^2/m_Z^2]\beta(m_Z,m_{h_3}) , \tag{B6}$$

where

$$\beta(x,y)=\left\{\left[1-\left[\frac{x+y}{2m_S}\right]^2\right]\left[1-\left[\frac{x-y}{2m_S}\right]^2\right]\right\}^{1/2} .$$

Finally, we show in Fig. 1(e) the h_1h_2 final state which again can contribute only when $m_S>(m_{h_1}+m_{h_2})/2>(m_Z+m_{h_2})/2$:

$$\sigma(\phi_S\phi_S\to h_1h_2)v\approx\frac{2(C/\beta_f)}{m_S^2}\beta(m_{h_1},m_{h_2})\{\sin 2\alpha+m_Z^2\sin(\alpha+\beta)P_{h_2}[2\sin 2\alpha\sin(\alpha+\beta)-\cos 2\alpha\cos(\alpha+\beta)]+2m_Z^2\cos 2\theta_S\sin(\alpha+\beta)\cos(\alpha+\beta)/[2m_S^2-(m_{h_2}^2+m_{h_1}^2)/2]+m_Z^2\cos 2\alpha\cos(\alpha+\beta)P_{h_1}[2\tan 2\alpha\cos(\alpha+\beta)-\sin(\alpha+\beta)]\}^2 . \tag{B7}$$

For the coannihilation cross section $\sigma_{SP}=\sigma(\phi_S\phi_P\to\text{all})$, there are four diagrams shown in Fig. 2, the most important of which is the annihilation into two fermions via s-channel Z exchange. Defining

$$D=\frac{\pi\alpha^2m_S^2P_Z}{12\sin^4\theta_W\cos^2\theta_W} , \tag{B8}$$

we have

$$\sigma_{SP}v(\phi_S\phi_P\to f\bar{f})\approx\sum_f Dc_f\beta_fv^2[(c_L^2+c_R^2)(4-m_f^2/m_S^2)+6c_Lc_Rm_f^2/m_S^2] , \tag{B9}$$

where $c_L=T_{3f}-Q_f\sin^2\theta_W$, $c_R=-Q_f\sin^2\theta_W$, Q_f is the charge of fermion f and T_{3f} is the third component of weak isospin. Note that the first term in the Taylor expansion for this cross section (and all coannihilation cross sections) is proportional to v^2. For the h_2h_3 final state we have

$$\sigma_{SP}v(\phi_S\phi_P\to h_2h_3)\approx\tfrac{1}{2}D\beta(m_{h_2},m_{h_3})^3v^2\cos^2(\alpha-\beta) . \tag{B10}$$

Likewise

$$\sigma_{SP}v(\phi_S\phi_P\to h_1h_3)\approx\tfrac{1}{2}D\beta(m_{h_1},m_{h_3})^3v^2\sin^2(\alpha-\beta) , \tag{B11}$$

and

$$\sigma_{SP}v(\phi_S\phi_P\to Zh_2)\approx\frac{3m_Z^2}{2m_S^2}D\beta(m_Z,m_{h_2})v^2\sin^2(\alpha-\beta)\left[1+\frac{m_S^2}{3m_Z^2}\beta(m_Z,m_{h_2})^2\right] . \tag{B12}$$

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[7]It is clear from Eq. (3) that if the Yukawa coupling matrices are proportional, then FCNC's vanish and such a basis can be chosen. This condition is certainly sufficient, but is it necessary? Strictly speaking, the Yukawa coupling matrices only need to commute for them to be simultaneously diagonalizable. In the quark-mass eigenstate basis, this is identical to saying that one could add, say, an $s\bar{s}H_2$ term; i.e., one can add couplings of the extra Higgs bosons to fermions as long as they are flavor diagonal. However, we are requiring that a *symmetry* eliminates FCNC's. The only way to have flavor-diagonal couplings to H_2 and not flavor-off-diagonal couplings is for different quark flavors to behave differently under this symmetry. This symmetry will generally eliminate Cabibbo mixing as well, and we thus do not consider this possibility here.

[8]If it is a global symmetry, then if only H_3 and $\bar{H}_3$ get vacuum values, they must each have a zero quantum number under the symmetry to avoid a massless Goldstone boson.

[9]If one considered a nonminimal model in which the scalar mass-squared parameters were not equal at the GUT scale, one could arrange the parameters so that the first-family fields could acquire VEV's at the electroweak scale. In this case, since there is no mixing with the third-family fields, one would have to fine-tune the parameters so that the first-family fields got VEV's at the same scale as the third-family fields; such fine-tuning is extremely unnatural and we assume that it does not occur.

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