PHYSICAL REVIEW D VOLUME 42, NUMBER 2 15 JUNE 1990

Baryogenesis in a baryon-symmetric universe

Scott Dodelson
Gordon McKay Laboratory, Harvard University Cambridge, Massachusetts 02138

Lawrence M. Widrow
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
and Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138
(Received 11 January 1990)

Baryon number is conserved in all interactions probed by present-day experiments. If baryon number is strictly conserved then currently popular scenarios for baryogenesis will have to be reexamined. We discuss a new paradigm for baryogenesis in which the fundamental Lagrangian is baryon conserving [invariant under $U(1)_B$]. At high temperatures, $U(1)_B$ is spontaneously broken and an excess of quarks over antiquarks of $10^{-10}s$ ($s \equiv$ entropy density) is produced. Today, $U(1)_B$ is restored. The most striking consequence of our assumptions is that the baryon number of the Universe is constant. During spontaneous symmetry breaking, the excess of baryons over antibaryons in the quark fields is exactly compensated by antibaryons hidden in the vacuum. Today, antibaryons appear either as massive $U(1)_B$ charged scalar particles or as stable, nontopological bubbles of antimatter. One intriguing possibility suggested by our scenario is that the dark matter may be antimatter.

I. INTRODUCTION

Experiments in terrestrial laboratories suggest that the laws of physics do not differentiate between matter and antimatter. (The single verified exception to this rule is *CP* violation in the K^0-$\bar{K}^0$ system.) Experiments further indicate that baryon number and lepton number are separately conserved in all physical processes. Our Universe on the other hand is very asymmetric, having far more protons, neutrons, and electrons than antiprotons, antineutrons, and positrons. This suggests one of two possibilities: either the Universe began with the very special initial conditions necessary to yield the baryon (and lepton) asymmetry observed today or new physics at high energies and early times allowed an initially baryon-symmetric universe to develop a baryon excess.

It is instructive to reexamine the evidence for the matter-antimatter asymmetry in the Universe.[1] There is clear evidence that sizable objects of antimatter do not exist in the solar system. If they did, then annihilations of particles in the solar wind striking their surfaces would produce γ rays that could be easily detected on Earth. High-energy (> 100-MeV) cosmic rays provide a sample of matter from distant regions of the Milky Way. To date, the only antiparticles detected in cosmic-ray experiments have been antiprotons: no heavy antinuclei have yet been seen. The measured ratio of antiprotons to protons with energies above 2 GeV is $\sim 10^{-4}$ and this measurement is consistent with the level of antiprotons expected to be produced as secondaries in cosmic-ray collisions (e.g., $p + p \rightarrow 3p + \bar{p}$). This strongly suggests that our Galaxy is composed almost entirely of matter with very little antimatter. Finally, there is indirect, though compelling, evidence that matter and antimatter galaxies do not coexist within clusters of galaxies. If they did, then nucleon-antinucleon annihilations in the intergalactic medium would lead to an observable flux of γ rays on Earth.

The baryon excess is usually characterized by the dimensionless ratio B where

$$B \equiv \frac{n_B - n_{\bar{B}}}{s} , \tag{1.1}$$

n_B ($n_{\bar{B}}$) is the number density in baryons (antibaryons), and s is the entropy density. If baryon number is conserved, then B is either constant or decreasing. Today $n_B \gg n_{\bar{B}}$ (at least on scales up to clusters of galaxies) and $B \simeq n_B/s \simeq 0.6-1.0\times 10^{-10}$ (Ref. 2).

In 1967 Sakharov[3] outlined the ingredients necessary for baryogenesis, the process whereby an initially baryon-symmetric universe develops an asymmetry. The most basic observation is that baryon number cannot be strictly conserved. If B is initially zero, then only baryon-violating interactions can generate a nonzero B. In addition, Sakharov points out that C and CP must be violated and that the interactions responsible for generating the baryon asymmetry must occur out of equilibrium. Grand unified theories[4] (GUT's) introduced in the 1970s provide a natural setting for baryogenesis.[5] GUT's generically predict baryon-violating interactions. In general, these interactions are mediated by a boson whose mass m_X is $\sim 10^{15}$ GeV, the GUT scale. In the early Universe, at temperatures above m_X, baryon-violating processes occur at rates comparable to the rates for other processes. However, today the rates for baryon-violating interactions (e.g., proton decay) would be highly suppressed ($\Gamma_{\text{proton decay}} \propto m_p^5/m_X^4$) so long as m_X is very large. *CP* violation can also occur quite naturally though GUT's shed little light on the origin of *CP* violation. Finally, GUT physics occurs at very early times when the expansion rate is very fast; it is therefore relatively easy to have processes occurring out of thermal equilibrium.

Baryogenesis at the GUT scale does have serious drawbacks. For example, an inflationary epoch produces a large entropy density thereby diluting B, and therefore baryogenesis must occur *after* reheating (or during reheating as in Ref. 6). However, having the Universe reheat to such high temperatures can be very difficult to arrange for in realistic models (see, for example, Ref. 7). Furthermore, GUT-scale baryogenesis requires that there be no baryon-violating interactions below 10^{15} GeV, for any such processes would wash out the asymmetry. This requirement appears particularly problematic in light of recent suggestions that quantum effects in the standard model may be powerful enough to wash out the baryon asymmetry at the weak scale.[8,9] Finally, it should be pointed out that there is no experimental evidence supporting GUT's; indeed the little evidence available rules out the simplest GUT's.[10]

In this paper we discuss a new paradigm for baryogenesis,[11] one which allows, perhaps even forces, the baryon asymmetry to be generated at relatively low temperatures[12] ($T \sim 1-10^5$ GeV). We assume that baryon number is conserved in all fundamental interactions. That is, we assume that the Lagrangian is invariant under the simultaneous [global $U(1)_B$] transformation

$$\psi_i \rightarrow e^{ib_i\theta}\psi_i \ , \tag{1.2}$$

where ψ_i are the fields in the Lagrangian and b_i are their charges under $U(1)_B$. We further assume that $U(1)_B$ is spontaneously broken at early times.[13] In general, this occurs when a scalar quantity (either a fundamental scalar or a quark bilinear) charged under $U(1)_B$ gets a nonzero vacuum expectation value (VEV). For definiteness let us assume that spontaneous symmetry breaking is accomplished by a complex scalar field ϕ charged under $U(1)_B$ but neutral under the standard model gauge group $[SU(3)_{\text{color}}\times SU(2)_L\times U(1)_Y]$. During spontaneous symmetry breaking, baryon-violating interactions among the quarks and leptons can occur. As long as Sakharov's other requirements are met, an excess of quarks over antiquarks can develop. Of course, if baryon number were spontaneously broken today we would see baryon-violating interactions occurring at very rapid rates. We therefore require that as the Universe expands and cools, the VEV's of the baryonic fields disappear and $U(1)_B$ is restored. We will discuss this rather unusual idea of high-temperature symmetry breaking and low-temperature symmetry restoration in more detail below. We note here that the phenomenon has been discussed by various authors.[14] Furthermore, high-temperature symmetry breaking is observed in the ferroelectric behavior of Rochelle salts.[15]

The astute reader may wonder how a baryon excess can develop in a theory in which baryon number is conserved in the fundamental Lagrangian. In truth, no baryon asymmetry has been generated. During spontaneous symmetry breaking, a baryon asymmetry in the quarks develops but this asymmetry is exactly compensated by antibaryons hidden in the vacuum. When the symmetry is restored, the antibaryons reappear, either as ϕ particles or as solitons containing large numbers of antibaryons.

The comments in the previous paragraph deserve further clarification. To this end we discuss the classical equations of motion. Let j_B^μ be the total baryon current: j_B^μ includes contributions from the quark fields, the scalar field ϕ, and all other fields charged under $U(1)_B$. In a baryon-symmetric theory, j_B^μ is conserved:

$$\partial^\mu j_\mu^B = 0 \ . \tag{1.3}$$

[For simplicity we will assume a flat (nonexpanding) background. The generalization to a Robertson-Walker spacetime is straightforward.[16]] During spontaneous symmetry breaking, $\langle\phi\rangle = v/\sqrt{2}$ and it is appropriate to write

$$\phi = \frac{v+\rho}{\sqrt{2}}e^{i\theta/v} \ , \tag{1.4}$$

where ρ is a heavy (real) scalar and θ is the massless Goldstone boson. The low-energy effective Lagrangian for θ is

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{v}\partial^\mu\theta\tilde{j}_\mu^B \ , \tag{1.5}$$

where $\tilde{j}_\mu^B$ is the baryon current from all fields except ϕ. The equation of motion for θ is, therefore,

$$\partial^\mu(v\partial_\mu\theta + \tilde{j}_\mu^B) = 0 \ . \tag{1.6}$$

Comparing Eq. (1.6) and Eq. (1.3) we see that during spontaneous symmetry breaking j_B^μ splits into a vacuum part (excitations of the Goldstone field θ) and a part containing ordinary matter fields. In a closed system (a closed universe, for example) we can derive a conservation law for baryonic charge by integrating Eq. (1.6) over a spacelike surface:[17]

$$\left.\frac{dQ_B}{dt}\right|_{\text{vacuum}} + \left.\frac{dQ_B}{dt}\right|_{\text{particles}} = 0 \ , \tag{1.7}$$

where

$$Q_B\Big|_{\text{vacuum}} = v\int d^3x\, \partial^0\theta \tag{1.8a}$$

and

$$Q_B\Big|_{\text{particles}} = \int d^3x\, \tilde{j}_B^0 \ . \tag{1.8b}$$

Once the symmetry is restored, the vacuum can no longer hide the charge and antibaryons must appear and must have a number density equal to the number density of baryons. This result is a direct consequence of our assumptions and has no analogue in scenarios with explicit baryon violation. The implications are rather striking. We immediately see that the energy per baryon number of the antimatter present today cannot exceed ~ 70 GeV: if it did, then antimatter would overclose the Universe. This does suggest the interesting possibility that the dark matter is actually antimatter. It further suggests that baryogensis in our scenario will occur at GeV and TeV energies and might therefore be testable in present-day particle accelerators.

In Sec. II we present a simple model which demonstrates the essential ideas of our scenario. In Sec. III we calculate the baryon asymmetry generated in this model. In addition, we demonstrate that inverse decays and baryon-violating scatterings do not wash out the asymmetry. In Sec. IV we discuss the possibility that the antibaryons which must be present today turn up as free ϕ particles. A somewhat more speculative idea is to have the antibaryons trapped in large regions of the false-vacuum energy. This possibility is discussed in Sec. V. A summary and conclusion are given in Sec. VI. Finally, an appendix is devoted to the question of how charge gets stored in the vacuum during spontaneous symmetry breaking.

II. BARYON VIOLATION AT HIGH TEMPERATURES

In this section we present a simple model in which $U(1)_B$ is unbroken at low temperatures but spontaneously broken at high temperatures. In addition to the ordinary fields and couplings of the standard model we introduce four new scalar fields—ϕ, ϕ_1, ϕ_2, and ϕ_3. Their couplings to ordinary quarks and leptons are

$$L=\lambda_2 M\phi^*\phi_1\phi_2^*+\lambda_3 M\phi^*\phi_1\phi_3^*+f_1\phi_1U^TCD+f_2\phi_2^*U^TCE+f_3\phi_3^*U^TCE+\text{H.c.} \quad (2.1)$$

Here the λ's and the f's are dimensionless coupling constants; M is a parameter with dimensions of mass— comparable to the masses of the ϕ_i's; C is the charge-conjugation matrix ($i\gamma^2\gamma^0$ in the Dirac representation); and U, D, and E refer to the ordinary quarks and leptons. For simplicity we assume that the ϕ_i's couple only to right-handed fermions so that, for example, U in Eq. (2.1) stands for $U_R\equiv[(1+\gamma_5)/2]U$. Color and generation indices have also been suppressed: ϕ_1, for example, couples to the top and bottom quarks as well as the up and down. Let us exhibit explicitly one of the Yukawa terms in Eq. (2.1):

$$f_1\phi_1U^TCD\to\epsilon^{\alpha\beta\gamma}f_1^{ij}\phi_{1,\alpha}U^T_{\beta,i}C\frac{1+\gamma_5}{2}D_{\gamma,j}\ . \quad (2.2)$$

Here i,j are indices labeling the three generations of fermions and α,β,γ are $SU(3)_{color}$ indices.

The interaction Lagrangian in Eq. (2.1) respects all of the symmetries of the standard model. Clearly it is invariant under the gauge group $SU(3)_{color}\times SU(2)_L\times U(1)_Y$. For example, ϕ_2 transforms as an $SU(3)_{color}$ triplet, a singlet with respect to weak interactions and carries electric charge $-\frac{1}{3}$. The transformation properties of the scalar fields are listed in Table I. The local symmetries of the standard model are not the only ones preserved by these interactions; the global symmetries—baryon number and lepton number—are also respected.

TABLE I. Transformation properties of the scalar fields introduced in the text.

	$SU(3)_{color}$	$U(1)_{em}$	$U(1)_B$	$U(1)_L$
ϕ_1	3	$-\frac{1}{3}$	$-\frac{2}{3}$	0
ϕ_2	3	$-\frac{1}{3}$	$\frac{1}{3}$	1
ϕ_3	3	$-\frac{1}{3}$	$\frac{1}{3}$	1
ϕ	1	0	-1	-1
$\tilde{\phi}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$

For example, the interaction term $\phi_2^*U^TCE$ is invariant under $U(1)_B$:

$$\phi_2^*U^TCE\to(e^{i\theta/3}\phi_2)^*(e^{i\theta/3}U)^TCE=\phi_2^*U^TCE\ , \quad (2.3)$$

if we assign ϕ_2 baryon number $+\frac{1}{3}$. Similar considerations applied to the other scalar fields allow us to fill out Table I. Of particular interest is the scalar field ϕ; it is a gauge singlet and has $B=L=-1$, just like a positron-antiproton combination (antihydrogen). Therefore, if ϕ gets a VEV, the global symmetries $U(1)_B$ and $U(1)_L$ will be broken, but the gauge symmetries will remain unbroken.

We note that ϕ_3 has the same quantum numbers as ϕ_2 and is apparently redundant. Baryogenesis is in fact possible in a model without ϕ_3 but achieving a final baryon-to-photon ratio of order 10^{-9} proves difficult. [This problem is reminiscent of one encountered in baryogenesis models based on the minimal SU(5) GUT.] We will return to this point below. Finally, we note that there is a simple change that can be made in Eq. (2.1) that maintains all of the features necessary for baryogenesis: we can replace the $\phi^*\phi_1\phi_2^*$ coupling by the term

$$\tilde{\phi}^*\tilde{\phi}^*\phi_1\phi_2^*\ . \quad (2.4)$$

$\tilde{\phi}$ like ϕ is a gauge singlet but carries baryon (and lepton) number $-\frac{1}{2}$. As with ϕ, if $\tilde{\phi}$ gets a VEV, then baryon number and lepton number will be spontaneously broken. The baryon asymmetry based on Eq. (2.4) is roughly the same as the asymmetry that develops in the model based on Eq. (2.1). However, as we shall see in Sec. IV, there are dramatic differences between the ϕ model and the $\tilde{\phi}$ model in the present Universe. For the time being we will stick with the $\phi^*\phi_1\phi_2^*$ coupling.

Our model Lagrangian also has a potential for ϕ: $V(\phi)$. The Lagrangian is symmetric under $U(1)_B$ and we therefore require that $V=V(|\phi|)$. Consider then the potential

$$V(\phi,\sigma)=m_\phi^2|\phi|^2+\alpha_1|\phi|^4+\alpha_2|\sigma|^4-2\alpha_3|\phi|^2|\sigma|^2\ , \quad (2.5)$$

where σ is another complex scalar field. The dimensionless couplings α_i are assumed real and positive. Furthermore,

$$\alpha_1\alpha_2>\alpha_3^2 \quad (2.6)$$

is required so that the potential is bounded from below. The values of $|\phi|$ and $|\sigma|$ in the ground state (the vacuum) are determined by minimizing the potential with respect to $|\phi|$ and $|\sigma|$:

$$\frac{\partial V}{\partial|\phi|}=0=2|\phi|(m_\phi^2+2\alpha_1|\phi|^2-2\alpha_3|\sigma|^2)\ , \qquad (2.7a)$$

$$\frac{\partial V}{\partial|\sigma|}=0=2|\sigma|(2\alpha_2|\sigma|^2-2\alpha_3|\phi|^2)\ . \qquad (2.7b)$$

The solution $|\sigma|=|\phi|=0$ is clearly an extremum, but is it the global minimum? Inspection of Eqs. (2.7) shows that

$$|\phi|=|\sigma|=0 \qquad (2.8)$$

is the only extremum and is therefore the global minimum of the potential. By definition this is the zero-temperature vacuum state. In it, ϕ has mass m_ϕ; σ is massless (this is not essential as we could have added an $m_\sigma^2|\sigma|^2$ term to the potential); and since ϕ does not have a VEV, baryon number and lepton number are unbroken symmetries.

At finite temperature there are corrections to the potential which are capable of shifting the ground state. We need consider only the one-loop corrections to the finite-temperature effective potential, $V(\sigma,\phi;T)$. Figure 1 shows typical diagrams that contribute to V. Summing all one-loop contributions (see, for example, Ref. 14) we find

$$V_{1\text{ loop}}(\sigma,\phi;T)=-\frac{\alpha_3-2\alpha_1}{6}T^2|\phi|^2+\frac{2\alpha_2-\alpha_3}{6}T^2|\sigma|^2 \qquad (2.9)$$

for temperatures above m_ϕ. If $2\alpha_1<\alpha_3<2\alpha_2$, then these corrections induce a negative mass-squared term for $|\phi|^2$ which depends on the temperature. Once again the vacuum state is found by minimizing $V(\sigma,\phi;T)=V(\sigma,\phi)+V_{1\text{ loop}}(\sigma,\phi;T)$ with respect to the fields. We therefore look for solutions to the equations

$$\frac{\partial V}{\partial|\phi|}=0=2|\phi|\left[m_\phi^2-\frac{\alpha_3-2\alpha_1}{6}T^2+2\alpha_1|\phi|^2-2\alpha_3|\sigma|^2\right], \qquad (2.10a)$$

$$\frac{\partial V}{\partial|\sigma|}=0=2|\sigma|\left[2\alpha_2|\sigma|^2+\frac{2\alpha_2-\alpha_3}{6}T^2-2\alpha_3|\phi|^2\right]. \qquad (2.10b)$$

Clearly $|\sigma|=|\phi|=0$ is a local extremum, but again we must check to see if it is the global minimum. It is straightforward to show that $|\sigma|\neq0, |\phi|=0$ and $|\sigma|\neq0, |\phi|\neq0$ are not solutions of Eqs. (2.10). However, $|\sigma|=0$ and

$$|\phi|^2=\frac{1}{2\alpha_1}\left[\frac{\alpha_3-2\alpha_1}{6}T^2-m_\phi^2\right] \qquad (2.11)$$

is a solution so long as the term in parentheses is positive.

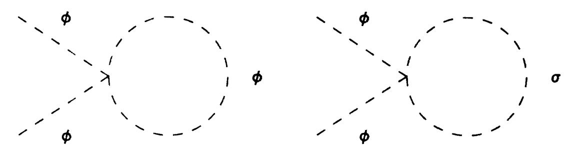

FIG. 1. Feynman diagrams for the one-loop correction to the finite-temperature effective potential. Shown here is the corrections to the $|\langle\phi\rangle|^2$ term in the effective potential. Similar diagrams can be drawn for the $|\langle\sigma\rangle|^2$ term.

Thus, for $T>T_c$ where

$$T_c\equiv\left[\frac{6}{\alpha_3-2\alpha_1}\right]^{1/2}m_\phi\ , \qquad (2.12)$$

the effective potential has two extrema. It is easily verified that in this case $|\sigma|=|\phi|=0$ is a local maximum

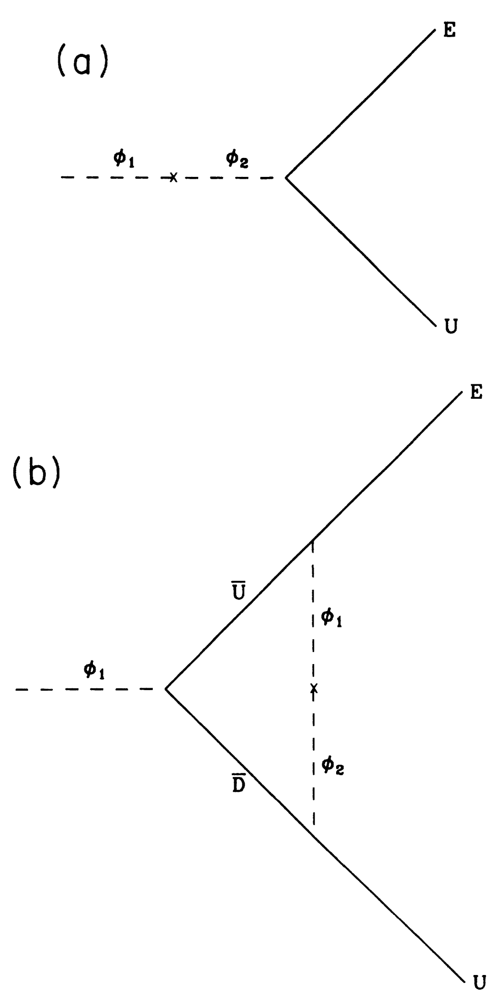

FIG. 2. Feynman diagrams for the $\phi_1\to UE$ decay. Scalar particles are represented by dashed line and fermions by solid lines; (a) is the tree-level graph and (b) is the one-loop graph. In addition, the Feynman diagrams with ϕ_2 replaced by ϕ_3 contribute to the decay rate.

and the true minimum is at $\sigma=0$ and

$$|\langle\phi\rangle|=\kappa(T^2-T_c^2)^{1/2} , \quad (2.13)$$

where

$$\kappa\equiv\left[\frac{\alpha_3-2\alpha_1}{12\alpha_1}\right]^{1/2} \quad (2.14)$$

is dimensionless and typically of order unity. To summarize, at low temperatures baryon and lepton numbers are unbroken symmetries. However, at temperatures above the critical temperature T_c, ϕ gets a VEV and baryon number and lepton number are spontaneously broken.

Spontaneously broken baryon number means that there are fundamental processes—decays, $(2\leftrightarrow2)$ scattering, etc.—in which the baryon number of the initial set of particles is different from the baryon number of the final set. For example, since ϕ_1 has baryon number $-\frac{2}{3}$, it normally decays into $\overline{U}\overline{D}$. When ϕ has a VEV, there are terms in the interaction Lagrangian [Eq. (2.1)] that mix ϕ_1 and ϕ_2:

$$\begin{aligned} L_{\rm mix}&=\lambda_2M\langle\phi\rangle^*\phi_1\phi_2^* \\ &=\lambda_2M\kappa\sqrt{T^2-T_c^2}\phi_1\phi_2^* . \end{aligned} \quad (2.15)$$

These terms facilitate the decay $\phi_1\to UE$, as shown in Fig. 2. Therefore ϕ_1 decays into two modes $(\overline{U}\overline{D},UE)$, that have different baryon numbers $(-\frac{2}{3},+\frac{1}{3})$; it is no longer meaningful to assign ϕ_1 (or ϕ_2,ϕ_3) distinct baryon numbers. This is analogous to the situation encountered in GUT-inspired theories in which the so-called X and Y bosons couple explicitly to the two different modes.[18] It is therefore reasonable to ask if a nonzero baryon number could build up at $T>T_c$ just as a nonzero baryon number emerged from GUT's. In the next section we address this question.

III. BARYON-SYMMETRIC BARYOGENESIS

A. General considerations

A net baryon asymmetry can be generated only if (a) there are baryon-nonconserving interactions, (b) CP is violated, and (c) the Universe is not in equilibrium. At temperatures above T_c we have seen that there are fundamental processes in which the baryon number of incoming particles is not equal to the baryon number of outgoing particles. CP violation is achieved by having complex Yukawa couplings in the interaction Lagrangian. We must be careful that the complex phases associated with the coupling constants cannot be rotated away by redefining the quark fields. In fact, without ϕ_3 and with only one generation of quarks and leptons, just such a rotation exists:

$$E\to e^{-i(\text{phase } f_2)}E,\quad D\to e^{-i(\text{phase } f_1)}D . \quad (3.1)$$

The new coupling constants would be completely real, and there would be no CP violation. There are two simple ways to avoid this: one is to rely on the fact that there are several generations of fermions. It can be shown that the CP-violating phases cannot be rotated away when we consider more than one generation. However, the calculation of the baryon asymmetry in this case is tedious and we have resorted to a less elegant albeit simpler way of maintaining CP violation: we have simply added ϕ_3. As we will see, with this addition CP is violated. We focus on one generation of fermions and assume that it is the heaviest generation so that U, D, and E become the top quark, bottom quark, and tau lepton, respectively. To be explicit, we are assuming that the Yukawa couplings to the heaviest generation are much larger than those to the lighter generations. We note that it is straightforward to generalize our results to the case in which all of the Yukawa couplings are comparable.

Finally, we need an out-of-equilibrium scenario. We assume that at some temperature T_d there is an equal number density $n\simeq T_d^3$ of ϕ_i particles and antiparticles. If T_d is much smaller than the ϕ_i mass M_i, then these heavy particles are out of equilibrium. [Recall that in equilibrium the abundances would be suppressed by the Boltzmann factor $\exp(-M_i/T_d)$.] Since their lifetimes are much shorter than the age of the Universe, they immediately decay. If T_d is above the critical temperature T_c, then each ϕ_i can decay into two channels: $\overline{U}\overline{D}$ and UE. The decay of n $\phi_i-\phi_i^*$ pairs produces a net baryon-number density in the quarks given by

$$\begin{aligned} n\epsilon_i=n\frac{1}{\Gamma_i}\{&\tfrac{1}{3}[\Gamma(\phi_i\to UE)-\Gamma(\phi_i^*\to\overline{U}\overline{E})] \\ &-\tfrac{2}{3}[\Gamma(\phi_i\to\overline{U}\overline{D})-\Gamma(\phi_i^*\to UD)]\} . \end{aligned} \quad (3.2)$$

Here the Γ's are the partial widths into the given channels and Γ_i is the total decay width of ϕ_i. Recall that while CPT constrains the total width of ϕ_i to be equal to that of ϕ_i^* the partial widths can differ if CP is violated.

We can obtain an order of magnitude estimate for the baryon asymmetry by examining the Feynman graphs that contribute to ϵ. In our model, CP violation manifests itself in the interference of the tree-level and one-loop graphs. For example, in the $\phi_1\to UE$ decay, one contribution is $M^{(0)*}M^{(1)}$ where $M^{(0)}$ is the tree-level graph depicted in Fig. 2(a) and $M^{(1)}$ is the one-loop graph in Fig. 2(b). For simplicity we take the masses of the $\phi_i[(M_1,M_2,M_3)]$ to be of the same order of magnitude M. We then estimate the net baryon-number density to be

$$\begin{aligned} n\epsilon_1&\sim n\frac{1}{|f_1|^2M^2}{\rm Im}[(f_2^*\lambda_2\langle\phi\rangle^*)^*(|f_1|^2f_3^*\lambda_3\langle\phi\rangle^*)] \\ &=n\frac{|\langle\phi\rangle|^2}{M^2}{\rm Im}(f_2f_3^*\lambda_2^*\lambda_3) . \end{aligned} \quad (3.3)$$

As we will see in the next section this estimate is a good one. Several points about Eq. (3.3) are readily apparent. First we note that if $U(1)_B$ were unbroken, then $\langle\phi\rangle=0$ and no asymmetry could develop. In our model $\langle\phi\rangle$ is of order T so that if a large enough asymmetry is to develop, then the decays must take place at temperatures not too much smaller than M. Third, the final asymmetry depends on the phase of $f_2f_3^*\lambda_2^*\lambda_3$; this confirms the fact that ϕ_3 is necessary for *CP* violation (at least if we consider only one generation of fermions). Finally, we note that the final asymmetry does *not* depend on the phase of $\langle \phi \rangle$. This is important since during symmetry breaking, the magnitude of $\langle \phi \rangle$ is fixed but the phase of $\langle \phi \rangle$ is completely undetermined. It is expected to vary from one causal domain to another. The fact that the final baryon asymmetry does not depend on the phase of $\langle \phi \rangle$ means that the asymmetry is the same everywhere, even in domains in which $\langle \phi \rangle$ has a different phase.

As discussed in the Introduction, the fundamental Lagrangian is baryon symmetric and therefore the total baryon number in the Universe must be conserved. This implies that any baryon violation in ordinary matter fields that arises during spontaneous symmetry breaking must be compensated by baryon number stored in the vacuum. To be more explicit, at temperatures above T_c, $\langle \phi \rangle \neq 0$ and we can parametrize ϕ using Eq. (1.4) with $v = \sqrt{2}\kappa(T^2 - T_c^2)^{1/2}$ [cf. Eq. (2.13)]. The low-energy effective theory for θ, the phase of ϕ, is then given by Eq. (1.5) where $\tilde{j}^B_\mu$ includes contributions from the ϕ_i as well as the ordinary quarks. In the above scenario $\tilde{j}^B_0$ starts out at 0 but builds up to some nonzero value as the ϕ_i's decay. This time dependence of $\tilde{j}^B_0$ causes θ to develop a finite velocity. Even after $\tilde{j}^B_0$ becomes constant, there is no force acting against the time dependence of θ so it continues to rotate. With this velocity, θ (or equivalently ϕ) can store charge (i.e., baryon number) and this charge exactly cancels any change in baryon number in the ordinary matter fields. In the Appendix we discuss the process of storing charge in the vacuum in more detail. The fate of the stored charge once the symmetry is restored is the subject of Secs. IV and V.

In the next section we will undertake a more careful computation of the final asymmetry. Before this, though, let us return to the question of nonequilibrium. We have assumed that the ϕ_i's decay when they are out of equilibrium. However, the simplest processes which govern their abundance—decays and inverse decays—have rates much greater than the expansion rate of the Universe. If the Universe evolved in a straightforward manner, then the number density of ϕ_i would decrease like $e^{-M/T}$ as the temperature dropped. At $T \ll M$, there would be essentially none of these particles left and therefore no asymmetry would result.

However, the early Universe holds many mysteries and there is no reason to expect it to evolve in a "straightforward" manner. In particular there are many violent phenomena that might produce ϕ_i's at temperatures far beneath their mass. For example, at the end of an inflationary era, the vacuum energy responsible for driving inflation is converted into particles. It is easy to envision ϕ_i-ϕ_i^* pairs produced at the end of such an era even if the "reheating" temperature is much less than M. The pairs would immediately decay producing the requisite asymmetry.[19] Alternatively, a very massive particle (mass $\gg M$) could have a very long lifetime and decay into ϕ_i-ϕ_i^* pairs only when the temperature has dropped beneath M. Another intriguing idea[20] is that the ϕ_i's get their mass from the vacuum expectation value of another field. If the Universe undergoes a phase transition wherein the VEV of this other field changes, then the mass of ϕ_i changes accordingly. Say, for example, the mass of ϕ_i is initially 0 but after the phase transition it becomes M_i. Initially the number density of ϕ_i (and ϕ_i^*) is $\sim T^3$; after the phase transition this is much larger than the equilibrium number density $[\sim (MT)^{3/2} e^{-M/T}]$. The ϕ_i-ϕ_i^* pairs then immediately decay producing the asymmetry. It is conceivable that one or another of these speculations can be incorporated into the phase transition at T_c, where ϕ loses its VEV. In this work, to make the new physics as transparent as possible, we will not rely on any specific way of attaining out of equilibrium conditions; we simply parametrize whatever process is responsible by saying that at some temperature T_D below the mass M, there is a number density n of $\phi_i - \phi_i^*$ pairs and this number density is considerably larger than the equilibrium number density.

B. Calculating the asymmetry

In this section we calculate the final baryon asymmetry produced when the ϕ_i's decay. To simplify the calculation we work in a basis in which the mass matrix for the ϕ_i's is diagonal. When $\langle \phi \rangle$ becomes nonzero, there are mass mixing terms in the Lagrangian. The mass terms are

$$L_{\text{mass}} = (\phi_1, \phi_2, \phi_3)^* \begin{bmatrix} M_1^2 & \lambda_2^* M \langle \phi \rangle & \lambda_3^* M \langle \phi \rangle \\ \lambda_2 M \langle \phi \rangle^* & M_2^2 & 0 \\ \lambda_3 M \langle \phi \rangle^* & 0 & M_3^2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} . \tag{3.4}$$

We can diagonalize this mass matrix by rotating Φ with a unitary matrix. Specifically, we define the rotated fields

$$(X_1, X_2, X_3) \equiv \mathcal{U}^\dagger \Phi \simeq \begin{bmatrix} 1 & \dfrac{\lambda_2^* M \langle \phi \rangle}{M_1^2 - M_2^2} & \dfrac{\lambda_3^* M \langle \phi \rangle}{M_1^2 - M_3^2} \\ -\dfrac{\lambda_2 M \langle \phi \rangle^*}{M_1^2 - M_2^2} & 1 & 0 \\ -\dfrac{\lambda_3 M \langle \phi \rangle^*}{M_1^2 - M_3^2} & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} . \tag{3.5}$$

This form of $\mathcal{U}$ is correct in the limit that the off-diagonal terms are small. Even if the λ's are of order 1, $\langle\phi\rangle \sim T \ll M$ so that this is the appropriate limit. In terms of the new fields X_i the mass matrix is diagonal; indeed to lowest order in $\lambda T/M$, it is simply $\mathrm{diag}(M_1^2, M_2^2, M_3^2)$. The coupling of the X_i's to ordinary fermions can be read off from the interaction Lagrangian (2.1) and the relationship between X_i and ϕ_i [Eq. (3.5)]. We find

$$L_{\mathrm{int}} = \sum_i X_i(c_i U^T C D + d_i \bar{U} E^c) , \qquad (3.6)$$

where $E^c \equiv C\gamma_0^T E^*$ is the conjugate of E and the new couplings written in terms of the old ones are

$$c_i \equiv \mathcal{U}_{1i} f_1, \quad d_i \equiv \mathcal{U}_{2i} f_2^* + \mathcal{U}_{3i} f_3^* . \qquad (3.7)$$

In this form the similarity between the spontaneously broken theory considered here and theories with explicit baryon violation becomes apparent. The X_i propagate without mixing. However, they do not have definite baryon (or lepton) number as can be seen from the fact that each couples to two modes that have different baryon number.

We are interested in the net baryon number produced when these heavy particles decay. However, it is simpler to calculate the net lepton number produced. These two quantities are equal since $B-L$ is *not* spontaneously broken: in each fundamental process the baryon number minus lepton number of the incoming set of particles is equal to that of the outgoing set. In analogy to Eq. (3.2) we have

$$\frac{n_B}{n} = \frac{n_L}{n} = \sum_i \frac{1}{\Gamma_i}[\Gamma(X_i \to UE) - \Gamma(X_i^* \to \bar{U}\bar{E})] , \qquad (3.8)$$

where Γ_i is the width of X_i.

We will restrict our analysis to the case of massless decay products. This is a good approximation as long as the decaying particles are much heavier than the quark and lepton masses. In this limit the kinematics is greatly simplified and the decay rates are related to the amplitudes by

$$\Gamma = \frac{1}{16\pi M_i} \sum_{\mathrm{spins}} |T|^2 , \qquad (3.9)$$

where T is the amplitude for the process. From this we get the full widths of the X_i:

$$\Gamma_i = \frac{M_i}{16\pi}(2|c_i|^2 + |d_i|^2) , \qquad (3.10)$$

where the factor of 2 accounts for the fact that there are really two different $\bar{U}\bar{D}$ final states with $\bar{U}$ and $\bar{D}$ color indices interchanged. In terms of the amplitudes, the final lepton number is

$$\frac{n_L}{n} = \sum_i \frac{1}{2|c_i|^2 + |d_i|^2} \frac{1}{M_i^2} \sum_{\mathrm{spins}} [\,|T(X_i \to UE)|^2 - |T(X_i^* \to \bar{U}\bar{E})|^2] . \qquad (3.11)$$

To calculate the difference between the amplitudes, we need to evaluate the interference between the tree graph in Fig. 3(a) and the one-loop graph in Fig. 3(b). The tree amplitude is

$$T_i^{(0)} = d_i S , \qquad (3.12)$$

where S is the spinor product

$$S = u_U^\dagger(q) C \frac{1+\gamma_5}{2} u_E^*(q') \qquad (3.13)$$

and the u's are the spinors for massless particles. The one-loop amplitude is

$$T_i^{(1)} = 2iS \sum_j c_i^* d_j^* c_j I_{ij} , \qquad (3.14)$$

where the loop integral is

$$I_{ij} = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - M_j^2)(q-k)^2(q'+k)^2} . \qquad (3.15)$$

The subscripts on I indicate that it depends on the masses of both the internal particle X_j and the decaying particle X_i (the latter since $2q \cdot q' = M_i^2$). Again the factor of 2 in

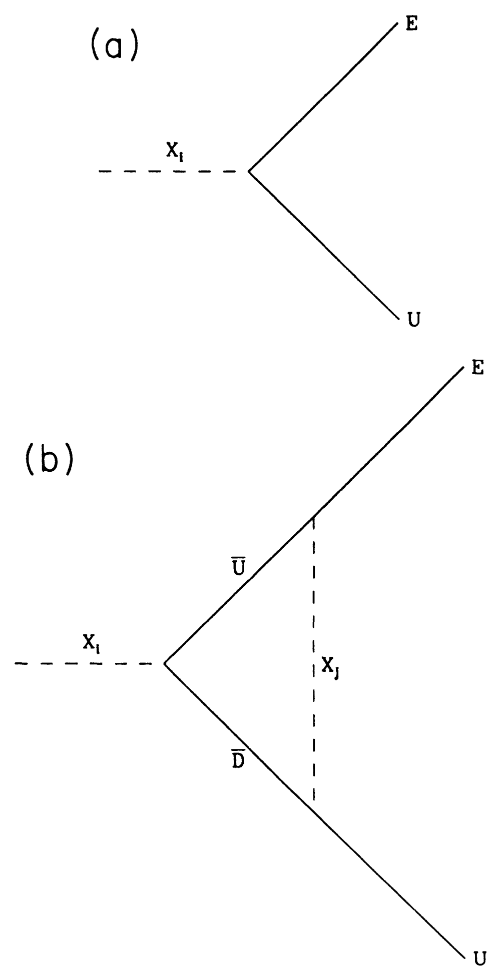

FIG. 3. Feynman diagrams for $X_i \to UE$ decay; (a) is the tree-level graph and (b) is the one-loop graph.

the one-loop amplitude accounts for the two different graphs with only internal color indices interchanged. So far we have focused only on the amplitude for X_i decay; the amplitude for X_i^* decay is exactly the same except that the coupling constants are replaced by their complex conjugates. A little algebra then leads to

$$\sum_{\text{spins}}[|T(X_i\to UE)|^2-|T(X_i^*\to \overline{U}\overline{E})|^2] =8\left[\sum_{\text{spins}} S^*S\right]\sum_j \mathrm{Im}(d_id_j^*c_i^*c_j)\mathrm{Re}(I_{ij}) . \quad (3.16)$$

The sum over the different spin states yields

$$\sum_{\text{spins}} S^*S=2q\cdot q'=M_i^2 . \quad (3.17)$$

The real part of the integral in Eq. (3.16) is not divergent and can be evaluated by standard techniques:

$$\mathrm{Re}(I_{ij})=-\frac{1}{16\pi}\left[1-\frac{M_j^2}{M_i^2}\ln(1+M_i^2/M_j^2)\right] . \quad (3.18)$$

We can use these last three equations to express the final lepton number as

$$\frac{n_L}{n}=\sum_i \frac{1}{2|c_i|^2+|d_i|^2}\sum_j I(M_j^2/M_i^2)\mathrm{Im}(d_id_j^*c_i^*c_j) , \quad (3.19)$$

where

$$I(x)\equiv-\frac{1}{2\pi}[1-x\ln(1+1/x)] . \quad (3.20)$$

The c's and d's can be written in terms of the fundamental parameters λ_i, M_i, and $\langle\phi\rangle$ using Eqs. (3.5) and (3.7). Substituting we find

$$\frac{n_L}{n}=\frac{|f_1|^2M^2|\langle\phi\rangle|^2}{(M_1^2-M_2^2)(M_1^2-M_3^2)}\mathrm{Im}(\lambda_2\lambda_3^*f_2^*f_3) \times\left[\frac{I(M_2^2/M_1^2)-I(M_3^2/M_1^2)}{2|f_1|^2}+\frac{I(M_3^2/M_2^2)-I(M_1^2/M_2^2)}{|f_2|^2}+\frac{I(M_1^2/M_3^2)-I(M_2^2/M_3^2)}{|f_3|^2}\right] \equiv\epsilon . \quad (3.21)$$

Equation (3.21) gives us an expression for the final baryon-number density produced in terms of the fundamental parameters of the theory, the number density of the heavy particles right before they decay, and the temperature at the time of decay (recall that $\langle\phi\rangle$ depends on the temperature). We now calculate B the ratio of the final baryon-number density over the entropy density. This ratio will remain constant once baryon-violating reactions have become ineffective (assuming of course that entropy is conserved). In the next section we demonstrate that for a reasonable set of parameters baryon-violating reactions are indeed ineffective immediately after the ϕ_i's decay. The baryon to entropy ratio today is then the same as it was immediately after decays. At that time the entropy density was

$$s_f=\frac{2\pi^2}{45}g_*T_f^3 , \quad (3.22)$$

where g_* counts the number of degrees of freedom. At temperatures above the electroweak scale g_* is roughly equal to 100. The subscript f on the temperature refers to the fact that the temperature is raised when the decay products of the ϕ_i's equilibrate. If we denote T_d as the temperature right before decays and if we make the approximation that all decays take place simultaneously, then T_f can be determined by energy conservation:

$$\rho_d=2n(M_1+M_2+M_3)+\frac{2\pi^2}{45}g_*T_d^4 =\frac{2\pi^2}{45}g_*T_f^4 =\rho_f . \quad (3.23)$$

With this expression we can determine T_f in terms of T_d and write the final baryon to entropy ratio as

$$B=\frac{n}{(2\pi^2/45)g_*T_d^3}\left[1+\frac{2n(M_1+M_2+M_3)}{(2\pi^2/45)g_*T_d^4}\right]^{-3/4}\epsilon . \quad (3.24)$$

From this expression we see that if the decay temperature is many orders of magnitude smaller than the M_i's, a lot of entropy will be produced, thereby diluting the final asymmetry. For our purposes, though, we will assume that the decay temperature is not too much lower than the heavy masses, so that the entropy increase factor (in large parentheses) is of order unity. In the next sections, then, we will use the approximate equality $B\sim\epsilon/100$. Since B today is approximately 10^{-10} we will require

$$\epsilon=10^{-8} . \quad (3.25)$$

C. Washing out the asymmetry

Once the heavy particles have decayed, baryon-nonconserving processes must become ineffective. If not,

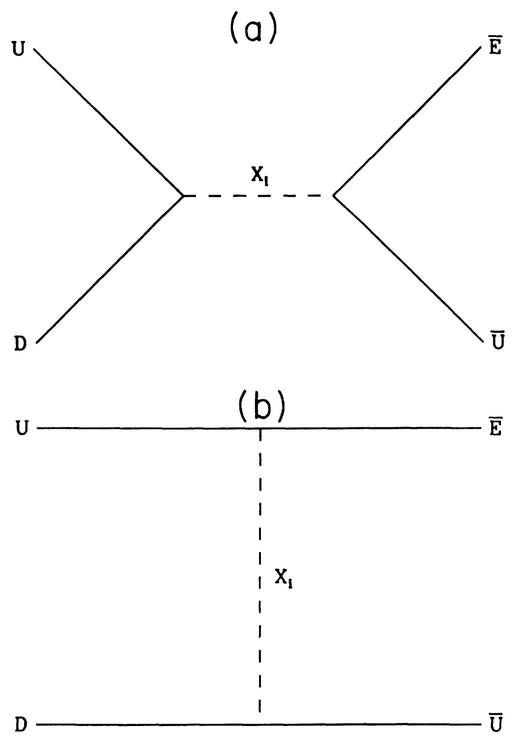

FIG. 4. Feynman diagrams for the baryon-violating scattering interaction $U+D\rightarrow\bar{U}+\bar{E}$; (a) is the diagram for s-channel scattering and (b) is the diagram for t-channel scattering.

then these processes will wash out any baryon asymmetry produced. Quantitatively, the requirement is that

$$\Gamma_{\Delta B} < H \ . \tag{3.26}$$

That is, the baryon-nonconserving rates must be less than the Hubble expansion rate.

There are two types of processes which are dangerous. The first is inverse decays: if the heavy particles can be reproduced rapidly, inverse decays will wash out the asymmetry. However, this process is suppressed by a Boltzmann factor. To see this, note that the total energy of two incoming fermions must exceed M_i if they are to produce an X_i. Since the decay products quickly equilibrate with the rest of the plasma, the number of any species having energy of order M_i is suppressed by e^{-M_i/T_d}. Even if the nominal rate for inverse decays is large compared to the Hubble rate, it is clear that for

$$T_d < M/25 - M/30 \tag{3.27}$$

inverse decays should not cause a problem.

The second type of dangerous reactions are $(2\leftrightarrow2)$ scattering processes mediated by one of the heavy particles. To get the rate for this process we first calculate the cross section due to both s- and t-channel graphs [Figs. 4(a) and 4(b)] and then integrate over the light-particle distributions. Explicitly,

$$\Gamma_{2\leftrightarrow2} = \frac{1}{\int dp\, p^2 e^{-p/T}} \int dp\, p^2 e^{-p/T} \int \frac{d^3p'}{(2\pi)^3} e^{-p'/T} \sigma(p,p')$$
$$= \frac{1}{2T^3} \int dp\, p^2 e^{-p/T} \int \frac{d^3p'}{(2\pi)^3} e^{-p'/T} \frac{1}{4pp'} \int \frac{d^3q}{(2\pi)^3 2q} \int \frac{d^3q'}{(2\pi)^3 2q'} (2\pi)^4 \delta^4(p+p'-q-q') \tfrac{6}{2} \sum_{\text{spins}} |\mathcal{T}|^2 \ . \tag{3.28}$$

Here the cross section $\sigma(p,p')$ has been expressed in terms of the amplitude $\mathcal{T}$. A number of approximations have been used to derive this expression. First, we have again taken all fermion species to be massless. We will briefly return to this point later; for now we note that this leads to an *over*estimate of the rate and will therefore give an overly conservative constraint for our theory. We have also used Boltzmann statistics; this should be a good approximation since the particles most likely to take part in such a reaction are those in the tail of the distribution $(p > T)$. For such particles, the difference between the Fermi-Dirac and the Boltzmann distributions is negligible $[1/(e^{p/T}+1) \rightarrow e^{-p/T}]$. The factor of 6 in the last line of Eq. (3.28) comes from summing over colors of the incident and outgoing particles and the factor of $\frac{1}{2}$ accounts for spin averaging.

First we evaluate the amplitude and sum over spins. In terms of the Lorentz invariants

$$s \equiv (p+p')^2, \quad t \equiv (p-q)^2 \tag{3.29}$$

we find

$$\sum_{\text{spins}} |\mathcal{T}|^2 = |G|^2 (s^2 + t^2 + 2st) \ . \tag{3.30}$$

G is the four-Fermi coupling for baryon-violating interactions

$$G = \sum_i \frac{d_i^* c_i}{M_i^2} \ . \tag{3.31}$$

To perform the q and q' integrations we notice that since the measures of integration are Lorentz invariant, the integrals over the Lorentz-invariant quantity s^2+t^2+2st must also be Lorentz invariant. The only such quantity that depends on p and p' is the center-of-mass energy squared s, so that

$$\int \frac{d^3q}{(2\pi)^3 2q} \int \frac{d^3q'}{(2\pi)^3 2q'} (2\pi)^4 \delta^4(p+p'-q-q') \times (s^2+t^2+2st) = \text{const} \times s^2 \ . \tag{3.32}$$

By choosing a simple reference frame, it is easy to verify that the constant is $1/24\pi$. Therefore the scattering rate is

$$\Gamma_{2\leftrightarrow 2}=\frac{|G|^2}{64\pi T^3}\int dp\, p e^{-p/T}\int \frac{d^3p'}{p'(2\pi)^3}e^{-p'/T_S 2} . \quad (3.33)$$

Performing the integrations and setting $T=T_d$ we find

$$\Gamma_{2\leftrightarrow 2}=\frac{3}{2\pi^3}|G|^2T_d^5 . \quad (3.34)$$

Again we can express the c's and d's in G in terms of the fundamental constants of the interaction Lagrangian (2.1):

$$|G|^2=\frac{|f_1|^2M^2|\langle\phi\rangle|^2}{M_1^4}\left|\frac{f_2\lambda_2^*}{M_2^2}+\frac{f_3\lambda_3^*}{M_3^2}\right|^2 . \quad (3.35)$$

To see that the constraint equation (3.26) can be satisfied while at the same time producing a large enough baryon asymmetry, we assume that all of the couplings are of the same order of magnitude ($f_i\sim f,\lambda_i\sim\lambda$) and all of the heavy masses are similar ($M_i\sim M$). Then, ϵ is of order

$$\epsilon\sim f^2\lambda^2\frac{T_d^2}{M^2} \quad (3.36)$$

and the scattering rate is

$$\Gamma_{2\leftrightarrow 2}\sim f^4\lambda^2\frac{T_d^7}{M^6}\sim\epsilon f^2\frac{T_d^5}{M^4} . \quad (3.37)$$

Note that we have used the fact that $\langle\phi\rangle\sim T_d$ to obtain these expressions. We must compare the scattering rate with the expansion rate:

$$H=\sqrt{8\pi/3}\frac{[(2\pi^2/45)g_*T_d^4]^{1/2}}{m_{\rm Pl}}=20\frac{T_d^2}{m_{\rm Pl}} . \quad (3.38)$$

Using the fact that ϵ must be 10^{-8}, the ratio of the rates is then

$$\frac{\Gamma_{2\leftrightarrow 2}}{H}\sim 6\times 10^6 f^2\left[\frac{T_d}{M}\right]^3\left[\frac{1\text{ TeV}}{M}\right] . \quad (3.39)$$

However, we have argued above [Eq. (3.27)] that T_d must be smaller than $M/25$, so

$$\frac{\Gamma_{2\leftrightarrow 2}}{H}<400f^2\left[\frac{1\text{ TeV}}{M}\right] . \quad (3.40)$$

To satisfy Eq. (3.26) we require that this be less than 1; this in turn places a constraint on the parameters f and M:

$$\left[\frac{f}{1/20}\right]^2\left[\frac{1\text{ TeV}}{M}\right]<1 . \quad (3.41)$$

We see that a significant asymmetry can develop without being erased by rescattering even if the heavy masses are $\sim 10^2-10^3$ GeV. Moreover, it is possible that ϕ_i couples strongly with the heaviest generation (top, bottom, tau) and weakly with the lighter generations. If this is the case then we have probably overestimated the scattering rate for

$$M\sim 100\text{ GeV} . \quad (3.42)$$

For then, by Eq. (3.27), the decay temperature is much smaller than the top-quark mass and this ensures that there are very few top quarks around (their number is suppressed by $e^{-m_{\rm top}/T_d}$) that can rescatter into ϕ_i's. In any event, it is clear that with natural choices of the parameter set, a large baryon asymmetry can emerge.

D. Baryon-number-$\frac{1}{2}$ scalars

Until now we have focused on a model in which the field which gets a vacuum expectation value is a gauge singlet and carries baryon and lepton number equal to -1. As mentioned in Sec. II, we could have substituted the scalar interaction $\tilde\phi^*\tilde\phi^*\phi_1\phi_2^*$ for the term $\phi^*\phi_1\phi_2^*$. Here we pause to point out that it is possible to achieve a final asymmetry of the requisite order in this alternative model. The only change in the calculations of the previous sections is in the off-diagonal mass terms of Eq. (3.4). The coupling $\lambda_2M\phi^*\phi_1\phi_2^*$ gives rise to the off-diagonal mass term

$$\lambda_2M\langle\phi\rangle^*\phi_1\phi_2^*=\lambda_2M\kappa(T^2-T_c^2)^{1/2}\phi_1\phi_2^* . \quad (3.43)$$

With $\tilde\phi$ we have

$$\lambda_2\langle\tilde\phi\rangle^{*2}\phi_1\phi_2^*=\lambda_2\kappa^2(T^2-T_c^2)\phi_1\phi_2^* . \quad (3.44)$$

Therefore, our final result for ϵ, Eq. (3.21), is exactly the same except that in the numerator we must make the substitutions

$$|\langle\phi\rangle|^2\to|\langle\tilde\phi\rangle|^2 , \quad (3.45a)$$

$$M^2\to|\langle\tilde\phi\rangle|^2 . \quad (3.45b)$$

Let us rework the order-of-magnitude estimates of the previous subsection for the $\tilde\phi$ model. The asymmetry is now of order

$$\epsilon\sim f^2\lambda^2\left[\frac{T_d}{M}\right]^4 . \quad (3.46)$$

Since T_d/M must be less than $\frac{1}{25}$ to suppress inverse decays, the couplings f and λ must be slightly larger to achieve a final asymmetry of order 10^{-10}. In fact we require

$$f\lambda>0.06 . \quad (3.47)$$

This is certainly not unreasonable but it is a little more stringent than in the ϕ model. The scattering rate in the $\tilde\phi$ model is different, but when written in terms of ϵ, it is again given by Eq. (3.37). Therefore, the final constraint on the coupling f and the heavy mass scale M, Eq. (3.41), holds in this model as well. To summarize, the final symmetry is relatively insensitive to the choice of model. However, we will see in Sec. IV that the later history of the Universe may be very sensitive to this choice.

IV. SCALAR ANTIBARYONS

During the baryogenesis epoch, the baryon asymmetry generated in the quark fields is exactly compensated by a density of antibaryons hidden in the vacuum. Once the symmetry is restored the vacuum can no longer hide the charge and antibaryons, in one form or another, must appear, being present today with a number density equal to the number density of ordinary baryons. In this section we discuss the possibility that the antibaryons present today are in the form of free ϕ particles.

At temperatures above the critical temperature T_c, the vacuum hides charge by having $\langle\phi\rangle$ develop a time-dependent phase; $\langle\phi\rangle = ve^{-i\omega t}/\sqrt{2}$. In this simple ansatz, ϕ is homogeneous; the magnitude of ϕ and the angular velocity are roughly constant on time scales short compared to the expansion time $[\dot{v}/v \sim \dot{\omega}/\omega = O(H)]$. v and ω are determined by minimizing the energy while holding the charge density fixed. As T drops below T_c, $\langle\phi\rangle$ is driven to zero. However, ϕ carries a net baryon number and therefore ϕ cannot be driven to zero everywhere. To see this, let us assume that $\langle\phi\rangle = ve^{-i\omega t}/\sqrt{2}$ at temperatures below T_c where again v and ω are slowly varying functions of time. The expression for the energy density of the ϕ field is

$$\rho_\phi = \tfrac{1}{2}\omega^2 v^2 + V(v) \ . \tag{4.1}$$

At low temperature, v will be small so it is sufficient to retain only the quadratic term in the potential, $V(v) \simeq m_\phi^2 v^2/2$. Substituting n_B/v^2 for ω and minimizing the result with respect to v we find

$$v = \left[\frac{n_B}{m_\phi}\right]^{1/12} , \tag{4.2a}$$

$$\omega = m_\phi \ , \tag{4.2b}$$

$$\rho_\phi = n_B m_\phi \ . \tag{4.2c}$$

These results indicate that the ϕ field is behaving like a condensate of zero momentum ϕ particles and this is precisely what it is.

It is useful to think of baryon-number density in the field ϕ as a type of angular momentum. [It is in fact just the angular momentum of ϕ rotating in its internal $U(1)_B$ space.] Above T_c, ϕ has a large VEV ($\langle\phi\rangle \sim T$) and can "spin up" to a charge density of $\sim 10^{-9}\ T^3$ with small angular velocity ($\omega \sim 10^{-9}T$) and relatively low rotational energy density ($\rho_{\text{vac}} \sim 10^{-18}\ T^4$). (The fact that the baryon number can be stored in the θ field with such low energy is further justification for our claim that charge is *hidden* in the vacuum.) As T drops below T_c, $\langle\phi\rangle$ is driven to zero and ω as well as ρ_{vac} must increase in order to conserve baryon number. We can in fact think of the $\omega^2 v^2/2 = n_B^2/2v^2$ term in the energy density as a centrifugal barrier which prevents $\langle\phi\rangle$ from reaching zero.

While at early times ϕ might well be in the coherent state described above, it is unlikely that this will be the case today. Gravitational interactions between the ϕ field and ordinary matter should be enough to endow the ϕ field with nonzero momentum. It is therefore useful to think of the field as free, nonrelativistic ϕ particles. The ϕ's in fact make a natural candidate for cold dark matter and would be expected to accrete into our galactic halo. (As a dark-matter candidate, the ϕ is in some respects similar to the axion).

The parameters governing ϕ are tightly constrained by both cosmology and particle physics. The constraints of course must be satisfied subject to the conditions that $\epsilon \sim \lambda^2 f^2 (T_d/M)^2 \sim 10^{-8}$ and that the asymmetry is not washed out [Eqs. (3.27) and (3.41)]. To simplify the discussion which follows we take $T_d/M \simeq 0.03$ so that $\epsilon \sim 10^{-3} f^2\lambda^2$ or equivalent $f^2\lambda^2 \sim 10^{-5}$.

Clearly, m_ϕ must be greater than the proton mass, else the proton would decay into a ϕ^* and a positron. Assuming that most of the ϕ's have not yet decayed, the energy density in ϕ's as compared to the critical energy density ρ_c is

$$\Omega_\phi = \frac{\rho_\phi}{\rho_c} = \frac{m_\phi}{m_p}\Omega_b \ , \tag{4.3}$$

where Ω_b is the density in baryons relative to the closure density. Since $0.014 \le \Omega_b \le 0.16$ we have that

$$0.94\ \text{GeV} < m_\phi < 6\text{–}70\ \text{GeV} \ . \tag{4.4}$$

We note here that as T drops below T_c, there may be many ϕ's and ϕ^*'s over and above the ϕ's necessary to compensate for the baryon excess in the quarks. That is, we might have $n_\phi - n_{\phi^*} = n_B$ but $n_\phi + n_{\phi^*} \gg n_B$. We assume that ϕ-ϕ^* annihilations [e.g., $\phi + \phi^* \to \sigma + \sigma^*$, cf. Eq. (2.5)] are very efficient so that at temperatures not too far below T_c, $n_{\phi^*} \simeq 0$ and $n_\phi = n_B$ (Ref. 21) (If this were not the case then ϕ's and ϕ^*'s would overclose the Universe.)

In the model described by Eq. (2.1), ϕ can decay into an antiproton and a positron. This process will wipe out the baryon asymmetry, since all antiprotons produced will annihilate with protons, unless the lifetime of the ϕ's is greater than the age of the Universe (i.e., $\tau_\phi > t_U$). The constraint imposed is actually much tighter than this since the decay products from the ϕ's can be observed on Earth.[22] For example, ϕ decay could lead to an observable flux on Earth of antiprotons in cosmic rays. Let us assume that ϕ particles make up the dark matter in the galactic halo. The number density of ϕ's is then $n_\phi \simeq 0.01\ (m_\phi/30\ \text{GeV})\ \text{cm}^{-3}$. Antiprotons produced by ϕ decays will be trapped in the halo of our Galaxy by galactic magnetic fields with a confinement time t_h of about 10^8 yr.[23] The number of $\bar{p}$'s found in the halo is therefore $n_{\bar{p}} \simeq n_\phi t_h/\tau_\phi$ and the flux of antiprotons measured on Earth will be $F_{\bar{p}} = c n_{\bar{p}}/4\pi \simeq 2\times 10^5 \times (m_\phi/30\ \text{GeV})(t_U/\tau_\phi)\ \text{cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1}$. For this to be consistent with observations, we require that $F_{\bar{p}} < 10^{-5}\ \text{cm}^{-2}\,\text{s}^{-1}\text{sr}^{-1}$ (see, for example, Refs. 22 and 24). This translates to a limit of $\tau_\phi > 10^{10} t_U$. In addition, it is possible that ϕ decays will produce an observable level of high-energy γ rays. For example, ϕ can decay into heavy quarks and leptons. These in turn decay into light quarks and leptons emitting pions in the process. 100-MeV γ rays are then produced in e.g., $\pi^0 \to 2\gamma$. Measurements by Fitchel, Simpson, and Thompson[25] show that the number density of 100-MeV γ rays is $\simeq 4\times 10^{-15}/\mathrm{cm}^3$ while the number density of ϕ's is $n_\phi = n_B \simeq 1.4-3.0\times 10^{-6}/\mathrm{cm}^3$. We find, then, as an overall limit, that less than 1 part in 10^{10} of the ϕ's should decay. That is, we require that $\tau_\phi > 10^{27}$ sec $\sim 10^{10} t_U$.

To implement this constraint we must calculate the ϕ decay rate. The decay proceeds through diagrams such as the one in Fig. 5. We can estimate the amplitude corresponding to this diagram by noting that the typical momentum transfers (of order m_ϕ) are much smaller than the masses the of the heavy internal particles. Thus, from coupling constants and heavy propagators alone, we find

$$\mathcal{T} \propto \frac{(\lambda_2 M) f_1 f_2}{M_1^2 M_2^2}\,, \tag{4.5}$$

where $\mathcal{T}$ is the amplitude. Now we assume that m_ϕ is sufficiently above m_p so that there is no phase-space suppression. The lifetime of the ϕ is then

$$\tau_\phi \simeq \frac{1}{\lambda^2 f^4}\left[\frac{M}{m_\phi}\right]^6 m_\phi^{-1}$$
$$\simeq 10^{22}\ \mathrm{sec}\, f^{-2}\left[\frac{M}{10^8\ \mathrm{GeV}}\right]^6\left[\frac{10\ \mathrm{GeV}}{m_\phi}\right]^7 . \tag{4.6}$$

We could, for example, have $\lambda \sim 1$, $f^2 \simeq 10^{-5}$, $m_\phi \simeq 10$ GeV, and $M > 10^5$ TeV. This places a rather stringent constraint on M. We can soften this constraint by making f smaller. For example, we can have a situation where the ϕ_i's couple strongly to the heavy generation quarks and leptons and weakly to the light generation.

Interactions such as $\phi p \to \pi^0 e^+$ are also possible and are potentially dangerous in wiping out the asymmetry. However, the rate for these reactions is very slow and in particular is much slower than the rate for ϕ decay.

The situation is very different in the $\tilde{\phi}^* \tilde{\phi}^* \phi_1 \phi_2^*$ model. In this case $b_{\tilde{\phi}} = -\frac{1}{2}$ and the $\tilde{\phi}$'s are stable. There are, however, scattering interactions that convert $\tilde{\phi}$'s to ordinary antibaryons. For example, the interaction $\tilde{\phi}+\tilde{\phi}\to \bar{p}+e^+$ can potentially wipe out the asymmetry. The cross section for this interaction is

$$\sigma v \simeq \lambda^2 f^4 \left[\frac{m_\phi}{M}\right]^8 m_\phi^{-2} \tag{4.7}$$

and the reaction rate is

$$\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+} = n_B \sigma v \simeq 10^{-9}\lambda^2 f^4 \left[\frac{m_\phi}{M}\right]^8 \frac{T^3}{m_\phi^2} . \tag{4.8}$$

To satisfy the constraint, Eq. (3.47), we take $f\lambda = 0.03$. Now we can compare $\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+}$ to the expansion rate H. During the radiation-dominated era, $H \simeq 10T^2/m_{\mathrm{Pl}}$ and

$$\frac{\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+}}{H} \simeq 10^{-8}\lambda^2 f^4 \left[\frac{m_\phi}{M}\right]^8 \frac{T m_{\mathrm{Pl}}}{m_\phi^2} . \tag{4.9}$$

The condition that $\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+}/H < 1$ is easily satisfied (e.g., take $m_\phi = 10$ GeV and $m_\phi < 0.1M$) even when the temperature is of order m_ϕ. Today, $\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+}/H$ is extremely small ($\Gamma_{\tilde{\phi}\tilde{\phi}\to\bar{p}e^+}/H \simeq 10^{-9} f^2$ with the above values) and the number of $\bar{p}$'s produced will be well below the threshold placed by cosmic-ray experiments.

To summarize, we have calculated the various depletion rates for the charge carried by the antibaryonic scalar fields. In the $b_\phi = -1$ model, the mode of depletion is decays; in the $b_{\tilde{\phi}} = -\frac{1}{2}$ model, scattering processes are needed to get rid of antibaryon number. In performing these calculations we have assumed we are dealing with particles; i.e., we calculated the decay rate of ϕ and the scattering rate of $\tilde{\phi}$. This is certainly justified today, since, as argued above, gravitational inhomogeneities will endow the coherent ϕ field with nonzero momenta. What about in the early Universe though, when the field configuration could well have been truly coherent? Abbott, Farhi, and Wise (Ref. 19) have examined this question in the context of inflation. They found that the damping rate for coherent field oscillations is indeed exactly equal to the decay rate of the corresponding free particles. This equality follows from a consideration of the imaginary part of the two-point function $\Gamma^{(2)}$ in the effective action. By the optical theorem, this imaginary part is equal (apart from some kinematic factors) to the decay rate. They proved their result for a field with Yukawa couplings to fermions (i.e., $\phi\bar{\psi}\psi$) corresponding to free particles which decay. This is our $b_\phi = -1$ model above. The $b_{\tilde{\phi}} = -\frac{1}{2}$ model is different in that we must consider the four-point function $\Gamma^{(4)}$ in the effective action. By the optical theorem again the imaginary part of this is the cross section. Therefore the rate of damping for the coherent oscillations of $\tilde{\phi}$ should be equal to the scattering rate we computed above. Work is now in progress to make these comments rigorous.

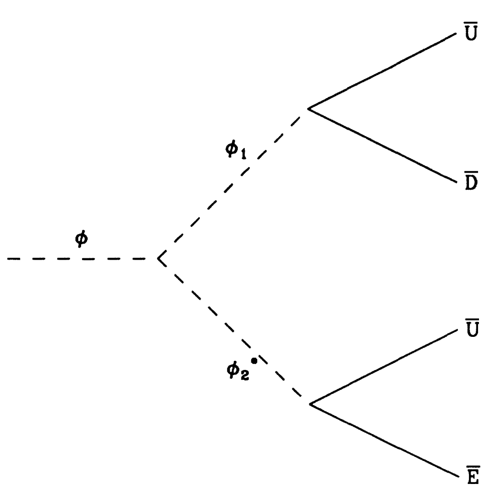

FIG. 5. Feynman diagram for ϕ decay into an antiproton and a positron.

V. BUBBLES OF ANTIMATTER

The baryon number hidden in the vacuum during an epoch of baryon-symmetric baryogenesis must be present today in some as of yet undetected form of matter. In the previous section we assumed that the vacuum baryon number was transferred to free ϕ particles. However, it is possible that the lowest-energy field configuration for ϕ (given fixed charge density) is very different from a free-particle configuration. In particular we may have $\phi=0$ everywhere except within small localized regions of very high charge density. That is, antimatter could be hidden in nontopological solitons[26,27] (NTS's) hereafter referred to as bubbles of antimatter (BAM's).

Before turning to BAM's in the context of baryon-symmetric baryogenesis, let us discuss some general properties of NTS's. NTS's are stable extended objects that arise in a variety of classical field theories. The common feature of these theories is the presence of an unbroken global symmetry. Associated with this symmetry is a conserved charge. [For BAM's the symmetry is just $U(1)_B$ and the conserved charge is baryon number.] A NTS is a localized, nondissipative field configuration that is, for fixed charge Q, the lowest-energy solution to the field equations. In particular, the NTS has lower energy than Q free particles. It follows that a necessary ingredient of NTS models is an attractive force that can bind the particles together.

Recently, a number of authors have considered cosmological NTS's. For example, Frieman, Gelmini, Gleiser, and Kolb[28] discuss the formation of NTS's in a second-order phase transition in the early Universe. Subsequent investigations have considered the possibility that NTS's are synthesized from free particles much in the way that heavy nuclei form during primordial nucleosynthesis.[29] A key assumption in most of these scenarios is that there is a charge asymmetry in whatever field forms the NTS's [typically a complex scalar field charged under a global U(1) symmetry]. Indeed, this is precisely the situation found after an epoch of baryon-symmetric baryogenesis.

We now consider some possible examples of BAM's. In what follows we will discuss the structure of these objects and how they interact with ordinary matter. We leave, for future investigations, the question of how the BAM's form. We note here that in general BAM's will coexist with free ϕ's. If both the ϕ's and the BAM's are stable, then the total antibaryon number density is

$$n_{\text{antibaryon}}=n_B=n_\phi+n_{\text{BAM}} , \tag{5.1}$$

where n_{BAM} is the total number density of antibaryons hidden in the BAM's. In the investigations of BAM formation mentioned above, n_ϕ/n_{BAM} can be large ($\gg 1$) and BAM's in such models would hold only a small fraction of the total antibaryon number in the Universe. In general, if ϕ's are stable (have a lifetime longer than the age of the Universe) then the BAM's will also be stable. However, the converse need not be true. As we shall see, it is possible for free ϕ's to decay while the ϕ's trapped in the BAM's are stable. In this case, the BAM's will hold *all* of the antibaryon number in the Universe.

The simplest example of a NTS is the Q ball first discussed by Coleman.[27] A Q ball involves a single scalar field whose Lagrangian is symmetric under a global U(1) symmetry. For the case at hand, the scalar field is ϕ and the U(1) symmetry is $U(1)_B$. We now discuss BAM's as Q balls. The first part of the discussion is fairly general in that it applies to all Q balls. (Our discussion, in fact, follows closely the discussion in Coleman's original paper.) Consider a BAM so large that surface effects can be neglected. We take, as an ansatz for the field,

$$\phi=\begin{cases}\phi_0 e^{-i\nu t}/\sqrt{2}, & r<R ,\\ 0, & r>R .\end{cases} \tag{5.2}$$

That is, the BAM is spherically symmetric with radius R. Inside the BAM, ϕ is homogeneous and rotates at a constant rate in its internal $[U(1)_B]$ space. Outside, $\phi=0$. The energy and charge (antibaryon number) of the BAM are

$$E=[\tfrac{1}{2}\phi_0^2\nu^2+V(\phi_0)]\Omega , \tag{5.3}$$

$$Q=\phi_0^2\nu\Omega , \tag{5.4}$$

where $\Omega=4\pi R^3/3$ is the volume. Q refers to antibaryonic charge so that a BAM of charge Q has baryon number $-Q$. Eliminating ν in the expression for E and minimizing the result with respect to Ω we find that

$$E=Q\left[\frac{2V(\phi_0)}{\phi_0^2}\right]^{1/2} , \tag{5.5}$$

where ϕ_0 is evaluated at the minimum of the function $V(\phi_0)/\phi_0^2$. A necessary condition for BAM solutions to exist is therefore that $V(\phi_0)/\phi_0^2$ have a minimum at $\phi_0\neq 0$. As noted by Coleman, this condition is never satisfied for renormalizable interactions. However, $V(\phi)$ may be an effective potential derived from a more fundamental theory operating at higher energies. Consider, for example,

$$V(\phi)=m_\phi^2|\phi|-\alpha|\phi|^4+\frac{\beta|\phi|^6}{m_\phi^2} , \tag{5.6}$$

where α and β are positive dimensionless coupling constants. This effective potential governs physics on energy scales lower than $m_\phi/\beta^{1/2}$. We note in passing that this potential also has the property mentioned in Sec. II: at intermediate temperatures $m_\phi \lesssim T \lesssim m_\phi/\beta^{1/2}$ the symmetry is spontaneously broken while at low temperatures, the symmetry is restored. In this case

$$\phi_0=\left[\frac{\alpha}{\beta}\right]^{1/2} m_\phi , \tag{5.7}$$

$$E=Q(1-b)m_\phi , \tag{5.8}$$

$$R=\left[\frac{3\beta Q}{4\pi\alpha(1-b)}\right]^{1/3} m_\phi^{-1} , \tag{5.9}$$

where $1-b\equiv(1-\alpha^2/4\beta)^{1/2}$. The binding energy per charge is therefore bm_ϕ. (We should note that BAM's of the type described here have a minimum allowable charge. BAM's with small Q have relatively large surface energy and this surface energy can destabilize the BAM. $Q_{\min}$ can be ~ 1 or larger depending on the parameters in the theory.)

Cosmological considerations place several constraints on the BAM's produced after an epoch of baryon-symmetric baryogenesis. Roughly speaking, the charge of a BAM cannot exceed the charge inside the horizon at the time when the BAM's form. That is, $Q<Q_{\max}$ where

$$Q_{\max}=n_B H^{-3}$$
$$=0.7\times 10^{47} g_*^{-1/2}\left[\frac{B}{10^{-10}}\right]\left[\frac{1\text{ GeV}}{T_f}\right]^3 \quad (5.10)$$

and T_f is the temperature of the Universe at the time when the BAM's form. Of course the exact constraint depends on the details of BAM formation, a complicated and model-dependent subject that will be left to future investigations. With this estimate we can give quantitative estimates for the maximum mass and radius of a Coleman-like BAM. With the nominal values in Eq. (5.10), the maximum radius is of order 1 cm with a corresponding mass of 10^{20} kg.

BAM's, being domains of antimatter should interact with ordinary matter (protons, neutrons, etc.). In particular, a BAM of charge Q and a nucleon can interact to yield a BAM of charge $Q-1$, a charged lepton, and radiation (pions, photons, etc), i.e.,

$$Q+p\rightarrow(Q-1)+e^{+}+\cdots \ . \quad (5.11)$$

Processes of this type can potentially wipe out both the BAM's and the proton-antiproton asymmetry. Let $\sigma(Q)$ be the cross section for the above process and v be the velocity of the nucleon relative to the BAM. $n_B\sigma(Q)v$ is then the reaction rate for the process in Eq. (5.11). If $n_B\sigma(Q)v/Q$ is greater than the expansion rate H, then we can expect a BAM of charge Q to disappear due to its interactions with nucleons. We can estimate $\sigma(Q)$ as follows. Inside the BAM, ϕ has a nonzero VEV and $U(1)_B$ is therefore broken. This implies that there are baryon-violating interactions among the quark fields inside the BAM. (The total baryon number of the quarks and the BAM together is of course conserved.) In particular, a proton inside the BAM can decay into a positron and radiation. Let τ_p be the lifetime of a proton inside a BAM. The probability $\mathcal{P}$, that a proton will decay while moving through a BAM of radius R is

$$\mathcal{P}(Q)=1-\exp(-R/\tau_p v) \quad (5.12)$$

since R/v is the amount of time the proton spends inside the BAM. The Q dependence in $\mathcal{P}$ enters through R [cf., Eq. (5.9)]. $\sigma(Q)$ is given by the product of the probability of proton decay inside the BAM and the geometric cross section for the BAM:

$$\sigma(Q)=\pi R^2\mathcal{P} \ . \quad (5.13)$$

We now calculate τ_p. Consider again, the Lagrangian equation (2.1). For simplicity we neglect ϕ_3. Isolating the mass terms for ϕ_1 and ϕ_2 we have

$$-\mathcal{L}_{\text{mass}}=M_1^2|\phi_1|^2+M_2^2|\phi_2|^2+\eta^2 e^{i\delta}\phi_1\phi_2^*+\eta^2 e^{-i\delta}\phi_1^*\phi_2 \ , \quad (5.14)$$

where $\eta^2=|\lambda_2 M\phi_0 e^{ivt}/\sqrt{2}|=\lambda_2 m_\phi M(\alpha/2\beta)^{1/2}$ and $\tan\delta=\text{Im}(\lambda_2 e^{ivt})/\text{Re}(\lambda_2 e^{ivt})$. We assume that $\eta^2<M_1^2,M_2^2$ and, for definiteness, take $M_1^2>M_2^2$. We can diagonalize $\mathcal{L}_{\text{mass}}$ by choosing the basis (X,Y) where

$$\phi_1=X\cos\theta+Ye^{-i\delta}\sin\theta \ , \quad (5.15a)$$

$$\phi_2=-Xe^{i\delta}\sin\theta+Y\cos\theta \ , \quad (5.15b)$$

with $\tan 2\theta=2\eta^2/(M_2^2-M_1^2)$. In this basis

$$-\mathcal{L}_{\text{mass}}=M_X^2|X|^2+M_Y^2|Y|^2 \ , \quad (5.16)$$

where

$$M_X^2=M_1^2+M_2^2+\sqrt{(M_1^2-M_2^2)^2+4\eta^4}$$

and

$$M_Y^2=M_1^2+M_2^2-\sqrt{(M_1^2-M_2^2)^2+4\eta^4} \ .$$

The interaction terms between the ϕ_i and the quarks now become

$$\mathcal{L}=f_1\cos\theta XU^TCD+f_1e^{-i\delta}\sin\theta YU^TCD$$
$$-f_2e^{-i\delta}\sin\theta X^*U^TCE+f_2\cos\theta Y^*U^TCE \ . \quad (5.17)$$

To simplify, we assume that $\eta^2\ll M^2$, i.e., $\lambda_2(\alpha/2\beta)^{1/2}(m_\phi/M)\ll 1$. Then, $M_X^2\simeq M_1^2\simeq M^2$, $M_Y^2\simeq M_2^2\simeq M^2$, $\cos\theta\simeq 1$, and $\sin\theta\simeq\lambda_2(\alpha/2\beta)^{1/2}(m_\phi/M)$. From these interactions we estimate the proton lifetime to be

$$\tau_p\simeq\frac{\beta}{f^4\alpha\lambda^2}\left[\frac{M^6}{m_p^4 m_\phi^2}\right]m_p^{-1} \ . \quad (5.18)$$

This expression can now be used to calculate $\sigma(Q)$ which in turn allows us to evaluate $n_B\sigma(Q)v/Q$ in terms of the parameters of the theory. The argument of the exponential in Eq. (5.12) is $R/v\tau_p$. To estimate this ratio take, for example, $m_\phi=10$ GeV, $M=10$ TeV, and $\lambda^2 f^2=10^{-5}$. Then

$$\frac{R}{v\tau_p}\simeq 10^{-28}\frac{Q^{1/3}}{v}\left[\frac{\alpha^2}{\beta^2(1-b)}\right]^{1/3}f^2 \ . \quad (5.19)$$

Even at very low temperatures, when the protons are traveling very slowly, $R/v\tau_p\ll 1$ for reasonable values of Q. Therefore, the probability that a proton traveling through a BAM decays is simply $\mathcal{P}\simeq R/v\tau_p$. This leads to a reaction rate

$$n_B\sigma v\simeq\frac{n_B R^3}{\tau_p} \ . \quad (5.20)$$

Since $R^3\sim Q$, $n_B\sigma(Q)v/Q$ is independent of Q. It is easy to check that $n_B\sigma(Q)v/Q$ is much less than H.

The BAM's described above can evaporate into antiprotons and positrons. (For a detailed discussion of Q-ball evaporation, see Ref. 31.) This occurs so long as the energy per charge inside the BAM is greater than the proton mass [i.e., $(1-b)m_\phi>m_p$]. Roughly speaking, the evaporation time scale is given by the decay time for the free ϕ's. It therefore follows that if the ϕ's are unstable, then the BAM's will also be unstable. We can solve this by requiring the energy per charge inside the BAM to be smaller than the proton mass. For the model in Eqs. (5.2)–(5.9), however, this requires rather fine-tuning.

A rather different situation is possible if one considers theories with more than one scalar field. BAM's in such theories are examples of NTS's of the type first discussed in Ref. 26. In these models the energy per charge inside the BAM can be well below the proton mass without fine-tuning. We can then have a situation where all of the free ϕ's decay and only the antibaryons in the BAM's survive.

Consider, for example, the Lagrangian[31]

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\rho)^2+|\partial_\mu\phi|^2-U(|\phi|,\rho)\ , \tag{5.21}$$

where

$$U(|\phi|,\rho)=\frac{\lambda}{4}(\rho^2-\rho_0^2)^2+m_{\phi\phi}^2|\phi|^2+\alpha|\phi|^4+m_{\phi\rho}^2|\phi|^2\frac{\rho^2}{\rho_0^2} \tag{5.22}$$

with ρ being a real scalar field. The vacuum of the theory is at $\langle\rho\rangle=\pm\rho_0$, $\langle\phi\rangle=0$. In this vacuum, the masses of the fields are $m_\rho^2=2\lambda\rho_0^2$ and $m_\phi^2=m_{\phi\phi}^2+m_{\phi\rho}^2$. Again, we require that $m_\phi^2>m_p^2$ in order to avoid proton decay. As in the simple Q ball, the ϕ field can carry a nonzero, conserved charge (baryon number for the case at hand). So long as $\alpha\neq 0$, the structure is similar to the Q ball.[31] Inside the BAM, $\phi=\phi_0 e^{-i\omega t}/\sqrt{2}$ and $\rho=\text{const}\neq\rho_0$. The mass as a function of charge is

$$M=Q(\lambda^{1/2}\alpha^{1/2}\rho_0^2+m_{\phi\phi}^2)^{1/2}\ . \tag{5.23}$$

The BAM's in this theory will be stable against evaporation into protons so long as $\lambda^{1/2}\alpha^{1/2}\rho_0^2+m_{\phi\phi}^2<m_p^2$ or $\lambda^{1/2}\alpha^{1/2}\rho_0^2<m_{\phi\rho}^2$.

VI. CONCLUSION

The simplest interpretation of proton decay experiments is that baryon number is conserved and therefore that the Lagrangian describing the interactions probed by these experiments has a global U(1) symmetry. It is possible that at very high energies, this symmetry is *explicitly* broken as in grand unified theories. A further possibility is that these hypothetical baryon-nonconserving interactions are responsible for the baryon asymmetry of the Universe. However, it is fair to say that at present no compelling model for either baryon-nonconserving interactions or baryogenesis at very high energies exists. In this paper we propose that the baryon asymmetry of the Universe can arise even if the fundamental Lagrangian is invariant under the global $\mathrm{U}(1)_B$ symmetry.

Our conclusion is that an asymmetry between protons and antiprotons can indeed develop so long as $\mathrm{U}(1)_B$ is *spontaneously* broken at some time in the early Universe. However, even during spontaneous symmetry breaking, the symmetry is realized. This fact leads to the most striking prediction of baryon-symmetric baryogenesis: there must be exactly as many antibaryons hiding in the Universe as there are baryons residing in protons and neutrons. During spontaneous symmetry breaking, the antibaryons are stored in excitations of the Goldstone field associated with the broken symmetry. Today, the antibaryons may be in the form of free scalar particles or in more exotic structures such as BAM's.

Our scenario should be testable. The energies in the theory naturally fall in the GeV and TeV ranges suggesting that there will be signatures for our model in particle accelerator experiments. Furthermore, since antibaryons are as abundant as baryons, the scenario will no doubt lead to important consequences for cosmology and astrophysics.

ACKNOWLEDGMENTS

We have benefited from discussions with many of our colleagues. Discussions with J. Bernstein, K. Benson, S. Coleman, S. Dimopoulos, J. Frieman, K. Griest, B. Green, L. Hall, S. Hsu, J. Primack, L. Randall, J. Silk, R. Watkins, and especially M. Turner, were particularly useful. This work was supported in part by the National Science Foundation (Grant No. PHY-8604396) and the U.S. Department of Energy (Grant No. DE-FG02-84ER40158 with Harvard University).

APPENDIX

In the Introduction we discussed classical equations of motion for the Goldstone field θ derivatively coupled to the baryon-number current $\tilde{j}^\mu_B$. These equations give us a conservation law for baryon number: any change in the baryon number of particles is exactly compensated by an equal and opposite change in the baryon number of the vacuum. By "baryon number in the vacuum" we mean baryon number stored in Goldstone mode (θ-field) excitations. At the classical level, θ stores charge by developing a velocity.

The arguments mentioned above are of course purely classical. On the other hand, the ϕ_i decays that give rise to the baryon asymmetry are quantum processes. In principle, we should treat the full system (Goldstone modes, ϕ_i's, and quark fields) using methods of quantum field theory. At present we do not have such a treatment though work is in progress.

Here we give a two-part argument that gives some justification to our assumptions. We first consider the quantum theory of the Goldstone field θ in the absence of sources [Eq. (1.5) with $\tilde{j}=0$]. We will show that even though the vacuum is not an eigenstate of the charge operator Q_B, the vacuum expectation value of Q_B is zero. We also calculate the quantum fluctuations in Q_B and show that in the context of baryogenesis, these fluctuations are extremely small and therefore irrelevant to the discussion in the text. In the second part of the Appendix we treat the θ field as a quantized scalar field interacting with a classical baryon current. The emission of θ particles interacting with a classical source can be calculated using standard techniques. The result, in a loose sense, is that the final state of Goldstone modes is precisely what we expect from classical physics.

Consider the low-energy effective Lagrangian for the Goldstone mode θ in the absence of external sources:

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\theta)^2 \ . \tag{A1}$$

Recall that θ is just the phase of ϕ and Eq. (A1) is relevant when $\langle\phi\rangle=v/\sqrt{2}\neq 0$. The $U(1)_B$ symmetry $\phi\rightarrow\phi e^{i\lambda}$ is equivalent to the symmetry

$$\theta\rightarrow\theta+\lambda v \ . \tag{A2}$$

If we view Eq. (A1) as the fundamental Lagrangian for a real, massless scalar field, then we see that the vacuum is not invariant under the transformation equation (A2) indicating that the symmetry associated with this transformation is spontaneously broken.[32] [This has to be since the symmetry, Eq. (A2) is equivalent to the $U(1)_B$ symmetry which is spontaneously broken.] The conserved current associated with Eq. (A2) is

$$j^\mu=v\partial^\mu\theta \ . \tag{A3}$$

In an infinite volume, the charge

$$Q_B=v\int d^3x\ \partial_0\theta(\mathbf{x},t) \tag{A4}$$

is ill defined. However, in a finite volume, the charge is well defined. Let us assume that the system is confined to a volume V and define the charge to be

$$Q_B=\pi^{-3/2}v\int d^3x\ \partial_0\theta(\mathbf{x},t)e^{-|\mathbf{x}|^2/V^{2/3}} \ . \tag{A5}$$

θ can be expanded in terms of creation and annihilation operators:

$$\theta(x)=\int\frac{d^3k}{2k^0(2\pi)^3}[a(k)e^{-ik\cdot x}+a^\dagger(k)e^{ik\cdot x}] \ , \tag{A6}$$

where

$$[a(k),a^\dagger(k')]=2k^0(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}') \ . \tag{A7}$$

Similarly, we can write the charge operator as

$$Q_B=-ivV\int\frac{d^3k}{2(2\pi)^3}[a(k)e^{-ik_0x_0}-a^\dagger(k)e^{ik_0x_0}]e^{-|\mathbf{k}|^2V^{2/3}/4} \ . \tag{A8}$$

It is easy to see that the expectation value of the charge operator vanishes:

$$\langle 0|Q|0\rangle=0 \ . \tag{A9}$$

However, the expectation value of Q^2 is nonzero,

$$\langle 0|Q^2|0\rangle=\frac{v^2V^{2/3}}{2\pi^2} \ , \tag{A10}$$

indicating that there are quantum fluctuations in the system. To see if these fluctuations are important we must compare the quantum fluctuations in Q with the total Q generated from our baryogenesis scheme. To do this we take V to be the Hubble volume at the time when the baryon asymmetry develops. We then find that

$$\frac{\sqrt{\langle Q^2\rangle}}{n_BV}\simeq\frac{v}{n_BV^{2/3}}\simeq 10^9\left[\frac{v}{m_{\rm Pl}}\right]^2 . \tag{A11}$$

For v in the TeV range we see that the quantum fluctuations are completely negligible. That is, quantum fluctuations cannot generate a baryon excess of sufficient magnitude, nor can they wash out an excess generated by some other mechanism.

We now discuss the emission of Goldstone bosons under the influence of an external classical source. The equation of motion

$$\partial_\mu\partial^\mu\theta=-\frac{1}{v}\partial_\mu\tilde{j}^\mu_B \tag{A12}$$

has the general solution

$$\theta(x)=\theta^{(0)}(x)+\int d^4y\ G(x-y)\partial_\mu\tilde{j}^\mu_B \ , \tag{A13}$$

where

$$\partial_\mu\partial^\mu G(x-y)=\delta^4(x-y) \tag{A14}$$

and $\theta^{(0)}(x)$ is a free quantum field. We assume that the source $\partial_\mu\tilde{j}^\mu_B$ is switched on for a finite period of time. [Indeed this is precisely what happens in baryon-symmetric baryogenesis. The net baryon density of quarks and antiquarks ($\tilde{j}^0_B$) starts out at 0 and builds up to some nonzero value. During the time in which this build-up takes place, $\partial_\mu\tilde{j}^\mu_B=\dot{n}_B$ is nonzero.] We can then calculate the emission of Goldstone particles due to the presence of the classical $\partial_\mu\tilde{j}^\mu_B$ source using standard S-matrix techniques. The problem is in fact analogous to one encountered in quantum electrodynamics: the emission and absorption of photons in the presence of a classical electromagnetic current (see, e.g., Ref. 33). The result is that the probability of Goldstone-particle emission as a function of the number of particles emitted is given by a Poisson distribution. That is, the final state does not have definite Goldstone-particle number. The final state is in fact a coherent state and the expectation value of θ, or more importantly, $\partial^0\theta$, is given by the solution to the classical equations of motion.

---

[1]An excellent review of the evidence for a baryon asymmetry in the Universe is given in G. Steigman, Annu. Rev. Astron. Astrophys. **14**, 339 (1976). For a guide to the more recent literature, see M. S. Turner and E. W. Kolb, *The Early Universe* (Addison-Wesley, Reading, MA, 1989).

[2]A. M. Boesgaard and G. Steigman, Annu. Rev. Astron. Astrophys. **23**, 319 (1985).

[3]A. D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 32 (1967)].

[4]J. C. Pati and A. Salam, Phys. Rev. D **8**, 1240 (1973); H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974). For a clear review and further references, see G. G. Ross, *Grand Unified Theories* (Benjamin/Cummings, Menlo Park, CA 1984).

[5]M. Yoshimura, Phys. Rev. Lett. **41**, 281 (1978); **42**, 740(E) (1979); S. Dimopoulos and L. Susskind, Phys. Rev. D **18**, 4500 (1978); D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, *ibid.* **19**, 1036 (1979); S. Weinberg, Phys. Rev. Lett. **42**, 850 (1979). For a clear review see, e.g., E. Kolb and M. Turner, Annu. Rev. Nucl. Part. Sci. **33**, 645 (1983), and references therein.

[6]R. Holman, P. Ramond, and G. G. Ross, Phys. Lett. **137B**, 343 (1984); D. Nanopoulos, K. Olive, and M. Srednicki, *ibid.* **127B**, 30 (1983).

[7]P. J. Steinhardt and M. S. Turner, Phys. Rev. D **29**, 2162 (1984).

[8]V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

[9]We caution, though, that this work has provoked much discussion and no concensus has yet been reached. See, for example, P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987); **37**, 1020 (1988); A. G. Cohen, M. J. Dugan, and A. V. Manohar, Phys. Lett. B **222**, 91 (1989). Moreover, if a nonzero $B-L$ is produced at energies above the weak scale then quantum effects in the standard model *cannot* wash out the asymmetry (see Ref. 8).

[10]For example, minimal SU(5) predicts proton decay via $p \rightarrow e^+\pi^0$ with a lifetime $\tau \leq 2.3\times10^{31}$ years [Ross (Ref. 4), Chap. 6]. The most recent experiments, T. J. Haines *et al.*, Phys. Rev. Lett. **57**, 1986 (1986), find at 90% confidence level $\tau(p\rightarrow e^+\pi^0)\geq 2.5\times10^{32}$ yr. Ross also documents the well-known discrepancy between the minimal theory's prediction of $m_d/m_s(\simeq\frac{1}{207})$ and experiment $(m_d/m_s\simeq\frac{1}{24})$. Recent measurements of the weak mixing angle appear too large to be consistent with the predictions of minimal SU(5), see, for example, P. Langacker and A. K. Mann, Physics Today **42** (12), 22 (1989).

[11]S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **64**, 340 (1990).

[12]Many authors have considered low-temperature scenarios for baryogenesis. A sample includes I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985); M. E. Shaposhnikov, *ibid.* **B287**, 757 (1987); **B299**, 797 (1988); S. Dimopoulos and L. J. Hall, Phys. Lett. B **196**, 135 (1987); A. Dannenberg and L. J. Hall, *ibid.* **198**, 411 (1987); R. N. Mohapatra and J. W. F. Valle, *ibid.* **186**, 303 (1987).

[13]Two previous papers, Affleck and Dine (Ref. 12) and A. G. Cohen and D. B. Kaplan, Phys. Lett. B **199**, 251 (1987) [see also A. G. Cohen and D. B. Kaplan, Nucl. Phys. **B308**, 913 (1988)], have also considered scenarios in which a $U(1)_B$ charged scalar acquires a VEV. However in both of these works, explicit baryon-violating interactions are introduced. That is, $U(1)_B$ is only an *approximate* symmetry whereas $U(1)_B$ is an *exact* symmetry in the model discussed here. As will be discussed in the text, this is a crucial difference.

[14]S. Weinberg, Phys. Rev. D **19**, 3357 (1974); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **42**, 1651 (1979); Phys. Rev. D **20**, 3390 (1979); P. Langacker and S. Pi, Phys. Rev. Lett. **45**, 1 (1980).

[15]See, for example, F. Jona and G. Shirane, *Ferroelectric Crystals* (Pergamon, Oxford, 1962).

[16]In a spatially flat Robertson-Walker spacetime, the metric is $g_{\mu\nu}=\mathrm{diag}(-1,a^2,a^2,a^2)$, where $a=a(t)$ is the scale factor. Equation (1.3) becomes $a^{-3}\partial_\mu(a^3j^{\mu B})=0$. That is, the current inside a comoving volume is conserved. Likewise, Eq. (1.6) becomes $a^{-3}\partial_\mu[a^3(v\partial^\mu\theta+\tilde{j}^{\mu B})]=0$. The charge-conservation laws Eqs. (1.7) and (1.8) now read that the charge inside a comoving volume is conserved (simply replace the volume element d^3x in the integrals by a comoving volume element).

[17]T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1981), Chap. 15.

[18]D. V. Nanopoulos and S. Weinberg, Phys. Rev. D **20**, 2484 (1979); E. W. Kolb and S. Wolfram, Nucl. Phys. **B172**, 224 (1980).

[19]This idea was first discussed in the context of new inflation, A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982); A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); L. F. Abbott, E. Farhi, and M. B. Wise, *ibid.* **117B**, 29 (1982).

[20]This idea initially arose in the context of intermediate scales in superstring models, G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, Phys. Rev. Lett. **56**, 557 (1986); K. Yamamoto, Phys. Lett. **168B**, 341 (1986). It has recently been invoked by Dimopoulos, Dannenberg, and Hall (Ref. 12), to design a low-temperature scenario.

[21]These assumptions are neither unusual nor unrealistic. Proton-antiproton annihilations, for example, are very efficient at getting rid of the antiprotons while leaving behind only the excess protons.

[22]See, for example, J. Silk and M. Srednicki, Phys. Rev. Lett. **53**, 624 (1984).

[23]V. Ginzburg and V. Ptuskin, Rev. Mod. Phys. **48**, 161 (1976).

[24]S. P. Ahlen *et al.*, Phys. Rev. Lett. **61**, 145 (1988).

[25]B. C. Fitchtel, G. Simpson, and D. Thompson, Astrophys. J. **222**, 833 (1978).

[26]R. Friedberg, T. D. Lee, and A. Sirlin, Phys. Rev. D **10**, 2739 (1976).

[27]S. Coleman, Nucl. Phys. **B262**, 263 (1985).

[28]J. A. Frieman, G. B. Gelmini, M. Gleiser, and E. W. Kolb, Phys. Rev. Lett. **60**, 2101 (1988).

[29]K. Griest and E. W. Kolb, Phys. Rev. D **40**, 3231 (1989); J. A. Frieman, A. V. Olinto, M. Gleiser, and C. Alcock, *ibid.* **40**, 3241 (1989). As noted by K. Griest, E. W. Kolb, and A. Massarotti, *ibid.* **40**, 3529 (1989). NTS's may form even if the overall charge asymmetry of the Universe is zero provided that there are large fluctuations in the charge distribution.

[30]A. Cohen, S. Coleman, H. Georgi, and A. Manohar, Nucl. Phys. **B272**, 301 (1986).

[31]J. Frieman and B. Lynn, Nucl. Phys. **B329**, 1 (1990).

[32]C. Itzykson and J. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), see especially the discussion on p. 521.

[33]Itzykson and Zuber (Ref. 32), Chap. 4-1.