

## Perturbative QCD analysis of pion and kaon form factors and pair production in photon-photon collisions using a frozen coupling constant

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Within the framework of leading-order perturbative QCD and using a frozen coupling constant, we calculate the pion and kaon form factors and the cross section of pion and kaon pair production in two-photon collisions. We use the same frozen coupling constant as taken in the nucleon Dirac-form-factor analysis and find that the results for the  $\pi$  and  $K$  form factors, the reactions  $\gamma\gamma \rightarrow \pi^+\pi^-$ ,  $K^+K^-$ , and the proton Dirac form factor are in fair agreement with the available experimental data. The cutoff value of the frozen coupling constant used in our analysis is consistent with the theoretical estimation presented by Cornwall.

### I. INTRODUCTION

For form-factor calculation in strong interactions, it has been shown<sup>1</sup> that the argument of the running coupling constant should be taken as the square of the momentum transfer of the exchanged gluon in order to make the perturbation theory meaningful. This was argued from the convergence of the perturbation series and can be justified in any exclusive process which does not involve triple or quartic vertices in the lowest order. In a recent leading-order perturbative QCD analysis of the nucleon Dirac form factor, we have shown<sup>2</sup> that it is possible to fit the data in the range of momentum transfer squared,  $10 < Q^2 < 30 \text{ GeV}^2$ , by evaluating the argument of QCD running coupling constant  $\alpha_s(Q^2)$  at the exact gluon momentum transfer for each of the diagrams contributing to the leading-order process. In this paper, we extend the same considerations to the pion and kaon form factors and their pair production in two-photon collisions,  $\gamma\gamma \rightarrow \pi^+\pi^-$ ,  $\gamma\gamma \rightarrow K^+K^-$ , and compare our results with experimental data. Our aim is to apply the same method and QCD running coupling constant adopted in the previous nucleon-form-factor calculation to these pion- and kaon-induced processes and to investigate whether the same method and cutoff value can give a consistent agreement with the available experimental data.

To illustrate our method and cutoff value, the simple example of the pion-form-factor analysis is summarized as follows. The factorized<sup>3</sup> QCD expression for the pion form factor (see Fig. 1) is given by

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \phi^*(y, \tilde{Q}_y) T_H(x, y, Q^2) \times \phi(x, \tilde{Q}_x), \quad (1)$$

where  $\tilde{Q}_x = \text{Min}(x, 1-x)Q$ ,  $\phi(x, \tilde{Q}_x)$  is the quark distribution amplitude of the pion, and the hard-scattering amplitude  $T_H$ , to the leading order in  $\alpha_s$ , is given by<sup>4</sup>

$$T_H(x, y, Q^2) = \frac{64\pi}{3Q^2} \left[ \frac{2}{3} \frac{\alpha_s[(1-x)(1-y)Q^2]}{(1-x)(1-y)} + \frac{1}{3} \frac{\alpha_s(xyQ^2)}{xy} \right]. \quad (2)$$

While the next-to-leading-order  $T_H$  has been calculated in Ref. 4, we are interested in keeping only the leading-order expression given by Eq. (2) to satisfy our aim mentioned above. In Eq. (2), the argument of  $\alpha_s$  is the momentum transfer of the exchanged gluon as shown in diagrams of Fig. 1. While the leading-order hard-scattering amplitude in Eq. (2) exhibits divergence at both end points of  $x$  and  $y$ , the bound-state quark distribution amplitude suppresses the end-point singularities.

In this case, however, an immediate problem arises if the calculation of Eq. (1) is attempted with the usual one-loop formula for the running coupling constant

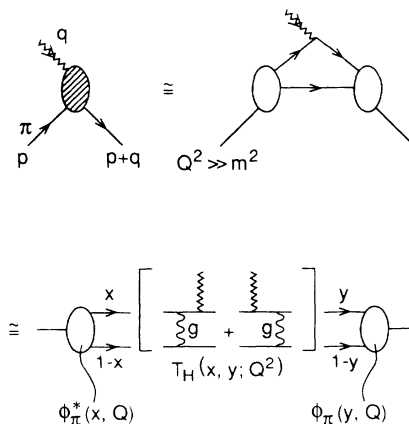


FIG. 1. Valence Fock-state contribution to the large-momentum-transfer meson form factor.  $T_H$  computed for zero-mass quarks  $q$  and  $\bar{q}$  parallel to the pion momentum.

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \ln(Q^2/\Lambda^2)} \quad (3)$$

( $\beta = 11 - \frac{2}{3}n_f$  and  $n_f$  is the number of flavors), since the integration in Eq. (1) allows  $\alpha_s$  to be evaluated near zero-momentum transfer. The same problem arises in the proton Dirac-form-factor analysis.<sup>2</sup> In Ref. 2, this problem was avoided by introducing<sup>5</sup> a cutoff in the formula for  $\alpha_s(Q^2)$  to prevent the coupling constant from becoming infinite for vanishing gluon momenta. In particular, in Ref. 2 the following modified relation for  $\alpha_s$ , as proposed in Ref. 6, was utilized:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \ln[(Q^2 + 4m_g^2)/\Lambda^2]}, \quad (4)$$

where  $m_g$  is interpreted as an ‘‘effective dynamical gluon mass’’ with a value of typically about 0.5 GeV and  $\Lambda$  is of order 100 MeV. For  $Q^2 \gg m_g^2$ , it coincides with the one-loop formula Eq. (3), but a very-low-momentum transfer, this formula ‘‘freezes’’ the coupling constant to some finite but not necessarily small value.

The physical meaning of frozen coupling constant<sup>5,6</sup> may be found in the confinement mechanism suggested by (1+1)-dimensional QED.<sup>7</sup> If one tries to elongate a positronium ( $e^+e^-$ ), it is energetically more favorable for the vacuum to create fermion and antifermion pairs so that the effective coupling between the original two charges  $e^+$  and  $e^-$  is frozen because of screening by vacuum condensates. In fact, the color confinement does not necessarily mean the divergence of  $\alpha_s(Q^2)$  at small momentum transfer. The idea of frozen coupling constant may be more natural to understand the color confinement problem. As an evidence of vacuum condensates in QCD, quark and gluon condensation order parameters are obtained by a QCD sum rule from PCAC (partial conservation of axial-vector current) and instanton solutions.<sup>8</sup>

$$\begin{aligned} \langle \text{vac} | : \bar{u}u : | \text{vac} \rangle &= \langle \text{vac} | : \bar{d}d : | \text{vac} \rangle \\ &\simeq -250 \text{ MeV}^3, \end{aligned} \quad (5a)$$

and

$$\langle \text{vac} | : \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} : | \text{vac} \rangle \simeq 0.012 \text{ GeV}^4. \quad (5b)$$

Using a special set of Schwinger-Dyson equations, the formation of dimensionful parameters, for example, given by Eq. (5), has been studied.<sup>6</sup> The numerical solution of the Schwinger-Dyson equation was consistent with the idea of frozen coupling constant given by Eq. (4) with  $m_g = 500 \pm 200$  MeV. In this way,  $m_g$  is related to  $\Lambda$ , which is order of 100 MeV, and the whole analysis still has only one QCD parameter.

Therefore, it is concluded that the QCD vacuum condensate affects not only the quark distribution amplitude,

but also the QCD running coupling constant at the small-momentum-transfer region.<sup>9</sup> Furthermore, since the value of  $m_g$  given by Ref. 6 freezes  $\alpha_s(Q^2)$  [Eq. (4)] to a value less than 1 even at  $Q^2=0$ , the perturbation series may be expanded in terms of the frozen coupling constant.

In this paper, we present the leading-order perturbative analysis of the pion and kaon form factors and the cross section of  $\gamma\gamma \rightarrow \pi^+\pi^-$  and  $\gamma\gamma \rightarrow K^+K^-$  using a frozen coupling constant given by Eq. (4). We follow the same method of calculations employed in Ref. 2 and use the same numerical values for  $m_g$  and  $\Lambda$  as introduced there. In Sec. II, we present the extended cross-section formula for  $\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-$ , considering the argument of the QCD running coupling constant as the gluon momentum transfer in the leading-order diagrams. In Sec. III, we present the quark distribution amplitudes for  $\pi$  and  $K$ , including its QCD evolution, which are used in this analysis. Numerical results and comparisons with experimental data are presented in Sec. IV, and conclusions are followed in Sec. V.

## II. PSEUDOSCALAR-MESON PAIR PRODUCTION IN PHOTON-PHOTON COLLISIONS

The perturbative QCD predictions for this process<sup>10</sup> including the next-to-leading-order calculation<sup>11</sup> are already presented. However, our purpose is to apply the same method and the frozen coupling constant used in our previous nucleon-form-factor calculations to  $\pi$ - and  $K$ -induced processes, including  $\gamma\gamma \rightarrow \pi^+\pi^-$  and  $\gamma\gamma \rightarrow K^+K^-$ . Thus, in this section, we summarize the extended leading-order formula to calculate the cross section of  $\gamma\gamma \rightarrow M^+M^-$  ( $M = \pi, K$ ) including the argument of the running coupling constant  $\alpha_s$  taken as the square of the momentum transfer of the exchanged gluon.

The spin-averaged cross section is given by

$$\frac{d\sigma}{dz} = \frac{s}{2} \left[ \frac{d\sigma}{dt} \right] = \frac{1}{32\pi s} \frac{1}{4} \sum_{\lambda\lambda'} \left| \mathcal{M}^{\lambda\lambda'} \right|^2, \quad (6)$$

where  $z$  is the cosine of the meson pair production angle in the  $\gamma\gamma$  center-of-mass frame (i.e.,  $z = \cos\theta_{\text{c.m.}}$ ) and  $s$  is the square of the c.m. energy  $W$  (i.e.,  $s = W^2$ ) of the  $\gamma\gamma$  system. The invariant amplitude  $\mathcal{M}^{\lambda\lambda'}$  for the initial helicities  $\lambda$  and  $\lambda'$  of two photons is given by

$$\mathcal{M}^{\lambda\lambda'} = \int_0^1 dx \int_0^1 dy \phi_M^*(x, \tilde{Q}_x) \phi_M^*(y, \tilde{Q}_y) T_H^{\lambda\lambda'}(x, y, Q^2), \quad (7)$$

where  $\tilde{Q}_x = \text{Min}(x, 1-x)\sqrt{s} |\sin\theta_{\text{c.m.}}|$ , similarly for  $\tilde{Q}_y$ , and  $\phi_M$  is the quark distribution amplitude of the meson (see Sec. III for details). The leading-order hard-scattering amplitude  $T_H^{\lambda\lambda'}$  including the argument of  $\alpha_s$  is given by

$$\begin{aligned}
\left. \begin{array}{l} T_H^{++} \\ T_H^{--} \end{array} \right\} &= \frac{16\pi}{3s} \frac{32\pi\alpha}{x(1-x)y(1-y)} \\
&\times \frac{a \left\{ e_1^2 \alpha_s ((1-x)ys) - e_1 e_2 \left[ \alpha_s \left[ \frac{-a+bz}{2} s \right] + \alpha_s \left[ \frac{-a-bz}{2} s \right] \right] + e_2^2 \alpha_s (x(1-y)s) \right\}}{1-z^2} \\
\left. \begin{array}{l} T_H^{+-} \\ T_H^{-+} \end{array} \right\} &= \frac{16\pi}{3s} \frac{32\pi\alpha}{x(1-x)y(1-y)} \\
&\times \left[ \frac{(1-a) \left\{ e_1^2 \alpha_s ((1-x)ys) - e_1 e_2 \left[ \alpha_s \left[ \frac{-a+bz}{2} s \right] + \alpha_s \left[ \frac{-a-bz}{2} s \right] \right] + e_2^2 \alpha_s (x(1-y)s) \right\}}{1-z^2} \right. \\
&+ \frac{e_1 e_2 \{x(1-x) + y(1-y)\}}{2} \left[ \frac{\alpha_s \left[ \frac{-a+bz}{2} s \right]}{a-bz} + \frac{\alpha_s \left[ \frac{-a-bz}{2} s \right]}{a+bz} \right] \\
&\left. + \frac{[e_1^2 \alpha_s ((1-x)ys) - e_2^2 \alpha_s (x(1-y)s)](x-y)}{2} \right], \tag{8}
\end{aligned}$$

where

$$\left. \begin{array}{l} a \\ b \end{array} \right\} = (1-x)(1-y) \pm xy,$$

and  $e_1, e_2$  are the quark charges [i.e., the mesons have charges  $\pm(e_1 - e_2)$ ]. One can easily show that if one neglects the difference in the argument of  $\alpha_s$ , Eq. (8) is reduced to the previous results obtained by Brodsky and Lepage<sup>10</sup> and Nizic.<sup>11</sup>

### III. QUARK DISTRIBUTION AMPLITUDES AND THEIR EVOLUTIONS

Useful constraints on the lowest moments of the distribution amplitude  $\phi(x, Q)$  can be obtained using the QCD sum-rule approach.<sup>12</sup> Although the numerical accuracy of this method is not known, the general agreement between its predictions and overall consistency with other hadron phenomenology<sup>13</sup> lends credence to its validity.

The distribution amplitudes of the pion and kaon at  $Q = \mu = 500$  MeV are given by<sup>12</sup>

$$\begin{aligned}
\phi_\pi(x, \mu) &= \frac{30f_\pi}{2\sqrt{3}} x(1-x)(2x-1)^2, \\
\phi_K(x, \mu) &= \frac{30f_K}{2\sqrt{3}} x(1-x) \\
&\times [0.6(2x-1)^2 + 0.25(2x-1)^3 + 0.08], \tag{9}
\end{aligned}$$

where  $\mu = 500$  MeV,  $f_\pi = 93$  MeV, and  $f_K = 112$  MeV. The normalization of  $\phi_M(x, \mu)$  ( $M = \pi, K$ ) is given by the condition

$$\int_0^1 dx \phi(x, \mu) = \frac{f_M}{2\sqrt{3}}. \tag{10}$$

Once the distribution amplitude is given at a certain value of  $Q$  ( $Q = \mu = 500$  MeV, for example), then  $\phi(x, Q)$  at other values of  $Q$  can be obtained by solving a Bethe-Salpeter-type evolution equation.<sup>3</sup> The result for valence-quark distribution amplitude of the meson is expanded in terms of Gegenbauer polynomials  $C_n^{3/2}(2x-1)$  and is given by

$$\phi_M(x, Q) = x(1-x) \sum_{n=0}^{\infty} a_n^{(M)} C_n^{3/2}(2x-1) \left[ \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{\gamma_n}, \tag{11}$$

where

$$a_n^{(M)} = \left[ \frac{f_M}{2\sqrt{3}} \right] \frac{15(2n+3)}{(2+n)(1+n)} I_n^{(M)}, \tag{12}$$

and

$$\gamma_n = \frac{\frac{4}{3}}{11 - \frac{2}{3}n_f} \left[ 1 + 4 \sum_2^{n+1} \frac{1}{k} - \frac{2}{(n+1)(n+2)} \right].$$

We found that  $I_n^{(\pi)} = 0$  for  $n \geq 3$  and  $I_n^{(K)} = 0$  for  $n \geq 4$ , and  $I_n^{(\pi)}$  and  $I_n^{(K)}$  for other values of  $n$  are given by

$$\begin{aligned}
I_0^{(\pi)} &= \frac{4}{15}, \quad I_1^{(\pi)} = 0, \quad I_2^{(\pi)} = \frac{16}{35}, \\
I_0^{(K)} &= \frac{4}{15}, \quad I_1^{(K)} = \frac{3}{35}, \quad I_2^{(K)} = \frac{48}{175}, \quad I_3^{(K)} = \frac{4}{63}.
\end{aligned}$$

At the boundary of  $Q = \mu$ , Eq. (11) reduces to Eq. (9), and in the limit  $Q \rightarrow \infty$ , Eq. (11) reduces to the asymptotic form in Ref. 3, as expected.<sup>14-16</sup>

### IV. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

Using the quark distribution amplitude of Eq. (11) and the frozen coupling constant of Eq. (4), we evaluated the

integrals given by Eq. (1) to calculate the pion (and similarly kaon) form factor. As we did in Ref. 2 for the nucleon Dirac-form-factor analysis, we included only the leading-order hard-scattering amplitude<sup>4</sup> as shown in Eq. (2). We emphasize again that our aim is to make predictions for meson form factors and meson pair production in  $\gamma\gamma$  annihilations using the same type of quark distribution amplitude determined by the QCD sum rule and the same  $m_g$  and  $\Lambda$  values in Eq. (4), and compare to experiment. In Ref. 2, it was shown that it is possible to fit the data for proton Dirac form factor  $F_1^p$  in the range of  $10 < Q^2 < 30 \text{ GeV}^2$  when one uses the distribution amplitudes proposed by the QCD sum-rule calculations<sup>12,14</sup> and a frozen coupling constant [Eq. (4)] with  $m_g^2$  between 0.1 and 0.5  $\text{GeV}^2$ .

The result for the pion form factor is shown in Fig. 2. Since the pion form factor  $F_\pi(Q^2)$  is multiplied by  $Q^2$  in Fig. 2(a), the numerical results seem to be sensitive to different values of  $m_g$  even at high  $Q^2$ . However, the pion form factor  $F_\pi(Q^2)$  itself is much less sensitive to variation of  $m_g$  [see Fig. 2(b)]. To investigate the sensitivity of the result by choosing another model wave function, we also calculate the pion form factor using the quark distribution amplitude obtained by Dziembowski and Mankiewicz:<sup>17</sup>

$$\phi_\pi(x) = N \exp\left[-\frac{m^2}{8x(1-x)\beta^2}\right] \times \left[\frac{(xM+m)(1-x)M+m}{4\beta^2} - 2x(1-x)\right].$$

Here  $N$  is determined by the normalization condition given by Eq. (10),  $M$  is the spin-averaged meson mass ( $M = 612.4 \text{ MeV}$  for the pion),  $m$  is the constituent quark mass ( $m = 330 \text{ MeV}$  for  $u$  and  $d$  quarks), and the Gaussian parameter  $\beta$  is chosen as  $\beta = 460 \text{ MeV}$  to reproduce the “double-hump” shape of the quark distribution amplitude similar to that provided by the QCD sum-rule method. To compare these two different quark distribution amplitudes, we summarize the first six moments<sup>18</sup> of the quark distribution amplitude in Table I; the  $n$ th moment is defined by

$$\langle \xi^n \rangle = \int_{-1}^{+1} d\xi \xi^n \tilde{\phi}_\pi(\xi),$$

where  $\xi = 2x - 1$  and the normalization of  $\tilde{\phi}_\pi(\xi)$  is fixed by the zeroth moment  $\langle \xi^0 \rangle \equiv 1$ .

The pion-form-factor result for these two different choices of quark distribution amplitude is presented in Fig. 3. While the difference in the moments (see Table I) is not negligible, the difference in the pion form factor

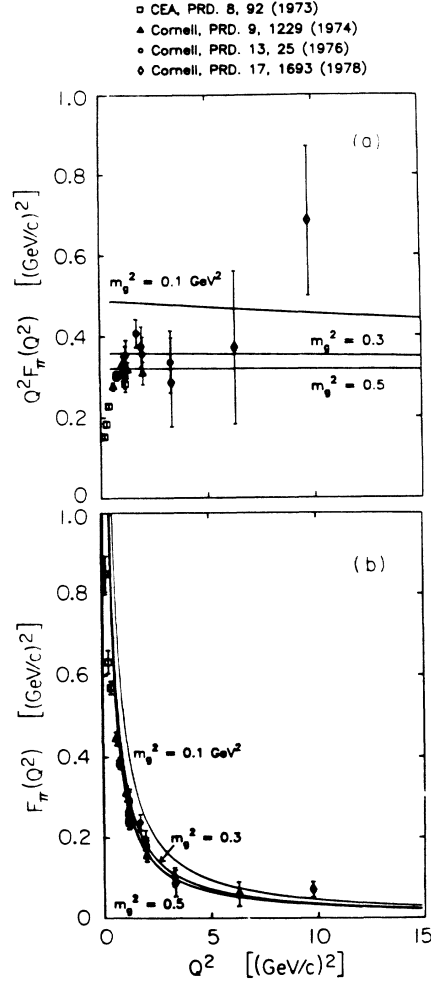


FIG. 2. Pion-form-factor calculation with the distribution amplitude of Chernyak and Zhitnitsky [Eq. (11)] and with the argument of  $\alpha_s(Q^2)$  evaluated at gluon momentum in Eq. (2). (a)  $Q^2 F_\pi(Q^2)$  and (b)  $F_\pi(Q^2)$ .

seems to be almost negligible for the two different recent quark distribution amplitudes available for the pion. Thus, in the pion-form-factor analysis, we have the same consistency with the available experimental data which was obtained in the proton Dirac-form-factor analysis even though further comparison with future experimental data is necessary at a higher- $Q^2$  region. In Fig. 4, we predict the kaon form factor at a high- $Q^2$  region using the quark distribution amplitude presented in Sec. III. The future experiment on the kaon-form-factor measurement at  $Q^2 \geq 1 \text{ GeV}^2$  is requested to compare with our prediction.

TABLE I. First six moments of the two different model  $\phi_\pi(\xi = 2x - 1)$  used in Fig. 3. The bottom line is the new lower and upper bounds for the first six moments presented recently by Narison (Ref. 18).

Model	$\langle \xi^1 \rangle$	$\langle \xi^2 \rangle$	$\langle \xi^3 \rangle$	$\langle \xi^4 \rangle$	$\langle \xi^5 \rangle$	$\langle \xi^6 \rangle$
CZ (Ref. 12)	0	0.43	0	0.24	0	0.15
Dziembowski (Ref. 17)	0	0.53	0	0.31	0	0.20
Narison (Ref. 18)	0	0.38–0.60	0	0.22–0.35	0	0.17–0.22

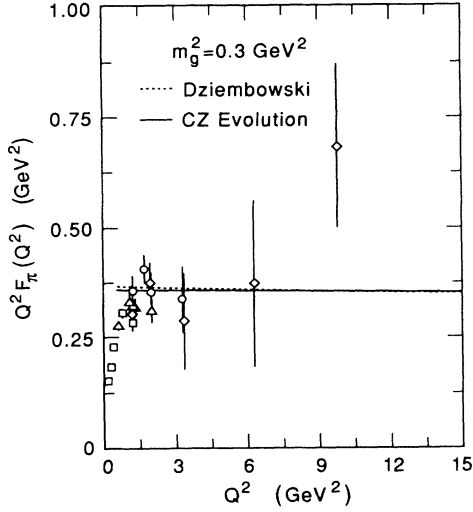


FIG. 3. Pion-form-factor calculations with two different distribution amplitudes of Chernyak and Zhitnitsky [Eq. (11)] and of Dziembowski and Mankiewicz (Ref. 17).

Next, we calculate the cross section given by Eqs. (6)–(8) for  $\gamma\gamma \rightarrow \pi^+\pi^-$  and  $\gamma\gamma \rightarrow K^+K^-$  processes using the same method and quark distribution amplitude considered in the calculation of  $F_\pi(Q^2)$  and  $F_K(Q^2)$ . The results for the pair production of  $\pi$  and  $K$  are presented in Figs. 5 and 6, respectively. The angular ranges for the  $\pi$  and  $K$  pair production cross section are restricted to  $|\cos\theta_{c.m.}| \leq 0.3$  and  $|\cos\theta_{c.m.}| \leq 0.6$ , respectively, to compare with available experimental data.<sup>19</sup> We also compared with other QCD calculations<sup>10,20</sup> in which they expressed  $\mathcal{M}^{\lambda\lambda'}$  [Eqs. (7) and (8)] in terms of the meson form factor. The results given by Benayoun and Chernyak<sup>20</sup> are obtained when the frozen coupling constant is taken as a fixed average value  $\bar{\alpha}_s = 0.36$ . While the normalization of their results is comparable to ours at  $m_g^2 = 0.1$  GeV<sup>2</sup> (from this, one can find the corresponding average

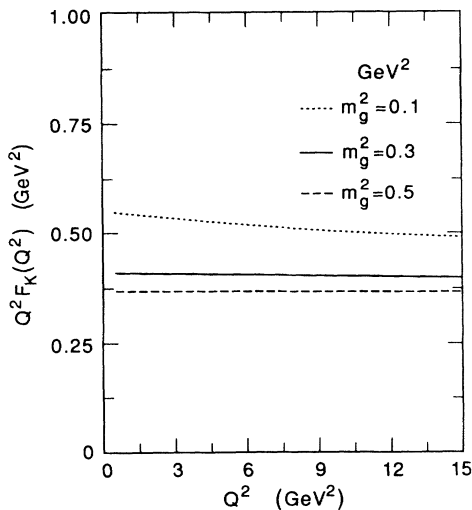


FIG. 4. Kaon-form-factor prediction [ $Q^2 F_K(Q^2)$ ] with the same method used for the pion form factor.

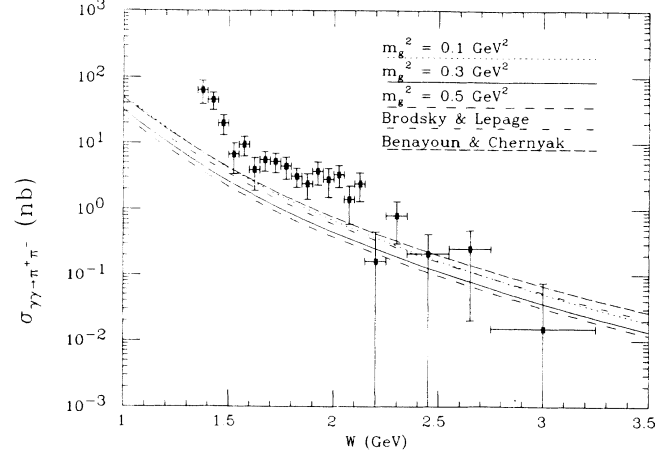


FIG. 5. Cross section for the pion pair production in two-photon collision. The angular range is restricted to  $|\cos\theta_{c.m.}| \leq 0.3$ .

momentum transfer of the virtual gluon, which is about 300 MeV), the slope (or energy dependence) of their curves is less steep than that obtained by using the gluon-momentum-dependent frozen coupling constant. The result for  $\gamma\gamma \rightarrow K^+K^-$  is obtained by taking the ratio  $(f_K/f_\pi)^4 = 2.37$  as discussed in Ref. 20. For the  $\gamma\gamma \rightarrow \pi^+\pi^-$  process, we also investigated the sensitivity of the results depending on the choice of different model  $\phi_\pi(x)$ , choosing again the model<sup>17</sup> wave function of Dziembowski and Mankiewicz. However, the deviation is even smaller than the pion-form-factor calculation (see Fig. 3) and cannot be observed in Fig. 5. It was argued by Nižić<sup>11</sup> that the poorer agreement between QCD calculations and data on pion pair production (Fig. 5) compared to kaon pair production (Fig. 6) can possibly be ascribed to the interference of the continuum with the  $f(1270)$  and other resonances, completely or incompletely reconstructed as  $\pi^+\pi^-$  final states.

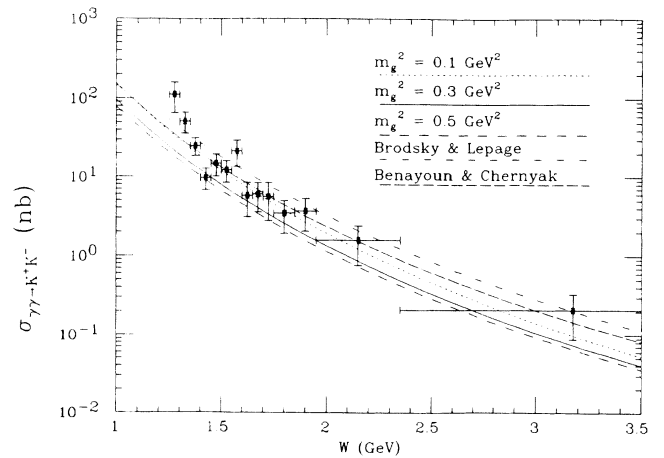


FIG. 6. Cross section for the kaon pair production in two-photon collision. The angular range is restricted to  $|\cos\theta_{c.m.}| \leq 0.6$ .

## V. CONCLUSIONS

In this paper we have analyzed the pion and kaon form factors and the pion and kaon pair production in  $\gamma\gamma$  annihilations within the framework of perturbative QCD using a frozen coupling constant. Our aim was to investigate whether the same type of agreement between experimental data and theoretical predictions achieved in the proton Dirac-form-factor analysis<sup>2</sup> can also be obtained in these meson-induced reaction analyses when the same types of wave functions and frozen-coupling-constant parameters are used. To make a fair comparison, we strictly restrict our calculations of hard-scattering amplitudes to the leading order<sup>4</sup> in  $\alpha_s$ . While similar results should be obtained using any form of cutoff which prevents  $\alpha_s(Q^2)$  from becoming infinitely large at small momentum transfers, we chose to use the formula of Eq. (4) because of its simple analytical form and its prior use in the nucleon form-factor analysis. Using the quark distribution amplitude of Eq. (9) constrained by the QCD sum rule,<sup>12</sup> we obtained numerical results shown in Figs. 2–6, including the sensitivity check by choosing other model

wave functions.<sup>17</sup> From the present investigation, we may conclude that our QCD predictions for the pion form factor and the cross sections of the pion and kaon pair production in  $\gamma\gamma$  annihilation are in fair agreement with available experimental data. It is also interesting to note that the numerical values of  $m_g$  used in this analysis are consistent with those of an “effective gluon mass” in the condensed vacuum obtained by QCD lattice calculations<sup>15</sup> and a recent discussion of dynamical mass generation in QCD.<sup>6,16</sup> Whether the same method would work for other baryon-induced reactions, such as  $N$ - $\Delta$  transition form factors<sup>21</sup> and  $\gamma\gamma \rightarrow p\bar{p}$ ,<sup>22</sup> is an interesting question which necessitates the application of this technique to other processes.

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