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One-loop P- and T-odd $W^{\pm}$ electromagnetic moments

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We compute the P- and T-violating electromagnetic moments (electric dipole and magnetic quadrupole) generated at one loop for the W boson. We argue that only models in which the physical W has both left- and right-handed fermion couplings at the tree level can generate these moments at one loop. The resulting magnetic quadrupole moment is quite generally related to the electric dipole moment (EDM) by the following one-loop relation: $2d_W+M_W\tilde{Q}_W=0$. On fairly general grounds $SU_L(2)\times U_Y(1)$ invariance relates the nonvanishing CP- and P-odd WWZ form factor to its electromagnetic counterpart: $f_Z(0)=-f_\gamma(0)\sin^2\theta_W$. A reasonable estimate of the resulting W EDM is 10^{-22}–10^{-23} e cm depending on values chosen for unknown parameters. Such a dipole moment would induce an EDM for electrons and neutrons of order 10^{-30}–10^{-31} e cm and 10^{-29}–10^{-27} e cm, respectively. For some corners of parameter space the fermion EDM's can be observable in current searches. The effects of CP violation from a heavy fourth generation can be typically ten times as large.

I. INTRODUCTION AND DISCUSSION

Observation of a permanent electric dipole moment (EDM) for an elementary particle would indicate the existence of interactions that are not invariant under both T (or CP) and P transformations. Since neither P nor T is a good symmetry of nature, it is probable that every particle has a permanent EDM at some level. It turns out, however, that because of the appearance of small couplings and mixing angles, the known CP- and P-odd interactions of the standard model predict particle EDM's that are many orders of magnitude smaller than the present experimental upper bounds. Current searches for these moments therefore probe any "new physics" that may ultimately replace the standard model at higher-energy scales than are presently directly accessible.[1,2]

Bounds on fermion EDM's, in particular those for the neutron[3] and electron,[4] presently furnish the strongest constraints on CP violation from new physics. Information concerning EDM's for other fundamental particles is also of interest, however, since it complements the neutron and electron results by probing different combinations of the underlying couplings. This will become particularly important, of course, should a nonzero EDM be discovered for the neutron or electron. For the purposes of disentangling the source of any CP violation, it is therefore of some theoretical importance to explore the comparative predictions of varying models. This is particularly pressing considering the remarkable improvements in experimental accuracy to be expected soon in EDM measurements.[5]

By contrast with the situation for fermion EDM's, little work[6,7] has been done to date on boson EDM's. Of the bosons of the standard model, all but the charged vector boson $W^{\pm}$ are self-conjugate particles and so cannot have electric dipole moments.[8] The $W^{\pm}$, on the other hand, can *a priori* have three distinct types of electromagnetic CP-odd couplings, of which one is CP odd and P even, while two are both CP and P odd. We focus in what follows exclusively on the CP- and P-odd interactions. These are given in terms of the matrix elements of the electromagnetic current J^λ by[9]

$$\langle W^-|J^\lambda|W^-\rangle \equiv -ie\varepsilon^*_\mu(p_1)\Gamma^{\mu\nu\lambda}(p_1,p_2)\varepsilon_\nu(p_2)\ ,$$
$$\Gamma^{\mu\nu\lambda}_{P,T\ \mathrm{odd}}=-f_\gamma(q^2)\epsilon^{\mu\nu\lambda\rho}q_\rho-\frac{1}{M_W^2}g_\gamma(q^2)p^\lambda\epsilon^{\mu\nu\sigma\rho}q_\sigma p_\rho\ . \tag{1}$$

$\varepsilon_\mu(p_i)$ denotes the polarization vector of a W boson whose four-momentum is p_i^ν. The second line of Eq. (1) gives the expression for the P- and CP-odd part of the matrix element in terms of the sum and difference, $p=p_1+p_2$ and $q=p_1-p_2$, of the initial and final W momenta.

The form factors $f_\gamma(q^2)$ and $g_\gamma(q^2)$ evaluated at zero momentum are related in the following way to the electric dipole (d_W) and magnetic quadrupole ($\tilde{Q}_W$) moment (MQM) of the W boson:

$$d_W=\frac{e}{2M_W}[f_\gamma(0)-4g_\gamma(0)] , \quad (2)$$

$$\tilde{Q}_W=-\frac{e}{M_W^2}f_\gamma(0) . \quad (3)$$

In the present article we investigate the contribution to f_γ and g_γ arising at one loop in various theories. Apart from their intrinsic interest, these moments are useful for the purposes of understanding fermion EDM's. This is because it is sometimes true that the dominant contribution to the EDM of a fermion (such as the electron) is the one induced by the EDM of the W. In fact, the standard model itself may be an example of one such model.[10] For these types of models an understanding of the graphs responsible for the W EDM is a prerequisite for a calculation of the fermion EDM.

Before describing them in detail, we briefly outline our main results. Our first conclusion is that relatively few models can generate a P- and CP-odd $WW\gamma$ vertex at the one-loop level. More specifically, we find that a one-loop EDM only arises in theories for which the W mass eigenstate acquires, at the tree level, a right-handed component in its coupling to fermions. Given such couplings, the form factor $f_\gamma(0)$ receives a one-loop contribution provided that either the fermion–gauge-boson couplings or the gauge-boson mixing matrix violate CP.

Remarkably the second CP-violating form factor $g_\gamma(0)$ vanishes for any model at one loop. A generic conclusion to be drawn, therefore, is that if the W electric dipole and magnetic quadrupole moments are generated at one loop, then both are determined by $f_\gamma(0)$ and so are related by

$$2d_W+M_W\tilde{Q}_W=0 . \quad (4)$$

Another general conclusion can be drawn if the fermion whose couplings are responsible for CP violation is heavy compared to the W-boson mass. In this case, integrating out this heavy fermion induces dimension-6 CP- and P-odd effective gauge-boson self-interactions in the low-energy effective Lagrangian at the W scale. There are two types of effective operators that arise in this way, and the heavy fermion's contribution to $f_\gamma(0)$ and $g_\gamma(0)$ is completely determined by their coefficients. It turns out, however, that $SU_L(2)\times U_Y(1)$ invariance implies that these same two operators also completely determine the corresponding form factors $f_Z(0)$ and $g_Z(0)$ that govern the WWZ vertex. This then implies the following one-loop prediction for the WWZ form factors in these theories: $g_Z(0)=0$ and $f_Z(0)=-f_\gamma(0)\sin^2\theta_W$.

The size of the W EDM that arises at one loop can be fairly close to the present bounds without extreme assumptions. To illustrate this point we consider for definiteness a left-right-symmetric model[11,12] in which we assume similar sizes for left- and right-handed Kobayashi-Maskawa matrix elements and coupling constants, and we take a W_L-W_R mixing angle[12] $\xi\approx10^{-3}$. These are fairly conservative assumptions, and for some parts of the parameter space of these models the induced EDM's can be much larger. With these choices, assuming the presence of a large CP-violating phase, we have $\text{Im}\,(a_{tb}^*b_{tb})\approx2\times10^{-4}$, and so a loop containing the t and b quarks gives $d_W\approx10^{-22}$ e cm (see below). This t-b quark loop contribution includes a suppression factor of $m_b/m_t<0.05$.

This estimate can be much larger for a heavy fourth generation. Even for equal-strength couplings, the induced fermion EDM can easily be 10 times larger than for a t-b loop simply because its effects are not suppressed by the small ratio m_b/m_t. Furthermore, such a fermion can have large right-handed couplings to the W and so $\text{Im}(a^*b)$ need not be small. In either case the resulting fermion EDM could be observable within current searches.

A W EDM of this size would most likely first be observable through the EDM's it would induce for the electron and neutron. We estimate the size of these induced EDM's with the result that a W EDM as large as 10^{-22} e cm would generate a neutron EDM of 10^{-29}–10^{-27} e cm (depending on unknown matrix elements). The corresponding electron EDM would be 10^{-31}–$10^{-30}e$ cm. Since the electron EDM induced in this way is not proportional to neutrino mass-matrix elements, for many models it can dominate the one-loop result.[13] Our results agree well with the estimates of Ref. 7 made using a cutoff theory.

Fermion EDM's of this size are probably too small to be seen in present-generation EDM searches. Being 10 times larger, the EDM induced by a heavy-fermion generation could potentially be observable, however. Unfortunately, such a small EDM is not likely to have observable consequences in accelerator experiments.[9,14]

We now turn to a derivation of these results.

II. CALCULATION OF d_W

We first justify the necessary conditions listed earlier that are required to generate a W EDM at one loop. We start with the observation that both form factors in Eq. (1) are proportional to the Levi-Civita tensor $\epsilon^{\mu\nu\lambda\rho}$. The only way to generate this tensor from a one-loop graph constructed from renormalizable scalar, spin-$\frac{1}{2}$, or gauge couplings is through the fermion loop of Fig. 1. For such a loop the Levi-Civita tensor arises from the trace over the Dirac matrices.

Imagine evaluating this graph using a basis of mass

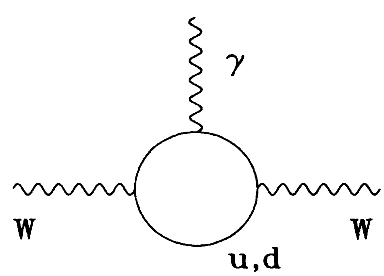

FIG. 1. Feynman graph responsible for generating P- and T-odd W electromagnetic moments at one loop.

eigenstates. In this basis, CP violation enters by way of the phases associated with the fermion–gauge-boson vertices. None of these phases arise from the fermion-photon vertex, however, since the invariance of the fermion mass matrix under electromagnetic gauge transformations precludes the generation of a Kobayashi-Maskawa-type matrix at the photon vertex. All CP-violating phases therefore appear in the W vertices. Taking the general form

$$\mathcal{L} = -iW_\mu \bar{u}_i \gamma^\mu (a_{ij} P_L + b_{ij} P_R) d_j + \text{c.c.} \ , \tag{5}$$

for the W-fermion interaction, in which P_L and P_R are the projection matrices onto left- and right-handed helicity, it is easy to show that the only phases which do not cancel between the two W-fermion vertices are those for which the left-handed coupling of one of the W's appears with the right-handed coupling of the other W. This is obviously only possible if the physical W boson has both left- and right-handed fermion couplings at the tree level. This kind of coupling arises naturally within left-right models in which the charged-gauge-boson mass eigenstates are linear combinations of the left- and right-handed weak-interaction eigenstates W_L and W_R.

These same arguments also explain the absence of the form factor $g_\gamma(0)$ from the results at one loop. A cursory inspection of Eq. (1) shows that the term involving $g_\gamma(q^2)$ contains a Levi-Civita tensor with two indices contracted into external momenta. The main point to be now made is that only terms in which precisely one momentum is contracted with the Levi-Civita tensor can arise from Fig. 1. This follows from the observation of the previous paragraph that all CP-violating phases cancel unless the left-handed coupling of one W vertex appears with the right-handed coupling of the other W vertex. But this combination of left- and right-handed projections can only survive the trace over the Dirac matrices if there are an odd number of γ matrices appearing between the two helicity projectors. It follows that the only Dirac trace capable of producing the Levi-Civita tensor involves a γ_5 together with exactly four (as opposed to six) other Dirac matrices. The important point is that three of these Dirac matrices come from the vertices of the graph, rather than from the propagators, and so must involve uncontracted Lorentz indices. This implies that at most one factor of external momenta can ever appear contracted into $\epsilon^{\mu\nu\lambda\rho}$ ruling out any contribution to $g_\gamma(q^2)$.

We now turn to the evaluation of the graph of Fig. 1 using the form of Eq. (5) for the fermion-W couplings. Within the left-right model the coefficients a_{ij} and b_{ij} appearing in this equation are given explicitly in terms of the model parameters by

$$a_{ij} = \frac{g_L \cos\xi}{\sqrt{2}} U_{ij} \ ,$$
$$b_{ij} = \frac{g_R e^{i\eta} \sin\xi}{\sqrt{2}} V_{ij} \ . \tag{6}$$

g_L and g_R are the left- and right-handed gauge couplings, U_{ij} and V_{ij} are the corresponding left- and right-handed Kobayashi-Maskawa matrices, the angle ξ parametrizes the mixing between W_L and W_R, and η is the complex phase associated with the vacuum expectation value (VEV) that is responsible for the W_L-W_R mixing.

Of the many sources of CP violation within these models, the ones potentially contributing to the W EDM are the phase η appearing through the gauge-boson mixing and the phases in the fermion Yukawa couplings. These latter phases show up in the Kobayashi-Maskawa (KM) matrices U_{ij} and V_{ij} that are generated once the left- and right-handed charged currents are expressed in terms of fermion mass eigenstates. In general, the left-handed mixing matrix U_{ij} need not be related to the right-handed matrix V_{ij}.

As usual, not all of the parameters of these Kobayashi-Maskawa matrices are physically significant since some may be absorbed by redefining the fields. For N families of quarks, all but $\frac{1}{2}(N-1)(N-2)$ of the phases in the left-handed KM matrix U_{ij} can be eliminated in this way. Once this has been done, however, no freedom remains to eliminate any of the $\frac{1}{2}N(N+1)$ phases in the right-handed matrix V_{ij}, and so these latter phases are all physically significant. The lepton Kobayashi-Maskawa matrix can potentially contain even more physical phases than appear for the quarks, depending on whether Majorana masses arise in the model in question.

Evaluating the graph of Fig. 1 gives the following results for the electromagnetic form factors of Eq. (1):

$$g_\gamma(0) = 0 \ , \tag{7}$$

and

$$f_\gamma(0) = -\frac{1}{8\pi^2} \sum_{ud} N_c \operatorname{Im}(a^*_{ud} b_{ud}) \left[\frac{m_d m_u}{M_W^2} \right] \times [q_d F(x_d, x_u) + q_u F(x_u, x_d)] \ . \tag{8}$$

In these equations u and d label the fermions (which can be leptons) circulating around the loop according to whether they are in the upper or lower component of a weak isodoublet. a_{ud} and b_{ud} denote the gauge couplings given in Eqs. (5) and (6). m_d, m_u, q_d, and q_u similarly represent the masses and electric charges of these fermions. x_i represents the mass ratio m_i^2/M_W^2, and N_c is a color factor equal to 1 if the internal fermions are leptons and equal to 3 for internal quarks.

The function $F(x,y)$ appearing in these formulas is defined by

$$F(x,y) = \frac{1+y-x}{S(x,y)} \ln \left[\frac{1-(x+y)-S(x,y)}{1-(x+y)+S(x,y)} \right] + \ln \left[\frac{x}{y} \right] \ ,$$
$$S(x,y) = [1-2(x+y)+(x-y)^2]^{1/2} \ . \tag{9}$$

We are interested in the behavior of $F(x,y)$ in the following three limits: (a) $x,y \ll 1$ (light generations), (b) $x \ll 1 \approx y$ and $y \ll 1 \approx x$ [a light and a heavy (e.g., t) quark], and (c) $x \approx y \gg 1$ (heavy generation). In these limits, $F(x,y)$ is well approximated by the asymptotic forms

$$F(x,y) \approx 2\ln(x) \quad \text{for } x,y \ll 1 \ , \tag{10a}$$

$$F(x,y)\approx\frac{1}{x} \quad \text{for } x\approx y \gg 1 , \tag{10b}$$

$$F(x,y)\approx\ln\left[\frac{x^2}{(x-1)^2}\right] \quad \text{for } y\ll 1,\ (x-1)^2 , \tag{10c}$$

$$F(x,y)\approx\left[\frac{y+1}{y-1}\ln\left[\frac{(y-1)^2}{xy}\right]+\ln\left[\frac{x}{y}\right]\right] \quad \text{for } x\ll 1,(y-1)^2 . \tag{10d}$$

From these expressions it is clear that the contribution of a light pair of fermions to the W EDM is suppressed by the factor $(m_u m_d/M_W^2)<10^{-4}$. For a loop involving one light (b, say) and one heavy (t) quark, the result varies as m_b/m_t for large m_t. If both quarks are heavy, the asymptotic behavior becomes $(M_W^2/m_U m_D)$. One should also bear in mind that although we take $V_{ij}\sim U_{ij}$ in making our estimates, in general the off-diagonal elements of the right-handed mixing matrix V_{ij} need not be small.

It is conventional to normalize the W EDM by the W mass according to $d_W=e\lambda_W/2M_W$, so that $d_W=\lambda_W(1.2\times10^{-16})$ e cm. With this convention, choosing $\xi\approx10^{-3}$, $|U_{ij}|\approx|V_{ij}|$, $g_L\approx g_R$, and assuming a nonzero phase for the t and b quark gives an estimate of the size of a typical contribution:

$$\lambda_W=f_\gamma(0)=+1.9\times10^{-6}\left[\frac{\mathrm{Im}(a_{tb}^*b_{tb})}{2.2\times10^{-4}}\right], \quad m_t=100\ \text{GeV} ,$$
$$\lambda_W=f_\gamma(0)=+7.1\times10^{-7}\left[\frac{\mathrm{Im}(a_{tb}^*b_{tb})}{2.2\times10^{-4}}\right], \quad m_t=200\ \text{GeV} . \tag{11}$$

The result varies smoothly for m_t between 100 and 200 GeV. The contribution of a heavy-fermion pair from a potential fourth generation contributes about 10 times this effect due to the loss of the suppression factor m_b/m_t.

III. INDUCED FERMION EDM'S

The best chance for observing a W EDM of this size is through its implications for fermion EDM's. We now compute the size of this induced EDM. It is convenient to treat the calculation differently, depending on whether or not the fermions traversing the loop in Fig. 1 are both heavy relative to the W scale.

A. Heavy fermions

If the fermion responsible for the W EDM is very heavy on the scale of the W mass, then it is useful to compute this bound within the framework of a low-energy effective theory involving just the light particles, obtained by integrating out the heavy particles. Phrasing the heavy-fermion loop in terms of such an effective Lagrangian pays two immediate dividends. First, the size of the fermion EDM obtained can be directly estimated within the effective theory without reference to the details of how the effective operator was produced. The connection between the W and fermion EDM's thus established may therefore be expected to hold in a much wider context than for just the present model.

The second bonus of working within the effective-Lagrangian approach is that we may take advantage of $SU_L(2)\times U_Y(1)$ invariance to relate the $WW\gamma$ and WWZ couplings in a model-independent way, thereby extending our results to a prediction for the WWZ vertex which will be extensively probed at CERN LEP 200. We discuss each of these points in turn.

The way that the heavy-fermion loop shows up in this effective theory is via nonrenormalizable P- and T-odd effective $WW\gamma$ interactions. The dominant contributions to light-fermion EDM's come from those operators having the lowest dimensions in powers of mass. The lowest-dimension contribution consists of operators with dimension 6. These operators must involve only the standard model fields (in models in which there are several light Higgs particles, more operators involving these other scalars can also appear) and must be $SU_L(2)\times U_Y(1)$ invariant. All operators of this type which can contribute to the $WW\gamma$ vertex may be written as a linear combination of the following two:

$$A\frac{g_2^3}{3!}\epsilon_{abc}\tilde{W}^{a\mu}{}_{\nu}W^{b\nu}{}_{\lambda}W^{c\lambda}{}_{\mu}+Bg_2g_1(\phi^\dagger\tau_a\phi)W^a_{\mu\nu}\tilde{B}^{\mu\nu} . \tag{12}$$

τ_a denote, as usual, the Pauli matrices, and ϵ_{abc} represents the completely antisymmetric symbol in the $SU_L(2)$ gauge indices a, b, c. g_i, $i=1,2$, are the gauge coupling constants for $U_Y(1)$ and $SU_L(2)$, respectively, while $W_{\mu\nu}$ and $B_{\mu\nu}$ are the corresponding field strengths with duals: $\tilde{W}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}W^{\lambda\rho}$. ϕ represents the usual Higgs doublet whose VEV is $(1/\sqrt{2})\binom{0}{v}$.

The constants A and B have dimensions of inverse mass squared and are given in terms of the fermion quantum numbers by evaluating the graph of Fig. 1. The result may be read off by comparing the tree-level contribution of these operators to the form factors $f_\gamma(0)$ and $g_\gamma(0)$,

$$f_\gamma(0)=g_2^2(\tfrac{1}{2}Bv^2-AM_W^2) , \tag{13}$$

$$g_\gamma(0)=-\tfrac{1}{2}g_2^2M_W^2A , \tag{14}$$

with the result of Eqs. (8) and (7). Clearly, the vanishing of $g_\gamma(0)$ implies through this comparison that $A=0$.

The estimate for the fermion EDM induced by this operator is now obtained by evaluating Fig. 2 using the

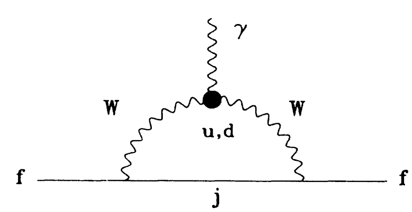

FIG. 2. Feynman graph that induces a fermion EDM from a W EDM. The solid circle represents the vertex with Feynman rule given by Eq. (1).

operators of Eq. (12). The loop integral diverges and must be cut off at a scale Λ, corresponding to the maximum momentum above which the effective operator description does not apply. In the present example, Λ is the smallest of the right-handed W mass or the masses m_U and m_D of the fermions in the loop. The result is[7]

$$\frac{d_f}{e}=\pm\frac{G_F m_f}{8\pi^2\sqrt{2}}f_\gamma(0)\left[\ln\left[\frac{\Lambda^2}{M_W^2}\right]+O(1)\right] . \quad (15)$$

The sign here is positive for the upper component of a weak doublet and negative for the lower component. The $O(1)$ term is finite as Λ tends to infinity. This equation may be used to place the numerical bounds discussed above, where we approximate the contents of the square brackets by one. The accuracy of this expression can be estimated by comparison with the more detailed calculation presented below. It is noteworthy that this expression is proportional to the mass of the fermion f, whose EDM is being computed, rather than to the mass of the internal fermion. Although this can be a bad thing when computing a light-quark EDM, it is potentially an advantage when considering the electron EDM since in this case it need not be suppressed if the neutrino mass-matrix elements are small.

The implications for the general WWZ couplings are immediate. The most general P- and CP-odd form factors that can arise in the WWZ vertex have the form[9]

$$\langle W^-|J_Z^\lambda|W^-\rangle=\frac{-ie}{\sin\theta_W\cos\theta_W}\varepsilon_\mu^*(p_1)\Gamma_Z^{\mu\nu\lambda}(p_1,p_2)\varepsilon_\nu(p_2) , \quad (16)$$

with the CP- and P-odd part of $\Gamma_Z^{\mu\nu\lambda}(p_1,p_2)$ given as in Eq. (1).

Now comes the main point. The only dimension-6 operators within the effective Lagrangian that can contribute at the tree level to the form factors $f_Z(0)$ and $g_Z(0)$ are precisely those of Eq. (12) that contribute to the W EDM and MQM. It follows that the one-loop prediction for the WWZ CP- and P-odd form-factors are

$$f_Z(0)=g_2^2(-\tfrac{1}{2}Bv^2\sin^2\theta_W-AM_W^2\cos^2\theta_W) , \quad (17)$$

$$g_Z(0)=-\tfrac{1}{2}g_2^2M_W^2 A\cos^2\theta_W . \quad (18)$$

The one-loop prediction for these form factors is then found by taking the result obtained above, $A=0$, and B as given by Eqs. (13) and (8). The result $A=0$ implies the particularly simple relations

$$f_Z(0)=-f_\gamma(0)\sin^2\theta_W ,$$
$$g_Z(0)=0 . \quad (19)$$

Unfortunately,[9] such small WWZ couplings would be unobservable at LEP 200.

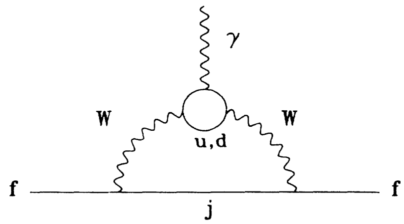

FIG. 3. Feynman graph responsible for the induced fermion EDM in the full theory.

B. Light fermions

In the event that the fermions whose CP-violating couplings are responsible for the W EDM are light, such as the ordinary quarks and leptons, this effective-field-theory analysis is not as useful. In this case we are in principle obliged to perform the full two-loop calculation of Fig. 3 in order to infer the size of the induced fermion EDM's. The calculation applies equally well, of course, to the case of heavy fermions and so furnishes an explicit physical example of how the heavy physics gets cut off in a specific model.

Evaluating the graph in Fig. 3 gives the contribution of the W EDM to the fermion EDM, defined as the coefficient d_f of the operator

$$\mathcal{L}=-i\frac{d_f}{2}\bar{f}\gamma_5\sigma_{\mu\nu}fF^{\mu\nu} , \quad (20)$$

in the theory at very low energies compared to the W mass. If the fermion f should be a quark, this low-energy scale cannot be taken to be lower than several GeV. The difficult part of the problem is then the evaluation of the matrix element of this operator within a nucleon state.

The result of this two-loop computation of d_f is

$$\frac{d_f}{e}=\mp\frac{2}{(4\pi)^4}\sum_{jud}N_c\mathrm{Im}(a_{ud}^*b_{ud})\left[\frac{m_u m_d}{M_W^4}\right](q_d\{(|a_{fj}|^2+|b_{fj}|^2)m_f[G(x_d,x_u,x_j)+x_jH(x_d,x_u,x_j)]$$
$$+\mathrm{Re}(a_{fj}^*b_{fj})m_j[G(x_d,x_u,x_j)+K(x_d,x_u,x_j)]\}+q_u[(x_d\leftrightarrow x_u)]) ,$$
$$\approx\mp\frac{2}{(4\pi)^4}\sum_{jud}N_c\mathrm{Im}(a_{ud}^*b_{ud})\left[\frac{m_u m_d m_f}{M_W^4}\right]|a_{fj}|^2[q_dG(x_d,x_u,x_j)+(u\leftrightarrow d)] . \quad (21)$$

Here, as before, $x_i=m_i^2/M_W^2$ and the W-fermion couplings a_{ij} and b_{ij} are as defined in Eqs. (5) and (6). The sign is $-$ $(+)$ according to whether f is the upper (lower) component of a weak isodoublet. The approximation used in the second line of Eq. (21) involves the neglect of terms proportional to either x_j or $\sin^2\xi$. Neglect of x_j is justified by the small size of the flavor-changing left-handed Kobayashi-Maskawa matrix elements that suppress the contribution of heavy quarks q_j in the light-quark EDM's.

The functions $G(a,b,c)$, $H(a,b,c)$, and $K(a,b,c)$ are known integrals expressed in terms of the function $z(x,a,b)=b/x+a/(1-x)$ by

$$G(a,b,c)=\frac{1}{(1-c)}\int_0^1\frac{dx}{1-x}\left[\frac{c}{z-c}\ln\left[\frac{c}{z}\right]+\frac{1}{1-z}\ln\left[\frac{1}{z}\right]\right], \tag{22}$$

$$H(a,b,c)=\frac{1}{(1-c)^2}\int_0^1\frac{dx}{1-x}\left[\frac{(c+1)z-2}{(1-c)(1-z)^2}\ln\left[\frac{1}{z}\right]-\frac{(c+1)z-2c}{(1-c)(z-c)^2}\ln\left[\frac{c}{z}\right]-\frac{1}{z-c}+\frac{1}{1-z}\right], \tag{23}$$

$$K(a,b,c)=\int_0^1\frac{dx}{1-x}\left[\frac{z}{(1-z)^2(c-z)}\ln(z)+\frac{c}{(1-c)^2(z-c)}\ln(c)+\frac{1}{(1-z)(c-1)}\right]. \tag{24}$$

Since the functions $H(a,b,c)$ and $K(a,b,c)$ do not contribute in the limit of small x_j and $\sin^2\xi$, we do not present analytic expressions for their limiting forms here. Some interesting limiting cases for the function $G(a,b,c)$ are

$$G(x_d,x_u,x_j)\approx\tfrac{1}{2}\ln^2(x_u) \quad \text{for } x_j\ll x_u\approx x_d\ll 1, \tag{25a}$$

$$G(x_d,x_u,x_j)\approx\frac{1}{2x_u}\ln(x_u) \quad \text{for } x_j\ll 1\ll x_u\approx x_d, \tag{25b}$$

$$G(x_d,x_u,x_j)\approx\frac{1}{x_d}\ln(x_d) \quad \text{for } x_j\ll x_u\ll 1\ll x_d, \tag{25c}$$

$$G(x_d,x_u,x_j)\approx\frac{1}{x_u}\ln(x_u)\ln\left[\frac{x_u}{x_d}\right] \quad \text{for } x_j\ll x_d\ll 1\ll x_u. \tag{25d}$$

Unlike the effective-Lagrangian result Eq. (15), the full expression, given by Eq. (21), applies equally well for both heavy and light fermions u and d. For heavy fermions, however, Eqs. (15) and (21) should both be accurate and must agree with one another. In this regime, Eq. (21) illustrates how underlying physics furnishes a physical cutoff for the effective theory. As is easily verified using the asymptotic forms presented in Eqs. (10b) and (25b), Eq. (21) agrees in the heavy-fermion limit with Eq. (15), provided that the cutoff Λ is evaluated at $\Lambda=m_u\approx m_d$.

It is noteworthy that the effective-Lagrangian expression, evaluated using $\Lambda=m_t$, also agrees unusually well with the full result for the case where the loop fermions are b and t quarks. This is so even though the b quark is quite light compared to the W mass. In fact, as is illustrated in Fig. 4, the numerical agreement between the effective-Lagrangian and two-loop expressions [Eqs. (15) and (21)] is even better for a t-b loop than it is for a loop for which both fermions are heavy, thus being an excellent approximation already for $m_t\approx 100$ GeV. This is, at first sight, surprising since the effective-Lagrangian approximation made in deriving Eq. (15) need not strictly apply when one of the fermions is light. The reason for this remarkable accuracy lies in the infrared mass singularities that arise as $m_b\to 0$. For large m_t/M_W the asymptotic form for both the effective-Lagrangian result [Eq. (15)] and the two-loop calculation [Eq. (21)] is

$$\frac{d_f}{e}=\pm\frac{g_L^2}{(4\pi)^4}\mathrm{Im}(a_{tb}^*b_{tb})\frac{m_b}{m_t}\frac{m_f}{M_W^2}\ln\left[\frac{m_t^2}{M_W^2}\right]\times\left[\ln\left[\frac{m_t^2}{m_b^2}\right]-2\right]. \tag{26}$$

This approaches its asymptotic form more quickly than with two heavy quarks because of the appearance of the extra large logarithm.

A graph of typical induced fermion EDM's, computed using the full two-loop result of Eq. (21), as functions of the masses of the internal particles is presented in Fig. 5.

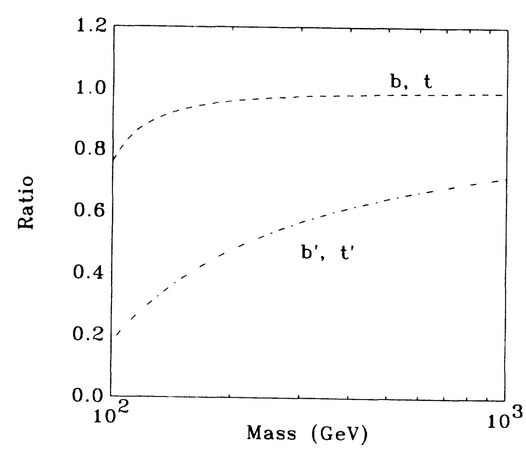

FIG. 4. Ratio of the down-quark EDM induced by the W EDM as calculated using the effective-Lagrangian approximation divided by that found from the full two-loop calculation. The dot-dashed line shows this ratio for an EDM induced by a loop of degenerate heavy quarks as a function of their mass. The dashed curve gives the same result for a b-t quark loop as a function of m_t.

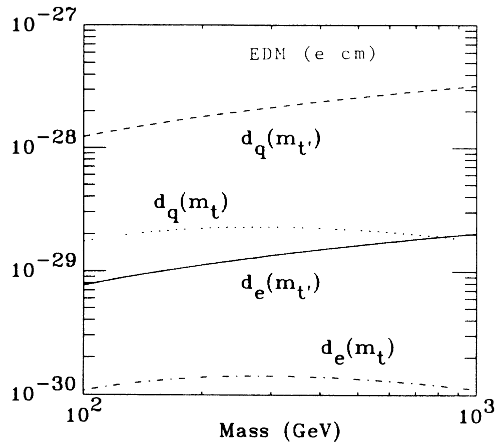

FIG. 5. Induced fermion EDM as functions of internal fermion masses. The solid curve gives an electron EDM induced by a heavy-quark loop b',t' as a function of $m_{b'}=m_{t'}$. The dashed curve indicates the same result for light-quark EDM's. The mass dependence for a heavy generation of degenerate leptons is identical. The dotted curve gives the light-quark EDM induced by the ordinary b and t quarks as a function of the t-quark mass and the dot-dashed curve is the same result for the electron EDM. All other numerical values are as in the text.

IV. SUMMARY

Our main results concern the perturbative determination of the *CP*- and *P*-odd electromagnetic moments of the *W* boson. We have argued that the only models that can produce these moments at the one-loop level are those (i) in which *CP* violation can be expressed through *CP*-violating phases in the Kobayashi-Maskawa matrix for the couplings of fermions to the *W*, and (ii) that predict right-handed couplings for the *W* mass eigenstate at the tree level. Perhaps the most natural such theories are left-right-symmetric models.

We compute, in these theories, the *W* electric dipole and magnetic quadrupole moments and find that they are always related at one loop by the simple relation $2d_W+\tilde{Q}_W M_W=0$.

If the fermions responsible for the *CP*-violating Kobayashi-Maskawa phases are very heavy compared to the *W*, then we show that $SU_L(2)\times U_Y(1)$ invariance relates the *CP*- and *P*-odd *WWZ* form factors to the *CP*- and *P*-odd electromagnetic ones. The relationship is $f_Z(0)=-f_\gamma(0)\sin^2\theta_W$, and $g_Z(0)=g_\gamma(0)=0$.

Assuming, for definiteness, that the *W* EDM is generated by a loop of t and b quarks whose *CP*-violating left- and right-handed couplings to the *W* are $\mathrm{Im}(a_{tb}^* b_{tb})\approx 2\times10^{-4}$, we find a predicted one-loop *W* EDM of around 10^{-22} e cm. The numbers used are motivated by a left-right model in which the left- and right-handed Kobayashi-Maskawa matrices have the same absolute size (but not equal phases) and the left-right mixing angle is $\xi\approx10^{-3}$. This would imply unobservable electromagnetic *CP* violation from this source in accelerator experiments. If the relation between *Z* and photon form factors holds, then the *CP*-odd *WWZ* couplings are orders of magnitude too small to be seen at LEP 200. We compute that a *W* EDM of this size induces an electron dipole moment of order $10^{-30}e$ cm and a neutron EDM of $10^{29}-10^{-27}e$ cm, depending on matrix-element uncertainties. Since the electron EDM induced by a *W* EDM is proportional to m_e (rather than m_ν as would be the one-loop result in left-right models, say), it can be the dominant contribution.

These estimates can be made larger in several ways. First, within left-right-symmetric models, less conservative corners of parameter space imply induced EDM's that can be much larger. Alternatively, if the fermions circulating the loop are members of a heavy generation, then their contribution, for equal couplings, is larger than the t-b loop since it is not suppressed by the small mass ratio m_b/m_t. It may also be larger because for a heavy generation right-handed currents may be large and so $\mathrm{Im}(a^*b)$ need not be small. In either case the induced fermion EDM's would be large enough to be observable in current searches.

Although EDM's this small are likely to be undetectable for the near future, the EDM's induced by a hypothetical fourth heavy generation of fermions are typically 10 times larger than the estimates just quoted. Induced EDM's from this source might therefore be observable in the not too distant future.

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