PHYSICAL REVIEW D VOLUME 42, NUMBER 9 1 NOVEMBER 1990

W-boson electric dipole moment

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(Received 29 May 1990)

The W-boson electric dipole moment is calculated in the $SU(3)_C\times SU(2)_L\times U(1)_Y$ model with several Higgs-boson doublets. Using the constraint on the CP-violating parameters from the experimental upper bound of the neutron electric dipole moment, we find that the W-boson electric dipole moment is constrained to be less than 10^{-4}.

The W-boson electric dipole moment (EDM) is an interesting quantity to study. It may act as a test of the compositeness of the W boson and also a measure of CP violation in the electroweak interaction. In 1965, it was conjecture that the W boson might have an EDM $d_W=\lambda_W e/2M_W$.[1] At that time it was speculated that if $\lambda_W\sim 1$, it could explain the observed CP violation in the neutral-kaon system.[1] However a recent nonrenormalizable calculation[2] showed that a nonzero λ_W will generate an EDM d_n for the neutron. Using the experimental upper bound on the neutron EDM [$d_n<1.2\times 10^{-25}\,e$ cm (Ref. 3)], λ_W is constrained to be less than 10^{-3}. If this is true, the detection of the W-boson EDM though $\bar{p}p\to W\gamma X$ (Ref. 4) will not be practical in the near future. However, it is still important to understand theoretically what EDM a W boson can have and how the experimental upper bound on the neutron EDM constrains λ_W in renormalizable theories. In this paper we will calculate the W-boson EDM in the $SU(3)_C\times SU(2)_L\times U(1)_Y$ model with several Higgs-boson doublets.[5] In this model the neutral Higgs scalars conserve neutral flavor current at the tree level and the exchange of Higgs scalars violates CP. In the standard model with one Higgs doublet, no W-boson EDM is generated even at the two-loop level and hence is very small. In the model we are considering, the W-boson EDM is generated at the two-loop level. The diagrams which generate a nonzero W-boson EDM are shown in Fig. 1.

The relevant terms in the Lagrangian for the calculation of the W-boson EDM is given by

$$\mathcal{L}=Q_q\bar{q}_i\gamma_\mu q_i A^\mu+\frac{g}{\sqrt{2}}(\bar{u}_{iL}\gamma_\mu d_{jL}W^{+\mu}+\text{H.c.})+(a_{hi}\bar{q}_iq_i+ib_{hi}\bar{q}_i\gamma_5q_i)H_h^0\,, \quad (1)$$

where A^μ, W^μ, H_h^0 are the photon, W boson, and neutral Higgs bosons, respectively. We will omit the index h of Higgs particles for simplicity of notation. q_i includes both up and down quarks. The parameters a_i,b_i are proportional to the masses of the corresponding quarks. A nonzero b is a measure of CP violation due to exchange of the neutral Higgs scalars.

The W-boson EDM is represented by a term in the effective Lagrangian of the form

$$\mathcal{L}_{\text{eff}}=e\lambda_W(q^2)\tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}W^{+\alpha}W^{-\beta}\,. \quad (2)$$

The quantity we are going to calculate is $\lambda_W(0)=\lambda_W$. At the tree and one-loop levels, $\lambda_W=0$. At the two-loop level, a nonzero λ_W is generated. Evaluating the diagrams in Fig. 1, we find, for small q_α,

$$\begin{aligned}\lambda_W(q^2)q_\alpha=|V_{ij}|^2\int\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\Bigg[&a_ib_jg^2Q_jm_im_j\\&\times\Bigg\{\frac{p\cdot q(p_\alpha-k_\alpha)}{(p^2-m_j^2)(k^2-m_j^2)[(k+l)^2-m_i^2]^2[(l+p)^2-m_i^2][(k-p)^2-m_H^2]}\\&\quad+\frac{2p\cdot qp_\alpha}{(p^2-m_j^2)[(p+l)^2-m_i^2]^2(k^2-m_j^2)(k^2-m_i^2)[(k-p)^2-m_H^2]}\Bigg\}\\&+a_ib_ig^2Q_im_i^2\frac{p\cdot qp_\alpha}{(p^2-m_j^2)[(p+l)^2-m_i^2]^2[(k+l)^2-m_i^2]^2[(k-p)^2-m_H^2]}\Bigg]\\&+(m_i\leftrightarrow m_j,Q_i\leftrightarrow Q_j)\,,\end{aligned} \quad (3)$$

where l_α is the W-boson momentum, $i=u,c,t$, $j=d,s,b$, and V_{ij} is the Kobayashi-Maskawa matrix element. Figure 1(a) does not contribute to λ_W. Since a_i,b_i are proportional to quark mass, it is clear that in Eq. (3) the dominant contribution to λ_W is from where i,j correspond to the third generation. Integrating over the internal momenta, we have

$$\lambda_W(0)=\sqrt{2}G_F\text{Im}ZM_W^2g^2\frac{1}{2(4\pi)^4}I(m_t,m_b,m_H) \quad (4)$$

with

$$I(m_t,m_H)=\frac{m_t^2}{m_W^2}m_t^2\int_0^1 dy\int_0^1 du\int_0^{1-u}dv\frac{y(1-u-v)}{v[(1-y)m_H^2+ym_t^2]+y(y-1)[u(1-u)m_W^2-m_t^2(1-u-v)-um_b^2]} .$$

In the above expressions, we have kept only the third term in Eq. (3) with $i=t$ and used $|V_{tb}|\approx 1$. It should be pointed out that the charged scalars also contribute to the W-boson EDM. However, since the CP-violating quantities due to exchange of the charged scalars are always proportional to the product of an up-type quark mass and a down-type quark mass, it may be suppressed by a factor of m_b/m_t compared to Eq. (4). In Eq. (4) we have written the product $a_t b_t$ in a notation used in Ref. 6, that

$$a_i b_i=\sqrt{2}G_F \text{Im}Z m_i^2 . \quad (5)$$

The function $I(m_t,m_H)$ is typically ~ 1. We have numerically evaluated $I(m_t,m_H)$. In the following we will discuss two representative cases: (a) $m_t \gg m_H$, I(200 GeV, 10 GeV)=3.4; (b) $m_H \gg m_t$, I(90 GeV, 1 TeV)=0.11.

Since the same parameter ImZ is related to both the W-boson EDM and the neutron EDM, λ_W is constrained by the experimental upper bound on the neutron EDM. There are different types of estimating the contribution of the neutral Higgs scalars to the neutron EDM. We now discuss the constraints on ImZ from the neutron EDM.

(i) The valence-quark-model calculation. This way of calculating the neutron EDM is to calculate the quark EDM d_q and then use the SU(6) model to relate it to the neutron EDM by $d_n=\frac{4}{3}d_d-\frac{1}{3}d_u$. One has[7]

$$d_q=Q_q\frac{1}{8\pi^2}\sqrt{2}G_F \text{Im}Z m_q\frac{m_q^2}{m_H^2}\ln\left[\frac{m_H}{m_q}\right]^2 . \quad (6)$$

Now using the neutron EDM upper bound from experiment to constrain ImZ and insert it in Eq. (4), we find λ_W can be of order 10^{-2}.

(ii) The neutron EDM can also be estimated by first calculating the Higgs-boson–nucleon couplings and then the neutron EDM. Using the calculations in Ref. 8, we have

$$d_n=0.45\times 10^{-3}\sqrt{2}G_F \text{Im}Z m_n^2\frac{m_n^2}{m_H^2}\mu_n , \quad (7)$$

where μ_n is the neutron magnetic dipole moment. Inserting the constraint on ImZ from Eq. (7) into Eq. (4), we have $\lambda_W<10^{-4}$.

(iii) Recently, it was pointed out that a dimension-6 operator $O=-\frac{1}{6}Cf_{abc}G^a{}_{\mu\nu}G^b{}_\alpha{}^\nu G^c{}_{\sigma\gamma}\epsilon^{\mu\alpha\sigma\gamma}$ can contribute to the neutron EDM and the neutral-Higgs-scalar-exchange contribution to C is significant. A two-loop calculation similar to the one for evaluating λ_W gives[6]

$$C=\frac{\sqrt{2}}{(4\pi)^4}G_F \text{Im}Z g_s^3 h(m_t/m_H) , \quad (8)$$

where $h(m_t/m_H)=1/8$, if $m_t\gg m_H$ and $h(m_t/m_H)=(m_t^2/2m_H^2)\ln(m_t/m_H)^2$, if $m_H\gg m_t$.

Using the "naive dimensional analysis," the neutron EDM is estimated to be

$$d_n=e\xi\frac{\sqrt{2}}{(4\pi)^2}G_F m_n \text{Im}Z h(m_t/m_H) , \quad (9)$$

where[9] $\xi=7.8\times 10^{-5}$ is a parameter which includes QCD corrections to d_n. We obtain $\sqrt{2}G_F \text{Im}Z m_W^2<5.1$, for $m_H\gg m_t$ and <0.6, for $m_t\gg m_H$. Putting these numerical numbers in Eq. (4), we have $\lambda_W<4\times 10^{-6}$, for $m_H\gg m_t$; and $\lambda_W<10^{-5}$, for $m_t\gg m_H$.

Among the three ways of constraining λ_W, (iii) gives the strongest constraint on the W-boson EDM. In all the above estimates of the neutron EDM there are uncertainties. Especially for (iii), in addition to the QCD correction uncertainties in evaluating the operator O, there are also uncertainties in the use of the "naive dimensional analysis." In the real calculation, the three gluons of the operator O must be included with quarks to form a neutron. The simplest way is to attach the three gluons to quarks. If more than one gluon is attached to one quark, it will result in a factor of light-quark mass in the neutron EDM. This reduces the bound from Eq. (9). If the three gluons are attached to different quarks, due to color factors, it becomes zero.[10] So the neutron EDM induced from the operator O will involve the portion of a neutron formed from gluon and quark bound state. This will again reduce the contribution of O to the neutron EDM. Because of these considerations, we conservatively conclude that the upper bound for the W-boson EDM is $\lambda_W<10^{-4}$.

Our constraint is more stringent than that of Marciano and Queijeiro.[2] This is to be expected since their constraint is model independent, the only assumption being that CP violation produces a nonzero λ_W, and the constraint comes from the induced neutron EDM. In our

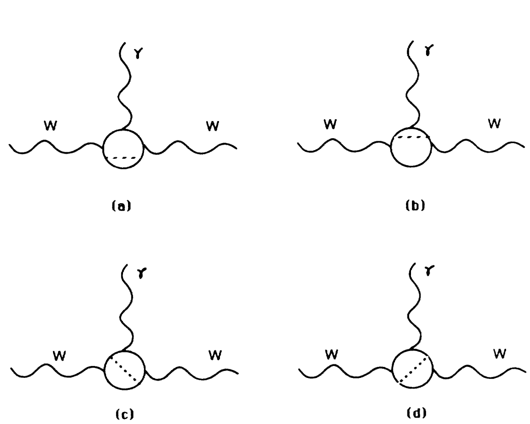

FIG. 1. The two-loop contributions to the W-boson EDM. The dashed lines indicate Higgs scalars and the solid circles represent quarks.

case we have made a specific model of *CP* violation, constraining the parameters of the model from the model prediction of the neutron EDM, and thus obtaining a constraint on the value of λ_W.

That our constraint respects the bound of Marciano and Queijeiro can be regarded as support for their analysis, indicating that the cutoff dependence of their result is consistently evaluated.

In conclusion, we reiterate that we have calculated the *W*-boson EDM in $SU(3)_C \times SU(2)_L \times U(1)_Y$ model with several Higgs-boson doublets. Using the constraints on the *CP*-violating parameters from the experimental upper bound on the neutron EDM, we find λ_W is constrained to be less than 10^{-4}.

We thank Professor S. Brodsky for useful discussions which initiated this investigation. X.G.H. thanks Dr. A. Davies for help in computer programming. This work was supported in part by the Australian Research Council.

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