Lattice QCD simulation of meson exchange forces

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We present the formalism for investigating the $\bar{Q}q\bar{Q}q$ system in lattice QCD. This system serves as a model for describing exchange forces between heavy, static hadrons. We use this formalism to calculate the exchange potential from gauge configurations which incorporate the effects of dynamical quarks. Our data can be interpreted as giving preliminary results on the range of the nuclear force.

I. INTRODUCTION

It is desirable to examine, within the framework of QCD, physical systems which contain more than one color singlet. These systems can serve as an introduction to problems which appear in nuclear physics. Since the long-distance properties of QCD are not describable by perturbation theory, we need a systematic, nonperturbative approximation to perform quantitative calculations. Lattice gauge theory provides such an approach.[1] We therefore describe here a simulation of a "flavor-changing" Green's function involving two light quarks and two heavy antiquarks ($\bar{Q}_a q_b \bar{Q}_c q_d$, where $a-d$ denote flavors). This Green's function displays evidence of meson exchange and allows us to extract information concerning an exchange potential.

Lattice gauge theory has already told us much about the low-energy properties of QCD. The spatial dependence of forces between massive quarks can be shown to be described[2,3,4] in terms of a set of potentials:

$$V=V_0(R)+\frac{\mathbf{S}_1\cdot\mathbf{L}_1+\mathbf{S}_2\cdot\mathbf{L}_2}{2m_Q^2R}\left[\frac{dV_0}{dR}+\frac{2\,dV_1}{dR}\right]+\frac{\mathbf{S}_1\cdot\mathbf{L}_2+\mathbf{S}_2\cdot L_1}{m_Q^2R}\frac{dV_2}{dR}+\frac{1}{m_Q^2}(\mathbf{S}_1\cdot\hat{R}\,\mathbf{S}_2\cdot\hat{R}-\tfrac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2)V_3+\frac{1}{3m_Q^2}\mathbf{S}_1\cdot\mathbf{S}_2\ . \quad (1.1)$$

Simulations in the quenched approximation of lattice QCD show that the spin-averaged potential is consistent with a linear plus Coulomb parametrization[5]

$$V_0(R)\cong KR+\frac{\alpha}{R} \quad (1.2)$$

used in phenomenological studies of the $c\bar{c}$ and $b\bar{b}$ spectra. Simulations also show that the spin-spin tensor potential V_3 and the spin-spin scalar potential V_4 have a shape consistent with a one-gluon contribution even though the magnitudes of these potentials are not calculable from a one-gluon-exchange mechanism.[6] In addition, the spin-orbit potentials have been extracted from simulations in the quenched approximation. In this work, the form of dV_2/dR can be seen to be consistent with one-gluon exchange while dV_1/dR is slowly varying with R and suggests the validity of the scalar confinement ansatz.[6]

The heavy-quark potentials discussed above provide a good introduction to the nonperturbative aspects of QCD. The apparent agreement with experiment found there provides the confidence necessary to tackle more difficult problems. Another aspect of the low-energy behavior of the theory which can be studied quantitatively with lattice simulations involves the hadron spectrum. The significance of spectral calculations for QCD is that they provide a calibration of numerical methods along with insight into the complexities of gauge theories. A recent review of the overall status of low-lying meson and baryon masses can be found in Ref. 7. Again, the message from the lattice calculations is that we can learn a great deal about nonperturbative dynamics from them. A slightly different role is played by the calculation of the glueball spectrum in lattice QCD. Since there is, as yet, no substantial experimental evidence for states without quark constituents, lattice simulations provide important predictions for the masses and couplings of low-lying glueball states. If these predictions can be confirmed experimentally it will justify the confidence expressed by lattice theorists.

The status of lattice-gauge-theory calculations discussed above is such that it makes sense to try to extend them into the realm of multihadron states. However, some fundamental problems emerge when we make this attempt to extend lattice calculations into the realm of nuclear physics. Nuclear physics is presumably described by the QCD Lagrangian. Yet nuclear interactions exhibit several features that would seem difficult to ascribe to QCD. First, why do nucleons in heavy nuclei retain their identity, even though the separations between the partons within a particular nucleon may be no greater than the separation between partons in two different nucleons? Second, how can QCD account for "color saturation," whereby the binding energy between nucleons, or between color-singlet states in general, is very much less than the typical QCD energy scale? Finally, we would also like to understand how QCD gives rise to the usual picture of internucleon interactions in which the force between color-singlet objects is mediated by the exchange of mesons and is attractive at large distances and repulsive at short distances.

Measurements of the potential between pairs of heavy quarks at large distances in lattice QCD discussed above suggest that the interquark force arises from a color flux tube joining the quarks with energy proportional to the separation, yielding a linear (confining) potential. This picture of the interquark potential gives rise to one of the most successful hadronic models, and has been widely applied to baryonic and mesonic spectroscopy.[8] It is tempting to apply this model to many quark systems, in which the quarks are expected to be configured into color-singlet hadrons, and there have recently been two attempts at such studies.[9,10]

In the study by Watson,[9] a system of quarks interacting through SU(2)-color flux tubes was investigated. Several simplifications are made: first, the spins of the quarks are ignored, and second, the quarks are associated with a particular color, so that the model describes mesonic rather than baryonic matter. An ensemble of up to seven pairs of quarks was simulated using a Monte Carlo procedure. The model displays color saturation, but unfortunately does not bind into "nuclei" at finite density.

The simplest multiquark system that can be treated as a nuclear molecule is the $\bar{Q}_a Q_b \bar{Q}_c Q_d$ system, where the subscripts denote (not necessarily distinct) quark flavors. The four partons can form color-singlet mesons in two ways: $(\bar{Q}_a Q_b)(\bar{Q}_c Q_d)$ and $(\bar{Q}_c Q_b)(\bar{Q}_a Q_d)$. For the case where the Q's are heavy quarks, this basic system has been investigated in the strong-coupling approximation by Matsuoka and Sivers.[10]

The results for the $Q\bar{Q}Q\bar{Q}$ system are instructive for our discussion. To simplify the geometry we assume that massive quarks are confined to the x-y plane at the corners of a rectangle as shown in Fig. 1. The labels a,b,c,d denote distinct flavors. There are two flux configurations, shown in Figs. 1(a) and 1(b), which associate the $Q\bar{Q}$ pairs into color-singlet mesons. The two possible orientations of the flux produce distinct quantum states:

$$|1\rangle = |(Q_a\bar{Q}_b)_1(Q_c\bar{Q}_d)_1\rangle \ ,$$
$$|2\rangle = |(Q_a\bar{Q}_d)_1(Q_c\bar{Q}_b)_1\rangle \ . \tag{1.3}$$

In a dynamical approximation consistent with confinement these are adiabatic states for the Hamiltonian at large distances.

The Euclidean space Green's function which corresponds to the creation of the $Q\bar{Q}Q\bar{Q}$ system at $t=0$ and its annihilation at $t=T$ is then a 2×2 matrix in color space. The generalization of a Wilson loop for a single $Q\bar{Q}$ pair is the 2×2 matrix W_{IJ}, $I,J=1,2$ whose components are indicated in Fig. 2.

Matsuoka and Sivers discussed the diagonalization of the matrix W_{IJ} in the strong-coupling limit. They were

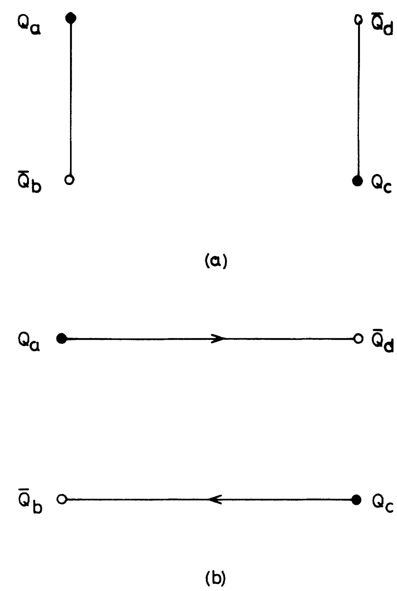

FIG. 1. The two possible orientations of the flux that bind the $Q\bar{Q}Q\bar{Q}$ system into two $Q\bar{Q}$ pairs are shown in (a) and (b), respectively.

able to show that the mixing between the two different color-singlet states was small and with $R_1=R_2=R$ corresponded to an energy shift

$$\frac{\Delta E}{E} = \frac{\exp(-\sigma R^2)}{2\sigma R a} \ , \tag{1.4}$$

where a is the lattice spacing, $\sigma(a)$ the string tension, and R is the quark separation.

Neither of the above models is consistent with the view that nucleons interact through meson exchange. Therefore before our simple models can accord with this picture we have to give them the flexibility to allow for quark exchange. We can incorporate this feature into the

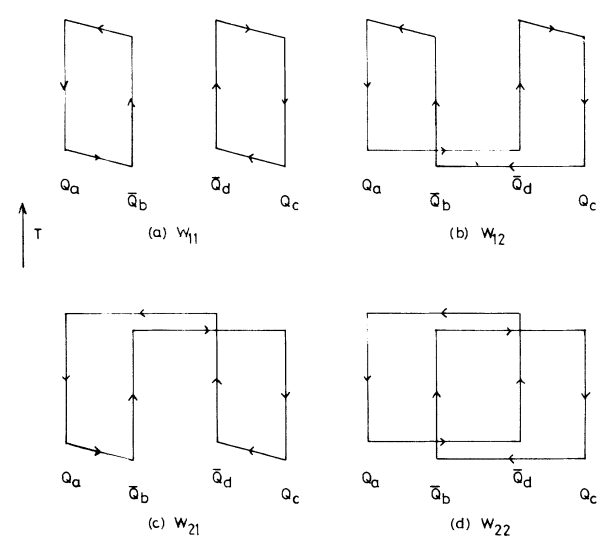

FIG. 2. The diagrams show the components W_{IJ} of the Euclidean Green's function for the $Q\bar{Q}Q\bar{Q}$ system.

approach of Matsuoka and Sivers by studying the $\bar{Q}_a q_b \bar{Q}_c q_d$ system, where Q and q represent heavy and light quarks, respectively, and the subscripts again denote the quark flavors. It is this system that we shall investigate in this paper, and in particular we shall aim to show that this model does indeed describe the interhadron force in terms of meson exchange. Such a four-quark system may be thought of as a two-hadron "molecule." It serves as an introduction to the nucleon-nucleon interaction in the quark-diquark picture, in which the heavy antiquarks assume the role of the diquarks. Since our treatment of the heavy $\bar{Q}$'s is such that they could equally well be thought of as heavy-QQ pairs our model could equally well be considered as a study of the interactions between two baryons each consisting of two heavy quarks and one light quark. In this interpretation we come even closer to the nuclear-physics problem.

The rest of this paper is organized as follows. In Sec. II we shall construct the formalism for studying $\bar{Q}q\bar{Q}q$ systems on the lattice, and discuss some simple expectations for the model. In Sec. III we shall describe the numerical simulations, and present results for the range of the interhadron force. We shall conclude by defining our strategy for future investigations.

II. $\bar{Q}q\bar{Q}q$ SYSTEMS ON THE LATTICE

$\bar{Q}q$ systems have been studied extensively by Eichten[11] in the regime $m_q \le \Lambda_{\text{QCD}} \ll m_Q$. Since the momentum transfer between the heavy quark and the light quark is typically of the order of Λ_{QCD}, and very much less than m_Q, the heavy quark can be treated nonrelativistically, and its propagator expanded as a power series in m_Q^{-1}. We shall apply the same techniques to the study of the $\bar{Q}q\bar{Q}q$ system, but shall always work to lowest order in m_Q^{-1} so that the heavy quarks represent fixed color sources.

The heavy-quark propagator is given by

$$S_H(r;s)=P(r;s)[\theta(s_0-r_0)e^{-m_Q(s_0-r_0)}\gamma_- +\theta(r_0-s_0)e^{-m_Q(r_0-s_0)}\gamma_+] , \qquad (2.1)$$

where

$$P(r;s)=U_4(\mathbf{r},r_0)\cdots U_4(\mathbf{r},s_0-1) \qquad (2.2)$$

for $r_0<s_0$, with a similar expression for $r_0>s_0$. In Eq. (2.1), γ_+ and γ_- denote the positive- and negative-energy projection operators, respectively, while in Eq. (2.2) $U(r,r_0)$ is an SU(3) matrix. For the case of a periodic lattice there is an additional contribution to the propagator that arises from the Wilson line that passes in the opposite direction around the lattice. For our work the contribution arising from the longer of the two lines is exponentially suppressed in m_Q and can therefore be ignored. The light-quark propagators are computed in the usual way.

The diagrams contributing to the interhadron force are shown in Figs. 3 and 4; the solid and curly lines denote the heavy- and light-quark propagators, respectively. The diagram of Fig. 4 corresponds to the off-diagonal elements in the Green's function of Fig. 2 for the static $Q\bar{Q}Q\bar{Q}$ system. Its evaluation allows us to extract the range of the nuclear force. In particular our simulation allows us to determine whether the range depends on the t-channel quantum numbers. Calculation of the diagonal components of the Euclidean space Green's function can, in principle, specify the strength of the nuclear potential. We employ local interpolating operators for the heavy-light mesons

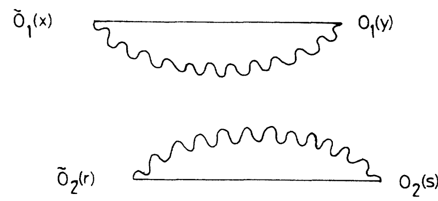

FIG. 3. Diagram contributing to flavor-nonexchange part of nuclear potential.

$$O(x)=\bar{\psi}_a(x)\Gamma_A\Psi_H(x) , \qquad (2.3)$$

where $\Gamma_A=\gamma_5$ for pseudoscalar mesons and $\Gamma_A=\gamma_i$ ($i=1,2,4$) for vector mesons. We begin with the construction of the correlation function

$$C(x,r;y,s)=\langle \tilde{O}_1^\dagger(x)\tilde{O}_2^\dagger(r)O_1(y)O_2(s)\rangle , \qquad (2.4)$$

where $y_0>x_0$ and $s_0>r_0$. The range of the force between the mesons can be extracted by measuring the falloff with z of the "z-sliced" correlation function

$$\tilde{C}(R,T)=\sum_{t_1,t_2,a_\perp} C(x,r;y,s) , \qquad (2.5)$$

where

$$x=(\mathbf{0},t_1),\quad y=(\mathbf{0},t_1+T) ,$$
$$r=(\mathbf{a}_\perp,R,t_2),\quad s=(\mathbf{a}_\perp,R,t_2+T) . \qquad (2.6)$$

The contribution of the flavor-exchange diagram to $\tilde{C}$ can be calculated by computing the quark propagator from one point on each time slice to every point on the lattice. However for the disconnected diagram the propagators from every point on one time slice to every point on the lattice have to be calculated. This is clearly a very

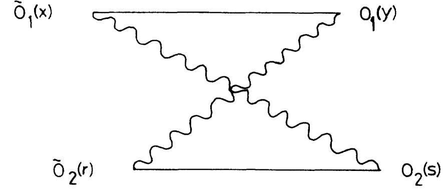

FIG. 4. Diagram contributing to flavor-exchange part of nuclear potential.

much more demanding task, and therefore as a first step we have restricted the calculation to the flavor-exchange contribution.

At large separations R the theoretical expectation is that the correlation function is dominated by the contribution of the lightest state $|n\rangle$ coupling to the operators in the t channel and becomes

$$\tilde{C}(R,T) \sim e^{-m_n R} \langle 0|\tilde{O}_1^\dagger(0,0) O_1(0,T)|n\rangle \times \langle n|\tilde{O}_2^\dagger(0,0) O_2(0,T)|0\rangle \,, \tag{2.7}$$

where m_n is the mass of this state. This correlation function therefore gives a measure of the range of the Yukawa potential between the two hadrons.

We conclude this section with a discussion of the expectations for the range of the force in the t channel. We shall use the notation V and P to denote the vector and pseudoscalar $\bar{Q}q$ mesons, respectively. The first process that we shall consider is

$$PP \to PP \quad (s \text{ channel}) \,. \tag{2.8}$$

In this process the $q\bar{q}$ particle exchanged in the t channel has natural parity, and hence we expect the mass of the intermediate state in Eq. (2.7) to be that of ρ. In order to see the exchange of a π meson we must consider the process

$$PV \to VP \quad (s \text{ channel}) \tag{2.9}$$

for which a particle of unnatural parity can be exchanged. The fact that the range of the correlator depends on the quantum numbers in the t channel provides an important theoretical constraint. In our numerical simulation we will look for this relationship.

III. NUMERICAL RESULTS

The gauge configurations for our simulations of the correlator (2.4) were generated by Grady, Sinclair, and Kogut[12] on an $8^3 \times 16$ lattice using the hybrid algorithm with four flavors of staggered fermions. However, we have chosen to use Wilson fermions for the calculation of the correlators of Eq. (2.4) since the Wilson formulation of the fermion propagator admits a straightforward flavor identification. The value of the hopping parameter is chosen so that the pion mass obtained using hadron correlators constructed from Wilson fermion propagators is the same (in units of the lattice spacing) as that obtained purely when using staggered fermions, and this value was determined in an earlier calculation of the hadron spectrum.[13] For this calculation the gauge configurations were replicated in the z direction to yield an $8^2 \times 16 \times 16$ lattice, since we are aiming to extract the lightest particle exchanged in that direction. Periodic boundary conditions were employed in the spatial directions, but an antiperiodic boundary condition was used in the time direction. The propagators were calculated using a conjugate residual algorithm preconditioned according to the prescription of Oyanagi.[14] On the Cray XMP/14 at Argonne National Laboratory the generation of the quark propagators required approximately 5 h per configuration, with a further hour required to construct the correlation functions.

The results presented in this paper are obtained from an analysis of 21 configurations at $m_q = 0.1$, $\beta = 5.4$, and of 29 configurations at $m_q = 0.05$, $\beta = 5.2$. The various parameters used in the simulation, together with the π and ρ masses obtained on the unreplicated lattice, are listed in Table I. We are obviously not in a range of parameters where the range of the pion-exchange force is vastly different from that of ρ. In fact even for lighter staggered quarks and larger lattice the Wilson ρ mass still remains relatively close to that of π.[15]

The correlation function corresponding to the processes of Eq. (2.8) and (2.9) were evaluated for all R and T. The effective masses of the exchanged particles obtained at $T = 1$, $T = 3$, and $T = 5$ are shown in Fig. 5 ($PP \to PP$) and Fig. 6 ($PV \to VP$). In each figure the masses of π and of ρ obtained from the falloff of the two-point correlator in the z direction on the *replicated* lattice are shown as the dashed and dot-dashed lines, respectively.

At both values of the quark mass the range of the force in the z direction is consistent with the exchange of a light meson. However, it is only for $m_q = 0.5$ that the separation between π and ρ masses is such that one can numerically discriminate between π and ρ exchange in the data. Even here, though, the closeness of the π and ρ masses makes analysis difficult. It should be kept in mind that there are considerable systematic errors in the calculation arising from the finite size of the lattice. The mass of the particle exchanged in the t channel is consistent with our expectation from Eqs. (2.8) and (2.9). Finally it should be noticed that the effective mass *increases* with increasing R at larger values of T. This may be the well-known artifact of lattice doubling,[16] or it may be indicative of a repulsive force present at short distances. Such a repulsive force is necessary in typical potential models to prevent a collapse of the system to $R = 0$.

Overall, the behavior of the correlation function in R provides a strong confirmation of the meson-exchange picture for nuclear forces, at least in a flavor-exchange channel.

IV. DISCUSSION AND CONCLUSIONS

We have discussed a model that forms a useful tool for studying the internucleon force within the framework of QCD. The internucleon force in this model is clearly mediated by meson exchange, and at the lower value of the quark mass the quantum numbers of the exchanged meson would appear in accordance with our naive expectations.

TABLE I. The various parameters used in the simulation are listed. Columns 4 and 5 show the pion and ρ masses, respectively, obtained on the unreplicated lattice.

m_q	β	No. of configurations	m_π	m_ρ
0.1	5.4	21	0.84(1)	0.90(2)
0.05	5.2	29	0.61(2)	0.74(5)

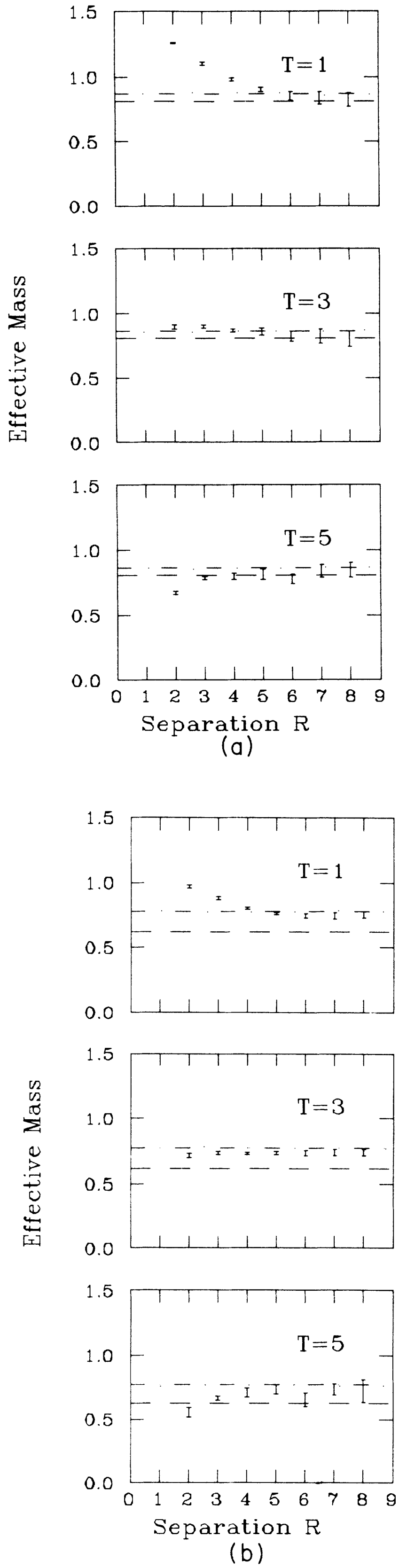

FIG. 5. The effective mass of the particle mediating the nuclear force in the process $PP \rightarrow PP$ at separations $T=1$, $T=3$, and $T=5$ is shown for $m=0.1$ and $m=0.05$ in (a) and (b), respectively.

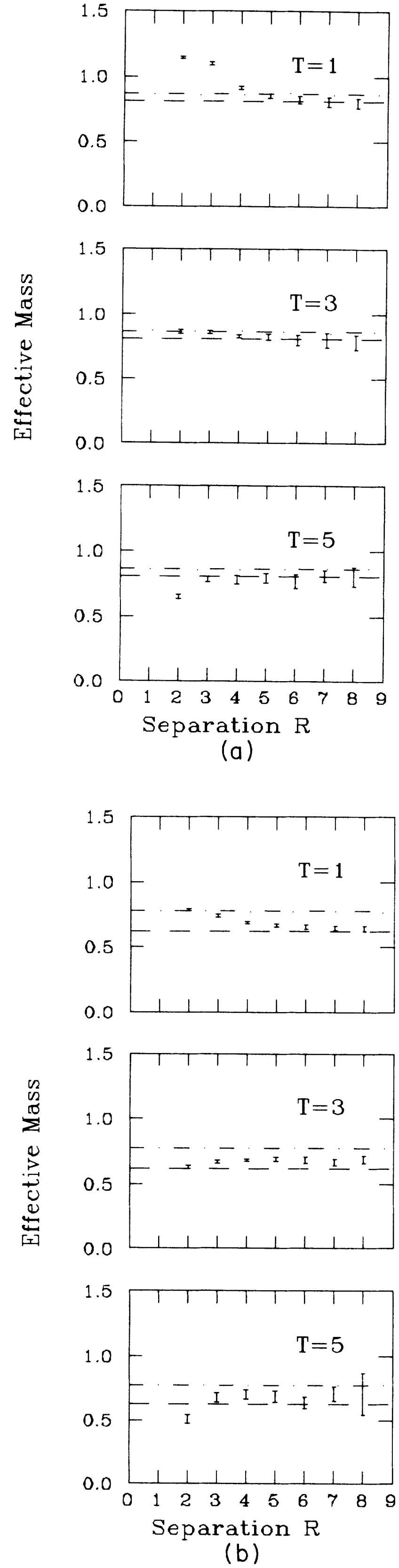

FIG. 6. The effective mass of the particle mediating the nuclear force in the process $PV \rightarrow VP$ at separations $T=1$, $T=3$, and $T=5$ is shown for $m=0.1$ and $m=0.05$ in (a) and (b), respectively.

It is instructive to compare the simulations discussed here with lattice calculations aimed at studying the H dibaryon proposed by Jaffe.[17] Two groups[18,19] have reached opposite conclusions concerning the mass of a $(udsuds)$ system which couples to two Λ's. Both groups have explicitly calculated the six-quark correlator at large T in an effort to distinguish the expected behavior

$$C_H \sim Ae^{-m_H T} \tag{4.1}$$

and

$$(C_\Lambda)^2 \sim Ae^{-2m_\Lambda T} . \tag{4.2}$$

In principle, such a calculation could hope to see a tightly bound H dibaryon ($m_H \ll 2m_\Lambda$) although one needs to worry whether the systematics in extrapolating in the H channel and the Λ channel are separately under control. Mackenzie and Thacker[18] reported no evidence for a tightly bound H dibaryon while Iwasaki *et al.*,[19] working on a larger lattice, reported a positive signal for binding. Note that, in order to study the weak binding typically associated with extended hadronic systems (such as nuclei), it is not practical to directly measure the T dependence as in (4.1) and (4.2), since the difference between the masses of the bound and unbound systems can be very small.

In order to study the general binding problem we need to incorporate the full scope of the multibody formalism. For the $\bar{Q}q\bar{Q}q$ system described here this means calculating both the flavor-exchange "off-diagonal" correlator discussed here and the corresponding diagonal entries in the Euclidean space Green's function. As mentioned above, because of computational limitations, we have not been able to do the more complete calculation at this time. With a full simulation of the 2×2 Green's function in flavor space a mixing formalism would enable us to extract a potential. This is the obvious next task in a program aimed at studying multihadron forces in lattice QCD.

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