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Rapid Communications

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Coulomb corrections for $\Upsilon_{4s} \to B\overline{B}$

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We show that meson structure strongly affects the magnitude of Coulombic final-state corrections to the decay rate for $\Upsilon_{4s} \to B^+B^-$. Model calculations indicate that the corrections are much smaller than they would be for pointlike mesons.

Recently it was suggested that the decay rate for $\Upsilon_{4s} \to B^+B^-$ is significantly enhanced relative to the rate for $\Upsilon_{4s} \to B^0\overline{B}^0$ by Coulomb interactions between the final-state B mesons.[1] Treating the mesons as pointlike particles, the leading correction to the decay rate is

$$\delta\Gamma = \frac{\pi\alpha}{2\beta}\Gamma_0 \ , \tag{1}$$

where α is the fine-structure constant, β is the B-meson velocity in the Υ's rest frame, and Γ_0 is the uncorrected decay rate. This formula predicts a large enhancement for decays into charged particles near threshold—an 18% enhancement in the case of $\Upsilon_{4s} \to B^+B^-$. However, as we show in this Rapid Communication, it is incorrect to treat the mesons as pointlike particles in analyzing this effect. The Coulomb corrections are highly sensitive to the structure of the Υ and B mesons, and model calculations suggest that the actual corrections are much smaller than those suggested by Eq. (1).

Assuming meson masses of

$$\begin{aligned} M(\Upsilon_{4s}) &= 10.580\,\text{GeV} \ , \\ m(B^+) = m(B^0) &= 5.279\,\text{GeV} \ , \end{aligned} \tag{2}$$

we see that the Υ_{4s} is only 0.022 GeV above the $B\overline{B}$ threshold. Thus the B mesons, having momentum $p_B = 0.34\,\text{GeV}$ and velocity $\beta = 0.065$, are highly nonrelativistic, and we can compute the dominant Coulomb correction using nonrelativistic time-dependent perturbation theory. Writing the uncorrected amplitude for $\Upsilon_{4s} \to B\overline{B}$ in the form

$$T_0 = \boldsymbol{\varepsilon} \cdot \mathbf{p}_B\, \Phi(\mathbf{p}_B), \tag{3}$$

where $\boldsymbol{\varepsilon}$ is the Υ's polarization vector and $\Phi(\mathbf{p}_B)$ is the Υ_{4s}–$B\overline{B}$ vertex function, the Coulomb correction is given by

$$\delta T = -e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{[F_B(\mathbf{q}^2)]^2}{\mathbf{q}^2} \frac{1}{\mathbf{p}_B^2/m - (\mathbf{p}_B+\mathbf{q})^2/m + i\epsilon}\, \boldsymbol{\varepsilon}\cdot(\mathbf{p}_B+\mathbf{q})\,\Phi(\mathbf{p}_B+\mathbf{q}) \ . \tag{4}$$

Here the first factor inside the integral is the potential for a Coulomb interaction between the B mesons, $F_B(\mathbf{q}^2)$ being the electromagnetic form factor of the B meson; the second factor is the nonrelativistic propagator for the B mesons during the time between the decay and the Coulomb interaction; and the remaining factors give the amplitude for the Υ_{4s} to decay into the B's. It is legitimate to formulate this amplitude in terms of the mesons, rather than their constituent quarks, since the energies involved ($\sim 0.02\,\text{GeV}$) are much smaller than the internal energies of the B mesons ($\sim 0.2\,\text{GeV}$).

The naive Coulomb correction [Eq. (1)] follows immediately from the equation for δT if we replace the form factors and vertex function by their values at zero momentum: $F_{B^+}=1$, $F_{B^0}=0$, and $\Phi(\mathbf{p}_B+\mathbf{q})=\Phi_0$. However, it is clear from the equation that the scale of the loop momentum q is set by $p_B=0.34$ GeV. Meson structure cannot be ignored at such momenta, and thus we require models for the form factors and vertex function if we are to compute the Coulomb correction.

In modeling the B form factor, we must take account of the asymmetry between the quarks comprising the meson. Typically the b quark carries almost all of the meson's momentum: if m_b is its mass, the fraction of the momentum carried by the b quark is $\tau\approx m_b/m(B)$, which should be of order 0.9–1.0. Thus, when the light quark in the meson absorbs a photon with a large momentum q, it transfers momentum $\tau q\approx q$ to the b quark. Such a large momentum transfer between constituents strains the meson, and leads to suppression of the amplitude by a form factor. On the other hand, when the b quark absorbs the photon, it need only transfer momentum $(1-\tau)q\ll q$ to the light quark, resulting in significantly less suppression of the amplitude. In general, the b quark can absorb momentum from a photon much more readily than the light quark can. A realistic model for the B-meson electromagnetic form factors is therefore

$$F_{B^+}(\mathbf{q}^2)=\tfrac{2}{3}F_\pi(\tau^2\mathbf{q}^2)+\tfrac{1}{3}F_\pi((1-\tau)^2\mathbf{q}^2)\ , \qquad (5)$$

$$F_{B^0}(\mathbf{q}^2)=-\tfrac{1}{3}F_\pi(\tau^2\mathbf{q}^2)+\tfrac{1}{3}F_\pi((1-\tau)^2\mathbf{q}^2)\ , \qquad (6)$$

where the first term in both cases is the contribution when the light quark is struck and the second term is the contribution when the heavy quark is struck. In these formulas, F_π is the pion form factor:

$$F_\pi(\mathbf{q}^2)\approx\frac{1}{1+\mathbf{q}^2/(0.77\ \mathrm{GeV})^2}\ . \qquad (7)$$

Note that the light-quark contribution to the B form factors becomes negligible at large $\mathbf{q}^2$, making the form factors for charged and uncharged B's roughly equal. Using these form factors together with the naive Υ_{4s}–$B\bar{B}$ vertex, $\Phi(\mathbf{p}_B+\mathbf{q})=\Phi_0$, one finds that the Coulomb enhancement is reduced from 18% to less than 14%—not a large difference.

Modeling the Υ_{4s}–$B\bar{B}$ vertex is less straightforward. Insofar as the B mesons have larger radii than the Υ_{4s}, one expects the Υ_{4s} wave function, rather than the B wave functions, to control the momentum flow between the B's. Thus one might model the vertex function by

$$\Phi(\mathbf{p})\propto\psi_{4s}(p)\ , \qquad (8)$$

where $\psi_{4s}(p)$ is the momentum-space wave function of the Υ_{4s}. With this choice of vertex function, the 18% enhancement becomes a 3% suppression of charged B's relative to neutral B's. The sign flip is due to a node in the wave function at $p\approx 0.5$ GeV. (The wave function was computed using the Cornell quark potential model.[2]) A different model for $\Phi(\mathbf{p})$ is given in Ref. 2. This model is less *ad hoc* than the last since it follows from a nonrelativistic analysis of the vertex function and includes effects from the B wave functions. The Coulomb correction is different with this model though still small, giving only a 4% enhancement in the rate. The p dependence of the vertex function in this second model can be approximated by

$$\Phi(\mathbf{p})\propto\frac{1}{p}\frac{d\psi_{4s}(p)}{dp}\ , \qquad (9)$$

from which we see that the nodes occur at larger momenta here than in the previous model, the first node being at $p\approx 0.8$ GeV. The Coulomb corrections for the neutral B's are negligible in all cases (although nonzero).

Unfortunately we are unable to compute the vertex function with great reliabiliy. Nevertheless it is clear that the correct vertex function has nodes starting at momenta of order a few hundred MeV, with the result that the Coulomb correction is greatly reduced from the result for pointlike particles. Any discrepancy larger than a few percent between the decay rates for the two channels would probably indicate a mass difference between the neutral and charged B's.[3] Finally note that the effects of meson structure can be reduced by moving off resonance towards the $B\bar{B}$ threshold. Thus one can vary the ratio of charged to neutral B's in the decay by measuring at different energies along the resonance curve: 12 MeV below the Υ_{4s} resonance peak, for example, the excess of the charged over neutral B's could be larger than 10%.

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[1] D. Atwood and W. J. Marciano, Phys. Rev. D **41**, 1736 (1990).

[2] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D **21**, 203 (1980).

[3] A mass difference would result in different momenta p_B. This would affect the rate through a phase-space factor ($\propto p_B^3$) and through a factor due to the vertex function [$\propto\Phi(p_B)^2$]. The second factor can be as important as the first and should not be neglected.