PHYSICAL REVIEW D VOLUME 42, NUMBER 9 1 NOVEMBER 1990

Problems with hadronic η_c decays and the perturbative QCD scheme for exclusive reactions

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(Received 9 April 1990)

We consider some observed decays of the η_c which should be forbidden in the perturbative QCD description of exclusive reactions: $\eta_c \to \rho\rho, K^*\bar{K}^*, \phi\phi$. It is shown explicitly that quark mass corrections do not help and still give a zero result. Also the experimental value for the decay rate $\Gamma(\eta_c \to p\bar{p})$ is argued to be anomalously large. Implications for the perturbative QCD scheme are discussed, and some other possible explanations of the data are suggested.

Heavy $Q\bar{Q}$ bound-state decays are supposed to be a good testing ground for perturbative QCD. The large values of the c and b-quark masses m_Q and the relatively small value of the strong coupling constant involved in these decays, $\alpha_s(Q^2 \simeq 4m_Q^2)$, should make the lowest-order perturbative QCD contributions the dominant ones. Even if higher-order corrections might be sizable, one expects the lowest-order terms to give at least the correct order of magnitude answers.

Many examples can be found in the literature of such decays: $\eta_c, \chi_{0,2} \to gg$; $J/\psi \to ggg$; $\chi_{0,2} \to \pi^+\pi^-, \rho^+\rho^-$; $J/\psi \to p\bar{p}$.[1-4] In all these computations the initial heavy meson is supposed to be well described by its nonrelativistic wave function, in the zero-binding-energy approximation. The hadronic decays are then dominated by the elementary annihilation of a free $Q\bar{Q}$ pair into light $q\bar{q}$ pairs; such annihilations are mediated by the exchange of hard (large-Q^2) gluons, thus justifying the use of lowest-order perturbative QCD diagrams only.

The elementary amplitudes are then convoluted with the heavy-meson and final-particle wave functions according to the usual perturbative QCD scheme:[2,3,5]

$$A(C \to M\bar{M}) = \sum \int dx\, dy\, d^3k\, T(Q\bar{Q} \to q_1\bar{q}_1 q_2\bar{q}_2) \times \Phi_M^*(x,Q^2)\Phi_{\bar{M}}^*(y,Q^2)\Psi_C(\mathbf{k})\ . \quad (1)$$

Equation (1) gives schematically the c.m. amplitude for the decay of a heavy $Q\bar{Q}$ meson C into two final mesons M and $\bar{M}$ in terms of the elementary amplitudes $T(Q\bar{Q} \to q_1\bar{q}_1 q_2\bar{q}_2)$; $\Psi(\mathbf{k})$ is the momentum, spin, flavor, and color wave function of the decaying C meson ($\mathbf{k}$ being the $Q\bar{Q}$ relative momentum) and analogously $\Phi_M(x,Q^2)$ and $\Phi_{\bar{M}}(y,Q^2)$ are the wave functions of the final $(q_1\bar{q}_2)$ and $(\bar{q}_1,q_2)$ mesons (x and y are the usual fractions of the hadron momenta carried by the quarks). We must sum all different elementary amplitudes contributing to the same decay process.

Whereas this scheme is certainly reliable in the large-Q^2 limit $[\alpha_s(Q^2) \ll 1]$, it seems to work already in the case of some $c\bar{c}$ state decays; it has been widely used in Ref. 3 to describe the $\chi_{0,2} \to \pi^+\pi^-, \rho^+\rho^-$ and to advocate the success of a particular hadronic wave function, which allows a good fit to the experimental data.

In this note we will consider the most simple case of a spinless particle, the η_c, and will show that some of its decays, namely, $\eta_c \to \rho\rho, K^*\bar{K}^*, \phi\phi, p\bar{p}$ cannot be described in the above scheme, even when taking into account some plausible corrections. In particular for the $\eta_c \to \rho\rho$, $K^*\bar{K}^*$, $\phi\phi$ decays Eq. (1) still gives a zero result even when taking into account mass corrections. Because all of the above processes have been experimentally observed and are well established, it is of the utmost importance to realize such a failure of the theoretical models and to discuss the possible reasons for it.

In the case of the η_c one has (omitting the color part)

$$\Psi(\mathbf{k}) = \frac{1}{\sqrt{4\pi}}\left[\frac{\pi}{2}\right]^{1/2} R(0)\frac{1}{k^2}\delta(k) \times \frac{1}{\sqrt{2}}(c_+\bar{c}_+ - c_-\bar{c}_-)\ , \quad (2)$$

and Eq. (1) reads simply

$$A_{\lambda_M\lambda_{\bar{M}}}(\eta_c \to M\bar{M}) = \frac{R(0)}{4}\int d^3k\, \delta(k)\frac{1}{k^2}[M_{\lambda_M\lambda_{\bar{M}};++}(\mathbf{k}) - M_{\lambda_M\lambda_{\bar{M}};--}(\mathbf{k})]$$
$$= \pi R(0)[M_{\lambda_M\lambda_{\bar{M}};++}(k=0) - M_{\lambda_M\lambda_{\bar{M}};--}(k=0)]\ , \quad (3)$$

with

$$M_{\lambda_M\lambda_{\bar{M}};\lambda_c\lambda_{\bar{c}}}=\sum_{\lambda_{q_1}\lambda_{q_2}}\int dx\,dy\,\Phi^*_{\lambda_M}\Phi^*_{\lambda_{\bar{M}}}T_{\lambda_{q_1},\lambda_{\bar{M}}-\lambda_{q_2},\lambda_{q_2},\lambda_M-\lambda_{q_1};\lambda_c\lambda_{\bar{c}}}\,. \tag{4}$$

In Eqs. (2)–(4) all indices label helicities, in the center-of-mass system of the decaying heavy meson. The Feynman diagrams corresponding to the elementary amplitudes $T(Q\bar{Q}\to q_1\bar{q}_1q_2\bar{q}_2)$ are given in Fig. 1, where all variables are also defined.

Their computation leads to

$$T_{\lambda_{q_1},\lambda_{\bar{q}_1},\lambda_{q_2},\lambda_{\bar{q}_2};\lambda_c\lambda_{\bar{c}}}=-64C_Fg_s^4m_c^2(m_c^2-m_M^2)^{1/2}\frac{(x-y)\lambda_c\lambda_{q_1}\delta_{\lambda_c\lambda_{\bar{c}}}\delta_{\lambda_{q_1}\lambda_{q_2}}\delta_{\lambda_{q_1},-\lambda_{\bar{q}_1}}\delta_{\lambda_{q_2},-\lambda_{\bar{q}_2}}}{g_1^2g_2^2d^2}\,, \tag{5}$$

where the three factors in the denominator come from the three propagators in the Feynman diagrams of Fig. 1:

$$\begin{aligned} g_1^2&=[(x-y)^2m_M^2+4xym_c^2]\,,\\ g_2^2&=[(x-y)^2m_M^2+4(1-x)(1-y)m_c^2]\,,\\ d^2&=[(x-y)^2m_M^2+2(2xy-x-y)m_c^2]\,. \end{aligned} \tag{6}$$

C_F is the color factor (when convoluting T with the hadronic wave functions it will turn out to be $C_F=(2\sqrt{3})/9$). In computing Eq. (5) we have kept all masses, including the final quark ones, by setting, for the four-momenta,

$$\begin{aligned} q_1&=xp_M,\quad \bar{q}_2=(1-x)p_M\,;\\ \bar{q}_1&=yp_{\bar{M}},\quad q_2=(1-y)p_{\bar{M}}\,. \end{aligned} \tag{7}$$

In the scheme of Refs. 2, 3, and 5, where all quarks are massless, we would get the same result as in Eq. (5), in the limit $m_M/m_c\to 0$. Let us notice that no term proportional to m_M appears in the numerator of Eq. (5): actually many of these terms are present at intermediate stages of the calculation, but cancel out in the final result. It is then clear that taking masses into account *does not* change the helicity structure of the elementary amplitudes; for example, helicities are conserved along the final quark lines, even if mass terms should allow helicity flips. Mass corrections only lead, in the elementary amplitudes, to numerical changes.

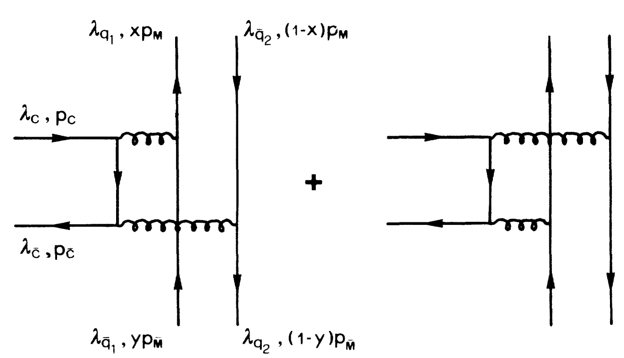

FIG. 1. Feynman diagrams contributing to the elementary processes $Q\bar{Q}\to q_1\bar{q}_1q_2\bar{q}_2$.

The wave functions for vector mesons (ρ,K^*,ϕ) are given by (again omitting the color part)

$$\begin{aligned} \Phi_{\lambda=\pm 1}&=q_\pm\bar{q}_\pm f_M\varphi_M(x)\,,\\ \Phi_{\lambda=0}&=\frac{1}{\sqrt{2}}(q_+\bar{q}_-+q_-\bar{q}_+)f_M\varphi_M(x)\,, \end{aligned} \tag{8}$$

where f_M is a dimensional constant related to the decay constant and the "hadronization amplitudes" $\varphi(x)$ are normalized as $\int dx\,\varphi(x)=1$. We neglect here the mild Q^2 dependence of $\varphi(x)$ deriving from QCD evolution, which is irrelevant for the subsequent discussion.

Inserting Eqs. (8) into Eq. (4) and using Eq. (5) we get $M_{\lambda_M,\lambda_{\bar{M}};\pm\pm}=0$, which implies

$$A_{\lambda_M\lambda_{\bar{M}}}(\eta_c\to M\bar{M})=0\,. \tag{9}$$

This result, in the $m_M/m_c\to 0$ limit only, was already stated in Ref. 3 and is implicit in the QCD helicity selection rules of the last of Refs. 5. We have shown here that it holds also when taking mass corrections into account. Notice that such corrections, in particular in the case of K^* and ϕ mesons, might have been, in principle, relevant. Of course, there might be other corrections, other than quark masses, that one could take into account, like a nonzero intrinsic $k_\perp$ of the quarks and orbital angular momentum in the wave functions, $q\bar{q}g$ components in the final mesons, etc. We have shown that the pure collinear quark configurations of the final mesons lead to a zero result which still holds true when quark masses are taken into account. The problem is that the η_c decays into the vector mesons ρ, K^*, and ϕ have all been experimentally observed,[6] with comparable decay rates; even if several corrections might still be important, one would expect the theory to be able to give at least a correct order-of-magnitude result.

Also the decay $\eta_c\to p\bar{p}$ shows unusual features. In the perturbative QCD scheme it is strictly forbidden by the helicity conservation of the gluon-quark couplings (in the limit of massless quarks) together with angular momentum and parity conservation.[7] Again, it does experimentally occur.

In such a case, however, one can still get a nonzero value for $\Gamma(\eta_c\to p\bar{p})$ by allowing for the presence of diquarks inside the protons, thus introducing helicity flipping gluon-vector diquark couplings.[7] The presence of diquarks inside baryons is supported by many experimental and theoretical arguments; using them as hadronic constituents, in a natural generalization of the pure quark scheme, can be considered as a way of modeling some nonpertubative corrections. Obviously, they are of no help with mesons.

Still, the $\eta_c \rightarrow p\bar{p}$ decay presents other unusual properties. The $\Gamma(\eta_c \rightarrow p\bar{p})$ is much larger than the $\Gamma(\chi_{0,1,2} \rightarrow p\bar{p})$ decay rates:[8] notice that the latter are not forbidden in the quark perturbative QCD scheme. In the quark-diquark model for the proton the four decay rates $\Gamma(\eta_c, \chi_{0,1,2} \rightarrow p\bar{p})$ have been computed, using some of the experimental data to fix the parameters of the model:[9] while the $\chi_{0,1,2}$ decay rates can easily be made to agree with the existing data, the η_c decay rate always turns out to be much too small. Thus the mystery of the large value of $\Gamma(\eta_c \rightarrow p\bar{p})$ still remains.

One might object that the values of Q^2 involved in the above processes are not large enough as to justify the use of lowest-order perturbative QCD; soft physics is probably still involved and there might be no reason to consider only tree diagrams. This might be true, although it is difficult to imagine how higher-order Feynman diagrams could change the helicity structure of $T(Q\bar{Q} \rightarrow q\bar{q}q\bar{q})$, Eq. (5), responsible for the zero result of Eq. (9). Also, one should not forget that the same scheme successfully describes the $\pi\pi$ decay channel of other $c\bar{c}$ mesons with masses very close to that of the η_c.[3]

The analogous decays in the case of $(b\bar{b})$ bound states, with much higher values of Q^2 involved, will certainly be a much more severe test: if the same dramatic discrepancies between the theoretical predictions and the experimental results remain true, then the whole perturbative QCD, hard exclusive reaction scheme will be in serious trouble.

We conclude by mentioning an alternative interesting possibility of explaining the "weird" decays of the η_c. Its quantum numbers, $J^{PC}=0^{-+}$, are the same as those of the $\iota(1456)$, which should have a significant gluonic component.[10] Might it be that the η_c decays receive a strong contribution from gluonic components or that they are mediated by glueballs? The idea that glueballs might play a role in $c\bar{c}$ meson decays has already been introduced in an attempt to explain another unexpectedly large decay of the charmonium family, $J/\psi \rightarrow \rho\pi$.[11]

We would like to thank E. Predazzi and E. Maina for useful discussions.

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