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Weak hyperon radiative processes and the asymmetry parameter in $\Xi^- \to \Sigma^- \gamma$ decay

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The calculation of the asymmetry parameter α_h in $\Xi^- \to \Sigma^- \gamma$ is argued to be less model dependent than in other similar decays. Including short- and long-distance contributions we find $\alpha_h = -0.13 \pm 0.15$.

Weak radiative decays of hyperons, which in principle are expected to provide a clear testing ground for electroweak phenomena, have instead been the source of some puzzling features.[1-11] The most outstanding one has been the large negative asymmetry[4] in Σ^+ radiative decay $\alpha_h(\Sigma^+ \to p\gamma) = -0.83 \pm 0.13$, which is to be contrasted with the "naive" expectation from U-spin considerations[5] of a vanishing asymmetry, or even of a positive one in a single-quark-transition picture.[6]

For a long time only the $\Sigma^+ \to p\gamma$ decay was well measured.[4,7] Recently, a flurry of new experimental results has improved the situation considerably, reporting data on $\Lambda \to n\gamma$,[8] $\Xi^- \to \Sigma^-\gamma$,[9] $\Xi^0 \to \Sigma^0\gamma$,[10] and $\Xi^0 \to \Lambda\gamma$.[11] During the last 20 years a large number of models have been constructed to treat these processes, many attempting to achieve a "unified picture" for all the radiative decays of hyperons. A condensed list of representative papers is given in Refs. 6, 12–17. Most of these models have optimized the fitting of the $\Sigma^+ \to p\gamma$ data and then presented predictions for the rates and asymmetries of other hyperon radiative decays. With the advent of the newest data,[8-11] one realizes that none of the existing models can reproduce simultaneously the measured features of the various decays. The long-standing $\Sigma^+ \to p\gamma$ puzzle is now embarrassingly reinforced by these new measurements, apparently throwing into open question the theoretical ability to treat this problem, and calling[11] for "precise measurements . . . necessary to guide the theory."

In this Rapid Communication, after commenting briefly on the disagreements among various model calculations for the measured $\Sigma^+ \to p\gamma$, $\Lambda \to n\gamma$, and $\Xi^0 \to \Sigma^0(\Lambda)\gamma$ transitions, we turn to a calculation of the asymmetry parameter for $\Xi^- \to \Sigma^-\gamma$. While the difficulties in accounting for the observed features in the above group of four decays are traceable, and do not imply at this stage a "threat" to the standard model, we shall argue that the asymmetry in $\Xi^- \to \Sigma^-\gamma$ is a more sensitive quantity. The measurement of a marked deviation from the value we calculate, $\alpha_h(\Xi^- \to \Sigma^-\gamma) = -0.13 \pm 0.15$, could indeed present a serious problem.

We start by emphasizing that the treatment of a weak radiative process in the (s,d,u) sector requires (see, e.g., Ref. 18) the consideration of two distinct types of contributions: (a) photon emission from short distances $(x \sim 1/M_W)$, which can be treated by the use of the electroweak standard-model Hamiltonian expressed in terms of local quark operators; (b) photon emission from intermediate hadronic states at "long distances" of the scale of the confinement radius. In addition to causing intermediate hadronization effects, the strong interactions affect the nonleptonic weak Hamiltonian also at short distances. This can be treated in a Wilson expansion by use of renormalization-group equations and we adopt for it the formulation of Ref. 19, with

$$H^{\text{nonlept}}_{\text{weak}} \equiv H^{\text{QCD}}_{\text{NL}} = \sqrt{2} G_F \sum c_i O_i \, ,$$

where c_i are constants and O_i are quark operators products.

Consider first the group of five decays $\Sigma^{+(0)} \to p(n)\gamma$, $\Lambda \to n\gamma$, $\Xi^0 \to \Sigma^0(\Lambda)\gamma$. The short-distance single-quark transition $s \to d\gamma$ turns out to contribute[18] 2×10^{-6} to the branching ratio of $\Sigma^+ \to p\gamma$ and similarly to the other decays in this group. This is 3 orders of magnitude below the observed rates. Two-quark transitions[13,20] are the next short-distance candidate; however, it appears that taken alone they cannot provide the observed large asymmetry in $\Sigma^+ \to p\gamma$. One is led to conclude that long-distance contributions, not unexpectedly, are also present. Examination of the structure of H_{NL} shows that the operator O_1, which has by far the largest coefficient, induces pole diagrams in this group of decays [e.g., $\Sigma \to (p,N^*) \to p\gamma$, etc.] which dominate over multiparticle intermediate states. Including[12] $\frac{1}{2}^+$ and $\frac{1}{2}^-$ intermediate pole states, one generates both parity-conserving and parity-violating amplitudes and a large negative asymmetry in $\Sigma^+ \to p\gamma$ is thus obtainable. However, the calculation of these five decays requires the use of couplings to the intermediate states for which good experimental information is presently unavailable. Accordingly, it is not surprising that using models to supply the missing information one arrives[12-16] at widely diverging results for the asymmetries. Thus, the predictions for a specific asymmetry, e.g., $\alpha_h(\Xi^0 \to \Lambda\gamma)$ might differ as violently as between (-0.8) (Ref. 12) and $(+0.95)$ (Ref. 16) (experimentally[11] one has 0.43 ± 0.44). Nevertheless, the source of the problem is traceable to the complexity of the transition, which possibly involves both short- and long-distance contributions; the latter are difficult to calculate reliably at present.

The situation is quite different in $\Xi^- \to \Sigma^-\gamma$ decay. Let us start with the short-distance contributions. Because of the quark content of the Ξ^-,Σ^- states, there is no W-exchange diagram (at least in the valence-quark picture) for this transition. The other possibilities are penguin-type transitions induced, e.g., by the O_5,O_6 operators, and the single-quark loop transition $s \rightarrow d\gamma$. The penguin contributions to this decay have been calculated[21] and found to give a negligible branching ratio of less than 10^{-7}, while the experimental figure is[9] $B(\Xi^- \rightarrow \Sigma^- \gamma) = (2.3 \pm 1.0) \times 10^{-4}$. On the other hand, the $s \rightarrow d\gamma$ loop transition corrected for gluonic effects gives a contribution[22] of the order of 10^{-5} to this branching ratio. Although not the principal agent to the rate, this contribution might well affect the asymmetry parameter. As to the long-distance component, the O_1 operator will now induce transitions via two-particle intermediate states such as $(\pi^-\Lambda)$. Kogan and Shifman[18] have shown that these contributions can be reliably calculated and lead to $B(\Xi^- \rightarrow \Sigma^- \gamma) = 1.7 \times 10^{-4}$. Pole contributions would enter only through the O_2-O_4 operators and because of the smallness of c_2-c_4 these are corrections of less than 10% in the amplitude. The success of the calculation of Ref. 18 and the relatively good control one has in this case[23] on both short- and long-distance contributions lead us to turn to the asymmetry parameter, which as we show here can be reliably computed in this case.

We define the matrix element for $\Xi^- \rightarrow \Sigma^- \gamma$ decay:

$$M(\Xi^- \rightarrow \Sigma^- \gamma) = ieG_F \bar{U}(\Sigma^-)(A + B\gamma_5)\sigma^{\mu\nu} q_\nu U(\Xi^-)\epsilon_\mu(\gamma) \, , \quad (1)$$

where q_μ, $\epsilon_\mu(\gamma)$ are the photon momentum and polarization vectors, and A (B) are the parity-conserving (-violating) amplitudes. The angular distribution of the decay is proportional to $1 + \alpha_h \mathbf{P} \cdot \mathbf{n}$, where $\mathbf{P}$ is the Ξ^- polarization in its rest frame and $\mathbf{n}$ is a unit vector in the direction of the Σ^- momentum. The asymmetry parameter α_h is given by

$$\alpha_h = \frac{2\mathrm{Re}(A^* B)}{|A|^2 + |B|^2} \, . \quad (2)$$

Both amplitudes could have short-distance (from $s \rightarrow d\gamma$) and long-distance (from two-particle intermediate states) parts and we denote

$$A = A^{\mathrm{SD}} + A^{\mathrm{LD}}, \quad B = B^{\mathrm{SD}} + B^{\mathrm{LD}} \, . \quad (3)$$

To calculate A^{SD}, B^{SD} we have to consider the single-quark transition. The relevant magnetic form factor of the $s \rightarrow d\gamma$ amplitude is known[6] to have a quadratic Glashow-Iliopoulos-Maiani cancellation in the free-quark model, and would be negligible but for the QCD corrections, which restores it to a logarithmic form.[25] This transition has the same general form as (1) and after including QCD corrections is given by

$$M(s \rightarrow d\gamma) = (-0.72)\frac{is_1 c_1 eG_F \sqrt{2} m_s}{16\pi^2} \times \bar{U}(d)\sigma^{\mu\nu} q_\nu (1+\gamma_5) U(s)\epsilon_\mu(\gamma) \, . \quad (4)$$

The coefficient (-0.72) is calculated from the expression of Ref. 25 in a four-quark approximation with their chosen mass scale $\mu = 0.3$ GeV at which $\alpha_s = 1$. Recently, McGuigan and Sanda[26] repeated this calculation with six quarks and have confirmed that the result is practically not affected by the inclusion of the t quark, due to the smallness of the relevant Cabibbo-Kobayashi-Maskawa matrix element.

In order to calculate the contribution of (4) to the hyperon decay amplitude we use the procedure of Gilman and Wise[6] whereby the baryons are described by SU(6) wave functions. The physical decay amplitudes of the hyperon are related to the quark transition by $M(B_i \rightarrow B_f \gamma) = (M_f/M_i)^{1/2} C_{fi} M(s \rightarrow d\gamma)$, where C_{fi} is a Clebsch-Gordan coefficient. We obtain[27]

$$\mathrm{Re}A^{\mathrm{SD}} = \mathrm{Re}B^{\mathrm{SD}} = 1.2 \text{ MeV} \, , \qquad \mathrm{Im}A^{\mathrm{SD}} = \mathrm{Im}B^{\mathrm{SD}} = 0 \, . \quad (5)$$

This value should be considered an upper limit since by taking larger values of μ, say $\mu = 0.7$–1 GeV/c^2, a value smaller by 2–3 is obtained. This is a reflection of the uncertainty in the QCD-correction procedure[25] in the low-mas (s,d,u) sector (in principle, of course, the result should not depend on μ).

For the long-distance contributions we use the formalism of Kogan and Shifman,[18] which we update with presently available data. Only the main results are presented here; Ref. 18 should be consulted for details. The imaginary part of the amplitude is generated by the real intermediate states, according to unitarity.[28] For $\Xi^- \rightarrow \Sigma^- \gamma$ there is only one state $(\Lambda\pi^-)$. Thus,

$$\mathrm{Im}M(\Xi^- \rightarrow \Sigma^- \gamma) = \tfrac{1}{2} \int \frac{d^4k}{(2\pi)^2} \delta(k^2 - m_\pi^2)\delta((p-k)^2 - M_\Lambda^2) M(\Xi^- \rightarrow \Lambda\pi^-) T(\pi^-\Lambda \rightarrow \gamma\Sigma^-) \, . \quad (6)$$

The amplitude $\Xi^- \rightarrow \Lambda\pi^-$ is very accurately known[29] and the photoproduction amplitude $T(\pi^-\Lambda \rightarrow \gamma\Sigma^-)$ is reliably calculated[18] within PCAC (partial conservation of axial-vector current), up to terms linear in the photon momentum. One finds

$$\mathrm{Im}A^{\mathrm{LD}} = 0.94 \text{ MeV}, \quad \mathrm{Im}B^{\mathrm{LD}} = -8.3 \text{ MeV} \, . \quad (7)$$

The real part of the amplitude, related to the imaginary one by a dispersion integral, is dominated by an infrared log divergence in the chiral limit (due to nonvanishing of $\mathrm{Im}B^{\mathrm{LD}}$ at threshold), which is cut off at m_π. The major contribution comes from the diagram with an $(\Lambda\pi)$ intermediate loop, with photon emission from the pion. Since the P-wave amplitude of $\Xi^- \rightarrow \Lambda\pi^-$ would not lead to a logarithmic term, the parity-conserving amplitude in $\Xi^- \rightarrow \Sigma^- \gamma$ which derives from it is vanishing in this approximation. Accordingly, one obtains

$$\begin{aligned} \mathrm{Re}A^{\mathrm{LD}} &= 0 , \\ \mathrm{Re}B^{\mathrm{LD}} &= \frac{-1.3 m_\pi^2 g_A^{\Sigma\Lambda}}{8\pi^2 f_\pi} \ln\frac{M_\Xi}{M_\pi} A(\Xi^- \to \Lambda\pi^-) \\ &= -6.9\ \mathrm{MeV} , \end{aligned} \tag{8}$$

where $A(\Xi^- \to \Lambda\pi^-) = 2.03 \pm 0.01$ (Ref. 29) is the S-wave part of that transition and $g_A^{\Sigma\Lambda} = 0.62$ is the axial-vector coupling in $\Sigma \to \Lambda e\gamma$ decay. The factor 1.3 is accounting for the additional intermediate states and is derived using $\mathrm{SU}(3)_f$ symmetry. Although $g_A^{\Sigma\Lambda}$ is measured only with 40% accuracy, it is causing no major uncertainty since it appears as a factor[18] in all amplitudes of (7) and (8).

We are now ready to calculate α_h from (2), but since (5) is obtained from a quark-level calculation while (7) and (8) derive from definitions at the hadronic level, there is a sign ambiguity in adding coherently the real parts of the short- and long-distance contributions; we thus take both possible signs for the short-distance contributions (5) when combining with (8) to get the full amplitude. As a result we find

$$\alpha_h(\Xi^- \to \Sigma^-\gamma) = -0.13 \pm 0.15 , \tag{9}$$

where $\alpha_h = -0.13$ comes from the long-distance part and ± 0.15 is the range allowed by a short-distance coherent addition with a maximal absolute value as given by (5). The rate obtained from these considerations is

$$\left.\frac{\Gamma(\Xi^- \to \Sigma^-\gamma)}{\Gamma(\Xi^- \to \mathrm{all})}\right|_{\mathrm{theor}} = (1.8 \pm 0.4) \times 10^{-4} , \tag{10}$$

in good agreement with the only (low-statistics) existing measurement[9] of $(2.3 \pm 1.0) \times 10^{-4}$.

There are two remarks to be made on the result (9). First, the negative sign is an outcome of the relative negative sign of the S- and P-wave amplitudes of $\Xi^- \to \Lambda\pi^-$, reflected via (6) into (7) and of the fact that $\mathrm{Re}A^{\mathrm{LD}} = 0$ in the approximation employed here. It may turn positive only if the short-distance amplitude contributes significantly being negative and of absolute value as in (4) or larger. Second, the small value of α_h we obtained is to be expected from symmetry considerations,[5] which would predict a zero value. However, in the physical case calculated here, it is actually the parity violating amplitude B which is the larger component, as seen in (7) and (8).

To summarize, we predict a small and negative asymmetry for $\Xi^- \to \Sigma^-\gamma$, based on a calculation which involves tested assumptions and measured inputs only. In fact, the uncertainty occurring in (9) is probably an upper limit as we explained; it is more likely that $\alpha_h(\Xi^- \to \Sigma^-\gamma)$ will be found in the narrower range between (-0.06) and (-0.20). An accurate experimental determination will thus assess the short-distance contribution and possibly "detect" indirectly the $s \to d\gamma$ transition. Before concluding, we remark that here are several calculations in the literature in which this asymmetry has been calculated. The values obtained for $\alpha_h(\Xi^- \to \Sigma^-\gamma)$ span the range -0.74,[30] -0.44,[13] -0.40,[16] -0.015,[17] 0,[15] 0.47,[31] and 0.8.[21] However, these calculations usually attempting a unified approach for the hyperon radiative decays have already failed in predicting correctly the recently measured[8–11] asymmetries and/or rates. The measurement of this quantity is highly desirable; if the result is markedly different from (9), one would think there is indeed a serious problem in understanding hyperon radiative decays.

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