RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 42, NUMBER 9 1 NOVEMBER 1990

On the $\phi \rightarrow K^0\bar{K}^0\gamma$ decay

J. L. Lucio M.
Departimento de Física, Centro de Investigación y Estudios Avanzados del Instituto Politécnico National,
Apartado Postal 14-740, Caixa Postal 07000 México, Distrito Federal, Mexico

J. Pestieau
Institut de Physique Théorique, Université Catholique de Louvain, Chemin du Cyclotron, 2-B 1348 Louvain-la-Neuve, Belgium

(Received 30 May 1990)

The branching ratio of $\phi \rightarrow K^0\bar{K}^0\gamma$ is reexamined, considering the chain reactions $\phi \rightarrow K^+K^- \rightarrow f_0(a_0)+\gamma \rightarrow K^0+\bar{K}^0+\gamma$.

In a recent Letter,[1] Nussinov and Truong have calculated the radiative decay $\phi \rightarrow K^0+\bar{K}^0+\gamma$ with $K^0\bar{K}^0$ in the S state. They find the branching ratio

$$\frac{\Gamma(\phi \rightarrow K^0\bar{K}^0\gamma)}{\Gamma(\phi \rightarrow K^0\bar{K}^0)} \simeq 10^{-7} . \tag{1}$$

In this Rapid Communication, we present a critical review of their calculations. We have found analytical errors in Ref. 1. Furthermore, we will comment on the assumptions involved.

Following Ref. 1, we use the model consisting of the following chain of decays:

$$\phi \rightarrow K^+ + K^- \rightarrow f_0(k)+\gamma \rightarrow K^0+\bar{K}^0+\gamma ,$$

where $K^{\pm}$ are real or virtual. $f_0(976)$ is the scalar meson $I=0$ (Ref. 2) with momentum k. We observe that $m_{f_0} < 2m_K < m_\phi$.

The amplitude describing the decay $\phi \rightarrow K^+K^- \rightarrow f_0(k)+\gamma$ can be written as

$$M(\phi(p,\eta) \rightarrow f_0(k)+\gamma(q,\epsilon)) = \frac{eg_\phi g}{2\pi^2 m_{K^+}^2} I(a,b)[(p\cdot q)(\epsilon\cdot\eta)-(p\cdot\epsilon)(q\cdot\eta)] . \tag{2}$$

Here g_ϕ and g stand for the ϕK^+K^- and $f_0K^+K^-$ coupling constants, related to the widths by

$$\Gamma(\phi \rightarrow K^+K^-) = \frac{g_\phi^2}{48\pi m_\phi^2}(m_\phi^2 - 4m_{K^+}^2)^{3/2}$$

and

$$\Gamma(f_0(k) \rightarrow K^+K^-) = \frac{g^2}{16\pi k^2}(k^2 - 4m_{K^+}^2)^{1/2} .$$

$\epsilon(q)$ and $\eta(p)$ denote γ and ϕ polarizations (momenta), respectively. a and b are defined as

$$a = \frac{m_\phi^2}{m_{K^+}^2}, \quad b = \frac{k^2}{m_{K^+}^2} .$$

Then $a-b = 2p\cdot q/m_{K^+}^2$ is proportional to the photon energy. $I(a,b)$ has been computed in different contexts[3] and is given by

$$I(a,b) = \frac{1}{1(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right] , \tag{3}$$

where

$$f(x) = \begin{cases} -\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2 , & x > \frac{1}{4} , \\ \frac{1}{4}\left(\ln\frac{\eta+}{\eta-} - i\pi\right)^2 , & x < \frac{1}{4} , \end{cases} \tag{4a}$$

$$g(x) = \begin{cases} (4x-1)^{1/2}\arcsin\left(\frac{1}{2\sqrt{x}}\right) , & x > \frac{1}{4} , \\ \frac{1}{2}(1-4x)^{1/2}\left(\ln\frac{\eta+}{\eta-} - i\pi\right) , & x < \frac{1}{4} , \end{cases} \tag{4b}$$

with

$$\eta\pm = \tfrac{1}{2}[1 \pm (1-4x)] .$$

Notice that Eq. (2) is in disagreement with Eq. (8) in the Erratum in Ref. 1, where Eq. (8) should be corrected as

$$H(m_\phi^2, m_\phi^2) = \frac{eg_\phi g}{4\pi^2}\frac{1}{m_\phi^2}\left(1 - \frac{i\pi}{4(1-4m_K^2/m_\phi^2)^{1/2}}\right)$$

(in the first approximation where $m_\phi = m_{f_0} = 2m_K$). Our Eqs. (2)-(4) give an exact result. We have

$$\frac{\text{Re}I(a,a)}{\text{Im}I(a,a)} = -\frac{1}{3} .$$

Using Eq. (2), we get

$$\Gamma(\phi \rightarrow f_0(k)+\gamma) = \frac{\alpha}{96\pi^4}\frac{g_\phi^2 g^2}{m_{K^+}^4}\frac{(m_\phi^2 - k^2)^3}{m_\phi^3}|I(a,b)|^2 . \tag{5}$$

With $m_{f_0} = 976$ MeV (Ref. 2) [i.e., $x > \frac{1}{4}$ in Eqs. (4a) and (4b)] and using $g^2/4\pi = 0.6$ GeV^2,[1] we find

$$\Gamma(\phi \to f_0 + \gamma) = 8.5 \times 10^{-4}\ \mathrm{MeV}\,. \tag{6}$$

But the scalar meson $f_0(976)$ is not on the mass shell. Equation (5) must be evaluated with $k^2 > 4m_K^2$ [or $x < \frac{1}{4}$ in Eqs. (4a) and (4b)]. Using a Breit-Wigner propagator to deal with the fact that f_0 is off-shell in the decay $f_0 \to K^- \bar{K}^0$, we obtain

$$\Gamma(\phi \to K^0 \bar{K}^0 \gamma) = \frac{\alpha}{24\pi^3} \frac{g_\phi^2}{4\pi} \left(\frac{g^2}{4\pi} \frac{1}{m_\phi^2} \right)^2 m_\phi P\,, \tag{7}$$

with

$$P = \int_4^a db\, |I(a,b)|^2 \frac{(a-b)^3 (1-4/b)^{1/2}}{(b - m_f^2/m_K^2)^2 + \Gamma_f^2 m_f^2/m_K^4}\,. \tag{8}$$

Using the numerical values

$$\frac{g_\phi^2}{4\pi} = 1.66, \quad \frac{g^2}{4\pi m_\phi^2} = 0.58\,,$$

we obtain $\Gamma(\phi \to K^0 \bar{K}^0 \gamma) = 6 \times 10^{-6}$ MeV, leading to the branching ratio

$$\frac{\Gamma(\phi \to K^0 \bar{K}^0 \gamma)}{\Gamma(\phi \to K^0 \bar{K}^0)} = 4 \times 10^{-6}\,, \tag{9}$$

which is 40 times bigger than the result given in Eq. (1) (Erratum to Ref. 1). Observe that, in the Erratum in Ref. 1, there are numerical mistakes in Eqs. (5)–(9). On dimensional grounds only, Eqs. (8) and (9) are wrong. Our main result is given in Eqs. (2)–(5) and (7)–(9). Equation (9) provides an important background for a precision test of *CP* violation in $\phi \to K_S K_L$. But, as explained in Ref. 1, we also have to include the contribution of the isovector scalar meson $a_0(980)$:[2]

$$\phi \to K^+ K^- \to a_0(k) + \gamma \to K^0 + \bar{K}^0 + \gamma\,.$$

The contribution of $a_0(980)$ is expected to be of the same order of magnitude as the $f_0(976)$ one in $\Gamma(\phi \to K\bar{K}\gamma)$. And if $a_0(980)$ and $f_0(976)$ interfere *destructively*, $\phi \to K^0 \bar{K}^0 \gamma$ decay could become a negligible background to $\phi \to K_S K_L$ decay.

We would like to thank G. López Castro for useful discussions.

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