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Thermal relics: Do we know their abundances?

Marc Kamionkowski and Michael S. Turner
Physics Department, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433
and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500
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The relic abundance of a particle species that was once in thermal equilibrium in the expanding Universe depends upon a competition between the annihilation rate of the species and the expansion rate of the Universe. Assuming that the Universe is radiation dominated at early times the relic abundance is easy to compute and well known. At times earlier than about 1 sec after the bang there is little or no evidence that the Universe *had* to be radiation dominated, although that is the simplest—and standard—assumption. Because early-Universe relics are of such importance both to particle physics and to cosmology, we consider in detail three nonstandard possibilities for the Universe at the time a species' abundance froze in: energy density dominated by shear (i.e., anisotropic expansion), energy density dominated by some other nonrelativistic species, and energy density dominated by the kinetic energy of the scalar field that sets the gravitational constant in a Brans-Dicke-Jordan cosmological model. In the second case the relic abundance is less than the standard value, while in the other two cases it can be enhanced by a significant factor. We also mention two other more exotic possibilities for enhancing the relic abundance of a species—a larger value of Newton's constant at early times (e.g., as might occur in superstring or Kaluza-Klein theories) or a component of the energy density at early times with a very stiff equation of state ($p>\rho/3$), e.g., a scalar field ϕ with potential $V(\phi)=\beta|\phi|^n$ with $n>4$. Our results have implications for dark-matter searches and searches for particle relics in general.

I. INTRODUCTION

The existence of the cosmic-microwave-background radiation (CMBR) with a temperature of 2.74 K is very strong evidence that the Universe was both radiation dominated [for times earlier than $t_{\rm EQ}\simeq 4.4\times 10^{10}(\Omega_0 h^2)^{-2}$ sec] and very hot [$T\sim \text{MeV}(t/\text{sec})^{-1/2}$] at early times.[1] Because the temperatures reached early on were so high, there is every reason to believe that essentially all the known particle species and possibly other species yet to be discovered were present in great numbers. If equilibrium thermodynamics were the entire story, these facts would be of little interest, as today the equilibrium abundance of any massive particle species would be exponentially small, proportional to $\exp(-m/T)$. However, it has long been realized that because of the expansion of the Universe, the actual abundance of a stable particle species cannot track its equilibrium value forever, and depending upon the strength of its interactions, the abundance per comoving volume eventually ceases to decrease and freezes in at some constant value. "Freeze-in" of the particle's abundance occurs when the annihilation rate can no longer keep pace with the expansion rate of the Universe: Roughly, the abundance ceases to decrease when the annihilation rate falls below the expansion rate—when annihilations are said to freeze out.[2] (The reactions that regulate the number of a particle species are pair production and annihilation; the pair creation rate is related to the annihilation rate by detailed balance, or time-reversal invariance.) Moreover, the weak shall dominate—the relic abundance of a particle species is inversely proportional to its annihilation cross section.

Calculating the relic abundance of a particle species that was once in thermal equilibrium is a routine chore for the particle cosmologist. The differential equation governing the abundance of a species follows from the Boltzmann equation and depends upon two pieces of input physics: the expansion rate as a function of temperature and the annihilation rate as a function of temperature.[2] Once the particle species and its interactions are specified, the annihilation rate is precisely determined in terms of the number density of the species and the temperature of the Universe. The expansion rate as a function of temperature is another matter. In the standard, radiation-dominated Friedmann-Robertson-Walker (FRW) cosmology, the expansion rate at early times ($t\lesssim t_{\rm EQ}$) is given by

$$H^2\equiv\left(\frac{\dot R}{R}\right)^2=\frac{8\pi G}{3}\rho\simeq\frac{4\pi^3 g_* T^4}{45 m_{\rm Pl}^2}\ , \tag{1}$$

where R is the scale factor of the Universe, ρ is the total energy (for a thermal bath of relativistic particles $\rho_r=g_*\pi^2T^4/30$), and g_* counts the effective number of ultrarelativistic degrees of freedom (one for each relativistic bosonic degree of freedom and $\frac{7}{8}$ for each relativistic fermionic degree of freedom). Having made this assumption, the path to determining the relic abundance—usually expressed as the ratio of the number density n of the species to the entropy density $s=2g_*\pi^2T^3/45$—is a tried and true one.

The crucial uncertainty in determining the relic abundance is the assumption that the Universe is radiation dominated at freeze-out. The concordance of the predictions of primordial nucleosynthesis with the observed light-element abundances provides strong evidence that the Universe was indeed radiation dominated at an age of about 1 sec.[3] However, there is little or no evidence that requires the Universe at times earlier than about 1 sec to be radiation dominated. Moreover, most of the interesting thermal relics would have decoupled at such early times.

Given the importance of relic particles both to cosmology and to particle physics (particle relics may provide the bulk of the mass density of the Universe thereby explaining the nature of the dark matter,[4] and they might provide the first evidence for new physics beyond the standard model of particle physics), we decided to study in detail three nonstandard, but plausible, possibilities for the expansion rate of the Universe around the time of freeze-out. Our first example involves the geometry of spacetime: If the expansion of the Universe is not isotropic, then the volume-expansion rate at fixed temperature exceeds that for the standard case and freeze-out occurs at higher temperature, leading to a larger relic abundance. Here we explore a particularly simple and interesting example: a Bianchi I model where the effects of the anisotropy on the volume-expansion rate can be quantified in terms of an anisotropy-energy density that decreases as R^{-6}. For this model (and other similar models) the anisotropy simply decays without leaving a trace, and the only lasting effect is to enhance the abundance of the thermal relic.

In the second example, at early times, the energy density is dominated by a massive, nonrelativistic particle species. Again, the expansion rate for fixed temperature is increased, leading to an earlier freeze-out and a larger relic abundance. Of course, there is every evidence that the Universe only became matter dominated relatively recently and so the nonrelativistic particles would eventually have to decay, producing entropy and diluting the relic abundance. As we shall show, the net effect is to *decrease* the relic abundance. (We note that this possibility is different from the one involving particle decays in which the only effect of the decaying species is to produce entropy, in which case the relic abundance is decreased precisely by the amount of the increase in entropy.[5])

In the third example we use the Brans-Dicke-Jordan theory of gravity instead of general relativity. Here the analog of the Friedmann equation contains the kinetic energy term for the Brans-Dicke scalar field, which decreases as R^{-6} and, of course, increases the expansion rate for fixed temperature. As in the case of anisotropic expansion, the only lasting effect is to enhance the abundance of the thermal relic.

The motivation of this work then is to assess the reliability of the standard estimate for the relic abundance of a stable particle species that was once in thermal equilibrium by considering three nonstandard possibilities for the evolution of the Universe at early times ($t \lesssim 1$ sec). The outline of our paper is as follows. In Sec. II we briefly review the formalism for calculating the relic abundance of a species and the standard result. In the following three sections we consider the nonstandard possibilities mentioned above and how they affect the relic abundance of a stable particle species. In the final section we put our work in perspective with some concluding remarks.

II. REVIEW OF THE STANDARD RESULT

To obtain quantitative results for the relic abundance of a stable particle species X (and its antiparticle $\bar{X}$), one solves the Boltzmann equation that governs the number density of the species:[2]

$$\frac{dn}{dt}+3Hn=-\langle\sigma v\rangle(n^2-n_{\rm EQ}^2)\ . \tag{2}$$

Here $n(t)$ is the number density of species X at time t, $n_{\rm EQ}$ is the equilibrium number density at time t when the temperature of the plasma is T, $\langle\sigma v\rangle$ is the thermally and spin-averaged cross section times relative velocity for $X\bar{X}$ annihilation, and H is the Hubble parameter. We assume that there is no particle-antiparticle asymmetry so that the number density of antiparticles is also n. (It is easy to extend this formalism to apply to the case where there is a particle-antiparticle asymmetry; see Griest and Seckel.[6]) Exact solutions of this particular example of the Ricatti equation do not exist; however, an approximate (accurate to $\sim 5\%$) analytical solution is easily obtained.[7] Since we will follow this approach in the nonstandard models, we will briefly review that solution here.

In the absence of entropy production the entropy per comoving volume ($S=R^3s$) is constant, and we use the entropy density

$$s=\frac{2\pi^2}{45}g_*T^3 \tag{3}$$

as a fiducial quantity and rewrite the Boltzmann equation in terms of $Y\equiv n/s$, which corresponds to the number of X particles per comoving volume. [Actually, the g_* in Eq. (1) is in principle different from that in Eq. (3); however, in practice they are very nearly equal at temperatures greater than 1 MeV. See Ref. 5, pp. 65–70.] Furthermore, since the quantities of interest depend explicitly on temperature rather than time, we use the quantity $x\equiv m/T$, instead of time as the dependent variable. Doing so, Eq. (2) becomes

$$\frac{dY}{dx}=-\frac{\langle\sigma v\rangle s}{xH}(Y^2-Y_{\rm EQ}^2)\ , \tag{4}$$

where, of course, $\langle\sigma v\rangle$, H, s, and $Y_{\rm EQ}$ are all functions of x. (Actually, there is another term involving the derivative of g_*;[6,8] however, this term is small and to a good approximation we can fix g_* at its value around freeze-out.) Here $Y_{\rm EQ}\equiv n_{\rm EQ}/s$ and in the nonrelativistic limit ($x \gg 3$) is given by

$$\begin{aligned}Y_{\rm EQ}(x)&=\frac{45}{2\pi^4}\left[\frac{\pi}{8}\right]^{1/2}\frac{g}{g_*}x^{3/2}e^{-x}\\&=0.145\frac{g}{g_*}x^{3/2}e^{-x}\ .\end{aligned} \tag{5}$$

In the case of interest, at freeze-out $X\bar{X}$ particles are moving at nonrelativistic velocities and the cross section is proportional to v^{2n} ($n=0$ corresponds to s-wave annihilation, $n=1$ corresponds to p-wave annihilation, etc.), and so we can write $\langle \sigma v \rangle = \sigma_0 x^{-n}$. Furthermore, $s \propto x^{-3}$ and $H \propto x^{-2}$ [cf. Eq. (1)], and so the Boltzmann equation becomes

$$\frac{dY}{dx} = -\frac{\lambda}{x^{n+2}}(Y^2 - Y_{EQ}^2) , \tag{6}$$

where we have defined

$$\lambda \equiv \left[\frac{\langle \sigma v \rangle s}{H} \right]_{x=1} = 0.264 g_*^{1/2} m_{\rm Pl} m_X \sigma_0 . \tag{7}$$

To solve Eq. (6) we follow Ref. 7 and consider the differential equation for $\Delta \equiv Y - Y_{EQ}$, the departure from equilibrium:

$$\Delta' = -Y'_{EQ} - \frac{\lambda}{x^{n+2}}\Delta(2Y_{EQ}+\Delta) , \tag{8}$$

where the prime denotes d/dx. At early times when the annihilation rate $\Gamma_{\rm ann}$ is much greater than the expansion rate H ($x \ll x_f$), the X abundance tracks its equilibrium value very closely so that $\Delta \ll Y_{EQ}$ and $\Delta' \ll Y'_{EQ}$, and

$$\Delta \simeq \frac{x^{n+2} Y'_{EQ}}{\lambda(2Y_{EQ}+\Delta)} . \tag{9}$$

At late times ($x \gg x_f$), Y tracks Y_{EQ} very poorly; therefore, $\Delta \simeq Y \gg Y_{EQ}$ and $Y'_{EQ} \ll \Delta'$, so that

$$\Delta' = -\frac{\lambda}{x^{n+2}}\Delta^2 . \tag{10}$$

Upon integrating Eq. (10) from $x=x_f$ to $x=\infty$, we obtain[7]

$$Y_\infty = \Delta_\infty = \frac{n+1}{\lambda} x_f^{n+1} + O(x_f^n) , \tag{11}$$

where x_f is determined from Eq. (9) by $\Delta(x_f) = cY_{EQ}(x_f)$ and c is a numerical constant of order unity that serves to define the epoch of transition between the regimes mentioned above. Solving for x_f iteratively, the result is

$$x_f \simeq \ln[(2+c)\lambda a c] - (n+\tfrac{1}{2})\ln\{\ln[(2+c)\lambda a c]\} , \tag{12}$$

where $a = 0.145(g/g_*)$. Note that the final abundance only depends logarithmically upon the value of c. The best agreement between the analytic result and a numerical integration of Eq. (4) obtains for $c(c+2) = n+1$.[7] To obtain the present mass density contributed by the relic, Y_∞ is multiplied by the mass of the relic and the present entropy density $s_0 = 2970 T_{2.75}^3\ {\rm cm}^{-3}$, where $T_{2.75}$ is the CMBR temperature in units of 2.75 K.

III. FREEZE-OUT IN A SHEAR-DOMINATED UNIVERSE

As stated earlier, there is little or no evidence that the Universe before the time of big-bang nucleosynthesis *had* to be homogeneous or isotropic, although this is the standard assumption and is certainly well motivated. The simplest nonstandard cosmological models are homogeneous but anisotropic; these are the Bianchi (and Kantowski-Sachs) models, which are classified according to the Lie algebras of their isometries.[9,10] The metric of the simplest of these models, the Bianchi type-I spacetime, is

$$ds^2 = -dt^2 + R_1^2(t)(dx^1)^2 + R_2^2(t)(dx^2)^2 + R_3^2(t)(dx^3)^2 , \tag{13}$$

where the R_i are the scale factors of the three principal axes of the Universe. The Einstein equations for this metric lead to the analog of the Friedmann equation for the volume expansion rate H of the Universe:

$$H^2 \equiv \frac{1}{9}\left[\frac{\dot V}{V}\right]^2 = \left[\frac{\dot{\bar R}}{\bar R}\right]^2 = \frac{8\pi}{3m_{\rm Pl}^2}(\rho_r + \rho_s) , \tag{14}$$

where we assume that the matter content is the usual thermal bath of radiation at temperature T and the shear "energy density" is defined to be

$$\rho_s \equiv \frac{m_{\rm Pl}^2}{48\pi}[(H_1-H_2)^2 + (H_1-H_3)^2 + (H_2-H_3)^2] . \tag{15}$$

Here $V = R_1R_2R_3$ is the "volume scale factor," $\bar R = V^{1/3}$ is the mean-scale factor, and the $H_i \equiv (\dot R_i/R_i)$ (no sum) are the expansion rates of the three principal axes. As is manifest from Eq. (15), the shear-energy density is proportional to the amount of anisotropic expansion. Note that since we are always free to relabel our comoving coordinates, differences between the various R_i's have no physical meaning; only differences in the expansion rates are meaningful. From Eq. (15) it also follows that $|H_i| \le 3H$ (we use absolute-value signs because at very early times, when ρ_r is negligible, the spacetime becomes the Kasner spacetime, in which one of the spatial dimensions must be contracting).

In general, the redshift suffered by a particle as the Universe expands will be direction dependent. For example, for a particle moving in the x direction, $p \propto R_1^{-1}$. Provided that the interaction rate of the thermal bath of particles is much larger than H, particle distributions will remain isotropic and the mean momenta will redshift as $\bar R^{-1}$. In this case, the remaining Einstein equations become (for $i \ne j$)

$$\frac{d}{dt}\ln|H_i - H_j| = -3H = -3\frac{d}{dt}(\ln \bar R) , \tag{16}$$

which implies that $\rho_s \propto \bar R^{-6}$. Therefore, the shear-energy density falls off faster than the radiation-energy density and the anisotropy in a Bianchi I universe simply decays without leaving a trace.[11]

For the freeze-out calculation we are interested only in the expansion rate H and not the detailed form of the anisotropy as given by the H_i; therefore, we use the fact that $\rho_s \propto \bar R^{-6}$ and constancy of the entropy per comoving volume ($g_* \bar R^3 T^3 = $const) to express the shear-energy density in terms of the plasma temperature T. We define the temperature at which $\rho_r = \rho_s$ to be T_e. For $T \gg T_e$ the Universe is shear dominated: $H \propto \bar R^{-3}$ and $\bar R \propto t^{1/3}$; for $T \ll T_e$ the Universe is radiation dominated:

$H \propto \bar{R}^{-2}$ and $\bar{R} \propto t^{1/2}$. The temperature T_e quantifies the size of the anisotropy energy: Smaller values of T_e correspond to larger anisotropy energy density at fixed temperature. We then write the shear-energy density in terms of the radiation-energy density:

$$\rho_s(T)=\rho_s(T_e)\left[\frac{g_* T^3}{g_*^e T_e^3}\right]^2=\rho_r\left[\frac{g_* T^2}{g_*^e T_e^2}\right], \tag{17}$$

where g_*^e is the value of g_* at T_e.

In order to avoid interfering with the successful predictions of big-bang nucleosynthesis, we must be sure that the shear-energy density is sufficiently small at the time of primordial nucleosynthesis. The shear contribution to the energy density would speed up the expansion rate, thereby increasing the ^{4}He production.[12] Assuming that the only contribution to the energy density comes from radiation, concordance of the outcome of nucleosynthesis with the observed abundance of ^{4}He requires that $g_*(T \sim 1\ \text{MeV}) \leq 12.5$.[13] To assess the effect of the shear-energy density, we write the total energy density as

$$\rho=\frac{\pi^2}{30}g_*^{\text{eff}}, \tag{18}$$

where

$$g_*^{\text{eff}} \equiv g_*\left[1+\frac{g_* T^2}{g_*^e T_e^2}\right]; \tag{19}$$

the primordial nucleosynthesis constraint is then $g_*^{\text{eff}} \leq 12.5$. [In terms of the number of light-neutrino species, this is equivalent to $N_\nu \leq 4$. Recent SLAC Linear Collider (SLC) and CERN LEP results have confirmed this constraint, determining that the number of neutrino species lighter than about 40 GeV is 3.2 ± 0.2.[14]] If the τ neutrino is light ($m_{\nu_\tau} \lesssim$ few MeV), we know that g_* is at least 10.75. Requiring that $g_*^{\text{eff}} \leq 12.5$ leads to the constraint $T_e \gtrsim 2.5$ MeV.

At this point, we can see how shear can increase the relic abundance of a particle species. In Fig. 1 we plot the expansion rate H in a shear-dominated universe as a function of temperature T. At low temperatures (late times), $H \sim T^2$, while at high temperatures (early times), $H \sim T^3$. The broken curve shows the expansion rate H_{std} with no shear. The equilibrium number density n_{EQ} of X is proportional to T^3 at high temperatures ($T \gg m$) and falls exponentially at low temperatures ($T \ll m$). For the case that the thermally and spin-averaged cross section times relative velocity, $\langle \sigma v \rangle$, for $X\bar{X}$ annihilation is constant, the annihilation rate $\Gamma_{\text{ann}}=n_{\text{EQ}}\langle \sigma v \rangle \propto n_{\text{EQ}}$ is also shown in Fig. 1. Roughly speaking, at the freeze-out temperature T_f, defined by $\Gamma_{\text{ann}}=H$, annihilations freeze out, and the number of X's per comoving volume "freezes in," at approximately its value at T_f. If $T_f > T_e$, the expansion rate in the shear-dominated universe is much greater than that in the standard radiation-dominated model, and the annihilations freeze out earlier when the abundance is greater. Since Γ_{ann} decreases exponentially around freeze-out, the freeze-out temperature for the two cases (shear and standard) is nearly the same. Moreover, because $n_{\text{EQ}} \propto \Gamma_{\text{ann}}$, the relic abundance in a shear-dominated model ($T_f > T_e$) is enhanced roughly by a factor $H(T_f)/H_{\text{std}}(T_f) \sim T_f/T_e$.

To obtain more quantitative results for the relic abundance, we must solve the Boltzmann equation using the

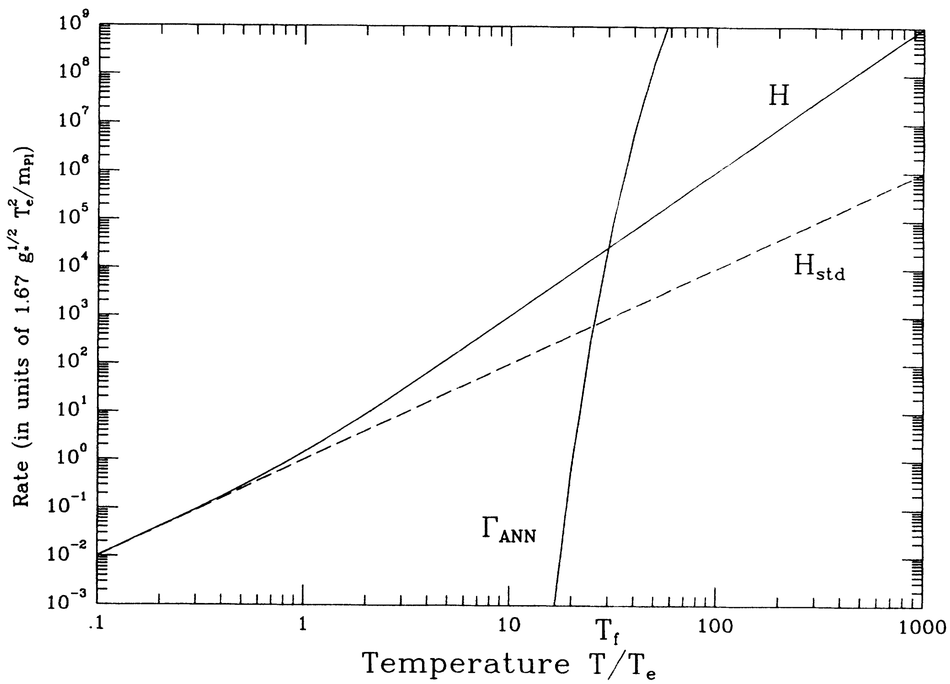

FIG. 1. Plot of the Hubble parameter in a shear-dominated universe (H) and in the standard model (H_{std}) as a function of temperature T. Also plotted is the annihilation rate Γ_{ann} of a particle species of mass m. The annihilation rate becomes equal to the expansion rate at a temperature T_f.

expression for H which includes the effects of anisotropic shear. In the Appendix we show that the Boltzmann equation used in the standard cosmological model [Eq. (2)] is also valid in the Bianchi I model as well. For an anisotropic-universe model the expansion rate is conveniently written as $H=H_{\rm std}(x^2+x_e^2)^{1/2}/x$, where $H_{\rm std}$ is the standard-model expansion rate and

$$x_e \equiv \frac{m}{T_e}\left[\frac{g_*}{g_*^e}\right]^{1/2} . \tag{20}$$

Doing so, the Boltzmann equation [cf. Eq. (6)] becomes

$$\frac{dY}{dx}=-\frac{\lambda}{x^{n+1}(x_e^2+x^2)^{1/2}}(Y^2-Y_{\rm EQ}^2) . \tag{21}$$

The differential equation for Δ then becomes

$$\Delta'=-Y'_{\rm EQ}-\frac{\lambda}{x^{n+1}(x^2+x_e^2)^{1/2}}\Delta(2Y_{\rm EQ}+\Delta) . \tag{22}$$

As before, at early times,

$$\Delta \simeq -\frac{x^{n+1}(x^2+x_e^2)^{1/2}Y'_{\rm EQ}}{\lambda(2Y_{\rm EQ}+\Delta)} , \tag{23}$$

and at late times

$$\Delta'=-\frac{\lambda}{x^{n+1}(x^2+x_e^2)^{1/2}}\Delta^2 . \tag{24}$$

For a model with no shear ($x_e=0$) we recover the results of the previous section. For a model universe where shear is important at freeze-out ($x_e>x_f$), an exact closed-form solution for Eq. (24) for arbitrary n is not simple to write down. However, if $x_e \gg x_f$, a good approximation for Y_∞ may be obtained by integrating in the interval $x_f \le x \le x_e$, assuming $H=H_{\rm std}/x$, and in the interval $x_e \le x \le \infty$, assuming $H=H_{\rm std}$. Doing so, we obtain

$$Y_\infty=\frac{nx_e x_f^n}{\lambda}\left[1+O\left[\frac{x_f}{x_e}\right]\right] , \tag{25}$$

for $n \neq 0$, and

$$Y_\infty \simeq \frac{x_e}{\lambda \ln(2x_e/x_f)} , \tag{26}$$

for $n=0$. Assuming that freeze-out occurs while the Universe is shear dominated ($x_f \ll x_e$), the equation for x_f is given by

$$x_f \simeq \ln[(2+c)ac\lambda x_e^{-1}] \\ -(n-\tfrac{1}{2})\ln\{\ln[(2+c)ac\lambda x_e^{-1}]\} . \tag{27}$$

We see that x_f decreases roughly by only an additive factor of $\ln(x_e/x_f)$, justifying our previous assertion that the freeze-out temperature is nearly the same in a shear- or radiation-dominated model.

Defining an enhancement factor

$$\xi \equiv \left[\frac{g_*}{g_*^e}\right]^{1/2}\frac{T_f}{T_e} , \tag{28}$$

we see that if the particle-antiparticle annihilation is primarily s wave, the relic abundance in a shear-dominated universe is increased roughly by a factor of $\xi/\ln\xi$ over that in the standard case. If the annihilation is primarily p wave, which is often the case for Majorana particles,[15] the enhancement is roughly 0.5ξ. This result is particularly interesting for Majorana particles (e.g., Majorana neutrinos, photinos, Higgsinos, etc.) since an enhancement in the relic abundance due to a particle-antiparticle asymmetry is not possible for self-conjugate particles.

As an example of current interest, we may apply our results to a Majorana neutrino of mass 60 GeV. Since such a neutrino is heavier than half the mass of the Z^0, the decay $Z^0 \to \nu\nu$ is kinematically forbidden. Thus such a fourth-generation neutrino is not excluded by the recent SLC-LEP results.[14] Furthermore, since it has only "spin-dependent" couplings to nuclei, its elastic scattering cross section is too small for it to be ruled out by the results of germanium ionization experiments.[16] Even so, it is not generally considered a candidate for the primary component of the dark matter in the galactic halo since its abundance, as determined by standard calculations, is small ($\Omega_\nu \ll 1$) and cannot be increased by introducing a particle-antiparticle asymmetry.[17]

In Fig. 2 we show the results of a numerical integration for Y as a function of x for the cases of $x_e=0$ (standard model), $x_e=1300$ (shear-dominated model with $T_e \simeq 120$ MeV), and $x_e=13\,000$ ($T_e \simeq 12$ MeV). For all three cases, the numerical results agree with the analytic results [Eqs. (25) and (27)] to within 5% [using $c(c+2)=n+1$]. In the standard cosmology, the resulting value of $\Omega_\nu h^2$ is 1.1×10^{-3}—too small for heavy neutrinos to be the primary component of the galactic halo. We find that for $T_e \simeq 120$ MeV the present mass density is increased to $\Omega_\nu h^2 \simeq 0.021$, a value comparable to that known to be contributed by the halos of spiral galaxies; and for $T_e \simeq 12$ MeV the present mass density is $\Omega_\nu h^2 \simeq 0.9$, which is about right to close the Universe.

From this we conclude that some stable particle species that have not been considered dark-matter candidates because of their small relic abundances could indeed still be dark-matter candidates. Only dark-matter search experiments, such as germanium ionization experiments or future bolometric detectors, can definitely rule out a particle species as being the primary component of the galactic halo. Moreover, if an "unlikely" particle relic is discovered, cosmologists would have to significantly alter their current notions of the first second of the Universe's history.

Finally, we mention a possibility suggested by Misner and others, the "decay" of anisotropy into radiation due to dissipative processes.[11] If this occurs, then the anisotropy-energy density at freeze-out could have been much larger than the upper bound imposed from nucleosynthesis ($T_e>2.5$ MeV), provided that the dissipation took place before the epoch of nucleosynthesis. Naively, one might expect that the enhancement of the

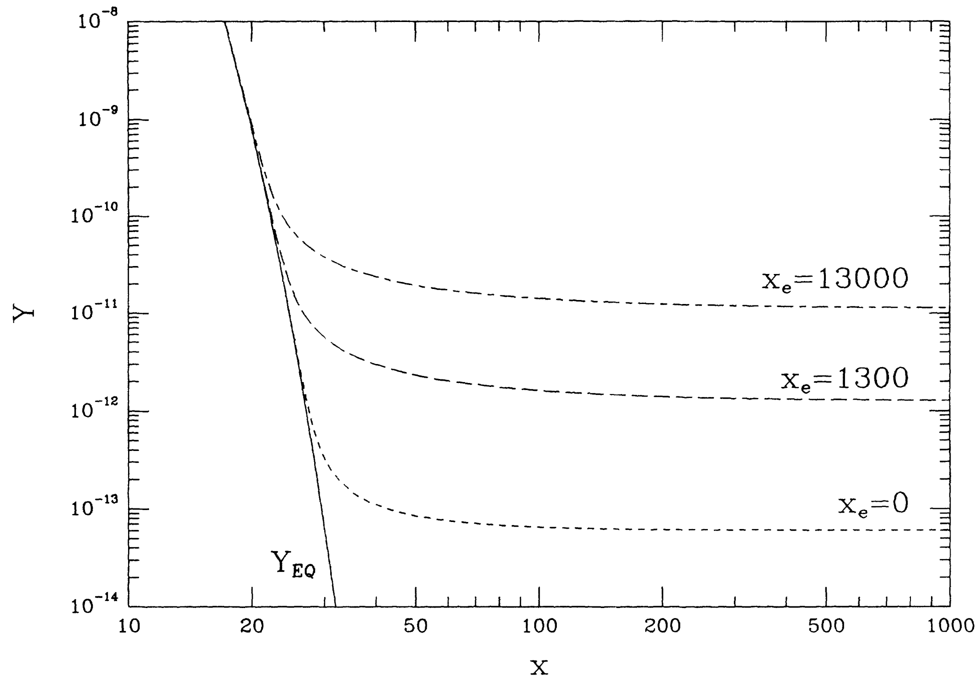

FIG. 2. Plot of the abundance Y of a Majorana neutrino of mass $m=60$ GeV as a function of $x=m/T$ in a radiation-dominated universe ($x_e=0$), and in shear-dominated models with $x_e=1300$ and 13 000.

relic abundance could be arbitrarily large; however, this is not correct. The entropy produced by the dissipation of the anisotropy will dilute the relic abundance. Ignoring factors of order unity if the anisotropy is dissipated at a temperature T_D, the ratio of entropy per comoving volume after dissipation to that before dissipation is $(T_D/T_e)^{3/2}$; the relic abundance is reduced by this factor. The net enhancement over the standard result is about

$$\left[\frac{T_e}{T_D}\right]^{1/2}\frac{T_f}{T_D} . \tag{29}$$

Since $T_e<T_D<T_f$, this factor can be greater than 1; however, since $T_D \gtrsim 1$ MeV, the enhancement can never be as great as the maximum enhancement allowed by our previous analysis where there was no dissipation and $T_e \gtrsim 1$ MeV.

IV. FREEZE-OUT IN A MATTER-DOMINATED UNIVERSE

Next, consider a model where the energy density of the Universe at the freeze-out of particle species X is dominated by some other massive particle species Θ which subsequently decays. Before the decay of Θ, the Friedmann equation is

$$H^2=\frac{8\pi^3 g_*}{90m_{\rm Pl}^2}(T^4+MT^3) , \tag{30}$$

where M is a very large ($M \gg T_f$) parameter with dimensions of mass: Specifically, $M=4m_\Theta n_\Theta/3s$, where m_Θ is the mass and n_Θ is the number density of Θ particles. Once again, since freeze-out occurs roughly when $\Gamma_{\rm ann}=H$ and the number density is proportional to $\Gamma_{\rm ann}$, the relic abundance is apparently enhanced by roughly $(M/T_f)^{1/2}$. To be more precise, since $M \gg T_f$, the expansion rate in this model is $H=H_{\rm std}(Mx/m)^{1/2}$, where $H_{\rm std}$ is the expansion rate in the standard cosmology; therefore, the Boltzmann equation for this case is given by Eq. (6) with the substitutions $\lambda \to \lambda(m/M)^{1/2}$ and $n \to n+\frac{1}{2}$. Making these substitutions, we can "read off" the solution from Eq. (11); the result is

$$Y_\infty=\frac{(n+\frac{3}{2})M}{\lambda m}x_f^{n+3/2}+O(x_f^{n+1/2}) , \tag{31}$$

which agrees with our rough guess of the enhancement factor.

However, this is not the whole story. The subsequent decays of Θ particles occur out of equilibrium and produce a large amount of entropy, thereby lessening the previous enhancement. In fact, the net result is a *reduction* in the relic abundance relative to the standard case. To see this, suppose that the temperature at which the Θ particles decay is T_D (which, of course, is less than T_f); then the ratio of entropy per comoving volume after decay to that before decay is roughly $(M/T_D)^{3/4}$ (see Ref. 5); therefore, the final relic abundance is roughly a factor

$$\left[\frac{T_D^3}{MT_f^2}\right]^{1/4} \tag{32}$$

times that in the standard case, given by Eq. (11). Since $T_D<T_f<M$, no enhancement in the relic abundance is possible; rather, the relic abundance is reduced. We could attempt to circumvent the entropy-production problem by supposing that Θ particles decay into some noninteracting, inert species that does not contribute to

the "visible" entropy density. However, in any interesting case, the additional relativistic degrees of freedom would exceed those allowed by primordial nucleosynthesis.

V. BRANS-DICKE-JORDAN COSMOLOGY

There has been renewed interest in alternative theories of gravity, particularly those in which the gravitational "constant" varies as it does in the Brans-Dicke-Jordan theory. Much of this interest is owed to the advent of extended inflation,[18] a variant of old inflation in which the "graceful exit" problem is solved. Although it now appears that extended inflation in the Brans-Dicke-Jordan[19] theory is not viable, as the isotropy of the microwave background requires the Brans-Dicke parameter ω to be *less* than about 30,[20] while solar-system experiments require that $\omega \gtrsim 500$,[21] variants of the Brans-Dicke-Jordan theory may still lead to successful inflationary scenarios.[22] In this section we will show that a cosmological model based on the Brans-Dicke-Jordan theory with $\omega \gtrsim 500$ allows for significant enhancement in the abundance of a thermal relic. Since many of the scalar-tensor theories currently under consideration resemble Brans-Dicke-Jordan theory (with a variable ω), we expect that our results may generalize to these theories as well.

The Brans-Dicke-Jordan theory of gravitation[19] is the scalar-tensor theory that can be derived from the action

$$S=\frac{1}{16\pi}\int d^4x\,\sqrt{-g}\left[-\Phi\mathcal{R}+\omega\frac{\partial^\mu\Phi\,\partial_\mu\Phi}{\Phi}+16\pi\mathcal{L}_{\rm matter}\right], \tag{33}$$

where $\mathcal{R}$ is the curvature scalar, and the real scalar field Φ has dimensions of mass squared and sets the value of the gravitational constant $G=\Phi^{-1}$; for this reason, Φ must necessarily be greater than zero.[23] Since Φ is a dynamical field one expects the gravitational constant to evolve with time. The quantity ω is the dimensionless Brans-Dicke parameter; in the limit that $\omega\rightarrow\infty$, the scalar-tensor theory reduces to general relativity. While the scalar-tensor theory becomes much less attractive for $\omega\gg 1$, it still provides a simple example of the kind of different gravitation theory that might arise as the low-energy limit of superstring models.[24]

Specializing to the Robertson-Walker line element and for simplicity to a spatially flat model, the equations of motion for the scale factor $R(t)$ and for Φ are

$$\frac{d}{dt}(\rho R^3)=-p\frac{d}{dt}R^3\ , \tag{34}$$

$$\frac{d}{dt}(\dot\Phi R^3)=\frac{8\pi}{3+2\omega}(\rho-3p)R^3\ , \tag{35}$$

$$H^2\equiv\left[\frac{\dot R}{R}\right]^2=\frac{8\pi\rho}{3\Phi}+\frac{\omega}{6}\left[\frac{\dot\Phi}{\Phi}\right]^2-H\left[\frac{\dot\Phi}{\Phi}\right], \tag{36}$$

$$H=-\frac{\dot\Phi}{2\Phi}+\left[\frac{2\omega+3}{3}\left[\frac{\dot\Phi}{2\Phi}\right]^2+\frac{8\pi\rho}{3\Phi}\right]^{1/2}, \tag{37}$$

where as usual ρ is the energy density (of all the fields other than Φ) and p the isotropic pressure. In going from Eq. (36) to Eq. (37), we have considered only the positive root, as we are interested in *expanding* universe models.

Note that in Brans-Dicke-Jordan cosmology there are two additional boundary conditions that must be specified: the values of $\dot\Phi$ and Φ at some epoch. Since the theory must closely resemble general relativity today, the present value of Φ must be equal to G^{-1} (for large ω; see Weinberg, Ref. 19): $\Phi_0=G^{-1}$. That effectively specifies one of the boundary conditions. The other, involving the value of $\dot\Phi$ at some epoch, still remains to be specified.

The Brans-Dicke-Jordan analog of the Friedmann equation [cf. Eq. (37)] differs from the usual one in two regards: First, the gravitational constant is given by Φ^{-1}; second, there is an additional contribution to the energy density that involves the kinetic energy of the Φ field. It will be useful to consider the ratio of the Φ-kinetic term to the usual energy density term:

$$r\equiv\frac{2\omega+3}{3}\left[\frac{\dot\Phi}{2\Phi}\right]^2\Big/\frac{8\pi\rho}{3\Phi}=\frac{(2\omega+3)\dot\Phi^2}{32\pi\Phi\rho}\ ; \tag{38}$$

as we shall see, the ratio r decreases with time: $r\propto R^{-2}$ (when ρ is radiation dominated), and $r\propto{\rm const}/(\ln t)^2$ (when ρ is matter dominated). Having defined r, we can rewrite Eq. (37) in a very suggestive form:

$$H=\left[\frac{8\pi\rho}{3\Phi}\right]^{1/2}\left[(1+r)^{1/2}\mp\left[\frac{3r}{2\omega+3}\right]^{1/2}\right], \tag{39}$$

where the upper sign applies for $\dot\Phi>0$ and the lower sign for $\dot\Phi<0$. In Eq. (39) the two modifications to the usual Friedmann equation are manifest: For $r\neq 0$ the presence of the Φ field speeds up the expansion rate; and if $\Phi\neq G^{-1}$, the expansion rate is also changed.

A. Energy density dominated by relativistic particles

To begin, let us consider the case where the energy density of the Universe is dominated by relativistic particles, which is what one expects at very early times. In this case, $p=\rho/3$, so that

$$\frac{d}{dt}(\dot\Phi R^3)=0\ \Longrightarrow\ \dot\Phi R^3=B\ ,$$

$$\frac{d}{dt}(\rho R^3)=-\frac{\rho}{3}\frac{d}{dt}R^3\ \Longrightarrow\ \rho R^4=A\ ,$$

where A and B are numerical constants. (We will neglect the slight variation of A that occurs because g_* evolves.) In terms of A and B, r is given by

$$r=\frac{(2\omega+3)B^2}{32\pi A\Phi R^2}\ . \tag{40}$$

We see that the $\dot\Phi$ boundary condition can be set by specifying the value of B, or equivalently the value of r, at some epoch. During the radiation-dominated epoch, the value of Φ does not change very much, so that $r\propto R^{-2}$; stated another way, the additional energy density associated with the Φ field redshifts as R^{-6}.

It is simple to integrate the equations of motion [Eqs. (35) and (37)] to obtain Φ as a function of R:[25]

$$\Phi(R)=C\left[\frac{(1+r^{-1})^{1/2}-1}{(1+r^{-1})^{1/2}+1}\right]^{\pm\sqrt{3/(2\omega+3)}}, \tag{41}$$

where the upper sign applies if $B>0$ and the lower sign if $B<0$, and the constant of integration C manifests the freedom one has to specify the value of Φ at some epoch. At early times, corresponding to small R and large r,

$$\Phi \rightarrow C(4r)^{\mp\sqrt{3/(2\omega+3)}},$$

while at late times, corresponding to large R and small r,

$$\Phi \rightarrow C(1-2\sqrt{r})^{\pm\sqrt{3/(2\omega+3)}}.$$

Since $\omega \gg 1$, at early times when $r \gg 1$, the value of Φ *slowly* increases for $B>0$ (decreases for $B<0$); once $r \sim 1$, the value of Φ asymptotes to the value $\Phi = C$ (regardless of the sign of B). When $r \ll 1$—dynamics controlled by the energy density in radiation—$\Phi \simeq C$, and $r \propto R^{-2}$. And, of course, the scale factor of the Universe grows as $t^{1/2}$. In this regime the expansion of the Universe behaves as if there is an additional form of energy density that decreases as R^{-6}, just like shear.

In order that the successful predictions of primordial nucleosynthesis not be upset, r must be less than about 0.2 when $t \sim 1$ sec and $T \sim 1$ MeV;[13] this constrains the initial value of $\dot{\Phi}$. (Moreover, we must also ensure that the value of Φ does not differ from its present value by more than about 20%; as we shall see below, this only requires that $\omega \gtrsim 50$.) The constraint is

$$\frac{\dot{\Phi}^2}{\Phi^2} \lesssim 0.2\left[\frac{12}{2\omega+3}\right]H^2_{\rm BBN} \sim \frac{t^{-2}_{\rm BBN}}{2(2\omega+3)}, \tag{42a}$$

or

$$|\dot{\Phi}_{\rm BBN}| \lesssim (\Phi_{\rm BBN}/t_{\rm BBN})/\sqrt{2(2\omega+3)}. \tag{42b}$$

B. $\dot{\Phi}$-dominated expansion dynamics

Since r evolves as R^{-2}, at early times the dynamics of the expansion will necessarily be dominated by the Φ field. For $r \gg 1$, the equations for the expansion rate of the Universe becomes

$$H \simeq \pm\frac{1}{2}\left[\left[\frac{2\omega+3}{3}\right]^{1/2} \mp 1\right]\frac{\dot{\Phi}}{\Phi} \equiv \pm\beta\frac{\dot{\Phi}}{\Phi}, \tag{43}$$

where $\beta \equiv [\sqrt{(2\omega+3)/3} \mp 1]/2 > 0$, and the upper sign applies if $\dot{\Phi}>0$, while the lower sign applies if $\dot{\Phi}<0$. Assuming that the energy density of the Universe is still dominated by relativistic particles, this equation is supplemented by $\dot{\Phi} = B/R^3$.

These equations are straightforward to solve:

$$R \propto \Phi^{\pm\beta}, \quad R \propto t^{\beta/(3\beta\pm1)}, \quad \Phi \propto t^{\pm1/(3\beta\pm1)},$$

$$\left[\frac{\dot{\Phi}}{\Phi}\right]^2 \propto t^{-2} \propto R^{-6\mp2/\beta}, \quad r \propto R^{-2\mp1/\beta}.$$

Since $\beta \sim \sqrt{\omega/6}$ is expected to be large (greater than about 10), it follows that $R \propto t^{1/3}$, $\dot{\Phi}^2/\Phi^2 \propto R^{-6}$, $r \propto R^{-2}$, and Φ increases slowly with time for $\dot{\Phi}>0$ (decreases for $\dot{\Phi}<0$). That is, during the $\dot{\Phi}$-dominated phase, the Universe behaves like a FRW model whose expansion dynamics are controlled by a form of energy density that decreases as R^{-6}, just as in a shear-dominated model.

C. Energy density dominated by nonrelativistic matter

At an age of about $t_{\rm EQ} \sim 4\times10^{10}$ sec and temperature of about $T_{\rm EQ} \sim 10$ eV, the Universe becomes matter dominated. Based upon the nucleosynthesis bound, we can infer that $r_{\rm EQ} \sim 10^{-10} r_{\rm BBN} \lesssim 10^{-11}$ and

$$\Phi_{\rm EQ} = C\left[1 \mp 2\left[\frac{3}{2\omega+3}\right]^{1/2} r_{\rm EQ}\right] \Longrightarrow \frac{|\Phi_{\rm EQ}-C|}{C} \lesssim \frac{10^{-11}}{\sqrt{\omega}}.$$

To a very good approximation, the Universe will behave as an ordinary matter-dominated FRW model and $R/R_{\rm EQ} = (t/t_{\rm EQ})^{2/3}$. During the matter-dominated epoch, $\rho R^3 = $ const, and it is convenient to express the value of that constant as $\rho_{\rm EQ} R^3_{\rm EQ}$. Thus the evolution of Φ is given by

$$\frac{d}{dt}(\dot{\Phi}R^3) = \frac{8\pi}{2\omega+3}\rho_{\rm EQ}R^3_{\rm EQ}, \tag{44}$$

$$\dot{\Phi}(t) = \left[\dot{\Phi}_{\rm EQ}t^2_{\rm EQ} - \frac{4\Phi_{\rm EQ}t_{\rm EQ}}{3(2\omega+3)}\right]\frac{1}{t_2} + \frac{4}{3}\frac{\Phi_{\rm EQ}}{2\omega+3}\frac{1}{t}, \tag{45}$$

$$\Phi(t) = \Phi_{\rm EQ} + \frac{4\Phi_{\rm EQ}}{3(2\omega+3)}\ln\left[\frac{t}{t_{\rm EQ}}\right] + \left[\dot{\Phi}_{\rm EQ}t_{\rm EQ} - \frac{4\Phi_{\rm EQ}}{3(2\omega+3)}\right]\left[1-\frac{t_{\rm EQ}}{t}\right]. \tag{46}$$

From Eq. (46) we can find the value of Φ at the present epoch ($t = t_0 \simeq 10^7 t_{\rm EQ}$):

$$\Phi_0 = \Phi_{\rm EQ} + \frac{4\Phi_{\rm EQ}}{3(2\omega+3)}\left[\ln\left[\frac{t_0}{t_{\rm EQ}}\right]-1\right] + \dot{\Phi}_{\rm EQ}t_{\rm EQ}; \tag{47}$$

from our constraint to $\dot{\Phi}_{\rm EQ}$ it is simple to show that the term involving $\dot{\Phi}_{\rm EQ}$ is negligible: $\dot{\Phi}_{\rm EQ}t_{\rm EQ} \lesssim 10^{-4}\Phi_{\rm EQ}/2(2\omega+3)^{1/2}$. In order that Φ_0 not differ from $\Phi_{\rm EQ}$ by more than about 20%, ω must be greater than about 50, which is not as stringent a bound as that provided by the solar-system experiments. Finally, it is simple to see from Eq. (45) that in the matter-dominated epoch, $r \propto {\rm const}/(\ln t)^2$, while Φ grows logarithmically with time.

To summarize, primordial nucleosynthesis constrains r to be less than about 0.2 at the epoch of nucleosynthesis and ω to be greater than about 50. The constraint to r provides information about the initial value of $\dot{\Phi}$. At very early times the dynamics of the Universe are necessarily controlled by $\dot{\Phi}$ since $r \propto R^{-2}$. The transition to $\dot{\Phi}$-dominated expansion dynamics will occur at a temperature of about $T_\Phi \simeq r_{\rm BBN}^{-1/2}$ MeV, which could be as low as 3 MeV. During the phase when $\dot{\Phi}$ controls the dynamics of the expansion, the Universe behaves like an ordinary FRW model whose energy density is dominated by a form of energy that decreases as R^{-6}: $R \propto t^{1/3}$. This has implications for the relic abundance of a thermal relic that freezes out at a temperature greater than about 3 MeV, which we will address below, as well as for coherent axion production and for baryogenesis, which we will address elsewhere.[26]

The analysis of the "freeze-in" of the relic abundance of a stable particle species that freezes out at a temperature $T_f > T_\Phi$ is identical to that in the previously discussed shear-dominated model. That is, the relic abundance is increased, relative to the standard case, by a factor of $\xi/\ln\xi$ for s wave, or 0.5ξ for p wave, where

$$\xi = (g_* / g_*^\Phi)^{1/2} \frac{T_f}{T_\Phi}$$

and g_*^Φ is the value of g_* when $r=1$. We should point out that for the simplest Brans-Dicke-Jordan extended-inflationary model, the Universe enters the radiation-dominated epoch directly at the end of inflation bypassing a $\dot{\Phi}$-dominated epoch so that no enhancement in the abundance of a thermal relic can occur.[25] However, the details of extended inflation are far from being completely understood, including whether or not inflation took place, and so a $\dot{\Phi}$-dominated epoch is an interesting cosmological possibility.

As is clear from this section and the previous two, what is required to enhance the relic abundance of a particle species is that the Universe at early times be dominated by a form of energy density that decreases faster than R^{-4}. In this case, this component of the energy density can dominate the energy density at freeze-out and then conveniently disappear before primordial nucleosynthesis without leaving a trace. Shear in a Bianchi I model and the kinetic energy of the Brans-Dicke field provide two examples where the additional energy density is proportional to R^{-6}.

There are more. The energy density of any fluid for which the pressure exceeds the energy density divided by 3 will decrease faster than R^{-4}. The extreme case is $p=\rho$, which used to be discussed as an equation of state for the Universe at very early times.[27] A homogeneous scalar field with Lagrangian density

$$\mathcal{L} = (\partial_\mu \phi)^2/2 - \beta|\phi|^n$$

behaves like a perfect fluid with equation of state $p=(n-2)\rho/(n+2)$ as it oscillates about the minimum of its potential ($|\phi|=0$). In so doing the associated energy density $\rho \propto R^{-6n/(n+2)}$.[28] For $n>4$, the energy density of such a field decreases faster than R^{-4}. For $\beta=0$ or $n \to \infty$, ϕ is a massless, free scalar field and $\rho \propto R^{-6}$. [An interesting example of a massless scalar field is a Goldstone mode. Consider a complex scalar field $\phi = \psi \exp(i\theta)$ with a "Mexican-hat" potential, where ψ and θ are real scalar fields. Suppose that the magnitude of ψ is fixed by spontaneous symmetry breaking; the phase θ is a massless Goldstone mode, which can spin around the brim of the hat. In fact, this is precisely what occurs in a recently suggested scenario for baryogenesis.[29]]

Along similar lines as the Brans-Dicke-Jordan theory is the possibility that the gravitational constant varies because it is related to the size of some extra, compactified dimensions. In many Kaluza-Klein and superstring theories, the gravitational constant G varies as $G = G_{\rm today}(L/L_{\rm today})^D$, where L is the scale factor of D compactified dimensions.[30] If one assumes that all $3+D$ spatial dimensions were of comparable magnitude at early time, then it is plausible to expect that early on the gravitational constant was larger than it is today. The strongest constraint to the variation of G is that imposed by primordial nucleosynthesis,[31] which implies that by the epoch of nucleosynthesis the value of G differed from that today by less than about 20%. However, there are no stringent constraints to the value of G at earlier times. If it were very different than its present value, and larger, then the expansion could have been faster than in the standard cosmology. Since $Y_\infty \propto m_{\rm Pl}^{-1} \propto G^{1/2}$ [cf. Eqs. (7) and (11)], we would expect the relic abundance to be increased.

VI. CONCLUDING REMARKS

While much of the activity in cosmology these days involves the study of the earliest moments of the Universe, we have precious few probes of those early times. A class of potential probes are thermal particle relics—stable particle species that were once in thermal equilibrium. Already such relics have received a great deal of attention, particularly as candidates for the dark matter. Thermal relic dark-matter candidates include heavy neutrinos, neutralinos, and light neutrinos, to mention three of the most interesting possibilities. It goes without saying that the discovery of such a relic would be of enormous importance to cosmology; in addition, the discovery of any of the aforementioned particle species would be of equal importance to particle physics, providing evidence for new physics beyond the standard model of particle physics.

The calculation of the relic abundance of a particle species has become a very routine task for the particle cosmologist. In this paper we have addressed the crucial and untested assumption in the calculation: the temperature dependence of the expansion rate of the Universe. In nonstandard cosmological models where the energy density of the early Universe is dominated by nonrelativistic matter, anisotropy, or the kinetic energy of a scalar field, the relic abundance can be significantly different. In the case of the energy density being dominated by nonrelativistic matter, the relic abundance is ultimately smaller than in the standard case, because of the entropy produced by the eventual decays of the nonrelativistic particles. In the case of a universe that is shear dominated, or $\dot{\Phi}$ dominated, early on, the relic abundance can be greatly enhanced owing to the fact that the expansion rate for a given temperature is larger, which leads to a freeze-out at a higher temperature and a larger abundance. We remind the reader that in spite of the fact that the standard, radiation-dominated FRW model is very well motivated, there is no direct evidence that excludes the possibilities that we have discussed here. (We do mention that the levels of shear that are interesting for our purposes are definitely incompatible with the inflationary universe.[32])

The fact that the relic abundance of a particle species can be greater than the canonical abundance is of no small interest to those involved in dark-matter searches. It is well known that the results of the standard relic abundance calculation can be decreased by phenomena, such as inflation, an electroweak or quark-hadron phase transition, or out-of-equilibrium decay of a massive particle, that produce a significant amount of entropy after freeze-out. If the canonical calculation indicates that the relic abundance is too small for the species to be the primary component of the galactic halo, any of these entropy-producing processes only make the conclusion that much stronger. However, if freeze-out occurs in a shear- or $\dot{\Phi}$-dominated epoch, the relic abundance is enhanced. Thus some particle species that are not interesting dark-matter candidates according to the standard calculations may indeed be interesting dark-matter candidates. Perhaps one should take the empirical view that a particle dark-matter candidate should only be ruled out by null results in dark-matter searches or accelerator searches. Our work also implies that the rates for indirect signatures, such as high-energy neutrinos from particle dark-matter annihilations in the Sun or Earth, or positron-line or γ-ray-line radiation from particle dark-matter annihilation in the halo, could be significantly larger than expected. Finally, the discovery of one of these "unlikely" particle relics in the galactic halo would force us to reconsider our current view of a radiation-dominated universe at time earlier than about 1 sec.

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APPENDIX

Here we show that the Boltzmann equation [cf. Eq. (2)] used to describe the evolution of the number density of a species in the FRW model is also valid for the Bianchi I model considered in this paper. To do so we follow the discussion in Ref. 5.

The evolution of a particle's phase-space distribution $f(p^u, x^u)$ is governed by the Boltzmann equation, which can be written as

$$\hat{\mathbf{L}}[f] = \mathbf{C}[f] , \tag{A1}$$

where $\mathbf{C}$ is the collision operator and $\hat{\mathbf{L}}$ is the Liouville operator and is given by

$$\hat{\mathbf{L}} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} , \tag{A2}$$

where $\Gamma^\alpha_{\beta\gamma}$ are the usual Christoffel symbols. For the Bianchi models, the phase-space density is spatially homogeneous, and so f is a function of time $t = x^0$ but not space. A crucial assumption is that scattering interactions are occurring rapidly enough so that the particle species remains in kinetic equilibrium. Provided that this is the case, the phase-space density f is isotropic and only depends on the magnitude of the momentum or, equivalently, the energy E, and the Boltzmann equation becomes

$$\hat{\mathbf{L}}[f(E,t)] = E\frac{\partial f}{\partial t} - [R_1\dot{R}_1(p^1)^2 + R_2\dot{R}_2(p^2)^2 + R_3\dot{R}_3(p^3)^2]\frac{\partial f}{\partial E} . \tag{A3}$$

Since the number density of the species is

$$n(t) = \frac{g}{(2\pi)^3}\int d^3p\, f(E,t) , \tag{A4}$$

the equation for the evolution of the number density is obtained by multiplying Eq. (A1) by $g\,d^3p/(2\pi)E$ and integrating. The first term on the left-hand side becomes dn/dt, and the right-hand side becomes the right-hand side of Eq. (2). To obtain the remaining term we note that the local three-momentum squared is $|\mathbf{p}|^2 = g_{ij}p^ip^j$ (i.e., the physical components of the momenta are $p_x = R_1 p^1$, etc.), and so the second term on the right-hand side is

$$\begin{aligned}\frac{-g}{(2\pi)^3}\int \frac{\partial f}{\partial E}(H_1p_x^2 + H_2p_y^2 + H_3p_z^2)\frac{d^3p}{E} &= \frac{-g}{(2\pi)^3}(H_1+H_2+H_3)\int \frac{\partial f}{\partial E}p_x^2\frac{d^3p}{E} \\ &= -\frac{1}{3}\frac{g}{(2\pi)^3}(H_1+H_2+H_3)\int\frac{\partial f}{\partial E}|\mathbf{p}|^2\frac{d^3p}{E} \\ &= (H_1+H_2+H_3)n = 3Hn ,\end{aligned} \tag{A5}$$

where we used the isotropy of f in the first two steps and integrated by parts in the third step. In doing so, we recover Eq. (2). Thus, although the form of the Liouville operator in the Bianchi I model differs from that in the FRW model, the Boltzmann equation for the evolution of the number density of a particle species is the same.

The crucial assumption made above is that the particle species is in kinetic equilibrium. Earlier than the time of freeze-out, annihilations are occurring rapidly ($\Gamma_{\rm ann} > H$), and they serve to maintain both kinetic and chemical equilibrium. In addition, if the species (X) annihilates into relativistic particles, then by crossing symmetry $X\bar{X}$'s can elastically scatter with particles in the thermal bath with a similar cross section. The relativistic particles in the thermal bath are always more abundant than $\bar{X}$'s, especially when $x \gg 1$ and $Y_{\rm EQ} \ll 1$, and so these elastic scattering processes will serve to keep $X\bar{X}$'s in kinetic equilibrium even after chemical equilibrium ceases to be maintained (i.e., after freeze-out).

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[25]See, e.g., the Appendix of Weinberg (Ref. 20); in this paper, Weinberg also considers the evolution of Φ in the case where the energy density is dominated by vacuum energy.

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