Proton and neutron structure functions in a Fermi-gas approximation

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The proton and neutron deep-inelastic scattering structure functions are found to be fit reasonably well by a finite-temperature Fermi-gas (effective) description of the confined quarks. Support problems at negative x are avoided completely and formal sum rules are satisfied. The characteristic difference between the u and d valence distributions, which is difficult to achieve in many models of nucleon structure, arises naturally as a consequence of the different densities of u and d quarks in the nucleon.

I. INTRODUCTION

The theoretical determination of deep-inelastic-scattering (DIS) structure functions is an outstanding problem in high-energy nuclear physics. While we believe that their Q^2 dependence is explained by perturbative QCD we must always refer to an empirical determination at some given scale Q_0^2. A theoretical understanding of what governs the shape of DIS structure functions has become a matter of increased importance with the discovery of apparent dependence of nucleon structure functions on the nuclear environment, i.e., the European Muon Collaboration (EMC) effect.[1] Our lack of understanding of structure functions hampers interpretation of this phenomenon.

Attempts[2-4] to calculate structure functions in quark models invariably encounter support problems. The region of physical support is $x \in [0,1]$ where $x = Q^2/2p \cdot q$ is the Bjorken scaling variable ($Q^2 = -q^2$ where q is the four-momentum transfer and p is the target four-momentum). The upper bound, $x=1$ is a kinematic limit determined by the requirement that the final-state invariant mass in inclusive scattering must be at least that of the (stable) target while the lower bound $x=0$ is determined by the requirement that the energy loss $\nu \equiv q^0$ be positive. In the parton model x may be interpreted as the fraction k^+/p^+ of momentum of the target carried by the struck quark, where $k^+ = (k^0 + k^3)/\sqrt{2}$ is the projection of the quark's momentum k along the light cone. Thus good support corresponds to $k^0 \geq k^3$ (since $p^+ > 0$ always) plus the statement that no quark can carry more momentum than that of the target as a whole. Unfortunately these conditions are found to be violated in naive quark model calculations. The calculated structure functions typically receive contributions over the entire region $-\infty < x < +\infty$.

There have been some calculations conducted in Bethe-Salpeter and light-cone formalisms which do avoid support problems.[5] However they suffer from loss of the insight afforded by quark models.

Our purpose here is to explore an approximation scheme for the nucleon which removes the negative x problem, and provides some insight into the shape of (spin-averaged) DIS structure functions. We find, in particular, that the spread of the structure functions is related to the fermionic density of the quarks and also that the shape is insensitive to small changes in the quark mass. In brief, our approximation scheme is to treat the quarks as part of an infinite free Fermi gas at finite temperature. This is consistent with the *inspiration* for the original MIT bag model where free quarks propagate in a confining cavity.[6] A similar picture has also been employed quite successfully for the quasielastic response of nuclei.[7] A subtle but important difference here is our use of a finite temperature. This mimics some of the volume-dependent effects of confinement.

It is obvious that such a model for nucleon structure allows great simplicity for calculations. First, a model with free quarks eliminates any problems with spurious contributions to structure functions at negative x. For any free (on-mass-shell) physical (i.e., timelike) particle the energy is greater than any component of momentum and thus $k^+ > 0$. The (approximate) validity of the free quark picture can be linked with the light-cone dominance of DIS and the asymptotic freedom property of the QCD interactions between quarks. Second, an infinite uniform medium eliminates problems that would otherwise arise from center-of-mass (c.m.) motion. The difficulties in extracting c.m. motion in many-body systems are well known.[8] These difficulties are compounded in the calculation of the nucleon structure functions because of the need for a relativistic description of the bound state for light quarks.[9] In a relativistic system even the definition of the c.m. is problematic.[10]

It should also be apparent that such a simple model has seemingly serious conceptual difficulties. First, what is the connection between an infinite-Fermi-gas approximation and a nucleon (presumably a system of three valence quarks, gluons, and an ocean of $q\bar{q}$ pairs all localized in space)? Second, why should the assumption of a finite-temperature Fermi gas be an appropriate parametrization of the effects of confinement in nucleon structure? And does not the use of free quarks trivialize the support problem? We shall provide some answers to these questions in this paper. We claim that the calculation of DIS in this model is actually a much closer representation of reality than one would naively expect.[11]

Given the simplicity of the physical picture employed, our results provide good qualitative agreement with experimental structure functions. In particular, we obtain remarkably good results for the proton and neutron electromagnetic structure functions F_2^p and F_2^n. In most quark-model calculations it is quite difficult to reproduce the rather different shape of these, given the near degeneracy of the proton and neutron masses and almost identical single-quark wave functions.[12] However, in the present model the different densities of u and d quarks in the nucleon lead, via the Pauli exclusion principle, to different quark momentum distributions for these two flavors; this produces a d-quark distribution which falls off faster than the u-quark distribution as x approaches 1—in qualitative agreement with experiment. The different charge weightings of the dominant quark species then lead to different F_2 structure functions for proton and neutron.

We commence the main body of this paper in Sec. II with discussion and motivation for our finite-temperature Fermi-gas approximation. We argue that a reasonable estimate of leading-twist effects in DIS can be obtained from a model with freely propagating positive-energy quarks. We also discuss our notion of a finite-temperature Fermi gas relative to the original MIT bag-model paper.[6] Our interpretation of the role of the finite temperature and indeed of the relevance of the Fermi-gas picture is noted to be very different from that espoused in the MIT paper. In Sec. III we derive the equations of state for the Fermi gas. We compare our results with those obtained in three different models of quark confinement. When one makes a uniform-medium approximation we show that our equations of state agree with those for quarks in the interior of the MIT bag, in soliton bag models, and in the color-dielectric model.

In Sec. IV we derive analytic formulas for the quark distributions in a Fermi gas. We also derive baryon number and momentum sum rules in this approximation. Our model is shown to reproduce successfully the baryon-number sum rule, in contrast to similar work by Cleymans and Thews.[13] Like several previous calculations,[3] our model predicts that the valence quarks saturate the momentum sum rule, in disagreement with experiment. We therefore evolve our quark distributions with lowest-order perturbative QCD up to a momentum scale where the quarks carry half of the nucleon momentum. In Sec. V we present our results for the proton and neutron structure functions. We conclude in Sec. VI with a discussion.

II. FINITE-TEMPERATURE FERMI-GAS ANSATZ

The notion that quarks inside a hadron behave like a free Fermi gas at finite temperature was discussed in some detail in the original paper on the MIT bag model.[6] The clear implication there is that the finite temperature is associated with a two-phase picture of QCD. At low temperature QCD is believed to be a confining theory which undergoes a deconfining phase transition at higher temperature. Since the observed scaling behavior of DIS structure functions indicates that quarks behave as if free inside a hadron, the MIT group argued that the interior region could be likened to the high-temperature deconfining phase. However, a rather different interpretation is advocated here.

The MIT group argued that for highly excited states the temperature T was approximately $B^{1/4}$ (of order 150 MeV) where B is the bag constant. This correspondence was drawn by assuming a high-temperature limit for the quark gas so that the T^4 radiation law applies. Now 150 MeV is a very high temperature (approximately 10^{12} K); but for the MIT argument to be valid, the temperature T must be much greater than the quark momenta. This is not the case: the quark momenta are of the same order as $B^{1/4}$ or even larger (particularly for high excitations). There is no physical regime in which the T^4 law is valid.

The temperature T in our model is an *effective temperature,* introduced to impart to the (infinite) Fermi gas some of the qualitative features of quarks confined to a small volume, e.g., the presence of $q\bar{q}$ pairs. This effective temperature in our model is determined self-consistently; in the following section we show that such temperatures are an order of magnitude smaller than suggested by the MIT group. There is no reason to believe this lower temperature is associated in any way with a deconfining phase transition.[14]

The MIT group also asserted that a statistical description of the quarks as a free Fermi gas would not be valid for low-lying states. They argued that the wavelength of the quark has to be much smaller than the bag dimensions for the interactions in the boundary region to be negligible. For low-lying states the quark three-momentum $|\mathbf{k}|$, is of order $1/R$ (where R is the bag radius) and so this condition is not met. Since we employ a Fermi-gas picture for the ground state of the nucleon we need to justify our procedure. The key lies in our interpretation of the Fermi gas and its correspondence with the nucleon. We note here that Cleymans and Thews[13] assumed the MIT interpretation to be relevant to DIS structure functions and were led by the above MIT argument to exclude low x from the region of validity of the Fermi-gas model. This is not the case with our interpretation.

It is appropriate to begin by explaining what we do *not* do. We do not claim that three quarks moving in orbits with discrete energies look anything like a Fermi gas with a continuous spectrum. The picture of quarks with discrete energies can in fact be very misleading when it comes to calculating a structure function. It implies solving the time-dependent Schrödinger equation for quarks in an effective potential to obtain localized wave functions and eigenenergies ϵ. Since the quarks are localized they are not in eigenstates of momentum. However, one can make a Fourier expansion of the wave function in $\mathbf{k}$ space. It is tempting to presume that the quark distributions in DIS are then obtained from the momentum distribution in $\mathbf{k}$ space by defining $mx=\epsilon+k^3$. Unfortunately the distribution in k^3 extends from $-\infty$ to $+\infty$ and the wrong support is obtained for the distribution in x (which is simply the distribution in k^3/m translated from the origin to ϵ/m). An essential problem here is that the energy eigenvalue ϵ fully incorporates the effect of the confining interaction. We suggest here that this is inappropriate. Also ϵ is not associated with an energyconserving (or, more to the point, a p^+-conserving) quark-hadron vertex. Thus there is no basis for associating the "quark-model quark" in its energy eigenstate with the quark of the parton model. Considering the nature of the quark-model quark,[15] it is little wonder that naive calculations result in unphysical support.

To proceed we need to understand what is measured in DIS. We take some guidance from experiment and QCD. DIS reveals pointlike constituents of the nucleon through scaling behavior of the structure functions. Slow logarithmic scaling violations may be ignored for the moment: to first approximation the (quark) constituents behave as if free. This does not mean that there are no gluons present, but merely that an impulse approximation for the struck quark is valid.

How can the quarks appear to be free in a confining theory? The answer[16] lies in the asymptotic-freedom property of QCD[17] wherein the interactions between quarks vanish at large spacelike momenta, and the nature of DIS which probes the (nucleon) target matrix element of a current commutator close to the light cone.[18] This matrix element is intimately related to the amplitude for forward virtual Compton scattering, depicted graphically in Fig. 1, where we have assumed that at high Q^2 the virtual photon interacts with a single quark constituent. Because of the high momentum flowing through this quark line it propagates (along the light cone) between 0 and ζ in a quasifree state, i.e., on its mass shell. This validates a free-field picture such as the parton model, modulo logarithms generated by gluon radiation and vertex corrections.[16] The struck quark is not really free, but its strong interactions with the spectator quarks are not sensed during the virtual Compton scattering. However, these strong interactions are present in the initial state (before the photon interaction) and must be included in a calculation of the structure function.[19] It is only the quark current which may be treated as if free.

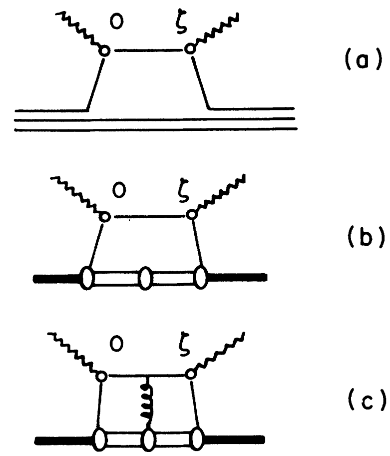

FIG. 1. Forward virtual Compton scattering. (a) Free-field theory; (b) parton-model approximation to QCD; (c) final-state QCD interactions, which generate higher-twist terms which vanish as a power of $1/Q^2$.

Since we cannot calculate the details of the confined nucleon state in QCD we must model it; we then calculate the DIS structure functions in a manner consistent with this model. As a preliminary consider free-field theory. At high momentum transfer only the leading light-cone singularity of the current commutator survives. In a free-field theory it is well known[20] that the relevant piece for an electromagnetic probe is

$$\tfrac{1}{2}\{[J_\mu(\zeta),J_\nu(0)]+[J_\nu(\zeta),J_\mu(0)]\}=[\bar\psi(\zeta)e_q^2\gamma^\beta\psi(0)-\bar\psi(0)e_q^2\gamma^\beta\psi(\zeta)]$$
$$\times(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta})\partial_\zeta^\alpha\frac{\epsilon(\zeta^0)}{2\pi}\delta(\zeta^2)+\cdots\ . \tag{2.1}$$

The resulting contribution to the electromagnetic structure functions F_1 and F_2 is often referred to as the leading-twist (specifically twist-2) piece and may be expressed[21] in terms of a light-cone Fourier transform of the correlation function

$$C(z)=\frac{2}{\rho}\overline{\sum_{\text{pol}}}\langle p|\psi_+^\dagger(\zeta)\psi_+(0)|p\rangle_c|_{\zeta=(z,0,0-z)}\ . \tag{2.2}$$

Here $\overline{\sum}_{\text{pol}}$ denotes an average over the polarization states of the nucleon target with four-momentum p, $\psi_+\equiv\frac{1}{2}(I+\alpha^3)\psi$ where ψ is the quark field operator, c denotes a connected matrix element, and ρ is the average density of the quarks. In an interacting theory the situation is considerably more complicated.[22] The usual procedure is to write an operator-product expansion for the current commutator. The twist-2 terms constitute an infinite series analogous to a Taylor expansion of the quark bilinears in (2.1). However, each term in this series is now an ultraviolet-divergent composite operator requiring renormalization. (This renormalization introduces momentum dependence and scaling violations.) Since each renormalized term has its own renormalization constant, the formal summation of the series is not normally possible.[22] Thus there is no immediate analogue of the quark correlation function in Eq. (2.2).

Fortunately, things are simplified in a quark model because there the quantum fluctuations are neglected. In other words, we are dealing with a mean-field theory. In this case the composite operators are rendered finite by application of standard mean-field normal-ordering prescriptions, and a quark correlation function may be properly defined. However, the resulting correlation function *is not* the same as in free field theory. In a mean-field approximation to QCD, the interactions modify the leading light-cone singularity such that a gauge factor $P\exp[-i\int_0^\zeta dz_\mu A^\mu(z)]$ appears in the correlation function (2.2). This gauge factor precisely cancels the effect of the vector gluon field on the space-time evolution of $\psi(\zeta)$. The quark correlation function may be evaluated (almost) as if the gluons were not there at all.[23] Thus all we need know is the Fourier expansion of the quark field at $t=0$. It is only here that the strong confining interactions enter the problem: the effect of confinement will dictate the quark and antiquark occupation numbers in **k** space.

It is possible that the above argument is incomplete because in soliton-bag[25] and color-dielectric[26] models of confinement, the strong confining forces of QCD appear via an effective *scalar* field. The effect of a confining scalar field on the leading-twist contribution to DIS is not well understood. However, in bag models the scalar potential is essentially uniform in the bag interior.[27] The effect of a uniform scalar potential on the leading-twist contribution is readily incorporated by rescaling the quark mass, or equivalently by rescaling the correlation length.[24] We may implement this in a local density approximation provided that the boundary region may be neglected on the scale of the quark correlation function. It is important to consider whether or not this is a valid approximation.

The strong scalar confining forces in the boundary region presumably affect the quark's propagation from 0 to ζ in the light-cone correlation function, either directly because of the strong confining forces[28] or indirectly through reconstructing the original momentum eigenstate of the target upon reinsertion of the quark line. Consider the MIT bag model[6] in which the boundary conditions exactly confine the quarks inside a sphere of radius R. After projecting out a momentum eigenstate (via the Peierls-Yoccoz procedure[8]) the maximum permitted quark correlation length (without invoking hadronization of the quark and spectators) is $z=4R$—corresponding to the situation depicted in Fig. 2.[29] The boundary region is expected to play a significant role in the quark correlation function well before this distance, because the correlation function must fall smoothly to zero.

Compare this with the light-cone correlation function for free massless quarks, which is readily determined[24] to be

$$C(z)=\frac{3}{2}\left[\left[\frac{\sin(2k_Fz)}{(k_Fz)^3}-\frac{\cos(2k_Fz)}{(k_Fz)^2}-\frac{1}{(k_Fz)^2}\right]+i\left[\frac{1}{(k_Fz)^3}-\frac{\cos(2k_Fz)}{(k_Fz)^3}-\frac{\sin(2k_Fz)}{(k_Fz)^2}\right]\right], \tag{2.3}$$

where k_F is the Fermi momentum of the quarks (in a zero-temperature Fermi gas). This is depicted in Fig. 3 where it can be seen that $C(z)$ becomes quite small for $z \gtrsim 4/k_F$ (this remains true at finite temperature). Using $\rho=\gamma k_F^3/6\pi^2$ and $\gamma=12$ (appropriate for three colors, two spins, and two flavors) we find $1/k_F \simeq \frac{2}{3}R$. Thus the correlation function is already damped on the same scale as the bag dimensions.[30] We conclude that the principal damping factor is the quark density rather than the confinement. Hence we expect that scalar confining interactions will only affect the free quark correlation function in a region where it is already quite small. So long as we are also ignoring quark hadronization, we may legitimately neglect the confining interactions.

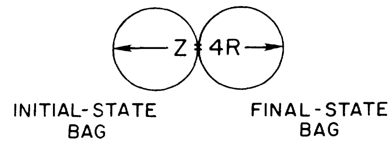

FIG. 2. The maximum allowed correlation length z in the center-of-mass corrected MIT bag model.

The situation here is somewhat reminiscent of that with the (equal-time) Pauli-exclusion-principle correlations encountered in nuclear physics. There one finds that if attention is restricted to the region of high nucleon density the infinite-Fermi-gas correlations are very similar to those calculated with nuclear shell-model wave functions.[32] (The correspondence is even closer if one makes c.m. corrections to the shell models.) This means that the important shorter-range correlations are dictated by the density of the fermions and are insensitive to details of the wave functions. Since these equal-time correlations are closely related to the light-cone correlations of interest here[24] it is not surprising that we find a similar phenomenon.

Therefore we conclude that it is reasonable to treat the quarks as propagating freely, for the purpose of evaluating the leading-twist contribution to DIS in baglike models. Specifically we may regard the difference between $\psi_+^\dagger(0)\psi_+(0)$ and $\psi_+^\dagger(\zeta)\psi_+(0)$ as due to the propagation of a packet of plane waves $e^{ik\cdot\zeta}$ with four-momenta obeying $k^2=m_q^2$ where m_q is the *effective* quark mass.[33] This guarantees that only positive-energy quarks contribute in the $k^+>0$ region.[24]

It remains to determine the occupation numbers of the quarks in **k** space. This is a highly nontrivial task in a relativistic (bag) model because, for example, we do not know how to make precise center-of-mass corrections. It

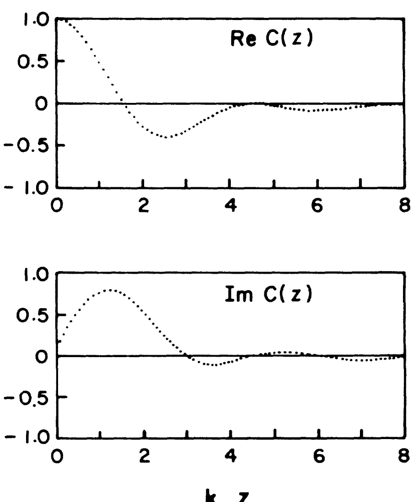

FIG. 3. The real and imaginary parts of the light-cone correlation function $C(z)$ for free massless quarks.

is at this stage that we introduce a simplifying ansatz. We assert that *the Fourier decomposition of the confined quark field will look something like a free*[34] *Fermi gas at finite temperature.* The finite temperature smears the unrealistic $T=0$ distributions and introduces antiquarks, which we know must be present if the confining volume is small. The value of T in our model will be determined self-consistently by solving the equations of state for physical values of the quark number and energy densities. Of course the "actual" temperature is essentially zero. To emphasize that we are dealing with an effective temperature we henceforth write it with a subscript, T_e. In the final analysis T_e will be eliminated as an independent degree of freedom and will become an implicit function of the confinement volume.

By introducing this model for the quark occupation numbers we circumvent the difficulties usually encountered with calculating structure functions in quark models.[35] Indeed in this model we are even able to obtain an analytic solution for the structure function. However, there is a potential problem with the support. Since the finite-temperature gas is an infinite system (of infinite mass) the physical region of support in x is 0 to ∞ rather than [0,1] as for a nucleon.[36] An important test of our model is that it not produce large contributions in the unphysical region $x>1$. In Sec. IV we show that for reasonable values of the parameters the structure function is numerically negligible for $x>1$. Since we completely avoid the problems typically encountered for $x<0$ in other calculations (these are usually a sizable fraction of the total structure function), we regard this state of affairs to be satisfactory.

III. EQUATIONS OF STATE

The thermodynamics of a finite-temperature Fermi gas has been extensively studied.[37] In this section we will review the equations of state for this model. In order to compare our resulting equations, we also present three conventional models of quark confinement: the MIT bag model, the soliton bag, and the color-dielectric model. Each of these models introduces a different mechanism for quark confinement. For each of these models we show that a uniform local density approximation for the interior of the bag gives equations of state which correspond directly to our Fermi gas model. We then solve the ensuing equations of state for realistic parameters.

We adopt the standard procedure of writing the stress-energy tensor for a uniform system as

$$T^{\mu\nu}=(\mathcal{E}+p)u^{\mu}u^{\nu}-pg^{\mu\nu}\ , \tag{3.1}$$

where $\mathcal{E}$ is the energy density, p is the pressure, and the velocity vector u is $(1,\mathbf{0})$ in the system's rest frame. Thus

$$\mathcal{E}=\langle T^{00}\rangle \tag{3.2}$$

and

$$p=\tfrac{1}{3}\sum_i \langle T^{ii}\rangle\ . \tag{3.3}$$

Let us now consider $T^{\mu\nu}$ for various systems.

A. The MIT bag model[6]

The MIT Lagrangian density is

$$\mathcal{L}_{\text{MIT}}=[\bar{\psi}(i\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}-m_q)\psi-B]\Theta_V-\tfrac{1}{2}\Delta_S\bar{\psi}\psi\ , \tag{3.4}$$

where $\Theta_V=1$ inside the bag and 0 outside while Δ_S is a surface delta function, and we have defined $\overleftrightarrow{\partial}\equiv\frac{1}{2}(\overrightarrow{\partial}-\overleftarrow{\partial})$. A uniform local density approximation to the interior region is obtained with

$$\mathcal{L}_U=\bar{\psi}(i\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}-m_q)\psi-B\ . \tag{3.5}$$

This leads to the free Dirac equations of motion for ψ and $\bar{\psi}$. The corresponding stress-energy tensor is

$$T^{\mu\nu}=\Theta^{\mu\nu}+Bg^{\mu\nu}\ , \tag{3.6}$$

where

$$\Theta^{\mu\nu}\equiv i\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi \tag{3.7}$$

is the quark contribution and we have used the equations of motion in (3.6). Thus

$$\mathcal{E}=B+\langle\Theta^{00}\rangle\ , \tag{3.8}$$

$$p=-B+\tfrac{1}{3}\sum_i\langle\Theta^{ii}\rangle\ . \tag{3.9}$$

B. Freidberg-Lee soliton model[25]

Ignoring gluon terms the Lagrangian density is

$$\mathcal{L}_{\text{FL}}=\bar{\psi}(i\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}-m_q-g\sigma)\psi+\tfrac{1}{2}(\partial_{\mu}\sigma)^2-U(\sigma)\ . \tag{3.10}$$

$U(\sigma)$ has a local minimum at $\sigma=0$ and a global minimum at $\sigma=\sigma_V$. A two-phase soliton solution connects these minima. Figure 4(a) shows a typical behavior of the σ field. The interior, where σ is small and slightly

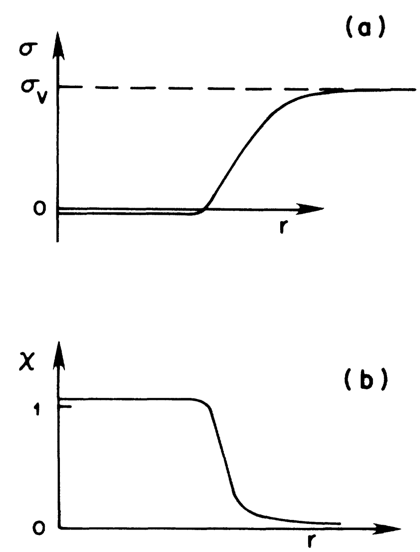

FIG. 4. The soliton fields σ and χ in (a) the Friedberg-Lee and (b) the color-dielectric models.

negative, corresponds to a baglike region. We can make a uniform local-density approximation in the interior by using

$$\mathcal{L}_U=\bar{\psi}(i\gamma^\mu\overleftrightarrow{\partial}_\mu-m_q-g\sigma)\psi-U(\sigma)\ , \tag{3.11}$$

where σ is a constant. The quarks in this approximation obey a free Dirac equation with effective mass

$$m_q^*=m_q+g\sigma \tag{3.12}$$

and

$$T^{\mu\nu}=\Theta^{\mu\nu}+g^{\mu\nu}U(\sigma)\ . \tag{3.13}$$

The equations of state are the same as those for the MIT model [Eqs. (3.8) and (3.9)] but with the substitutions $B\rightarrow U(\sigma)$ and $m_q\rightarrow m_q^*$.

C. Color-dielectric model[26]

After rescaling the quark fields $\chi^{1/2}\psi\rightarrow\psi$ the Lagrangian density is

$$\mathcal{L}_{\rm CD}=\bar{\psi}\left[i\gamma^\mu\overleftrightarrow{\partial}_\mu-\frac{m_q}{\chi}\right]\psi+\tfrac{1}{2}\sigma_V^2(\partial_\mu\chi)^2-U(\chi)\ . \tag{3.14}$$

$U(\chi)$ has a local minimum at $\chi=1$ and a global minimum at $\chi=0$. A two-phase soliton solution connects these minima. The interior region, where χ slightly exceeds 1 [see Fig. 4(b)], corresponds to a perturbative phase in which the quarks are localized as in a bag. We make a uniform-local-density approximation to this region by taking

$$\mathcal{L}_U=\bar{\psi}\left[i\gamma^\mu\overleftrightarrow{\partial}_\mu-\frac{m_q}{\chi}\right]\psi-U(\chi)\ . \tag{3.15}$$

The quarks obey a free Dirac equation with effective mass

$$m_q^*=m_q/\chi \tag{3.16}$$

and

$$T^{\mu\nu}=\Theta^{\mu\nu}+g^{\mu\nu}U(\chi)\ . \tag{3.17}$$

Again, it is clear that the equations of state are the same as in the MIT bag but with the substitutions $B\rightarrow U(\chi)$ and $m_q\rightarrow m_q^*$.

Thus each of these three models leads to essentially the same equations of state. Recalling our ansatz that the volume-dependent effects of confinement are to be mimicked in the uniform local density approximation by invoking an effective finite temperature T_e we obtain[37]

$$\mathcal{E}=B+\sum_j\gamma_j\int\frac{d^3k}{(2\pi)^3}E^*[n_k(T_e)+\bar{n}_k(T_e)]\ , \tag{3.18}$$

$$p=-B+\tfrac{1}{3}\sum_j\gamma_j\int\frac{d^3k}{(2\pi)^3}\frac{\mathbf{k}^2}{E^*}[n_k(T_e)+\bar{n}_k(T_e)]\ , \tag{3.19}$$

$$\rho_j=\gamma_j\int\frac{d^3k}{(2\pi)^3}[n_k(T_e)-\bar{n}_k(T_e)]\ , \tag{3.20}$$

where $\gamma_j=6$ is the degeneracy factor of quarks of flavor $j=\{u,d\}$, $E^*=(\mathbf{k}^2+m_q^{*2})^{1/2}$, and n_k and $\bar{n}_k$ are the fermion and antifermion occupation number distributions

$$n_k(T_e)=\{\exp[(E^*-E_F^*)/T_e]+1\}^{-1}\ , \tag{3.21}$$

$$\bar{n}_k(T_e)=\{\exp[(E^*+E_F^*)/T_e]+1\}^{-1}\ . \tag{3.22}$$

Here E_F^* is a characteristic energy for the fermion distributions. In the absence of a vector potential it is equal to the chemical potential μ.[37,38] We shall frequently express the energies E_F^* in terms of an equivalent "Fermi momentum" k_F defined through

$$E_F^*\equiv(k_F^2+m_q^{*2})^{1/2}\ . \tag{3.23}$$

Equations (3.18)–(3.22) are solved self-consistently by noting that for equilibrium $p=0$. Moreover, if (as in the MIT bag) there are no surface contributions, the nucleon mass is just

$$M=\mathcal{E}V\ , \tag{3.24}$$

where $V=\frac{4}{3}\pi R^3$ is the volume of the spherical bag. In principle we can also solve for m_q^* in terms of m_q but since m_q is just a parameter in this calculation, we simply treat m_q^* as the parameter. Thus our parameters are R, M, m_u^*, and m_d^*.

We first pick a trial value for T_e. The energies E_F^{*u} and E_F^{*d} are determined from

$$\rho_j=\frac{N_j}{V}\ , \tag{3.25}$$

where $N_u=2$ and $N_d=1$ are the numbers of quarks in a proton.[39,40] With these values we calculate M from Eq. (3.24) and compare it with our input value. We then modify T_e and repeat the above procedure until our calculated mass agrees with our input.

As an immediate consequence of the equilibrium condition, we see from (3.18), (3.19), and (3.20) that, in the MIT bag,

$$M=\sum\left\langle E^*+\frac{\mathbf{k}^2}{3E^*}\right\rangle\ , \tag{3.26}$$

where the sum is over each quark. This relation will be seen in the next section to play an important role in the momentum sum rule in DIS.

It is useful to consider the case of massless quarks both for orientation purposes and because this is a popular choice for the MIT model. In this case (3.26) reduces to the well-known virial theorem

$$M=\tfrac{4}{3}(N_u\langle E^u\rangle+N_d\langle E^d\rangle)=4BV\ , \tag{3.27}$$

where for each quark flavor we have defined

$$\langle E\rangle\equiv\frac{\gamma}{\rho}\int\frac{d^3k}{(2\pi)^3}E[n_k(T_e)+\bar{n}_k(T_e)] \tag{3.28}$$

and $\langle E^u\rangle\neq\langle E^d\rangle$ because of the different densities and quark distributions. Also we can analytically determine that for massless quarks

$$\rho_j=\frac{\gamma_jk_F^3}{6\pi^2}[1+(\pi T_e/k_F)^2] \tag{3.29}$$

and

$$\langle E\rangle=\frac{3}{4}k_F\frac{1+2(\pi T_e/k_F)^2+\frac{7}{15}(\pi T_e/k_F)^4}{1+(\pi T_e/k_F)^2}\ . \tag{3.30}$$

[Remember that k_F (and $\langle E \rangle$) in the previous equations are understood to be dependent upon the quark flavor j.] Note that for $T_e=0$, $\langle E \rangle$ reduces to $\frac{3}{4}k_F$ and thus we deduce

$$M \geq N_u k_F^u + N_d k_F^d \ . \tag{3.31}$$

Since the physical region is a low-T_e region, this sets the scale for the Fermi momenta at about one-third of the nucleon mass. The equilibrium condition also generates the inequalities

$$B \geq \frac{\gamma_u (k_F^u)^4 + \gamma_d (k_F^d)^4}{24\pi^2} \tag{3.32}$$

and

$$B \geq \left[\frac{\gamma_u + \gamma_d}{12} \right] \frac{7\pi^2 T_e^4}{30} \ . \tag{3.33}$$

The equilibrium surface generated by the self-consistent solution of these equations is depicted in Fig. 5. For small masses M and small radii R there is no solution. The boundary is a straight line at $T_e=0$ corresponding to the mass

$$M_Z = \sum_j \left[\frac{9N_j^4 \pi}{2\gamma_j} \right]^{1/3} \frac{1}{R} \ . \tag{3.34}$$

Altering the bag pressure B (at $T_e=0$) moves us along this line. For large R the surface again drops to $T_e=0$ (and $B=0$) and the mass is ill defined (because although the energy density is zero volume is infinite). Despite T_e being near zero the large R (and M) region is actually a high-temperature region in the sense that $T_e \gg k_F$; thus (for R large and fixed) M varies as T_e^4.

For finite quark masses we evaluate the integrals in Eqs. (3.18)–(3.20) numerically. The resulting equilibrium surface is not markedly different from that in Fig. 5 if the quark masses are kept small (i.e., no more than several tens of MeV). We shall note here only that the $T_e=0$ boundary is now given by [cf. (3.31)]

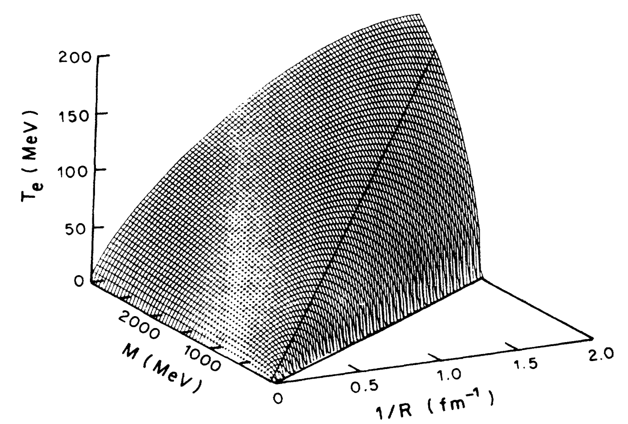

FIG. 5. Equilibrium surface generated by self-consistent solution of thermodynamic equations. The physical nucleon is constrained to lie on a line such as that shown.

$$M = N_u E_F^{*u} + N_d E_F^{*d} \tag{3.35}$$

which is not quite linear in $1/R$ and tends to the sum of the quark masses as $1/R \rightarrow 0$.

These equilibrium surfaces are equivalently obtained by maximizing the entropy with respect to volume change at fixed M and T_e. The entropy on the $p=0$ surface is obtained directly by integrating the first law of thermodynamics. One finds

$$S = \frac{M}{T_e} + S_0 \ . \tag{3.36}$$

If T_e were a genuine temperature, then the system would further maximize its entropy by cooling to $T=0$ at constant B. However, in our case $T_e \neq 0$ is only a device for introducing some of the properties of the confined states. Unlike an actual temperature, the effective temperature is not an independent dynamical degree of freedom and there must exist an explicit relationship between T_e and R. On dimensional grounds we must have

$$T_e = \frac{c}{R} \tag{3.37}$$

since R is the only dimensional parameter in the simplest bag model with massless quarks. Thus the physical nucleon state is constrained to lie on a line such as that shown in Fig. 5. Along this line, where also MR is constant (in the case of massless quarks), there exists an approximate dynamic equivalence between a bag of confined quarks at zero temperature and a free quark gas at finite temperature and the same density.

One may estimate the constant c in (3.37) by comparing with the analogous simple bag model result

$$M = \tfrac{4}{3}(N_u + N_d)\frac{\omega_0}{R} \ , \tag{3.38}$$

where the eigenfrequency $\omega_0=2.04$ (independent of R for massless quarks) is fixed by the (confining) bag boundary conditions.[41] However, Eq. (3.38) is subject to center-of-mass corrections. A simple estimate of these leads to[42]

$$M \simeq \left[\frac{4}{3}(N_u + N_d) - \frac{9}{8(N_u + N_d)} \right] \frac{\omega_0}{R} \ . \tag{3.39}$$

Since in this simple model the nucleon and Δ are degenerate we may take the "nucleon" mass as 1086 MeV. This leads to $R=1.25$ fm and suggests

$$c \simeq 85.8 \text{ MeV fm} \ . \tag{3.40}$$

This describes the line depicted in Fig. 5. We anticipate that relativistic center-of-mass corrections and other effects may produce significant deviations from the simple mass formula (3.39). Therefore, we have investigated points on the equilibrium surface in the region 900 MeV $\leq M \leq 1300$ MeV and 0.5 fm $\leq R \leq 1.5$ fm.[43] Small values of c are interpreted as implying weak confining interactions and large values, strong interactions. Note that in the physical region T_e is typically only a few tens of MeV and less than the Fermi momenta.

In Table I we display some values of the effective tem-

TABLE I. Equilibrium values of the Fermi variables for some proton masses and radii and quark masses.

R (fm)	M (MeV)	m^u (MeV)	m^d (MeV)	k_F^u (MeV)	k_F^d (MeV)	T_e (MeV)	$B^{1/4}$ (MeV)
0.9	1086	0	0	358.3	280.0	32.3	161.7
1.0		0	0	304.2	229.1	51.8	149.4
		20	20	304.6	229.6	51.4	149.2
		18	23	304.6	229.5	51.4	149.2
1.1		0	0	259.4	187.2	61.7	139.1
1.2		0	0	222.8	153.9	67.1	130.3
1.0	939	0	0	328.6	259.7	15.1	144.1
1.2		0	0	247.6	183.6	48.6	125.6
0.9	1232	0	0	333.6	249.1	61.7	166.8
1.0		0	0	278.9	198.3	72.6	154.2

perature, Fermi momenta, and bag constant for various values of M and R. We use these values as input to the formulas for the structure functions.

IV. LEADING-TWIST STRUCTURE FUNCTIONS

The Lagrangians of Sec. III in the uniform media approximation may all be classed as a special case of the Lagrangian studied in Ref. 24 where it was shown how the leading-twist structure function (per particle) could be calculated in mean-field approximation for fermions moving in uniform scalar and vector potentials. In summary, one quantizes the fermions in the theory, taking care to preserve Poincaré symmetry, and then extracts the leading light-cone singularity of the current commutator. No approximation is necessary other than those approximations standard in mean-field theory.

It is found that in mean-field approximation the structure functions scale and F_2 is given by the standard parton-model relation

$$F_2(x)=x\sum_j e_j^2[q_j(x)+\bar{q}_j(x)]\ , \tag{4.1}$$

where e_q is the charge of the jth quark. The quark and antiquark distributions are[24]

$$\begin{aligned} q_j(x)&=N_j[h^j(x)\Theta(x)+\bar{h}^j(x)\Theta(-x)]\ ,\\ \bar{q}_j(x)&=N_j[\bar{h}^j(x)\Theta(x)+h^j(x)\Theta(-x)]\ , \end{aligned} \tag{4.2}$$

with

$$h^j(x)=\frac{\gamma_j}{\rho_j}\int\frac{d^3k}{(2\pi)^3}\left[1+\frac{k^3}{E^*}\right]\times\left[n_k(T)\delta\left[x-\frac{E^*+k^3}{m_N}\right]-\bar{n}_k(T)\delta\left[x+\frac{E^*+k^3}{m_N}\right]\right] \tag{4.3}$$

[in Eq. (4.3) we have specialized to the case of no vector potential] and

$$\bar{h}^j(x)\equiv -h^j(-x)\ . \tag{4.4}$$

E^* is conventionally defined as the positive square root

$$E^*\equiv+\sqrt{k^2+m_q^{*2}} \tag{4.5}$$

for both quarks and antiquarks. Thus the first δ function in Eq. (4.3) can only be satisfied for $x>0$ and the second only for $x<0$. Performing the angular integrations we find

$$h(x)=\frac{\gamma m_N^2 x}{4\pi^2\rho}\int_{E_{\min}(x)}^{\infty}dE^*[n_k(T)\Theta(x)+\bar{n}_k(T)\Theta(-x)]\ , \tag{4.6}$$

where

$$E_{\min}(x)\equiv\frac{m_N}{2|x|}\left[x^2+\left[\frac{m_q^*}{m_N}\right]^2\right]\ .$$

Inserting the occupation numbers for a finite-temperature Fermi gas [Eqs. (3.21) and (3.22)] we obtain[24]

$$h(x)=\frac{\gamma m_N^2 x T_e}{4\pi^2\rho}[\Theta(x)\ln(1+\exp\{[E_F^*-E_{\min}(x)]/T_e\})+\Theta(-x)\ln(1+\exp\{-[E_F^*+E_{\min}(x)]/T_e\})]\ . \tag{4.7}$$

It is noteworthy that for $T_e=0$, where there are no antiquarks, this reduces to

$$h(x)=\frac{3}{4}\left[\frac{m_N}{k_F}\right]^3\left[\left[\frac{k_F}{m_N}\right]^2-\left[x-\frac{E_F^*}{m_N}\right]^2\right]\times\Theta\left[\frac{k_F}{m_N}-\left|x-\frac{E_F^*}{m_N}\right|\right]\ , \tag{4.8}$$

where E_F^* is now recognizable as the quark energy at the Fermi surface. The free-field limit is obtained by taking $m_q^*=m_q$. It is not immediately obvious but along a line such as described by Eq. (3.37) the distributions are invariant, if m_N is calculated self-consistently. The essential point to note is that E_F^*/T_e is also constant along such a line.

Some typical distributions are shown in Fig. 6 for the

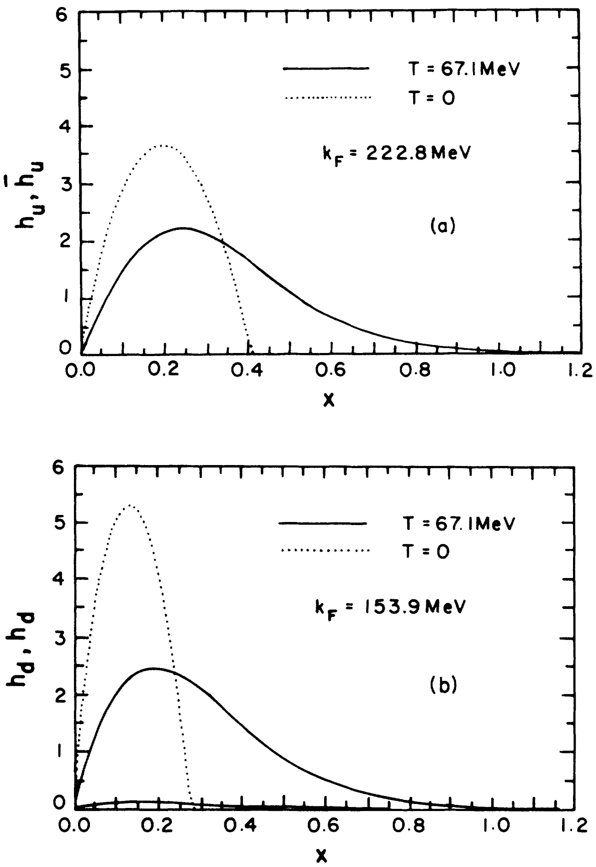

FIG. 6. The distributions h^j and $\bar{h}^j$ for (a) u and (b) d quarks in the Fermi-gas approximation to the proton [note that in (6a), the antiquark distribution $\bar{h}_u$ is so small that it is indistinguishable from the x axis]. The $T=0$ curves are shown for the same value of k_F and do not correspond to equilibrium states. The finite-temperature curves are for $M=1086$ MeV and $R=1.2$ fm.

case of massless quarks. The $T_e=0$ curves illustrate the difference between the full result Eq. (4.7), and the $T_e=0$ limit Eq. (4.8) for the same value of E_F^*. For small values of T_e, such as illustrated here, the antiquark distribution is quite small. Notice that both h and $\bar{h}$ vanish at $x=0$. The effect of including finite quark masses is to displace the $T_e=0$ curves to higher x, by an amount $(E_F^*-k_F)/m_N$, without altering the shape of $h(x)$. Clearly, the quark mass has to be comparable with k_F to have significant effect.

The structure function F_2 obtained from the above in the massless case agrees with that of Cleymans and Thews[13] who used a quite different derivation.[44] Their interpretation though and their numerical results differ markedly from ours. In particular they find that they "underestimate the baryon content by a substantial fraction." It appears that this is due to a failure on their part to determine the chemical potential and temperature self-consistently with the nucleon mass and volume.[45] Without a self-consistent formalism one will not obtain a properly normalized distribution (and the momentum sum rule will also be violated). A problem with the baryon-number sum rule was also encountered in an attempt to use a Boltzmann gas to describe the quarks in the nucleon.[46] In fact this is a problem rife in quark-model calculations of structure functions,[2] with many authors arbitrarily renormalizing their distributions to obey this sum rule. In our approach such arbitrary normalization changes are neither necessary nor allowed. We have from (4.2) and (4.3) that

$$\int_0^\infty dx[q^j(x)-\bar{q}^j(x)]$$
$$=N_j\int_0^\infty dx[h^j(x)+h^j(-x)]$$
$$=N_j\frac{\gamma_j}{\rho_j}\int\frac{d^3k}{(2\pi)^3}\left[1+\frac{k^3}{E^*}\right][n_k(T_e)-\bar{n}_k(T_e)] .$$

Noting that the k^3 term averages to zero in the integral and comparing with Eq. (3.20), wherein arises the need for self-consistency, we trivially obtain

$$\int_0^\infty dx[q^j(x)-\bar{q}^j(x)]=N_j . \tag{4.9}$$

(This result justifies our assertion that we have no spurious negative-x contributions.) The upper limit of integration here is infinity because we are dealing with an infinite system. In approximating the finite mass nucleon by this system we require that the contribution to this integral from the region beyond $x=1$ be small. We have checked this numerically and find that for realistic parameters the region $x>1$ typically contributes much less than 1%.

Similarly we may obtain an expression for the momentum sum rule. Again noting that $\langle k^3\rangle=0$ and also that $\langle (k^3)^2\rangle=\frac{1}{3}\langle \mathbf{k}^2\rangle$, we obtain

$$\int_0^\infty dx\, x[q^j(x)+\bar{q}^j(x)]=\frac{N_j}{m_N}\left\langle E^*+\frac{\mathbf{k}^2}{3E^*}\right\rangle . \tag{4.10}$$

Recalling Eq. (3.26) we thus have the result that the quarks saturate this sum rule, i.e., when summed over all quarks the right-hand side (RHS) of (4.10) equals $+1$. This result for the MIT bag was first noted by Jaffe.[3]

Unfortunately this prediction for the momentum sum rule is in disagreement with experiment, which measures only about one-half for the fraction of momentum carried by the quarks. The present model prediction can be modified readily by several means. The most obvious is to include a vector gluon field in the equation of state. This alters the equations of state for nonzero temperatures.[37] However even for very high temperatures this mechanism seems insufficient to transfer 50% of the nucleon's momentum away from the quarks.[47] We note that in the present model we can also modify the momentum sum rule by including surface terms in Eq. (3.24) for the nucleon mass.

In subsequent work we intend to take a closer look at the thermodynamics of interacting fermions, but for the present we adopt a procedure advocated by De Rújula and Martin[4] and Jaffe and Ross[48] several years ago (though the genesis of the idea appeared even earlier[49]). In this approach, the quark distributions are evolved with perturbative QCD until the quarks carry only 50% of the nucleon momentum. This procedure was implemented with a measure of success[48] for Jaffe's attempted calculation[3] of the MIT bag structure functions. (See also Close *et al.*[50] for an updated analysis.) The basic idea is that because gluons have not been explicitly included in the model we should regard the quarks as being dressed with gluons. In this respect the idea is the same as in the valon model.[51] Specifically, it is postulated in Refs. 4, 48, and 50 that the quark-model matrix elements correspond to those of field operators renormalized at some (low-) momentum scale. By evolving with perturbative QCD the gluon dressing is removed.

In what follows we shall be content with leading-order (LO) evolution. This may disturb some readers because we have to evolve a long way in $\ln Q^2$ to remove 50% of the momentum. However beyond LO it is not clear how to define the quark distributions. This is a well-known problem in the literature. To specifically relate it to our case, consider the prescription followed recently by Martin *et al.*,[52] i.e., scheme S of Furmanski and Petronzio.[53] In scheme S one evolves quark, antiquark, and gluon distributions in next-to-leading order and then calculates the leading-twist structure functions in terms of these using various coefficient functions. The relationship between the structure functions and the quark distributions can be approximated by parton-model relations such as Eq. (4.1) only at high Q^2. However, in the mean-field theory the parton-model relations are valid independent of Q^2 and in particular must be taken as valid at the starting scale μ^2 for the evolution procedure [where, for example, the sum rule (4.9) is valid]. Until we better understand the connection between the mean-field structure functions and those of the full theory we should be reluctant to go beyond LO.[54] One advantage of our model is that this problem can be systematically studied.

We conclude this section by noting that the valence distributions $u_v \equiv u - \bar{u}$ and $d_v \equiv d - \bar{d}$ are pure nonsinglet as is

$$F_2^p - F_2^n = x[\tfrac{1}{3}(u-d) + \tfrac{1}{3}(\bar{u}-\bar{d})] \; . \tag{4.11}$$

The sum of these, on the other hand, is

$$F_2^p + F_2^n = \tfrac{5}{9}x\Sigma - \tfrac{1}{3}x[(s-c)+(\bar{s}-\bar{c})] \; , \tag{4.12}$$

where the dominant piece

$$\Sigma = \sum_j (q_j + \bar{q}_j) \; , \tag{4.13}$$

is a pure singlet and mixes with the gluon distribution G under evolution. At the starting scale μ^2 the sum over j includes just u and d; s and c are zero. Because the nonsinglet distributions $s-c$ and $\bar{s}-\bar{c}$ are initially zero, they remain so under evolution and the second term in Eq. (4.12) may be discarded, in our model, at any Q^2.[55]

The evolution equations may be found in Ref. 56. There are several numerical methods available in the literature for implementing them. We have tried most and found quite a few to be unsatisfactory. The method of Jacobi polynomials[57] has been found adequate provided very careful attention is paid to the choice of weight function.

V. RESULTS

The input distributions to the QCD evolution equations are those such as in Fig. 6(b). Any contribution in the region $x > 1$ is discarded. (In most cases this results in less than 1% of the total momentum being lost. If more than 2% is lost we regard the infinite Fermi-gas approximation as having failed.) After evolution we compare with data, primarily from the European Muon Collaboration[58,59] (EMC), at $Q^2 = 15$ GeV^2. At this scale the EMC finds that the valence quarks carry $39.1 \pm 2.0\%$ of the nucleon momentum and all quarks and antiquarks together carry $46.5 \pm 2.3\%$. Gluons are presumed to carry the remaining 53.5%.

The fraction of momentum carried by gluons is fairly well agreed upon. However, the division of the balance of the momentum between valence quarks and ocean quarks appears to be less certain with neutrino experiments finding a significantly lesser amount to be carried by valence quarks than that found by the EMC. This raises the issue of whether we should evolve the model distributions until the nonsinglet valence distribution $u_v + d_v$ carries 39.1% of the momentum or until the singlet distribution Σ carries 46.5%. In general it is not possible to obtain both simultaneously. We choose to be governed by the valence distribution since that is the more reliably determined distribution in our model. Thus the amount of evolution is determined by the QCD expression for the second moments of $xu_v + xd_v$:

$$\frac{M_2^{\text{val}}(Q^2)}{M_2^{\text{val}}(\mu^2)} = \exp(-d_2^{\text{NS}} s) \; , \tag{5.1}$$

where

$$s = \ln \left[\ln \left[\frac{Q^2}{\Lambda_{\text{LO}}^2} \right] \Big/ \ln \left[\frac{\mu^2}{\Lambda_{\text{LO}}^2} \right] \right] \; . \tag{5.2}$$

We have $M_2^{\text{val}}(Q^2) = 0.391$; $M_2^{\text{val}}(\mu^2)$ is a model-dependent number close to 1 and d_2^{NS} is the second nonsinglet anomalous dimension in LO. The value of Λ_{LO} chosen then determines the starting scale μ^2. The actual value of μ^2 is of little interest; we typically find values about 1.5 times Λ_{LO}^2 for reasonable values of Λ_{LO} (say 200 MeV). Note that the amount of evolution is determined entirely by the combination $d_2^{\text{NS}} s$ and our results for the valence distributions are independent of both the choice of Λ_{LO} and the number of flavors. In contrast the total quark distribution Σ does depend on the number of effective flavors used. We have chosen to evolve all the way from μ^2 to Q^2 with just two effective flavors because on a logarithm scale the bulk of the evolution is below all higher flavor thresholds. We do not believe that it is sensible to use more flavors without a proper treatment of the thresholds.[60]

Following the above procedure then, we obtain the valence distributions for the proton shown in Figs. 7 and 8 where in the first we have varied the equilibrium radius, R, at fixed proton mass M and in the second we have varied M at fixed R. (In both figures we take the quark masses to be zero.) In addition to the EMC data we have shown some neutrino data from the CERN-Dortmund-Heidelberg-Saclay (CDHS) Collaboration[61] and the WA25 experiment.[62] The differences between these data sets are greater than the acknowledged experimental er-

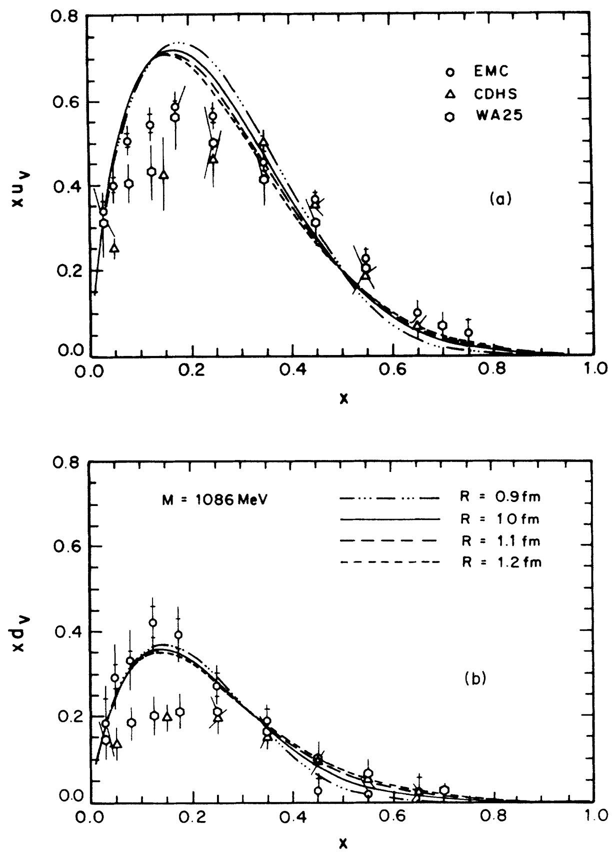

FIG. 7. The valence distributions in the proton at $Q^2=15$ GeV^2 as a function of Bjorken x for $M=1086$ MeV and various bag radii. The EMC data are from Ref. 58, the CDHS data are from Ref. 61, and the WA25 data of Ref. 62 were extrapolated to $Q^2=15$ GeV^2 in Ref. 58. The CDHS and WA25 errors are statistical only. (a) $xu_v(x)$, (b) $xd_v(x)$.

rors so some caution is required in judging the model results.

We see that very good agreement with the EMC data on xd_v is obtained but that our model xu_v distribution is too narrow. The agreement with the neutrino data looks worse though it is to be remembered that we evolved our distributions so that the sum of the areas under the xu_v and xd_v curves is 0.391 as required by the EMC data. We have shown the neutrino data to emphasize the difficulty in obtaining these measurements. In fact Martin *et al.*[63] raise further uncertainty over the data in drawing attention to differences between the EMC muon data and preliminary muon data from the Bologna-CERN-Dubna-Munich-Saclay (BCDMS) collaboration. In their analysis of the respective distributions derived from the experiments Martin *et al.* find that the BCDMS u_v distribution exceeds that of the EMC at low x (less than about 0.45) and is less than EMC at high x. The ratio peaks at about 1.25 at $x=0.2$. On the other hand the BCDMS d_v distribution is up to 5% less than EMC at $x<0.2$ and exceeds EMC at large x. If we were to accept this analysis then we could claim excellent agreement with both u_v and d_v data.[64] Given the approximations in our model though this would undoubtedly be fortuitous.

As we increase the equivalent bag radius at fixed M our distributions become broader,[65] though the incremental difference becomes less as R increases. This latter feature is to some extent an artifact of the evolution ansatz. At large R there is an appreciable ocean-quark distribution at the starting scale and also there is a loss of momentum from the support mismatch so that the amount of evolution involved becomes progressively less. [In the case $R=1.2$ fm in Fig. 7 the amount of lost momentum has grown to 2% (the maximum we allow) and the ocean quarks carry just over 1% at the starting scale whereas for $R=0.9$ fm the valence quarks begin with 100% of the momentum.]

In Fig. 8 the case $M=939$ MeV leads to a distribution which is far too narrow. This is because the equilibrium value of the effective temperature is too small. Since the nucleon and Δ are degenerate in our model we prefer to use their average mass of 1086 MeV. The agreement with data is preferable to using the physical nucleon mass.

Taking any set of parameters the following features are evident in our distributions. (i) The xd_v distribution is narrower than the xu_v distribution, and (ii) the xd_v distribution peaks at a slightly, but noticeably, smaller value of x. Both of these features are due to the d quarks having a lower value of k_F than the u quarks and this in turn stems from their differing densities: there are two u quarks in the proton but only one d quark. It is well-

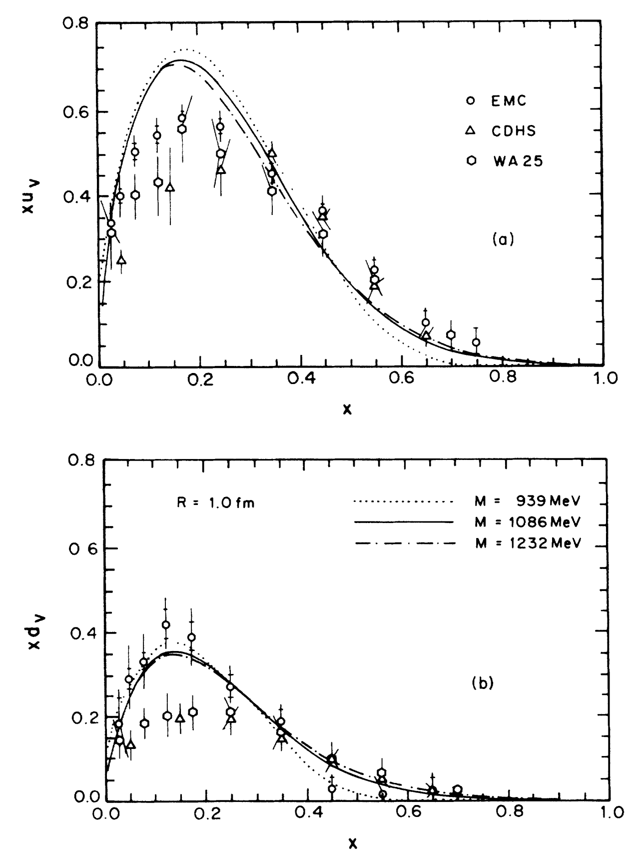

FIG. 8. The valence distributions in the proton at $Q^2=15$ GeV^2 as a function of Bjorken x for $R=1.0$ fm and various masses. The data is the same as in Fig. 7. (a) $xu_v(x)$, (b) $xd_v(x)$.

known experimentally that the first of these features is true and it has been a long-standing puzzle. We believe that our model goes some way toward explaining it. The data are not good enough to confirm the second feature beyond doubt but it does favor it.

The Fermi-gas approximation could be used to make predictions for the structure functions of the Δ, provided we overlook the fact that it is an unstable particle. In our approximation it is degenerate with the nucleon and the Δ^+ and Δ^0 quark distributions and structure functions will be the same as for the proton and neutron, respectively. In the Δ^{++} there are three u quarks and the u-quark distribution is predicted to be even broader than that in the proton. The d distribution in the Δ^- will be the same as the u distribution in the Δ^{++}. The possibility of scattering off of Δ constituents of nuclei has been the subject of some speculation in connection with the EMC effect.[1] The present theoretical predictions though are possibly the first for Δ structure functions.

We have tried using nonzero quark masses. The spectroscopic splitting of isospin multiplets suggests that $m_d - m_u$ is about 4 to 5 MeV in bag models.[66] The quark masses in these models are generally considered to be the same as the quark masses in current algebra and chiral perturbation theory requires the current quark masses to be small. Indeed it predicts the ratio m_u/m_d to be about one-half.[67] Using the most lax limit of 0.78 (Ref. 68) we note that $m_u = 18$ MeV and $m_d = 23$ MeV are about the heaviest quark masses that we can sensibly entertain. The distributions obtained with these masses, particularly after the evolution procedure, are not appreciably different from those for zero quark masses. As noted in the last section the important parameter in these distributions is E_F^* and this is an order of magnitude greater than these masses. Furthermore the difference between E_F^{*u} and E_F^{*d} is dictated by the differing densities of the u and d quarks and is hardly affected by their mass difference. Thus current quark masses have a negligible effect on the quark distributions. Henceforth we shall content ourselves with using zero quark masses.

The total ocean distribution, obtained by subtracting the valence contribution from the singlet-quark distribution Σ is shown in Fig. 9. The ocean for $R = 1.0$ and 1.1 fm is indistinguishable from that for 0.9 fm on the scale of the plot. The small differences for $R = 1.2$ fm we attribute to the fact that for this value of R, T_e is sufficiently large that there is a significant nonperturbative ocean present. We compare our result with the total ocean distribution $5x\bar{q}$ extracted by the EMC. The two are very similar though our ocean distribution is more singular at low x. The EMC ocean is itself though rather different from the more familiar distributions obtained from neutrino experiments. In particular it is very soft and falls much more rapidly with x than normally accepted. In fact they find a $(1-x)^{15}$-type behavior rather than the usual $(1-x)^7$. This clearly deserves to be resolved.

The electromagnetic structure functions F_2 for the proton and neutron in our Fermi-gas approximation are shown in Fig. 10 for $M = 1086$ MeV and a variety of radii. The structure at low x is genuine and results from combining an ocean distribution, which probably is too

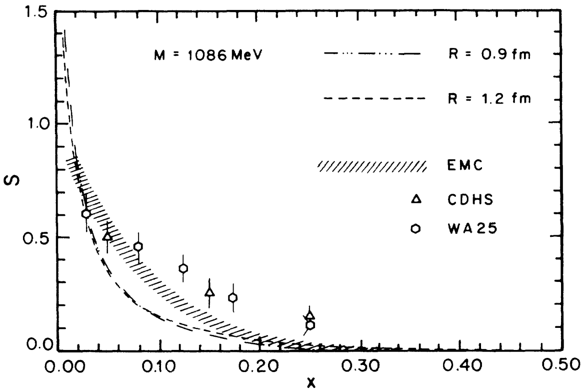

FIG. 9. The total ocean distribution $S(x) = x\Sigma - xu_v - xd_v$.

singular, with a valence contribution which is possibly too narrow. It is not clear to what extent this structure would survive in an improved model. Its simple origin does suggest that careful measurements are required in the low-x region. Although present data shows no sign of such a structure our model is in remarkably good agreement with the data especially in the case of the neutron. That we fare better for the neutron than the proton is re-

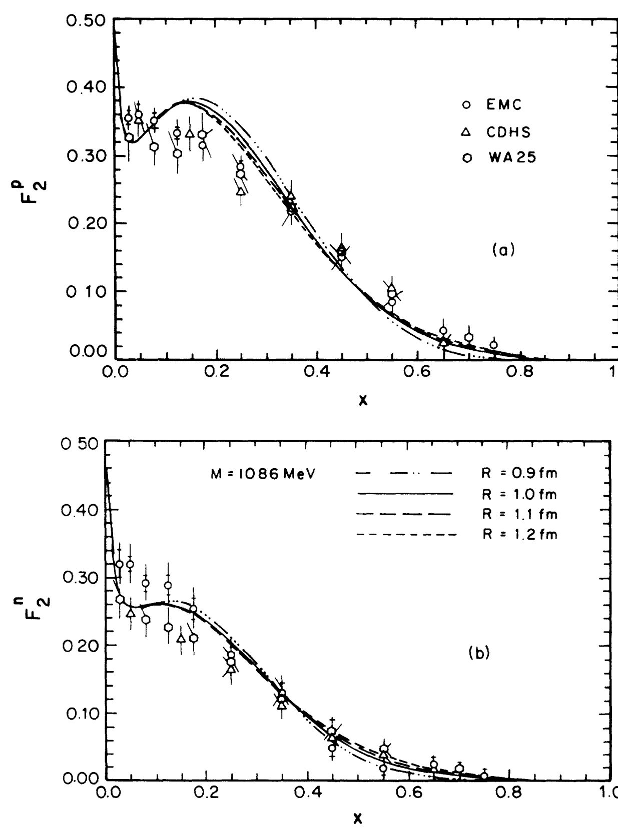

FIG. 10. The proton and neutron structure functions at $Q^2 = 15$ GeV^2 as a function of Bjorken x. The EMC muon data is from Ref. 58; the CDHS and WA25 structure functions have been calculated from the quark distribution functions measured in the respective neutrino experiments, see Ref. 58. (a) $F_2^p(x)$, (b) $F_2^n(x)$.

lated to the fact that the xd_v distribution is in better agreement with experiment than the xu_v distribution.

Our proton and neutron structure functions are compared directly in Fig. 11. The combination F_2^p-F_2^n is of particular interest since it is a pure nonsinglet. The Fermi-gas approximation is unsuccessful here, being clearly too narrow and highly peaked. This feature is also evident in the ratio F_2^n/F_2^p though in line with earlier remarks preliminary BCDMS data[59] is in closer correspondence with the model than the EMC data. A major problem with the data is the size of systematic errors. The New Muon Collaboration (NMC) claim that they will have much smaller systematic errors. Preliminary data from this group[69] is shown in Fig. 11(b) with statistical errors only. It does tend to confirm that there is a minor systematic problem with the model structure functions; this is most evident in the low-x region. Actually we find this encouraging because there are several features lacking in our model, such as a pionic component, which can be expected to affect primarily that region.

The behavior of the ratio F_2^n/F_2^p as x tends to 1 is of some theoretical interest. Since the ocean distributions are negligible at large x the charge weightings of the quarks yield

$$\frac{F_2^n}{F_2^p}=\frac{1+4d/u}{4+d/u} \tag{5.3}$$

and so the ratio of structure functions must lie between 0.25 and 4. The most simple quark model would have $d/u=\frac{1}{2}$ and thus $F_2^n/F_2^p=\frac{2}{3}$. However, there is no reason why the d/u ratio should be constant in x and it certainly is not in our model. Close and Thomas[12] have suggested d/u falls to 0 as x approaches 1 whereas Farrar and Jackson[70] suggest $\frac{1}{5}$. Because of the support mismatch the Fermi-gas approximation is not reliable at $x=1$. Nonetheless, before evolution we can straightforwardly obtain this ratio in the limit $x\rightarrow\infty$ from Eqs. (4.2) and (4.7). Since at large x the quark distribution is very small the value of the exponential in Eq. (4.7) is also small and a standard expansion for the logarithm may be used. Neglecting $m_d^*-m_u^*$ on the scale of the nucleon mass we find

$$d/u=\exp[(E_F^{*d}-E_F^{*u})/T_e] \tag{5.4}$$

which is independent of x. Indeed, because we have restricted our choice of parameters so that the calculated structure functions are negligible beyond $x=1$ we reach this asymptotic region even before $x=1$. Examination of the evolution equations[56] shows that the ratio at $x=1$ is independent of Q^2. Thus Eq. (5.4) applies to Fig. 11(b) where the asymptotic behavior is clearly evident (apart from oscillations at large x which arise from severe numerical problems[71]). Of course the exponential form of Eq. (5.4) is a remnant of the support mismatch and also its value is clearly sensitive to the model parameters so that a reliable quantitative prediction is not possible. What we do consider noteworthy is the asymptotic behavior of the ratio which implies that an extrapolation of the current data to $x=1$ is fraught with uncertainty.

The gluon distribution that we obtain, which is generated entirely by the evolution procedure, is shown in Fig. 12. It is somewhat more singular at low x than that usually extracted from data. Experimental knowledge of the gluon distribution is though very poor.

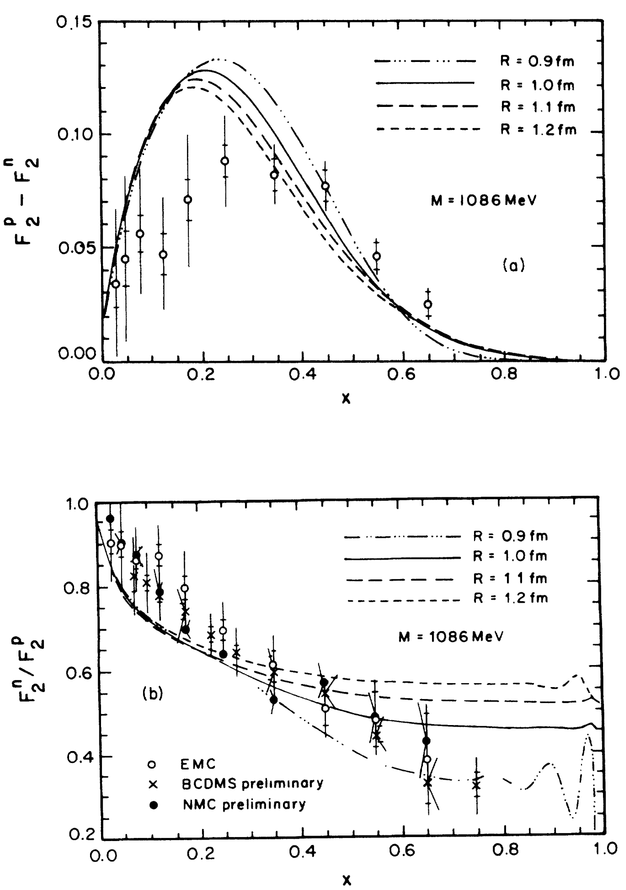

FIG. 11. Comparison of proton and neutron structure functions at $Q^2=15$ GeV^2. The data has been averaged over the Q^2 range of the experiments (Refs. 58, 59, and 69). (a) $F_2^p-F_2^n$, (b) F_2^n/F_2^p.

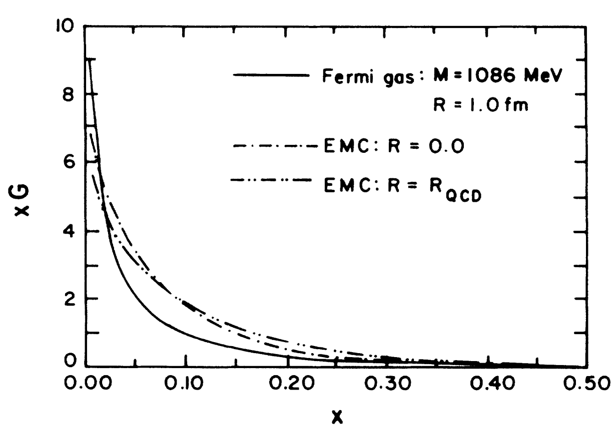

FIG. 12. The gluon distribution implied by LO QCD evolution with two effective flavors. The EMC curves are their fits (Ref. 58) at 5 GeV^2 evolved to 15 GeV^2.

VI. DISCUSSION

We have shown that a finite-temperature Fermi-gas approximation to the nucleon leads to quite good results for the nucleon structure functions and quark distributions. Of course the connection between the physical nucleon and an infinite Fermi gas is rather tenuous. However we do not consider that the reasonable agreement we obtain with data is coincidental. We have argued that a finite temperature Fermi gas should indeed give a fair first approximation to baryon structure functions in the Bjorken region of high-momentum transfer. The essential reasons are that in the Bjorken limit DIS measures an *effective* free-field decomposition of the confined quarks. The Fourier components in this decomposition dictate the shape of the structure functions. Because of the fermionic property of the quarks a detailed knowledge of the confined quark wave functions does not appear to be essential to obtain an approximate knowledge of these Fourier components (or occupation numbers). The prime factor is the density of the quarks and that is almost trivially calculated. By comparing with the interior region of bag models of hadron structure we have been able to choose (self-consistent) parameters of a Fermi gas so that the gas mimics the effective free-field decomposition seen in DIS. The finite temperature is particularly important in this regard. It has nothing to do though with a phase transition to a deconfining phase of QCD as in a popular interpretation of the MIT bag. On the contrary an essential ingredient in the calculation of the structure functions is the fact that the quarks are confined.

Although our model is simple it provides some useful insight into the shape of nucleon structure functions. Most importantly we find a possible explanation for the well-known mysterious differences in the shape of the electromagnetic structure functions of the proton and neutron and associated differences in the xu_v and xd_v distributions in the proton. In the Fermi-gas approximation this arises as a simple consequence of Fermi statistics and the fact that the proton contains two u valence quarks and just one d valence quark. (Note that the spin averaging is important here.) We also find that the quark mass difference $m_d - m_u$ plays an insignificant role in the difference between xu_v and xd_v.

However, perhaps the greatest significance of this work is that it provides a model in which structure functions may be calculated without the theoretical difficulties that have plagued past attempts to study quark-model structure functions. This permits us to introduce complications such as pion fields and to evaluate their importance in preparation for a more realistic model of the nucleon structure functions. One particularly important problem is to better understand the role of gluons in the momentum sum rule. We have followed the almost universal trend in taking a model with no gluons and introducing them via QCD evolution. This procedure has received much criticism—and justifiably so. Unfortunately, previous quark-model calculations have not been sufficiently well understood to avoid this approximation. The presence of gluons in the Fermi gas does not directly affect the structure functions.[24] However they will modify the equations of state and consequently change the appropriate gas parameters. By then introducing the effect of quantum fluctuations we would have a model with scaling violations built in but with the advantage that a direct comparison with mean-field results would be possible over a range of Q^2. This model could then be compared with the results given here and the utility of the evolution ansatz judged. This line is currently being pursued.

One of the motivations for this work was the EMC effect in which an apparent dependence of the nucleon structure functions on the surrounding media is observed. The Fermi-gas model does offer some insight into various proposed mechanisms for explaining this which we will discuss elsewhere.

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[1]For reviews, see C. Barone and E. Predazzi, Ann. Phys. (Leipzig) **12**, 525 (1987); K. Rith, Z. Phys. C **38**, 317 (1988); R. P. Bickerstaff and A. W. Thomas, J. Phys. G **15**, 1523 (1989).

[2]See, e.g., A. Le Yaouanc *et al.*, Phys. Rev. D **12**, 2137 (1975); Y. S. Kim and M. E. Noz, Prog. Theor. Phys. **60**, 801 (1978); L. S. Celenza and C. M. Shakin, Phys. Rev. C **27**, 1561 (1983); R. P. Bickerstaff and A. W. Thomas, University of Adelaide Report No. ADP-87-1/T29, 1987 (unpublished); A. W. Thomas, Prog. Part. Nucl. Phys. **20**, 21 (1988); C. J. Benesh and G. A. Miller, Phys. Rev. D **36**, 1344 (1987); Phys. Lett. B **215**, 381 (1988); A. I. Signal and A. W. Thomas, *ibid.* **211**, 481 (1988).

[3]R. L. Jaffe, Phys. Rev. D **11**, 1953 (1975).

[4]A. De Rújula and F. Martin, Phys. Rev. D **22**, 1787 (1980).

[5]See, e.g., T. N. Pham, Phys. Rev. D **19**, 707 (1979); A. Dhaul, A. N. Mitra, and A. Pagnamenta, Z. Phys. C **36**, 115 (1987); M. V. Terentev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)].

[6]A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974).

[7]E. J. Moniz, Phys. Rev. **184**, 1154 (1969); R. A. Smith and E. J. Moniz, Nucl. Phys. **B43**, 605 (1972).

[8]R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. London **A70**, 381 (1957); J. J. Griffin and J. A. Wheeler, Phys. Rev. **108**, 311 (1957); R. E. Peierls and D. J. Thouless, Nucl. Phys. **38**, 154 (1962); H. Rouhaninejad and J. Yoccoz, *ibid.* **78**, 353 (1966).

[9]In our calculation the lower components in the quark wave functions contribute 25% of the momentum sum rule and thus play an important role in the description of DIS.

[10]M. H. L. Pryce, Proc. R. Soc. London **A195**, 62 (1948); for attempts at c.m. projection in relativistic bag models, see, e.g., J.-L. Dethier, R. Goldflam, E. M. Henley, and L. Wilets, Phys. Rev. D **27**, 665 (1983); M. Betz and R. Goldflam, *ibid.* **28**, 2848 (1983); E. G. Lubeck, M. C. Birse, E. M. Henley, and L. Wilets, *ibid.* **33**, 234 (1986).

[11]The validity of this claim arises from an approximate equivalence between the structure functions (per unit baryon number) for two distinct systems: a confined gas of three (interacting) quarks at zero temperature and an infinite noninteracting gas of the same density at finite temperature.

[12]F. E. Close and A. W. Thomas, Phys. Lett. B **212**, 227 (1988).

[13]J. Cleymans and R. L. Thews, Z. Phys. C **37**, 315 (1988).

[14]See F. Takagi, Phys. Rev. D **35**, 2226 (1987).

[15]The quark-model quark is actually a quasiparticle. Let us make the problem explicitly relativistic. Then one can take any localized quark wave function at a given instant in time and expand it in a complete set of plane-wave solutions of the free Dirac equation. The time development is then described by the Heisenberg equation. It is well known that both positive- and negative-energy components will be found. Thus the quark-model quark is a linear combination of quark and antiquark states, possibly even dressed with gluons. Notice that in making such an expansion one must use the n-quark wave function if there are n quarks of the same type. Replicating the single-quark expansion n times will lead to violations of the Pauli principle. In practice one will encounter additional problems in making an accurate expansion because of the need for relativistic center-of-mass corrections. Furthermore, after quantization—which is actually a delicate matter in a confining theory—the time-dependent expansion coefficients do not retain the simple interpretation (valid in free-field theory) as creation and destruction operators for quarks and antiquarks of given four-momentum and mass.

[16]C. G. Callan and D. J. Gross, Phys. Rev. D **8**, 4383 (1973); D. J. Gross and F. Wilczek, *ibid.* **8**, 3633 (1973); **9**, 980 (1974); H. Georgi and H. D. Politzer, *ibid.* **9**, 416 (1974); see also J. Kogut and L. Susskind, *ibid.* **9**, 3391 (1974).

[17]D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, *ibid.* **30**, 1346 (1973).

[18]R. A. Brandt, Phys. Rev. Lett. **23**, 1260 (1969); Phys. Rev. D **1**, 2808 (1970); B. L. Ioffe, Pis'ma Zh. Eksp. Theor. Fiz. **9**, 163 (1969) [JETP Lett. **9**, 97 (1969)]; Phys. Lett. **30B**, 123 (1969); L. S. Brown, in *Proceedings of the Boulder Conference on High Energy Physics,* edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Associated University Press, Boulder, Colorado, 1970); H. Leutwyler and J. Stern, Nucl. Phys. **B20**, 77 (1970); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D **2**, 2473 (1970).

[19]Strong interactions also take place amongst the spectators but do not enter the calculation.

[20]C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[21]J. M. Cornwall and R. Jackiw, Phys. Rev. D **4**, 367 (1971); see also V. De Alfaro *et al., Currents in Hadron Physics* (North-Holland, Amsterdam, 1973); R. L. Jaffe, in *Relativistic Dynamics and Quark-Nuclear Physics,* proceedings of the Workshop, Los Alamos, New Mexico, 1985, edited by M. B. Johnson and A. Picklesimer (Wiley, New York, 1985).

[22]C. H. Llewellyn Smith, in *Particles and Fields 3,* proceedings of the Banff Summer Institute, Banff, Alberta, Canada, 1988, edited by A. N. Kamal and F. C. Khanna (World Scientific, Singapore, 1989).

[23]A simple demonstration of this effect may be found in Ref. 24.

[24]R. P. Bickerstaff, in *Medium and High-Energy Nuclear Physics,* proceedings of the International Conference, Taipei, Taiwan, 1988, edited by W.-Y. P. Hwang, K.-F. Liu, and Y. Tzeng (World Scientific, Singapore, 1989), p. 99; University of Idaho report, 1990 (unpublished).

[25]R. Friedberg and T. D. Lee, Phys. Rev. D **15**, 1694 (1977); T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1981); R. Goldflam and L. Wilets, Phys. Rev. D **25**, 1951 (1982).

[26]H. B. Nielson and A. Patkos, Nucl. Phys. **B195**, 137 (1982); G. Chanfray, O. Nachtmann, and H. J. Pirner, Phys. Lett. **147B**, 249 (1984); L. R. Dodd, A. G. Williams, and A. W. Thomas, Phys. Rev. D **35**, 1040 (1987).

[27]The scalar potential in bag models is also weak; indeed in the MIT bag the quarks are entirely free in the interior.

[28]It could be argued that the strong confining scalar forces in these models are merely a phenomenological representation of higher-twist effects in QCD and therefore do not enter the leading-twist calculation.

[29]See R. L. Jaffe and A. Patrascioiu, Phys. Rev. D **12**, 1314 (1975). These authors claim to prove that the bag boundary does not affect the leading-twist distribution. However, they do not consider center-of-mass corrections which means that their maximum correlation length is $2R$ rather than the expected $4R$. Also, like us, they ignore the complexities associated with quantizing fermions constrained within a cavity. Their proof should be regarded as incomplete.

[30]The correlation function of Eq. (2.3) does not reproduce the $x^{-\alpha}(\alpha>0)$ singularity associated with empirical distribution functions. Correlation functions which reproduce this have a larger spatial extent (Ref. 22). However, this may be associated with nonperturbative effects such as pion clouds (Ref. 31) not included in either the present model or in the $z \leq 4R$ constraint.

[31]V. Sanjosé, V. Vento, and S. Noguera, Nucl. Phys. **A470**, 509 (1987).

[32]See, e.g., M. A. Preston and R. K. Bhaduri, *Structure of the Nucleus* (Addison-Wesley, Reading, MA, 1975).

[33]The k in this sentence is the remnant of the full momentum after including the effects of the gauge factor (see Ref. 24).

[34]In later work we will admit perturbative (but not confining) QCD interactions. However, we will always disallow flavor-changing weak interactions in order to maintain the differing densities of u and d quarks.

[35]The real advantage here, over an arbitrary form for the occupation numbers, is that we have a self-consistent quantized theory for the quark density, current commutator, and stress-energy tensor.

[36]This is an inherent defect in applying a Fermi-gas local-density approximation and cannot be cured. Clearly, it means that our approximation for the nucleon structure functions will not behave as $(1-x)^3$ as $x \rightarrow 1$ as expected from quark counting rules (note that there are only two spectator quarks in the case of a struck valence quark in the nucleon but an infinite number in the case of a Fermi gas).

[37]For relevant discussions and further details, see, e.g., J. Cleymans, R. V. Gavai, and E. Suhonen, Phys. Rep. **130**, 217 (1986); B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[38]From Eq. (3.25), the energies E_F^*, and hence also the fermion and antifermion distributions $n_k(T_e)$ and $\bar{n}_k(T_e)$, will be different for u and d quarks. We have suppressed the flavor indices on these quantities in most of our equations.

[39]It should be emphasized that we are only interested in scattering from a spin-averaged proton state where the (average) number of spin-up and spin-down quarks of flavor j is the same. Hence, further density dependence arising from the usual color-flavor-spin wave functions for a nucleon of fixed spin is irrelevant here.

[40]In bag models the confinement volume is obviously the same for u and d quarks. In the real nucleon this may not be exactly true, but since QCD is flavor independent any difference in the confinement volumes due to quark mass effects should be quite small.

[41]A. Chodos *et al.*, Phys. Rev. D **10**, 2599 (1974).

[42]A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1983).

[43]We might even determine the constant c in Eq. (3.37) by choosing those mass and radius parameters which yield the best fit to the empirical structure functions; we have not pursued this idea.

[44]Cleymans and Thews used a parton-model argument. Our results are obtained as a special case of a uniform-mean-field calculation which we believe enables us to make some sensible comments on the role of realistic interactions and the validity of the present approximation.

[45]It is easy to check that their parameter set is far from equilibrium. Their relationship between k_F^u and k_F^d is also only valid at $T=0$.

[46]C. Angelini and R. Pazzi, Phys. Lett. **113B**, 343 (1982).

[47]The problem appears to be that a gluon field in the mean-field model represents valence gluons, and not the perturbatively generated gluons of QCD field theory which are generally accepted as carrying the remaining 50% of the proton momentum.

[48]R. L. Jaffe and G. G. Ross, Phys. Lett. **93B**, 313 (1980).

[49]See, e.g., G. Altarelli, N. Cabibbo, L. Maiani, and R. Petronzio, Nucl. Phys. **B69**, 531 (1974); G. Parisi and R. Petronzio, Phys. Lett. **62B**, 331 (1976); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Ann. Phys. (N.Y.) **105**, 276 (1977); N. Cabibbo and R. Petronzio, Nucl. Phys. **B137**, 395 (1978); T. DeGrand, *ibid.* **B151**, 485 (1979); F. Martin, Phys. Rev. D **19**, 1382 (1979).

[50]F. E. Close, R. L. Jaffe, R. G. Roberts, and G. G. Ross, Phys. Rev. D **31**, 1004 (1985).

[51]R. C. Hwa, Phys. Rev. D **22**, 759 (1980); **22**, 1593 (1980); R. C. Hwa and M. S. Zahir, *ibid.* **23**, 2539 (1981); see also P. E. Hussar, *ibid.* **23**, 2781 (1981).

[52]A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Rev. D **37**, 1161 (1988).

[53]W. Furmanski and R. Petronzio, Z. Phys. C **11**, 293 (1982).

[54]Suppose however that we postulated that the calculated distributions are scheme-S quark distributions and we attempted to calculate higher-order evolution. Then another problem arises: the scale μ^2 implied by the momentum sum rule is so low that the running coupling constant $\alpha_s(\mu^2)$ does not have any solution. (This happens for $\mu^2 \lesssim 2.6\Lambda^2$, the exact coefficient depending on the number of flavors.) Nevertheless we believe that use of LO evolution is a justifiable first approximation because it introduces gluons and additional antiquarks in a manner consistent with the baryon number and momentum sum rules. Similar qualitative changes must be anticipated in going to a better theory where these are included in the equations of state.

[55]We ignore flavor thresholds, in the region of which the new flavor evolves with higher-twist contributions. This effect is actually very important in generating flavor-symmetry violations in the quark ocean so we will not be justified in decomposing the total ocean distribution into its separate flavor components. In fact we may evolve the distributions legitimately in an effective theory with just two flavors.

[56]G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977); A. J. Buras, Rev. Mod. Phys. **52**, 199 (1980); G. Altarelli, Phys. Rep. C **81**, 1 (1982).

[57]G. Parisi and N. Sourlas, Nucl. Phys. **B151**, 421 (1979); J. Chyla and J. Rames, Z. Phys. C **31**, 151 (1986); V. G. Krivokhizhin *et al.*, *ibid.* **36**, 51 (1987).

[58]J. J. Aubert *et al.*, Nucl. Phys. **B293**, 740 (1987); V. Gibson, Ph.D. thesis, Oxford University, 1986.

[59]T. Sloan, G. Smajda, and R. Voss, Phys. Rep. **162C**, 45 (1988).

[60]If one were to evolve the whole way with three or more active flavors then one would find a larger fraction of momentum carried by the ocean quarks at the expense of the gluons. We emphasize though that this is unreasonable. With just two effective flavors we do indeed obtain about the right proportion of momentum on the gluons. Of course at $Q^2=15$ GeV^2 there are four active flavors and we would not be justified in making a flavor decomposition of the ocean.

[61]H. Abramowicz *et al.*, Z. Phys. C **25**, 29 (1984).

[62]D. Allasia *et al.*, Z. Phys. C **28**, 321 (1985).

[63]A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Lett. B **206**, 327 (1988).

[64]The distributions of Ref. 63 are constrained by neutrino data. Since much of the difference between EMC and CDHS/WA25 valence distributions can be attributed to the difference between their ocean distributions (Ref. 59) we assume that the EMC ocean is to be preferred to the CDHS and WA25 oceans.

[65]This because T_e/k_F is increasing (see Table I). Some authors, e.g., Ref. 14, have interpreted this increase of R with temperature as leading eventually to deconfinement. Recall however that our effective temperature is not an independent degree of freedom and, having chosen a value of c in Eq. (3.37), nature does not permit us to move in this direction. This corresponds to the fact that in the simple bag model modifying R at fixed M is not physically possible.

[66]R. P. Bickerstaff and A. W. Thomas, Phys. Rev. D **25**, 1869 (1982).

[67]See, for example, S. Weinberg, Trans. N.Y. Acad. Sci. **38**, 185 (1977).

[68]K. Maltman, T. Goldman, and G. J. Stephenson, Jr., Phys. Lett. B **234**, 158 (1990).

[69]New Muon Collaboration, D. Allasia *et al.*, Report No. CERN-PPE/90-103, 1990 (unpublished).

[70]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **35**, 1416 (1975).

[71]The difficulty arises because at large x the function being reconstructed from its moments is very small; small errors in the reconstruction get magnified when taking the ratio of two such small quantities. We have found it very difficult to eliminate this inaccuracy with the reconstruction procedure. The correct value is clearly obtained by averaging out the oscillations.