

Analyses of the nonleptonic charmed-meson decays of the B meson

Morimitsu Tanimoto*

*Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg-Institut für Physik,
P.O. Box 40 12 12, Munich, Federal Republic of Germany*

Kazunori Goda and Kei Senba

*Department of Physics, University of Ehime, 790 Matsuyama, Japan
(Received 3 July 1990; revised manuscript received 28 August 1990)*

We analyze the weak hadronic matrix elements in the nonleptonic charmed-meson decays of the B meson based on the factorization assumption. We derive the relevant form factors from the new experimental data of CLEO and ARGUS. Factorization with the large- N limit works successfully for single-charmed-meson decays. For double-charmed-meson decays, it may be consistent with the CLEO data if a large final-state phase shift is taken in the $B \rightarrow D^* \pi$ decay.

I. INTRODUCTION

In the standard model for the electroweak interaction, the magnitude of a physical amplitude is controlled by the Kobayashi-Maskawa (KM) mixing matrix.¹ However, any nonleptonic weak process is characterized by some hadronic matrix elements because quarks are confined into hadrons. Exclusive nonleptonic decays require the knowledge of the overlap form factors, which are given by the strong interaction. Therefore, the study of nonleptonic decay is important to improve our present understanding of both QCD and the electroweak standard theory.

The nonleptonic weak decays of the heavy quarks have been studied by using the assumption of factorization of the hadronic matrix elements,² which is based on the $1/N$ expansion, N being the color number.³ Factorization with the large- N limit works successfully for D -meson decays.⁴⁻⁶ It is also expected to be successful for B -meson decays.

We have recently analyzed the nonleptonic decays of the B meson in order to test the factorization assumption

with the large- N limit.⁷ In this paper we reanalyze the decay amplitudes of the charmed two-body nonleptonic decay of the B meson by using new ARGUS data⁸ and CLEO data.^{9,10} Our number result is somewhat changed from the previous one.⁷ We do not use the form factors given by Bauer, Stech, and Wirbel (BSW model²), but use the experimental data of $B \rightarrow D\pi, D^*\pi$, because the predicted form factors depend on the model of the quark potential.

We show the decay amplitudes for single-charmed-meson decays in Sec. II and for double-charmed-meson decays in Sec. III. In Sec. IV, the numerical results are given. Section V is devoted to summary.

II. SINGLE-CHARMED-MESON DECAYS

In this section we present the single-charmed-meson decay amplitude of the following processes: $B_d^0 \rightarrow D^- \pi^+$, $B^+ \rightarrow \bar{D}^0 \pi^+$, $B_d^0 \rightarrow D^{*-} \pi^+$, $B^+ \rightarrow \bar{D}^{*0} \pi^+$, $B_d^0 \rightarrow D^- \rho^+$, $B^+ \rightarrow \bar{D}^0 \rho^+$. Those weak amplitudes are given by using the factorization of the hadronic matrix elements, respectively, as

$$A^w(B_d^0 \rightarrow D^- \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [a_1 f_\pi (m_B^2 - m_D^2) F_0^{BD}(m_\pi^2) + a_2 f_B (m_D^2 - m_\pi^2) \hat{F}_0^{D\pi}(m_B^2)], \quad (2.1)$$

$$A^w(B^+ \rightarrow \bar{D}^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [a_1 f_\pi (m_B^2 - m_D^2) F_0^{BD}(m_\pi^2) + a_2 f_D (m_B^2 - m_\pi^2) F_0^{B\pi}(m_D^2)], \quad (2.2)$$

$$A^w(B_d^0 \rightarrow D^{*-} \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* 2m_B p_D [a_1 f_\pi A_0^{BD*}(m_\pi^2) + a_2 f_B \hat{A}_0^{D*\pi}(m_B^2)], \quad (2.3)$$

$$A^w(B^+ \rightarrow \bar{D}^{*0} \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* 2m_B p_D [a_1 f_\pi A_0^{BD*}(m_\pi^2) + a_2 f_D \hat{F}_1^{B\pi}(m_D^2)], \quad (2.4)$$

$$A^w(B_d^0 \rightarrow D^- \rho^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* 2m_B p_\rho [a_1 f_\rho F_1^{BD}(m_\rho^2) + a_2 f_B \hat{A}_0^{D\rho}(m_B^2)], \quad (2.5)$$

$$A^w(B^+ \rightarrow \bar{D}^0 \rho^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* 2m_B p_\rho [a_1 f_\rho F_1^{BD}(m_\rho^2) + a_2 f_D A_0^{B\rho}(m_D^2)], \quad (2.6)$$

where V_{ij} denotes the relevant KM matrix elements,¹ and we used the definitions

$$a_1 \equiv C_1 + \frac{C_2}{N}, \quad a_2 \equiv C_2 + \frac{C_1}{N}, \quad (2.7)$$

with $C_1 \simeq 1.11$ and $C_2 \simeq -0.25$ at the b -quark mass scale.⁶ The couplings f_P and f_V (P denotes pseudoscalar mesons, V denotes vector mesons) are defined as

$$\langle P | \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 | 0 \rangle \equiv f_P q_\mu, \quad (2.8)$$

$$\langle V | \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 | 0 \rangle \equiv f_V m_V \epsilon_\mu. \quad (2.9)$$

The form factors $F_0^{BD}(q^2)$, $F_1^{BD}(q^2)$, $A_0^{BD*}(q^2)$, $A_0^{B\rho}(q^2)$, etc., are defined in the form-factor decompositions as in the BSW model.¹¹ All of the second terms of Eqs. (2.1), (2.3), and (2.5) are negligibly small, because $a_1 \gg a_2$ is satisfied and the annihilation form factors $\hat{F}_0^{D\pi}(m_B^2)$, $\hat{A}_0^{D^*\pi}(m_B^2)$, $\hat{A}_0^{D\rho}(m_B^2)$ are damped at large q^2 ($=m_B^2$). For example, the second term of Eq. (2.1) contributes at most 1% in the BSW model; therefore, we neglect these terms in our analyses.

In the BSW model the values of the form factors at $q^2=0$ were calculated by use of the relativistic harmonic-oscillator potential model.¹¹ However, in order to test the factorization assumption, we do not use their model-dependent values, but use ones derived from the experimental data. The relatively precise experimental data have been given for the $B \rightarrow D\pi, D^*\pi$ processes. We show the branching-ratio data of CLEO⁹ and ARGUS,⁸ and the simple averaged values as follows (in percent):

	CLEO	ARGUS	Average
$B_d^0 \rightarrow D^- \pi^+$	0.25±0.07	0.48±0.16	0.37±0.12
$B^+ \rightarrow \bar{D}^0 \pi^+$	0.44±0.10	0.20±0.10	0.32±0.10
$B_d^0 \rightarrow D^{*-} \pi^+$	0.36±0.11	0.28±0.11	0.32±0.11
$B^- \rightarrow D^{*0} \pi^-$	0.7±0.3	0.40±0.18	0.55±0.24

(2.10)

We have studied the form factors in the three cases of using the CLEO data, the ARGUS data, and their averaged data, respectively. Although the numerical results are somewhat different from each other, the qualitative conclusions are the same ones in the three cases. Therefore, we show the result in the case of using the simple averaged data in the following.

The q^2 dependence of the form factors⁵ is approximated by a single pole for $F_0(q^2)$, $F_1(q^2)$, and $A_0(q^2)$ according to the power-counting laws of QCD¹² such as

$$F_0^{BD}(q^2) = \frac{F_0^{BD}(0)}{1 - q^2/M^2} \quad \text{with } M \simeq 6.80 \text{ GeV, etc.,} \quad (2.11)$$

where M is the relevant energy scale of the transition.¹¹

Using $V_{cb} \simeq 0.046$ and $V_{ud} \simeq 1$ we may get the values of $F_0^{BD}(0)$, $F_1^{B\rho}(0)$, $A_0^{BD*}(0)$, and $F_1^{B\pi}(0)$. However, we should take into consideration the final-state phase shifts of the $D\pi$ and $D^*\pi$ systems. The physical amplitude of

the $B_d^0 \rightarrow D^- \pi^+$ process is given by the isospin decomposition as¹³

$$A^{\text{phys}}(B_d^0 \rightarrow D^- \pi^+) = \frac{1}{\sqrt{3}} [a_{3/2}(D\pi) e^{i\delta_{3/2}} + a_{1/2}(D\pi) e^{i\delta_{1/2}}], \quad (2.12)$$

where $a_{3/2}$ and $a_{1/2}$ are the weak amplitudes (with KM factors included) for $I = \frac{3}{2}$ and $\frac{1}{2}$, respectively. After some calculations we get

$$A^{\text{phys}}(B_d^0 \rightarrow D^- \pi^+) = e^{i\delta_{1/2}} \left[A^w(B_d^0 \rightarrow D^- \pi^+) + \frac{A^w(B^+ \rightarrow \bar{D}^0 \pi^+)}{3} (e^{i\delta_{D\pi}} - 1) \right], \quad (2.13)$$

where $\delta_{D\pi} = \delta_{3/2} - \delta_{1/2}$. In deriving Eq. (2.13) we used the following relation, which is given by the factorization assumption:

$$A^w(B^+ \rightarrow \bar{D}^0 \pi^+) = A^w(B_d^0 \rightarrow D^- \pi^+) + \sqrt{2} A^w(B_d^0 \rightarrow \bar{D}^0 \pi^0). \quad (2.14)$$

As seen in Eq. (2.13), the physical amplitude deviates from the weak one in the case of $\delta_{D\pi} \neq 0$. It is noted that we have $|A^{\text{phys}}(B^+ \rightarrow \bar{D}^0 \pi^+)| = |A^w(B^+ \rightarrow \bar{D}^0 \pi^+)|$, because the final state $\bar{D}^0 \pi^+$ is the pure $I = \frac{3}{2}$ one. Since the absolute value of the physical amplitude is given by the experimental data, we can get the constraint for the phase shift $\delta_{D\pi}$ and the form factors $F_0^{BD}(0), F_1^{B\rho}(0)$. Once we know the values of these form factors, we can give the prediction of the branching ratios of the $B_d^0 \rightarrow D^- \rho^+$ and $B^+ \rightarrow \bar{D}^0 \rho^+$ processes as seen in Eqs. (2.5) and (2.6) because of $F_0^{BD}(0) \equiv F_1^{B\rho}(0)$.¹¹ Furthermore, we can predict the double-charmed-meson decay such as $B_d^0 \rightarrow D^- D_s^+$ and $B_d^0 \rightarrow D^- D_s^{*+}$.

The physical amplitude of the $B_d^0 \rightarrow D^{*-} \pi^+$ process is also given in the same way. The new data of $B^- \rightarrow D^{*0} \pi^-$ by ARGUS⁸ and CLEO⁹ make it possible to investigate the phase shift $\delta_{D^*\pi}$ in the final $D^*\pi$ system. This is one of the new points of our reanalysis.

III. DOUBLE-CHARMED-MESON DECAYS

In this section we discuss the double-charmed-meson decay such as $B_d^0 \rightarrow D^- D_s^+$, $B_d^0 \rightarrow D^{*-} D_s^+$, and $B_d^0 \rightarrow D^- D_s^{*+}$. These predictions are the good test for the factorization assumption.

The penguin process¹⁴ contributes to the double-charmed-meson decays¹⁵ in contrast with the single-charmed ones. We include the lowest-order penguin amplitude without the leading-logarithmic QCD corrections in this process, because the penguin one is not dominant amplitude, at most 10% in the double-charmed-meson decays. These amplitudes are given by using the factorization assumption and including the timelike penguin amplitude as

$$A(B_d^0 \rightarrow D^- D_s^+) = \frac{G_F}{\sqrt{2}} \left[a_1 V_{cb} V_{cs}^* + \frac{\alpha_s}{2\pi} \left[\sum_i V_{ib} V_{is}^* I_i \right] a_3 \left[1 + 2 \frac{m_{D_s}^2}{(m_s + m_c)(m_b - m_c)} \right] \right] f_{D_s} (m_B^2 - m_D^2) F_0^{BD}(m_{D_s}^2), \quad (3.1)$$

$$A(B_d^0 \rightarrow D^{*-} D_s^+) = \frac{G_F}{\sqrt{2}} \left[a_1 V_{cb} V_{cs}^* + \frac{\alpha_s}{2\pi} \left[\sum_i V_{ib} V_{is}^* I_i \right] a_3 \left[1 - 2 \frac{m_{D_s}^2}{(m_s + m_c)(m_b + m_c)} \right] \right] 2im_B f_{D_s} p_{D^*} A_0^{BD*}(m_{D_s}^2), \quad (3.2)$$

$$A(B_d^0 \rightarrow D^- D_s^{*+}) = \frac{G_F}{\sqrt{2}} \left[a_1 V_{cb} V_{cs}^* + \frac{\alpha_s}{2\pi} \left[\sum_i V_{ib} V_{is}^* I_i \right] a_3 \right] 2m_B f_{D_s} p_{D_s^*} F_1^{BD}(m_{D_s}^2), \quad (3.3)$$

with

$$a_3 = 1 - \frac{1}{N^2}. \quad (3.4)$$

The function I_i is the penguin loop function in the lowest order.¹⁶ We do not take account of the final-state interaction in these double-charmed-meson decay.

We comment on the final-state interactions in these processes. There may be channel mixing with $B \rightarrow K\psi$ and $K^*\psi$, but its effect is very small because these decay amplitudes are suppressed in an order of a_2/a_1 . There may also be the rescattering effect, which we cannot estimate reliably. However, we have found that the factorization works successfully for these processes without taking account of the rescattering effect in the following analyses.

IV. NUMERICAL RESULTS

Let us begin with showing the numerical results of the single-charmed-meson decays. By taking the simple averaged data of Eq. (2.10) we get the constraint for $\delta_{D\pi}$,

$F_0^{BD}(0)$, and $F_0^{B\pi}(0)$. Once the value of $F_0^{BD}(0)$ is fixed we get the value of $\delta_{D\pi}$, $F_0^{B\pi}(0)$ and the predicted branching ratios of the $B_d^0 \rightarrow D^- \rho^+$ and $B^+ \rightarrow \bar{D}^0 \rho^+$ decays as shown in Table I, where error bars are due to the used experimental data. We take tentatively $F_0^{B\pi}(0) \simeq A_0^{B\rho}(0)$ in predicting the $B^+ \rightarrow \bar{D}^0 \rho^+$ branching ratio. Since the contribution of $A_0^{B\rho}(0)$ is very small, this approximation is not crucial for our numerical results. The experimental values of these branching ratios have been given by the ARGUS group, such as $B(\bar{B}_d^0 \rightarrow D^+ \rho^-) = (0.9 \pm 0.6)\%$ and $B(B^- \rightarrow D^0 \rho^-) = (1.3 \pm 0.6)\%$. In Table I, we show these predicted values in both cases of $N=3$ and ∞ . Allowing the experimental values in the region of $0.3\% \leq B(B_d^0 \rightarrow D^- \rho^+) \leq 1.5\%$ and $0.7\% \leq B(B^+ \rightarrow \bar{D}^0 \rho^+) \leq 1.9\%$, and imposing the reasonable constraint $F_0^{BD}(0) > F_0^{B\pi}(0) > 0$, we get the allowed region of $F_0^{BD}(0)$: $0.60 \leq F_0^{BD}(0) \leq 0.75$ with $N=\infty$ as seen in Table I. It is remarked that there is no allowed region of $F_0^{BD}(0)$ in the case of $N=3$. These results seem to support the BSW model, in which $F_0^{BD}(0)=0.690$ and $F_0^{B\pi}(0)=0.333$ are used with $N=\infty$.² But, it is emphasized in our results that the final-state phase shift $\delta_{D\pi}$

TABLE I. The allowed values of $\cos\delta_{D\pi}$ and $F_0^{B\pi}(0)$ for fixed values of $F_0^{BD}(0)$ in the case of $N=\infty$. The values in parentheses correspond to the case of $N=3$. The predicted branching ratios are also shown for $B_d^0 \rightarrow D^- \rho^+$ and $B^+ \rightarrow \bar{D}^0 \rho^+$.

$F_0^{BD}(0)$	$\cos\delta_{D\pi}$	$F_0^{B\pi}(0)$	$B(D^- \rho^+)$	$B(\bar{D}^0 \rho^+)$
0.55	1.261±0.842 (1.663±0.971)	0.020±0.173 (0.134±0.361)	0.86% (0.73%)	0.84% (0.78%)
0.60	0.850±0.737 (1.217±0.828)	0.123±0.173 (-0.064±0.361)	1.02% (0.87%)	0.92% (0.85%)
0.65	0.496±0.695 (0.840±0.736)	0.226±0.173 (-0.262±0.361)	1.20% (1.03%)	1.00% (0.92%)
0.70	0.184±0.668 (0.513±0.697)	0.329±0.173 (-0.460±0.361)	1.39% (1.19%)	1.08% (0.99%)
0.75	-0.098±0.650 (0.222±0.670)	0.431±0.173 (-0.657±0.361)	1.60% (1.36%)	1.17% (1.07%)
0.80	-0.356±0.640 (-0.043±0.653)	0.534±0.173 (-0.855±0.361)	1.82% (1.55%)	1.25% (1.15%)

TABLE II. The allowed values of $\cos\delta_{D^*\pi}$ and $F_1^{B\pi}(0)$ for fixed values of $A_0^{BD^*}(0)$ in the case of $N = \infty$. The values in parentheses correspond to the case of $N = 3$.

$A_0^{BD^*}(0)$	$\cos\delta_{D^*\pi}$	$F_1^{B\pi}(0)$
0.60	0.728±0.637 (1.031±0.710)	-0.305±0.367 (0.851±0.764)
0.65	0.446±0.590 (0.721±0.636)	-0.191±0.367 (0.630±0.764)
0.70	0.204±0.563 (0.460±0.592)	-0.076±0.367 (0.409±0.764)
0.75	-0.011±0.549 (0.233±0.566)	0.039±0.367 (0.188±0.764)
0.80	-0.205±0.543 (0.031±0.551)	0.153±0.367 (-0.032±0.764)
0.85	-0.384±0.543 (-0.153±0.544)	0.268±0.367 (-0.253±0.764)
0.90	-0.552±0.548 (-0.323±0.543)	0.382±0.367 (-0.474±0.764)
0.95	-0.710±0.556 (-0.482±0.545)	0.497±0.367 (-0.695±0.764)
1.00	-0.861±0.566 (-0.632±0.551)	0.612±0.367 (-0.916±0.764)
1.05	-1.006±0.578 (-0.775±0.560)	0.726±0.367 (-1.136±0.764)

around 90° should be taken in order to explain the single-charmed-meson decays.

On the other hand, taking the averaged data of $B \rightarrow D^*\pi$ in Eq. (2.10), we get the constraint for $\delta_{D^*\pi}$, $A_0^{BD^*}(0)$, and $F_1^{B\pi}(0)$. We show the values of $\cos\delta_{D^*\pi}$ and $F_1^{B\pi}(0)$ for the fixed $A_0^{BD^*}(0)$ in Table II. Imposing the reasonable constraint $A_0^{BD^*}(0) > F_1^{B\pi}(0) > 0$, we get the allowed region of $A_0^{BD^*}(0)$: $0.75 \leq A_0^{BD^*}(0) \leq 1.05$ in the case of $N = \infty$, and $0.60 \leq A_0^{BD^*}(0) \leq 0.75$ in the case of $N = 3$. Although this result is not inconsistent with the one in the BSW model, $A_0^{BD^*}(0) = 0.623$,² both cases of $N = \infty$ and 3 are still allowed.

Next, we have calculated the branching ratio for the double-charmed-meson decays of the B meson by using Eqs. (3.1), (3.2), and (3.3) with our obtained values of $F_0^{BD}(0)$ and $A_0^{BD^*}(0)$. These results are shown for $f_{D_s} = 0.22$ GeV in $B(DD_s)$ and $f_{D_s^*} = 0.221$ GeV in $B(DD_s^*)$ with $m_t = 150$ GeV in the case of $N = \infty$ in Table III. The predicted values of $B(D^*D_s)$ and $B(D^*D_s)/B(DD_s)$ are shown in Table IV for

TABLE III. The predicted values of $B_d^0 \rightarrow DD_s$ and $B_d^0 \rightarrow DD_s^*$ in the case of $N = \infty$ and $m_t = 150$ GeV.

$F_0^{BD}(0)$	$B(DD_s)$ $\times (f_{D_s}/0.22 \text{ GeV})^2$ (%)	$B(DD_s^*)$ $\times (f_{D_s^*}/0.221 \text{ GeV})^2$ (%)
0.60	0.82	0.54
0.65	0.96	0.63
0.70	1.12	0.73
0.75	1.28	0.84

TABLE IV. The predicted values of the branching ratio of $B_d^0 \rightarrow D^*D_s$ and the ratio of the branching ratios of $B_d^0 \rightarrow D^*D_s$ to $B_d^0 \rightarrow DD_s$ in the case of $F_0^{BD}(0) = 0.7$, $N = \infty$, and $m_t = 150$ GeV.

$A_0^{BD^*}(0)$	$B(D^*D_s)$	
	$\times (f_{D_s}/0.22 \text{ GeV})^2$ (%)	$B(D^*D_s)/B(DD_s)$
0.75	0.86%	0.77
0.80	0.98%	0.88
0.85	1.11%	0.99
0.90	1.24%	1.12
0.95	1.39%	1.24
1.00	1.54%	1.38
1.05	1.69%	1.52

$F_0^{B\pi}(0) = 0.7$ and $A_0^{BD^*}(0) = 0.75 \sim 1.05$ in the case of $N = \infty$. Since the ratio $B(D^*D_s)/B(DD_s)$ is independent of the coupling f_{D_s} and very weakly depends on the form of the q^2 dependence in the form factor, this ratio is an important quantity to test the factorization assumption.

The penguin process contributes to the total decay amplitude in the negative sign as $-(9.5-10.0)\%$ for $B_d^0 \rightarrow DD_s$, $-(0.89-0.94)\%$ for $B_d^0 \rightarrow D^*D_s$, and $-(4.0-4.2)\%$ for $B_d^0 \rightarrow DD_s^*$ in the region of $m_t = 100-200$ GeV. We used here $\alpha_s = 0.23$, $m_s = 0.17$ GeV, $m_c = 1.4$ GeV, and $m_b = 4.95$ GeV. Thus, the contribution of the penguin one is the next leading one.

Recently, the CLEO group have reported the branching ratio of the double-charmed-meson decays as^{9,10}

$$\begin{aligned}
 B(\bar{B}_d^0 \rightarrow D^+ D_s^-) &= (0.75 \pm 0.38)\% , \\
 B(B^- \rightarrow D^0 D_s^-) &= (1.8 \pm 1.1)\% , \\
 B(\bar{B}_d^0 \rightarrow D^{*+} D_s^-) &= (1.5 \pm 1.1)\% .
 \end{aligned} \tag{4.1}$$

The predicted branching ratio is consistent with one for the $B_d^0 \rightarrow D^- D_s^+$ decay, and may be consistent with one for the $B_d^0 \rightarrow D^{*-} D_s^+$ decay in the case of $A_0^{BD^*}(0) = 0.75-1.05$ if $f_{D_s} \simeq 0.22$ GeV is used. The experimental ratio $B(D^*D_s)/B(DD_s) = 2.0 \pm 1.8$ is also consistent with the predicted one, $0.77-1.52$, which is independent of the value of f_{D_s} , as seen in Table IV. We need the confirmed experimental branching ratios of the $B_d^0 \rightarrow D^{*-} D_s^+$ and $B^+ \rightarrow \bar{D}^{*0} \pi^+$ processes to test clearly the factorization assumption with $N = \infty$. Furthermore, the predicted branching ratio of the $B_d^0 \rightarrow D^- D_s^{*+}$ decay will be compared with the experimental one in the future.

V. SUMMARY

We have analyzed the nonleptonic decay of the B meson based on the factorization assumption. The form factors at $q^2 = 0$ being derived from the single-charmed-meson decays of the B meson are consistent with ones in the BSW model.² Moreover, it is found that N should be taken to be ∞ . But, the large phase shifts should be taken in the $D\pi$ and $D^*\pi$ final states, probably, $\delta_{D\pi} \simeq 90^\circ$

and $\delta_{D^*\pi} > 90^\circ$. By using these values of the form factors, we have predicted the double-charmed-meson decays, and then found that the predicted ratio $B(D^*D_s)/B(DD_s)$ is consistent with the experimental one. Since this ratio does not depend on the value of f_{D_s} , this numerical value is important to test clearly the factorization assumption. We expect the precise measurement of the $B \rightarrow D^*D_s$ decay with the $B \rightarrow DD_s^*$ one.

ACKNOWLEDGMENTS

I thank Professor B. Stech and Professor J. G. Körner for helpful comments and discussions and thank Professor S. Wakaizumi for the information of new CLEO data. I also thank Professor T. Morii and Professor Y. Koide for useful comments as to the form factor and the final-state phase shifts. M.T. thanks the Alexander von Humboldt Foundation for financial support.

*On leave of absence from Science Education Laboratory, University of Ehime, 790 Matsuyama, Japan.

- ¹M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
²M. Bauer, B. Stech, and M. Wirbel, *Z. Phys. C* **34**, 103 (1987).
³A. J. Buras, J. M. Gérard, and R. Rückl, *Nucl. Phys.* **B268**, 16 (1986).
⁴J. M. Gérard, Max-Planck-Institute Report No. MPI-PAE/PTh 32/89, 1989 (unpublished).
⁵J. G. Körner, in *Proceedings of the International Symposium on Production and Decay of Heavy Hadrons*, Heidelberg, Germany, 1986, edited by K. R. Schubert and R. Waldi (DESY, Hamburg, 1986), p. 279.
⁶B. Stech, University of Heidelberg Report No. HD-THEP-89-29, 1989 (unpublished).
⁷M. Tanimoto, K. Goda, and K. Senba, Report No. MPI-PAE/PTh 31/90, 1990 (unpublished).
⁸ARGUS Collaboration, H. Albrecht *et al.*, DESY Report No. DESY 90-046, 1990 (unpublished).
⁹CLEO Collaboration, R. Poling, talk presented at the XXV International Conference on High Energy Physics, Singapore, 1990 (unpublished).
¹⁰CLEO Collaboration, D. Bortoletto *et al.*, *Phys. Rev. Lett.* **64**, 2117 (1990).
¹¹M. Wirbel, B. Stech, and M. Bauer, *Z. Phys. C* **29**, 637 (1985).
¹²S. J. Brodsky and G. P. Lepage, *Phys. Rev. D* **22**, 2157 (1980).
¹³D. Du, I. Dunietz, and D.-di. Wu, *Phys. Rev. D* **34**, 3414 (1986).
¹⁴A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, *Zh. Eksp. Teor. Fiz.* **72**, 1275 (1977) [*Sov. Phys. JETP* **45**, 670 (1977)].
¹⁵L.-L. Chau and H.-Y. Cheng, *Phys. Lett.* **165B**, 429 (1985).
¹⁶S. P. Chia, *Phys. Lett.* **130B**, 315 (1983); P. A. S. De Sousa Gerbert, *Nucl. Phys.* **B272**, 581 (1986).