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Determination of pseudoscalar-charmed-meson decay constants from B-meson decays

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
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Recent progress in understanding form factors for weak $B \to D$ and $B \to D^*$ transitions, combined with a factorization hypothesis, allows one to extract the pseudoscalar-meson decay constant $f_{D_s} = 259 \pm 74$ MeV, implying that the branching ratios for $D_s \to \tau \bar{\nu}_\tau$ and $D_s \to \mu \bar{\nu}_\mu$ are $(4.6 \pm 2.6)\%$ and $(0.5 \pm 0.3)\%$, respectively. A scaling law for nonrelativistic wave functions then implies $f_D = 207 \pm 60$ MeV, $f_B = 140 \pm 40$ MeV, and $f_{B_s} = 175 \pm 50$ MeV. Other consequences of the approach include a prediction that the branching ratio of $\bar{B}^0$ to $D^{*+} D^{*-}$ is about 0.06%, of which 94% occurs in the $CP = +$ final state.

I. INTRODUCTION

The determination of heavy-meson decay constants is of interest for several reasons. From the standpoint of strong interactions, the system of a heavy quark Q and a light antiquark $\bar{q}$ approaches that of the relativistic one-body problem in the limit $m_Q \to \infty$. The decay constant measures the probability that the light antiquark and heavy quark sit on top of one another, and thus reflects a useful dynamical property of the system related to its size. In studies of mixing and CP violation, meson decay constants relate matrix elements between hadron states to those between quark states, and thus are crucial in establishing the magnitudes and phases of Cabibbo-Kobayashi-Maskawa (CKM) quark mixing-matrix elements. Finally, heavy-meson decay constants govern purely leptonic decays of pseudoscalar mesons whose rates are often of interest: The branching ratio for $D_s^- \to \tau^- \bar{\nu}_\tau$ is crucial in estimating ν_τ fluxes in proposed accelerator experiments, while the rate for $B^- \to \tau^- \bar{\nu}_\tau$ could help measure the CKM element V_{ub} if f_B is known.

In this article we describe a method for measuring the decay constant f_{D_s} which makes use of the first data[1] on the decays $\bar{B} \to D D_s^-$ and $\bar{B} \to D^* D_s^-$. We assume factorization, so that the decays are taken to occur via a $\bar{B} \to D^{(*)} +$(weak-current) subprocess, in which the weak current materializes into a D_s^-. The new ingredient which makes this analysis possible is the hypothesis[2-6] that all heavy-meson decays are governed by a single universal form factor. We compare data on $\bar{B} \to D l \nu$, $\bar{B} \to D^* l \nu$, and $\bar{B}^0 \to D^{(*)} \pi$, assuming the nonleptonic decays satisfy factorization, and find satisfactory agreement with this universality hypothesis. We then are justified in employing the universal form factor at the value $q^2 = m_{D_s}^2$ in order to estimate f_{D_s} from the observed[1] branching ratio. There are several consistency checks of our approach which appear to be satisfied by present data; more accurate tests are expected in the near future as a large sample of B pairs becomes available from $e^+ e^-$ collision experiments.

In Sec. II we calculate decay rates and branching ratios in terms of a single universal form factor. These are then compared with experiment in Sec. III in order to specify the form factor. The meson decay constants are determined in Sec. IV. Further predictions are noted in Sec. V. Section VI contains a discussion and summary.

II. DECAY RATES AND BRANCHING RATIOS

The diagram describing every process which will be considered here is shown in Fig. 1(a). A $\bar{B}$ meson (containing a b quark) decays to a meson M_1 (containing a c quark) and a color-singlet charged current, which in turn materializes into the meson M_2. The decay constant is involved in the matrix element of the axial-vector or vector current between M_2 and the vacuum:

$$\langle 0 | A_\mu | P(q) \rangle = i q_\mu f_P \tag{1}$$

for a pseudoscalar meson P with four-momentum q and

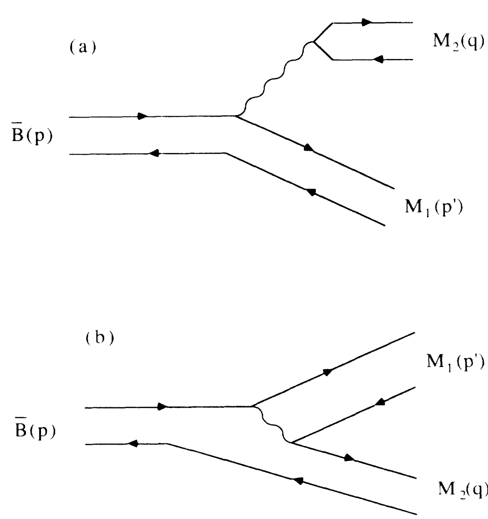

FIG. 1. Diagrams describing two-body decays of $\bar{B}$ mesons in the factorization hypothesis. (a) Color-favored diagram; (b) color-suppressed diagram.

$$\langle 0|V_\mu|V(q,\epsilon)\rangle=\epsilon_\mu M_V f_V \tag{2}$$

for a vector meson V with four-momentum q and polarization vector ϵ_μ. It will be important to consider vector mesons as well as pseudoscalar ones since in the heavy-quark limit both f_P and f_V are given by the nonrelativistic quark-model formula

$$f_P^2=f_V^2=12|\Psi(0)|^2/M\ , \tag{3}$$

where $\Psi(0)$ is the wave function of the light antiquark $\bar{q}$ and heavy quark Q at zero relative separation, and M is the heavy-meson mass ($\equiv m_Q$). Thus, information gained when M_2 is a vector meson will be applicable to the case when it is a pseudoscalar.[7]

We do not consider here the related diagrams of Fig. 1(b), which are subject to a color-suppression factor. In principle they are also amenable to the factorization hypothesis, and were in fact explicitly discussed in that context in Ref. 8. However, phenomenological parameters appear to be needed to account for the relative strengths of the graphs in Figs. 1(b) and 1(a), a delicacy which we avoid for the present discussion.

The mesons which we shall consider for M_1 consist of D and D^*, while those for M_2 consist of π^-, ρ^-, D_s^-, D_s^{*-}, D^-, and D^{*-}. For an extensive list of other possible choices, one may refer to those processes listed in Ref. 8 governed by the graph of Fig. 1(a) (whose amplitudes are proportional to a_1 in their notation). Thus, we shall be concerned with a total of twelve nonleptonic decays for present purposes. The universality of form factors for $\bar{B}\to D^{(*)}$ weak transitions may be understood from the fact that in the heavy-quark limit, the spins of b and c decouple from those of the light-antiquark spectator, while all that governs the form factor is the velocity change that the antiquark must undergo in order to pass from the meson $\bar{B}$ to the meson M_1. One can take "antiquark" in this context to mean not only a specific $\bar{u}$ or $\bar{d}$, but also any accompanying gluons or light quark-antiquark pairs. Convenient expressions for the matrix elements of vector and axial-vector currents are then[6]

$$\langle D(v')|V_\mu|B(v)\rangle=\sqrt{m_c m_b}\,\xi(w^2)(v+v')_\mu\ , \tag{4}$$

$$\langle D^*(v',\epsilon)|A_\mu|B(v)\rangle=\sqrt{m_c m_b}\,\xi(w^2)\times[\epsilon_\mu^*(1+v\cdot v')-\epsilon^*\cdot v\, v'_\mu]\ , \tag{5}$$

$$\langle D^*(v',\epsilon)|V_\mu|B(v)\rangle=-i\sqrt{m_c m_b}\,\xi(w^2)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v^\alpha v'^\beta\ . \tag{6}$$

For future purposes we shall take m_b equal to the mass of the B meson, $m_b=m_B\equiv 5.28$ GeV, and m_c equal to the spin-weighted average of the D and D^* masses, $m_c=m_D\equiv 1.97$ GeV. A more precise treatment[9] is possible in which one uses the actual particle masses, but we are neglecting all hyperfine effects here. The four-velocities are given by $v=p/m_B$ and $v'=p'/m_D$, and we have $q\equiv p-p'$. The universal velocity transfer $w=v-v'$ has invariant square

$$w^2=\frac{q^2-q^2_{\max}}{m_B m_D}=\frac{q^2-(m_B-m_D)^2}{m_B m_D}\ . \tag{7}$$

The universal form factor $\xi(w^2)$ may be parametrized, if we incorporate a QCD correction,[10] as

$$\xi(w^2)=\left[\frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)}\right]^{-6/(33-2n_f)}\frac{1}{1-w^2/w_0^2}\ . \tag{8}$$

We take[11] $\alpha_s(m_b^2)=0.189$, $\alpha_s(m_c^2)=0.29$, $n_f=4$, and find the corresponding QCD enhancement factor to be 1.11. The value of w_0 will be determined by reference to the data.

Straightforward calculations lead to the following differential decay widths with respect to the parameter $y\equiv m_{e\nu}^2/m_B^2=q^2/m_B^2$ for semileptonic processes:

$$\frac{d\Gamma}{dy}=\frac{\Gamma_0\lambda^{1/2}(1,\zeta,y)f(y)}{(1-w^2/w_0^2)^2}\ , \tag{9}$$

where

$$\Gamma_0\equiv\frac{G_F^2|V_{cb}|^2m_B^5}{192\pi^3}\left[\frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)}\right]^{-12/(33-2n_f)}=2.23\times10^{-13}\ \text{GeV} \tag{10}$$

for[12] $|V_{cb}|=0.044$ and $m_B=5.28$ GeV. Here $\lambda(a,b,c)\equiv a^2+b^2+c^2-2ab-2ac-2bc$, while

$$f(y)=\begin{cases}(1+\sqrt{\zeta})^2\lambda(1,\zeta,y)/4\sqrt{\zeta}\quad (Dl\nu)\ , & \text{(11a)}\\ y[(1+\sqrt{\zeta})^2-y](1+\zeta-y)/\sqrt{\zeta}\quad (D_T^*l\nu)\ , & \text{(11b)}\\ (1-\sqrt{\zeta})^2[(1+\sqrt{\zeta})^2-y]^2/4\sqrt{\zeta}\quad (D_L^*l\nu)\ . & \text{(11c)}\end{cases}$$

Here we have defined $\zeta=m_D^2/m_B^2$. For $\tau_B=1.18\times10^{-12}$ s, the corresponding spectra in y for branching ratios B are $dB/dy=0.40\lambda^{1/2}(1,\zeta,y)f(y)/(1-w^2/w_0^2)^2$. Examples of spectra are shown in Fig. 2. These spectra may then be integrated with respect to y to obtain predicted branching ratios as functions of the single parameter w_0.

A corresponding approach to the decays $\bar{B}^0\to P^-D^+$ leads to the expression

$$\Gamma(\bar{B}^0\to D^+P^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ij}|^2m_B^3f_P^2|\xi(w_P^2)|^2(1-\sqrt{\zeta})^2\times\lambda^{1/2}(1,\zeta,y_P)\frac{[(1+\sqrt{\zeta})^2-y_P]^2}{4\sqrt{\zeta}} \tag{12}$$

for a pseudoscalar meson P composed of a charge $-2/3$ antiquark $\bar{i}$ and a charge $-1/3$ quark j. Here $y_P\equiv m_P^2/m_B^2$, while $w_P^2\equiv[m_P^2-(m_B-m_D)^2]/m_Bm_D$. The rates for decays involving one or two vector mesons in the final state can be expressed as ratios with respect to the process (12). Comparing a process in which the current produces a vector meson V with one in which it produces a pseudoscalar P, we employ Eq. (3) to find

$$\frac{\Gamma(\bar{B}^0\to D^+V^-)}{\Gamma(\bar{B}^0\to D^+P^-)}=\left|\frac{\xi(w_V^2)}{\xi(w_P^2)}\right|^2\left[\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\right]^2\times\frac{\lambda^{3/2}(1,\zeta,y_V)/\lambda^{1/2}(1,\zeta,y_P)}{[(1+\sqrt{\zeta})^2-y_P]^2}\ , \tag{13}$$

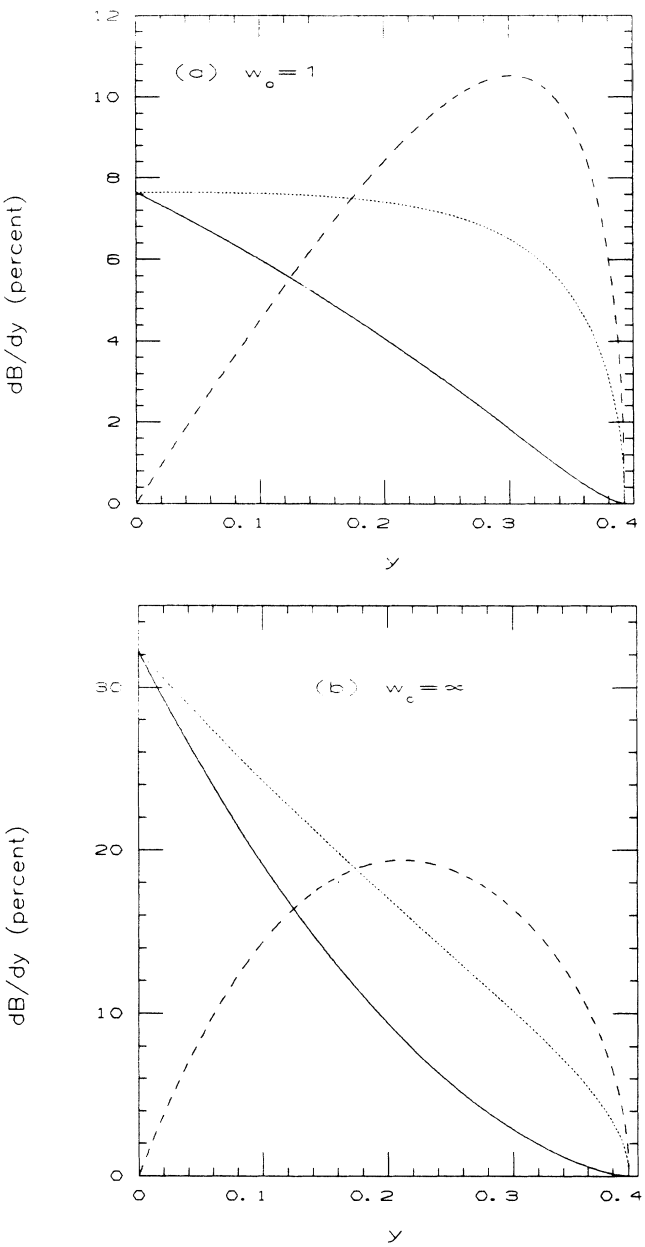

FIG. 2. Spectra with respect to the variable $y \equiv m_{l\nu}^2/m_B^2$ for the decays $\bar{B} \to D l \bar{\nu}_l$ (solid line), $\bar{B} \to D_T^* l \bar{\nu}_l$ (dashed line), and $\bar{B} \to D_L^* l \bar{\nu}_l$ (dotted line). The form-factor parameter is taken to have the values (a) $w_0 = 1$ and (b) $w_0 = \infty$.

where $w_V^2 \equiv [m_V^2 - (m_B - m_D)^2]/m_B m_D$. Using the universality of the form factor in Eqs. (4)–(6), we find

$$\frac{\Gamma(\bar{B}^0 \to D^{*+} P^-)}{\Gamma(\bar{B}^0 \to D^+ P^-)} = \left[\frac{1+\sqrt{\xi}}{1-\sqrt{\xi}}\right]^2 \frac{\lambda(1,\xi,y_P)}{[(1+\sqrt{\xi})^2 - y_P]^2} \tag{14}$$

(to the extent that we neglect $M_D^* - M_D$) and

$$\frac{\Gamma(\bar{B}^0 \to D^{*+} V^-)}{\Gamma(\bar{B}^0 \to D^+ P^-)} = \left|\frac{\xi(w_V^2)}{\xi(w_P^2)}\right|^2 \times \frac{N(\xi,y_V)[(1+\sqrt{\xi})^2 - y_V]}{(1-\sqrt{\xi})^2[(1+\sqrt{\xi})^2 - y_P]^2} \times \frac{\lambda^{1/2}(1,\xi,y_V)}{\lambda^{1/2}(1,\xi,y_P)} , \tag{15}$$

where

$$N(\xi,y_V) \equiv (1-\sqrt{\xi})^2[(1+\sqrt{\xi})^2 - y_V] + 4y_V(1+\xi-y_V) . \tag{16}$$

In decays involving only one vector meson, the vector meson is necessarily longitudinally polarized with respect to the decay axis. When two vector mesons are emitted, we distinguish among the states in which both are longitudinal (L), or both are transverse with linear polarization vectors parallel ($\parallel$) or perpendicular ($\perp$) to one another. The partial rates into these states are related to the total by

$$\frac{\Gamma_L}{\Gamma} = (1-\sqrt{\xi})^2[(1+\sqrt{\xi})^2 - y_V]/N(\xi,y_V) , \tag{17}$$

$$\frac{\Gamma_\parallel}{\Gamma} = 2y_V[(1+\sqrt{\xi})^2 - y_V]/N(\xi,y_V) , \tag{18}$$

$$\frac{\Gamma_\perp}{\Gamma} = 2y_V[(1-\sqrt{\xi})^2 - y_V]/N(\xi,y_V) . \tag{19}$$

The dependence of some predicted branching ratios for $\bar{B}^0$ decays on the parameter w_0 is shown in Table I. Because different processes probe different weighted averages of q^2 (as one can see from Fig. 2), ratios of branching ratios are sensitive to the shape of the form factor. The decay $\bar{B}^0 \to D^+ \pi^-$ is especially useful in constraining the form factor since it probes it at a value of w^2 farthest from the normalization point. We shall see that values of w_0 near 1 are favored, corresponding to a decay rate for

TABLE I. Dependence of branching ratios for $\bar{B}^0$ decays on parameter w_0 in form factor.

w_0	$B(Dl\nu)$[a] (%)	$B(D^*l\nu)$[a] (%)	$B(D\pi)$[a] (%)	$\frac{B(D_L^*l\nu)}{B(D_T^*l\nu)}$[b]	$\frac{B(Dl\nu)}{B(D\pi)}$	$\frac{B(D^*l\nu)}{B(D\pi)}$	$\frac{B(D^*l\nu)}{B(Dl\nu)}$
0.7	0.79	3.17	0.11	0.90	6.9	27.8	4.0
1.0	1.56	5.34	0.27	1.01	5.8	19.8	3.4
1.4	2.44	7.51	0.48	1.09	5.1	15.7	3.1
2.0	3.27	9.43	0.71	1.16	4.6	13.3	2.9
3.0	3.93	10.85	0.91	1.21	4.3	12.0	2.8
∞	4.62	12.30	1.13	1.25	4.1	10.9	2.7

[a]For $|V_{cb}| = 0.044$.

[b]The subscripts L and T denote longitudinal and transverse D^* polarizations.

$\bar{B}^0 \to D^+\pi^-$ only about $\frac{1}{4}$ of that for a pointlike form factor ($w_0=\infty$).

The use of ratios of branching ratios to constrain w_0 is important since the value of $|V_{cb}|$ quoted in Ref. 12 was obtained with the help of the exclusive semileptonic decay $\bar{B}\to Dl\nu$. An estimate[13] of $|V_{cb}|$ using only inclusive semileptonic decays is subject to uncertainty associated with quark masses, but yields the same central value: $|V_{cb}|=0.044\pm0.010$, if one assumes the average $b\to cl\nu$ semileptonic branching ratio is 11% rather than the 12.1% taken in Ref. 13. One should then regard all branching ratios in Table I as uncertain to about $2(\delta|V_{cb}|/|V_{cb}|)\approx 45\%$ of their values.

III. EXTRACTION OF FORM-FACTOR PARAMETER

For numerical estimates of w_0 we compare the predictions of Sec. II with the following measured branching ratios:

$$B(\bar{B}\to Dl^-\bar{\nu}_l)=\begin{cases}(1.7\pm0.6\pm0.4)\% \quad (D^+,\ \text{Ref. } 14)\ , & (20\text{a})\\ (1.6\pm0.6\pm0.3)0003 \quad (D^0,\ \text{Ref. } 15)\ , & (20\text{b})\\ (1.8\pm0.6\pm0.3)\% \quad (D^+,\ \text{Ref. } 15)\ , & (20\text{c})\\ (1.70\pm0.40)\% \quad (\text{average})\ , & (20\text{d})\end{cases}$$

$$B(\bar{B}\to D^*l^-\bar{\nu}_l)=\begin{cases}(5.4\pm0.9\pm1.3)\% \quad (D^{*+},\ \text{Ref. } 16)\ , & (21\text{a})\\ (4.1\pm0.8^{+0.8}_{-0.9})\% \quad (D^{*0},\ \text{Ref. } 15)\ , & (21\text{b})\\ (4.6\pm0.5\pm0.7)\% \quad (D^{*+},\ \text{Ref. } 17)\ , & (21\text{c})\\ (7.0\pm1.8\pm1.4)\% \quad (\text{Ref. } 18)\ , & (21\text{d})\\ (4.76\pm0.61)\% \quad (\text{average})\ , & (21\text{e})\end{cases}$$

$$B(\bar{B}^0\to D^+\pi^-)=(0.279\pm0.074)\% \quad (\text{Ref. } 19)\ , \tag{22}$$

$$B(\bar{B}^0\to D^{*+}\pi^-)=(0.284\pm0.077)\% \quad (\text{Ref. } 19)\ . \tag{23}$$

The best estimate of w_0 which is independent of $|V_{cb}|$ comes from comparing the $D^*l\nu$ rate with those of $D^+\pi^-$ and $D^{*+}\pi^-$, which are predicted to be equal. We are then justified in taking the average of Eqs. (22) and (23) to obtain

$$B(\bar{B}^0\to D^+\pi^-)=B(\bar{B}^0\to D^{*+}\pi^-)=(0.281\pm0.053)\% \tag{24}$$

or

$$\frac{\Gamma(\bar{B}\to D^*l\bar{\nu}_l)}{\Gamma(\bar{B}^0\to D^+\pi^-)}=16.9\pm3.9\ . \tag{25}$$

Equation (25) corresponds to

$$w_0=1.24^{+0.86}_{-0.29}\ . \tag{26}$$

Other quantities such as Γ_T/Γ_L (the ratio of rates for longitudinal and transverse D^* polarizations) in $D^*l\bar{\nu}$ decays or ratios involving the rate for $\bar{B}\to Dl\bar{\nu}$ are not well enough known at present to provide a constraint comparable to Eq. (26). In the long run, the measurement of spectrum shapes such as those illustrated in Fig. 2 will provide a model-independent determination of the universal function $\xi(w^2)$. (See note added in proof at end of this article.)

Estimates of w_0 based on the value $|V_{cb}|=0.044\pm0.010$ obtained from inclusive semileptonic b decays[13] and the averages (20d), (21e), and (24) are

$$w_0=\begin{cases}1.06^{+0.42}_{-0.34} \quad (Dl^-\bar{\nu}_l)\ , & (27)\\ 0.92^{+0.37}_{-0.32} \quad (D^*l^-\bar{\nu}_l), & (28)\\ 1.02\pm0.26 \quad (D^{(*)+}\pi^-)\ . & (29)\end{cases}$$

These are clearly consistent with one another and with (26). Comparing the values (20d), (21e), and (24) with those listed in the last row of Table I for $w_0=\infty$ (no form-factor suppression), we find that the form factors account for a suppression of approximately (0.37, 0.39, 0.25) for $(Dl\bar{\nu}, D^*l\bar{\nu}, D\pi)$. The first two suppression factors are considerably smaller than the corresponding values in Ref. 4, and the pole position in the form factor is considerably lower. In Ref. 4 a simple pole at $q^2\simeq(m_b+m_c)^2$ was adopted, corresponding to $w_0^2=4$. Our pole at $w_0^2\simeq1$ corresponds to $q^2\simeq(m_b-m_c)^2+m_bm_c$, not a physical particle mass. The reason for this may be that the form factor is dominated by many $b\bar{c}$ poles rather than the single one assumed in Ref. 4.

It is not permitted simply to average Eqs. (27)–(29), since the errors are dominated by the common error in $|V_{cb}|$. Instead, we shall take

$$w_0 = 1.12 \pm 0.17 \tag{30}$$

as expressing the range which is compatible with the constraints (26)–(29).

IV. DETERMINATION OF MESON DECAY CONSTANTS

We may now use the value (30) of w_0 to estimate branching ratios for the remaining states under discussion. For $\bar{B}^0 \to D^+ D_s^-$ we find

$$B(\bar{B}^0 \to D^+ D_s^-) = (0.265 \pm 0.053)0002(f_{D_s}/f_\pi)^2 , \tag{31}$$

where we have normalized to the branching ratio $B(B^0 \to D^{(*)+}\pi^-) = 00.281 \pm 0.053)\%$.

The process $B^- \to D^0 D_s^-$ should have the same branching ratio if $\tau_{B^0} = \tau_{B^-}$, which appears true to the required level of accuracy.[20] Here as well, only the graph of Fig. 1(a) contributes, given our factorization assumption. Thus we are justified in averaging the branching ratios for $\bar{B}^0 \to D^+ D_s^-$ and $B^- \to D^0 D_s^-$ quoted in Ref. 1. The results are[1]

$$B(\bar{B}^0 \to D^+ D_s^-) = (1.2 \pm 0.7)\% , \tag{32}$$

$$B(B^- \to D^0 D_s^-) = (2.9 \pm 1.3)\% , \tag{33}$$

leading to an average of $(1.6 \pm 0.6)\%$. However, these results were quoted for $B(D_s^- \to \phi\pi^-) = 2.0\%$. With a more recent measurement[21]

$$B(D_s^- \to \phi\pi^-) = (3.1 \pm 0.6^{+0.9}_{-0.6} \pm 0.6)\% , \tag{34}$$

one has

$$B(\bar{B}^0 \to D^+ D_s^-) = B(B^- \to D^0 D_s^-) = (1.02 \pm 0.55)\% . \tag{35}$$

When combined with the prediction (31), this leads to the result

$$f_{D_s} = 259 \pm 74 \text{ MeV} . \tag{36}$$

The accuracy of this determination should improve rapidly as more B decays are studied; the numbers in Eqs. (32) and (33) are based on a total of 3 and 5 events, respectively.

With the expression[22]

$$\Gamma(D_s^- \to l^- \bar{\nu}_l) = \frac{G_F^2 f_{D_s}^2 m_l^2 M_{D_s}}{8\pi} \left[1 - \frac{m_l^2}{m_{D_s}^2} \right]^2 |V_{cs}|^2 \tag{37}$$

and the observed D_s lifetime,[12] one estimates

$$B(D_s^- \to \tau^- \nu_\tau) = 2.74\% \quad (f_{D_s}/200 \text{ MeV})^2 , \tag{38a}$$

$$B(D_s^- \to \mu^- \bar{\nu}_\mu) = 0.30\% \quad (f_{D_s}/200 \text{ MeV})^2 , \tag{38b}$$

or

$$B(D_s^- \to \tau^- \bar{\nu}_\tau) = (4.6 \pm 2.6)\% , \tag{39a}$$

$$B(D_s^- \to \mu^- \bar{\nu}_\mu) = (0.5 \pm 0.3)\% , \tag{39b}$$

for the value (36). Estimates of the $\tau^- \bar{\nu}_\tau$ branching ratio are of interest in plans to construct an accelerator-based source of ν_τ using a beam dump,[23] while both the $\tau^- \bar{\nu}_\tau$ and $\mu^- \bar{\nu}_\mu$ branching ratios are predicted to be large enough that their observation may well be feasible in the near future.

We may relate f_{D_s} to an expected value for f_D using Eq. (3). We have some evidence that $|\Psi(0)|^2$ is smaller for D^+ $(=c\bar{d})$ than for D_s^+ $(=c\bar{s})$. The hyperfine splittings between pseudoscalar and vector mesons in both systems are approximately equal: $m(D_s^{*+}) - m(D_s^+) \approx m(D^{*+}) - m(D^+) \simeq 141$ MeV/c^2. These should behave as $|\Psi(0)|_D^2/m_c m_d$ and $|\Psi(0)|_{D_s}^2/m_c m_s$, respectively. Thus we can expect that

$$\frac{f_D}{f_{D_s}} \simeq \frac{|\Psi(0)|_D}{|\Psi(0)|_{D_s}} \simeq \left[\frac{m_d}{m_s} \right]^{1/2} \simeq 0.8 , \tag{40}$$

where we have used constituent-quark masses[24] of $(m_d, m_s) = (310, 485)$ MeV/c^2, respectively, and have neglected the D-D_s mass difference. Applying this result to Eq. (36), we estimate

$$f_D = 207 \pm 60 \text{ MeV} . \tag{41}$$

This is to be compared with the present upper limit[25] of

$$f_D < 290 \text{ MeV (90\% C.L.)} . \tag{42}$$

One might use the relation (3) to scale[26] from D to B and from D_s to B_s, assuming $|\Psi(0)|_B = |\Psi(0)|_D$ and $|\Psi(0)|_{B_s} = |\Psi(0)|_{D_s}$. Taking account of an additional QCD correction[10] of 1.11, equal to the first term on the right-hand side of Eq. (8), we find

$$f_B = 140 \pm 40 \text{ MeV} , \tag{43}$$

$$f_{B_s} = 175 \pm 50 \text{ MeV} . \tag{44}$$

However, in one relativistic model of $Q\bar{q}$ bound states,[27] the scaling law (3) is not necessarily so precise, leading to an enhancement of f_B/f_D by at least a factor of 1.45 with respect to the naive estimate $(M_D/M_B)^{1/2} \approx 0.6$.

There are many calculations of heavy-meson decay constants in the literature. In Table II we compare our results with a sample of these,[27–40] including most recent results and some older ones. A more complete compilation may be found in Ref. 36. There seems to be nearly universal consensus that f_{D_s} lies between 200 and 300 MeV, while f_D lies between 150 and 250 MeV. The question of whether the scaling relation $f_P \sim M_P^{-1/2}$ can be used to extrapolate these results to systems containing b quarks remains open, and accounts for the greater spread of values for f_B and f_{B_s}.

V. FURTHER PREDICTIONS

So far we have discussed decays such as $\bar{B}^0 \to D^+ \pi^-$ (governed by $b \to c\bar{u}d$) and $\bar{B}^0 \to D^+ D_s^-$ (governed by $b \to c\bar{c}s$) which are favored in terms of CKM elements. An important class of decays governed by the CKM-suppressed subprocess $b \to c\bar{u}s$ may be important for the study of CP violation: the process $\bar{B}^0 \to D^+ D^-$ and

TABLE II. Comparison of present results for meson decay constants with some previous estimates. Values are in MeV. Here $f_\pi = 132$ MeV.

Reference	f_D	f_{D_s}	f_B	f_{B_s}
Present	207±60	259±74	140±40[a]	175±50[a]
27[b]	225		195	
28[c]	174±26±46	234±46±55	105±17±30	155±31±48
29[c]	181±27[d]	218±27[d]	≈120	≈150
29[c]	197±14[e]	214±19[e]		
30[c]	190±33	222±16		
31[c]	180±25±30			
32[c]			233±19±38[f]	
33[g]	150	210	125	175
34[g,h]	117	129	75	86
35[g]	182	199	231	245
36[g]	240±20	290±20	155±15	210±20
37[i]	226±21	274±17	147−212	$\geq (m_{B_s}/m_B) f_B$
38[i]	173±16	217±20	187±24	200±19[j]
39[i]			170±20	
40[i]	165±15	200±15	115±15	

[a]Based on scaling via Eq. (3), assuming equal wave functions for D and B and for D_s and B_s, and including the QCD correction of Ref. 10.
[b]Relativistic-model estimate.
[c]Lattice estimate.
[d]Paris value.
[e]Bologna value.
[f]We have multiplied the results quoted in Ref. 32 by a factor of 0.75 in accord with the estimate of nonscaling contributions presented there.
[g]Potential-model estimate.
[h]$\alpha_s = 2.8$ assumed in extracting $|\Psi(0)|^2$ from $m_{B^*} - m_B$. For another value of α_s, multiply these values by $\sqrt{2.8/\alpha_s}$.
[i]QCD sum-rule estimate.
[j]Based on our estimate of $m_{B_s}/m_B = 1.017$.

those related to it by substituting one or two vector mesons for the pseudoscalar meson(s). Neglecting differences between form factors for $q^2 = m_D^2$ and $m_{D_s}^2$, we expect

$$\frac{B(\bar{B}^0 \to D^+ D^-)}{B(\bar{B}^0 \to D^+ D_s^-)} \simeq \tan^2\theta_C \left[\frac{f_D}{f_{D_s}}\right]^2 \simeq 0.05 \left[\frac{f_D}{f_{D_s}}\right]^2, \quad (45)$$

or an expected branching ratio of $B(\bar{B}^0 \to D^+ D^-) \simeq 3 \times 10^{-4}$ using central values quoted above.

We now present numerical estimates for branching ratios for the remaining processes itemized earlier. For this we need a value of f_ρ, since we do not necessarily trust Eq. (3) for light-quark systems. Independently, from vector meson dominance, we estimate $m_\rho f_\rho \simeq \sqrt{2} m_\rho^2 / g_\rho$, and using the relation of Refs. 41 for $m_\rho / g_\rho = f_\pi$, we find $f_\rho = \sqrt{2} f_\pi$. This value is smaller than the value $f_\rho = 221$ MeV taken in Ref. 8, but agrees with the observed branching ratio for $\tau \to \rho\nu$ (see Ref. 42 for a discussion). For f_{D^*} and $f_{D_s^+}$ we use Eq. (3). We have taken the central value of Eq. (30) for w_0.

The results are summarized in Table III. We have taken experimental numbers from Refs. 1 and 19, applying the correction of Ref. 21 for the $D_s^- \to \phi\pi^-$ branching ratio where necessary. [The value for $B(\bar{B}^0 \to D^{*+} D_s^{*-})$ was obtained from numbers quoted in Ref. 1 via a subtraction.] Given the present large experimental errors, the agreement is satisfactory.

The ratios of rates into different polarization states of pairs of vector meson are of interest in the study of CP violation. In particular, states of $D^{*+}D^{*-}$ with different polarization have different CP properties. L and $\parallel$ come from the axial-vector current, consist of superpositions of S and D waves, and have $CP = +$, while $\perp$ comes from the vector current, consists of a P wave, and has $CP = -$. In Table IV we summarize the relative branching ratios into each polarization state for $D^{*+}\rho^-$, $D^{*+}D_s^{*-}$, and $D^{*+}D^{*-}$ final states.

The small amount of the $CP = -$ ($\perp$) contribution predicted for $D^{*+}D^{*-}$ final states is good news for the study of CP asymmetries. It means that a decay $\bar{B}^0 \to D^{*+}D^{*-}$ will lead to a dominantly $CP = +$ final state, so that any asymmetry between $\bar{B}^0$ and B^0 decays to $D^{*+}D^{*-}$ should be readily observable even before helicity analyses to separate out opposite-CP final states[43] are performed.

With the estimate $(f_D/f_{D_s})^2 \approx 2/3$, the ratio of branching ratios quoted in Table III, and the branching ratio $B(\bar{B}^0 \to D^+ D_s^-) \approx 1\%$, one estimates

TABLE III. Predictions for ratios of rates in $\bar{B}^0$ decays.

Subprocess	$b\to c\bar{u}d$	$b\to c\bar{c}s$	$b\to c\bar{c}d$
Ratio	$D^{*+}\pi^-/D^+\pi^-$	$D^{*+}D_s^-/D^+D_s^-$	$D^{*+}D^-/D^+D_s^-$
Observed	1.02 ± 0.37	1.5 ± 1.4	
Predicted	1	0.70	$0.036(f_D/f_{D_s})^2$
Ratio	$D^+\rho^-/D^+\pi^-$	$D^+D_s^{*-}/D^+D_s^-$	$D^+D^{*-}/D^+D_s^-$
Observed			
Predicted	1.9	0.70	$0.036(f_D/f_{D_s})^2$
Ratio	$D^{*+}\rho^-/D^+\pi^-$	$D^{*+}D_s^{*-}/D^+D_s^-$	$D^{*+}D^{*-}/D^+D_s^-$
Observed	6.8 ± 6.0	3.2 ± 2.7	
Predicted	2.2	1.81	$0.092(f_D/f_{D_s})^2$

$$B(\bar{B}^0\to D^{*+}D^{*-})\simeq 0.06\% \ . \tag{46}$$

The observation of the $D^{*+}D^{*-}$ final state should be possible with a data sample not much greater than that available at present from e^+e^- collisions at the energy of the $\Upsilon(4S)$, if each D^* can be identified with at least 10% efficiency.

VI. DISCUSSION AND SUMMARY

We compare our procedure briefly with that of Ref. 8. Both approaches utilize factorization, and both require form-factor information. Whereas Ref. 8 takes this information from oscillator wave functions, we have used the hypothesis of a universal form factor and some properties of observed decays to estimate the form factor directly. At present we do not extend our discussion to the case in which the spectator antiquark must combine with a light quark (u, d, or s) from the four-quark operator, since the limits of validity of the heavy-quark approach[3] are exceeded. No such limitation is encountered in the approach of Ref. 8. It has often been noted (see, e.g., Refs. 6 and 44–46) that some related results are possible in decays such as $B\to\pi\pi$ and $B\to\pi l\nu$, but those lie beyond the scope of the present discussion.

The original motivation for our investigation was the strong dependence of conclusions about the CKM matrix and *CP*-violating observables on decay constants such as f_D and f_B. Expectations for *CP*-violating asymmetries in the decays $(B^0,\bar{B}^0)\to J/\psi+K_S$ and related processes (including the $D^{*+}D_s^{*-}$ final state discussed earlier) contain an uncertainty which at present is *dominated* by the uncertainty in f_B.[47,48] If β is the angle in the unitarity triangle corresponding to $-\mathrm{Arg}(V_{td})$ (in the phase convention of Ref. 12), it was found in Ref. 48 that

TABLE IV. Relative branching fractions in $\bar{B}^0\to VV$ decays for polarization states L (both longitudinal), $\parallel$ (transverse, linear polarization states parallel to one another), and $\perp$ (transverse, linear polarization states perpendicular to one another).

Polarization state	$D^{*+}\rho^-$	$D^{*+}D_s^{*-}$ or $D^{*+}D^{*-}$
L	88%	55%
$\parallel$	10%	39%
$\perp$	2%	6%

$$\sin(2\beta)=(0.35\pm0.09)\frac{2/3}{B_K}\left[\frac{f_B}{140\ \text{MeV}}\right]^2 . \tag{47}$$

In turn, the magnitude of B_s-$\bar{B}_s$ mixing can be expressed in terms of a mass shift:

$$\frac{\Delta m(B_s-\bar{B}_s)}{\Delta m(B-\bar{B})}=\left[\frac{f_{B_s}}{f_B}\right]^2\frac{B_{B_s}}{B_B}\left|\frac{V_{ts}}{V_{td}}\right|^2 , \tag{48}$$

where the factors B reflect corrections of the vacuum-saturation hypothesis. We have argued here that $(f_{B_s}/f_B)^2\approx m_s/m_u\approx 1.6$, which is right in the middle of the range (1.2–2.0) quoted in Table II for various estimates.

The values employed in Secs. III and IV for branching ratios were based on the assumptions that (1) the $\Upsilon(4S)$ decays equally to charged and neutral $\bar{B}B$ final states, and (2) non-$\bar{B}B$ final states provide a negligible contribution to $\Upsilon(4S)$ decays. The second assumption may be questionable in light of the recently found decay[49] $\Upsilon(4S)\to J/\psi+\cdots$. There is an independent way of estimating the behavior of the form factor which circumvents many systematic errors, including the one just mentioned and ones associated with uncertainties in D meson branching fractions.

It is estimated in Ref. 15 that $Dl^-\bar{\nu}_l$ and $D^*l^-\bar{\nu}_l$ final states account for 0.64 ± 0.10 of the total $\bar{B}$ semileptonic decay rate. Let us attribute the remainder of the semileptonic rate to form-factor effects, i.e., to excitation of charmed states other than D and D^*. Then the above ratio should be equal to the ratio of the predicted sum of the $Dl^-\bar{\nu}_l$ and $D^*l^-\bar{\nu}_l$ decay rates with respect to this sum for a pointlike form factor ($w_0=\infty$). If one estimates w_0 directly using this condition, one finds $w_0=1.55^{+0.40}_{-0.27}$. This is a determination with at least as good an accuracy as that of Eq. (26), and with less model dependence. It is consistent with Eq. (30) for $w_0\simeq1.3$, but if substantial non-$B\bar{B}$ decays of the $\Upsilon(4S)$ are found to occur, all B branching ratios must be revised upward, and Eq. (30) will have to be revised in any case. Turning the argument around, a more precise measurement of the fraction of $\bar{B}$ semileptonic decays to $Dl^-\bar{\nu}_l$ and $D^*l^-\bar{\nu}_l$ final states may be the best prospect in the near term for specifying form-factor effects [assuming that the parametrization (8) is a reasonable approximation], and thus settling the question of non-$B\bar{B}$ decays of the $\Upsilon(4S)$.

To conclude, we have found directly from the decays $\bar{B}^0 \to D^+ D_s^-$ and $B^- \to D^0 D_s^-$ that $f_{D_s} = 259 \pm 74$ MeV, and have extrapolated this result to estimate $f_D = 207 \pm 60$ MeV, $f_B = 140 \pm 40$ MeV, and $f_{B_s} = 175 \pm 50$ MeV. The value for f_{D_s} implies $B(D_s \to \tau \bar{\nu}_\tau) = (4.6 \pm 2.6)\%$ and $B(D_s \to \mu \bar{\nu}_\mu) = (0.5 \pm 0.3)\%$. Prospects for improving the accuracy of these results lie most immediately with increasing the sample of $\bar{B} \to D^{(*)} D_s^-$ decays, and with increasing the accuracy of measurements of the semileptonic and nonleptonic branching ratios from which form-factor information was extracted. We have also estimated that the $\bar{B}^0 \to D^{*+} D^{*-}$ decay occurs 94% of the time to final states with even *CP*, which enhances the attractiveness of this process for the study of *CP*-violating asymmetries.

Note added in proof

(1) Updated values[50] for the branching ratios in Eqs. (22) and (23) are $B(\bar{B}^0 \to D^+ \pi^-) = (0.35 \pm 0.06 \pm 0.08)\%$ and $B(\bar{B}^0 \to D^{*+} \pi^-) = (0.30 \pm 0.06 \pm 0.06)\%$. (2) A recent analysis[50] makes use of the measured q^2 spectrum[51,52] in the decays $\bar{B} \to D^* l^- \bar{\nu}_l$ to extract f_{D_s} directly from the measured values of $B(\bar{B} \to D^{(*)} D_s^-)$ and $dB(\bar{B} \to D^* l^- \bar{\nu}_l)/dq^2$. Factorization[53] implies $6\pi^2 f_\pi^2 = B(\bar{B}^{*0} \to D^{*+} \pi^-)/[dB(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l / dq^2]_{q^2 = m_\pi^2}$, while a corresponding relation between $f_{D_s}^2$ and $B(\bar{B} \to D^* D_s^-)$ involves the kinematic factors given in Sec. II. The assumption of form-factor universality allows one to relate $B(\bar{B} \to D^* D_s^-)$ to $B(\bar{B} \to D D_s^-)$ and thus to make use of a larger data sample from Ref. 1 (11 events instead of 8) than we used in determination of f_{D_s}. A new ARGUS result[21] for $B(D_s \to \phi \pi)$ was incorporated into the analysis, permitting further error reduction. The result of Ref. 50 for f_{D_s} is $276 \pm 45 \pm 44$ MeV. This method has the advantage that many systematic errors

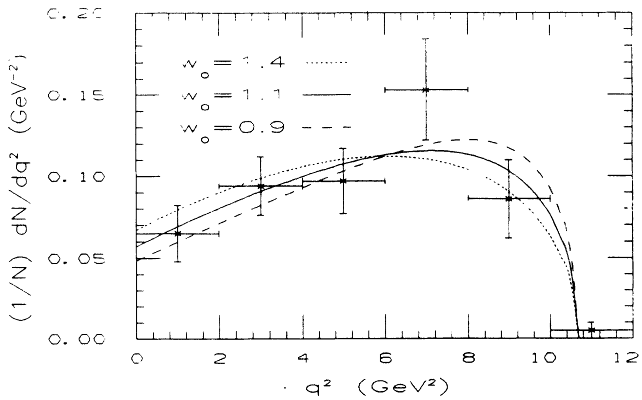

FIG. 3. Normalized spectrum in q^2 (the squared effective mass of the lepton pair) for the decay $\bar{B} \to D^* l^- \bar{\nu}_l$. Data points are based on our average of results in Refs. 51 and 52. Curves denote predictions based on the form factor (8) for various values of w_0.

(for example, those due to uncertainties in $|V_{cb}|$ and in D^* branching ratios) cancel one another out.

The consistency of our parametrization of the form factor $\xi(q^2)$ with the q^2 spectrum[51,52] in $\bar{B} \to D^* l^- \bar{\nu}_l$ is shown in Fig. 3. A satisfactory fit is obtained with $w_0 = 1.1^{+0.3}_{-0.2}$, consistent with our estimate (30).

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