Kaon excitation in the SU(3) Skyrme model

H. Weigel, J. Schechter, and N. W. Park
Physics Department, Syracuse University, Syracuse, New York 13244-1130

Ulf-G. Meissner
Universitat Bern, Institut für Theoretische Physik, Sidlerstrasse 5, CH-3012 Bern, Switzerland

(Received 18 December 1989)

We attempt to improve the description of the strange baryons in the "collective" approach to quantizing the SU(3) Skyrme model. We go beyond the recent work of Yabu and Ando by employing a kind of "cranking" procedure for the strange fields, which allows them to contribute to the corresponding moment of inertia. A better overall fit to the mass differences of the low-lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons is achieved using values of parameters fixed from the meson sector. The picture of the baryons that emerges is one in which they contain appreciably fewer "$s\bar{s}$ current quark pairs" than in a naive perturbation approach. At the constituent-quark level there is significant mixing of different SU(3) multiplets. This and a number of related points are treated.

I. INTRODUCTION

The SU(3) Skyrme model of pseudoscalars[1] has recently generated a lot of interest because it seems to predict[2,3] a large "strange content" of the proton and also because it gives[4,5] a pattern of axial-vector-current matrix elements similar to the one implied by analysis of the recent European Muon Collaboration (EMC) experiment.[6] We should say immediately that we do not feel the model of pseudoscalars alone is detailed enough to give a precise description of the properties of the baryons (for example,[7] the neutron-proton mass difference cannot be understood in this model), but nevertheless it seems an ideal beginning one for understanding various aspects of the baryons that cannot be easily explained in other ways. The first step, of course, is to find out just what the model really does predict. The answer to this question, both from conceptual and calculational standpoints, has undergone a certain amount of refinement in the literature. In the present paper we wish to discuss a new conceptual refinement that seems to also improve the predictions of the model.

In the limit when all light (u,d,s) quark masses vanish, the SU(3) Skyrme model is easy to understand. There is a classical soliton solution lying, for example, in the isospin subspace and eight zero modes which can be regarded as flavor rotations around this solution. Their collective quantization generates, among other things, eight baryons which can be thought of as containing not only valence quarks but also many quark-antiquark pairs. In particular the proton wave function contains[2] a relatively large amount of $s\bar{s}$ pairs. However, life is not so simple: the strange-quark mass is not at all negligible on the characteristic QCD scale and only a limited number of $s\bar{s}$ pairs can fit into a physical proton. The first workers[1] on the SU(3) Skyrme model treated the symmetry breaking to first order. They assumed that the baryon wave functions were unchanged from the zero-quark-mass limit. This is why large "strange" matrix elements of quantities such as $\bar{s}s$ and $\bar{s}\gamma_\mu\gamma_5 s$ were obtained. Perhaps it is thereby understandable why other predictions of the model treated in this way are not so realistic. For example, the pattern of octet-baryon mass splittings is not accurate and g_A is even smaller than the too small SU(2) value. One approach[8] to this problem involves quantizing only the "true" nonstrange SU(2) zero modes and treating the strange baryons as bound states of a kaon and an SU(2) Skyrmion. Here we want to follow the standard approach of quantizing all the SU(3) modes so as to be able to answer questions about the strangeness content of the proton and so as to be able to make contact with the ordinary successful low-energy phenomenolgy based on SU(3) multiplets.

Following along this standard line, Yabu and Ando[9] showed that the predictions of the model could be greatly improved by treating the same collective Hamiltonian obtained by everyone else[1] exactly (in a numerical approach) rather than by first-order perturbation theory. In fact, it turns out[10] that second-order perturbation theory is sufficient to explain the new qualitative features. Higher-order perturbation theory also enables one to develop some intuition about the model. As the strength of the symmetry breaker increases the baryon-octet wave function develops nontrivial admixtures of the SU(3) $\overline{10}$ and 27 representations. This leads, as expected, to greatly reduced matrix elements for "strange" operators and implies that the number of $s\bar{s}$ "current-type" quark-antiquark pairs in the proton has correspondingly decreased. At the "constituent-quark" level the $\overline{10}$ and 27 states cannot be of the form qqq but must be objects such as $qqq\bar{q}q$. This is very interesting since it implies a modified constituent-quark picture for the proton itself.

There seems to be a natural identification of the proton-like members of $\overline{\mathbf{10}}$ and **27** with the Roper $N(1440)$ and with the $N(1710)\,P_{11}$ resonances, respectively.

The collective Lagrangian [see Eq. (4.5)] contains a term corresponding to rigid rotations in space (with moment of inertia α^2) and a term corresponding to rotations into the "strange" direction of "internal space" (with moment of inertial β^2). One would expect that β^2 should be sensitive to the strange-quark mass (as revealed to us by the K-meson mass). However in the Yabu-Ando approach,[9] β^2 is computed at a stage before the model knows anything about symmetry breaking and is thus independent of symmetry breaking. The refinement of the model that we propose is to allow the moment of inertia for "strange rotations" to depend on the K-meson mass. This is implemented by introducing a cranking-type ansatz for the K-meson field; if Ω_k is an "angular velocity" isospinor with K-meson flavor quantum numbers, we set a suitable K field to equal $w(r)\tau\cdot\hat{r}\Omega_k$. Here $w(r)$ is a complex function that is determined as a solution of the equation arising from maximizing $\beta^2[w(r)]$. It is amusing that the existence of nonzero w depends on the presence of the Wess-Zumino term in the model's action. The resulting value of β^2 is substantially larger than that obtained by Yabu and Ando. This feature increases the strength of the *effective* symmetry breaker with the consequence that the strange content of the proton is reduced even further.

In this paper we shall use the experimental values of all physical quantities (F_π, F_k, m_π, m_k); the only adjustable parameter is the Skyrme[11] constant e. As is well known, using realistic values for these basic constants results in *absolute* masses that are much too high. But since for the SU(3) case there are many splittings available with which to compare we may get a feeling for the accuracy of our predictions without solving this problem. While the hope has been expressed that renormalization effects in the soliton sector might lower the effective F_π for baryons this is at present pure speculation.

The model Lagrangian is written in Sec. II and the classical soliton solution briefly discussed in Sec. III. Our method of deriving the parameters of the by now standard collective Hamiltonian are set out in detail in Sec. IV. Some related formulas and arguments are contained in Appendixes A and B. Section V contains the predictions of the model for mass splittings, excited levels, and some matrix elements relevant for discussing the interpretation of the results. A brief summary and discussion of some additional points are given in Sec. VI.

II. MODEL LAGRANGIAN

The underlying SU(3) chiral action we consider has three parts:

$$\Gamma=\int d^4x(\mathcal{L}_0+\mathcal{L}_{\rm SB})+\Gamma_{\rm WZ}\ . \tag{2.1}$$

$\mathcal{L}_0$ consists of the standard SU(3) nonlinear σ model supplemented by an *ad hoc* term introduced by Skyrme[11] to enforce the stability of the soliton:

$$\mathcal{L}_0=-\frac{F_\pi^2}{8}{\rm Tr}(\partial_\mu U\partial_\mu U^\dagger)+\frac{1}{32e^2}{\rm Tr}([\partial_\mu UU^\dagger,\partial_\nu UU^\dagger]^2)\ , \tag{2.2}$$

where F_π is the bare pion decay constant and e is the dimensionless "Skyrme constant." Typical values for e quoted in the literature are around 5. The dynamical variables are contained in the 3×3 unitary matrix U, which may be expressed in terms of the pseudoscalar octet fields φ as

$$U=\exp\left[\frac{2i\varphi}{F_\pi}\right]\ . \tag{2.3}$$

It is sometimes convenient to define a square root of U by

$$\xi\equiv U^{1/2}=\exp\left[\frac{i\varphi}{F_\pi}\right]\ . \tag{2.4}$$

$\Gamma_{\rm WZ}$ is the Wess-Zumino term:[12]

$$\Gamma_{\rm WZ}=\frac{-iN_c}{240\pi^2}\int{\rm Tr}(\alpha^5)\ , \tag{2.5}$$

the integral being over a five-dimensional manifold whose boundary is Minkowski space and where the one-form α is defined by $\alpha=dUU^\dagger$. $N_c=3$ is the number of colors. Finally the symmetry-breaking terms are

$$\mathcal{L}_{\rm SB}={\rm Tr}[(\beta'T+\beta''S)(\partial_\mu U\partial_\mu U^\dagger U+U^\dagger\partial_\mu U\partial_\mu U^\dagger) +(\delta'T+\delta''S)(U+U^\dagger-2)]\ . \tag{2.6}$$

Here the "spurions" T and S are defined by

$$T={\rm diag}(1,1,0)\ ,\qquad S={\rm diag}(0,0,1)\ . \tag{2.7}$$

Equation (2.6) mocks up the fundamental mass terms of the QCD quark Lagrangian to second order in derivatives[13] but with neglect of Okubo-Zweig-Iizuka- (OZI-) rule violation. We expect the coefficients to obey

$$\beta':\beta''=\delta':\delta''=\frac{m_u+m_d}{2}:m_S\ , \tag{2.8}$$

in terms of the usual quark "current" masses. From the physical values of m_π, m_k, $F_{\pi p}$, and F_{kp} we find (this analysis is discussed in Ref. 7, for the case where vector mesons are present)

$$\beta'\approx-26.4\ {\rm MeV}^2,\quad \beta''\approx-985\ {\rm MeV}^2\ ,\qquad \delta'\approx4.15\times10^{-5}\ {\rm GeV}^4,\quad \delta''\approx1.55\times10^{-3}\ {\rm GeV}^4\ . \tag{2.9}$$

Note that

$$F_\pi^2=F_{\pi p}^2+16\beta'\ , \tag{2.10}$$

where the physical pion decay constant $F_{\pi p}\approx132$ MeV. We have not included the η' field and a term needed to satisfy the $U(1)_A$ anomaly. This will not be relevant for our present purpose.

III. CLASSICAL SOLITON

SU(3)-symmetry breaking actually plays an important role in the determination of the classical soliton solution. The usual[1] "hedgehog" *Ansatz* is

$$U_0(\mathbf{r})=\begin{bmatrix} e^{i\hat{r}\cdot\tau F(r)} & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0 & 0\end{matrix} & 1\end{bmatrix}, \tag{3.1}$$

where $F(r)$ is to be determined to minimize the static energy of (2.1). If the symmetry-breaking terms $\mathcal{L}_{SB}$ in (2.6) were neglected one could equally well choose another *Ansatz* such as

$$\begin{bmatrix} 1 & \begin{matrix}0 & 0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & e^{i\hat{r}\cdot\tau F(r)}\end{bmatrix} \tag{3.2}$$

and the results for the classical mass and profile $F(r)$ would be the same. This is not true when $\mathcal{L}_{SB}$ is present. Then for (3.1), $F(r)$ will fall off like $(1/r)\exp(-m_\pi r)$ at large distances while for (3.2), $F(r)$ would fall off like $(1/r)\exp(-m_k r)$. The classical masses would differ in the two cases and further properties related to $F(r)$ like the "moments of inertia" would also differ. We see that already at the very start there is some ambiguity for the SU(3) Skyrme model. It is clear that physically motivated assumptions are required. Now if we were to allow a fast fall off like $(1/r)\exp(-m_k r)$, the nucleon charge radius would turn out to be too small. Hence we shall adopt the *Ansatz* (3.1) which also gives a lower energy. Substituting this into the Hamiltonian derived from (2.1) gives the classical mass:

$$M_{\rm cl}=4\pi\int dr\left[\frac{F_{\pi p}^2}{4}(F'^2r^2+2\sin^2F)+\frac{1}{2e^2}\sin^2F\left[2F'^2+\frac{\sin^2F}{r^2}\right]+4\delta' r^2(1-\cos F)+4\beta'(1-\cos F)(F'^2r^2+2\sin^2F)\right], \tag{3.3}$$

where $F'=dF/dr$. As usual $F(r)$ is found by minimizing $M_{\rm cl}$ with the boundary conditions $F(0)=\pi$, $F(\infty)=0$ appropriate to the baryon-number-one sector of the theory.

IV. COLLECTIVE QUANTIZATION

The analog of the Adkins-Nappi-Witten[14] quantization for the SU(3) Skyrme model without $\mathcal{L}_{SB}$ would be based on the recognition that the choice $AU_0A^\dagger$, where A is a unitary unimodular 3×3 matrix, leads to another equivalent classical solution with the same energy. Quantization then proceeds by elevating A to a function of time $A(t)$ and considering it as the collective dynamical variable. Now when $\mathcal{L}_{SB}$ is included A must be restricted to be an SU(2) rather than an SU(3) matrix in order to give a different classical solution with the same energy. Hence a collective quantization along the same lines would give SU(2) rather than SU(3) multiplets of baryons. This is the path taken by Callan and Klebanov in the "bound-state approach[8] to strangeness." However, it is clearly desirable to make contact with the great quantity of low-energy phenomenology, which is based on SU(3) multiplets, and to recognize that flavor SU(3) is a good approximate symmetry in nature. Hence we shall deal with the collective degrees of freedom contained in the 3×3 matrix $A(t)$. This is, of course, the usual approach to the present model. We would like to stress that, as just described, unlike the SU(2) case, it is not strictly based on quantizing the "zero modes." The initial work by many people[1] showed that this approach, when supplemented by a first-order perturbation theory treatment of $\mathcal{L}_{SB}$, gave rather poor results on comparison with experiment. More recently Yabu and Ando[9] have shown that these results could be noticeably improved by an *exact* treatment of $\mathcal{L}_{SB}$. Further interpretation from the present point of view is contained in Ref. 10.

The motivation of the present work is to attempt to improve the Yabu-Ando results which achieve a fit only by using unphysical values of $F_{\pi p}$ and/or m_k. The new feature is that we will take "cranking-type" corrections into account. Specifically, the quantized collective Hamiltonian of the SU(3) Skyrme model corresponds to a rigid object rotating in both ordinary as well as flavor space. The object, in turn, is made out of various field excitations. When the assemblage starts rotating we would expect these excitations to adjust themselves to maximize the moments of inertia. It is especially important that the moment of inertia for internal space rotations of nonstrange into strange flavors be maximized. This type of approach has been successful[15,16] in treating the contributions of the vector mesons to the moment of inertia in the SU(2) Skyrme model.

To accomplish our goal we introduce collective coordinates for the field ξ in (2.4) in the following manner:

$$\xi(\mathbf{r},t)=A(t)\xi_k U_0^{1/2}(\mathbf{r})\xi_k A^\dagger(t)\ . \tag{4.1}$$

We have chosen to work with ξ instead of U and have taken the particular parametrization (which is of course arbitrary) shown with an eye to extending this model to include vector mesons. As it stands, the only difference between (4.1) and the usual approach is the presence of the two factors of ξ_k. In order to explain our *Ansatz* for ξ_k it is useful to define the "angular velocity" matrix

$$A^\dagger\dot{A}=\frac{i}{2}\sum_{a=1}^{8}\lambda_a\Omega_a=i\begin{bmatrix}\Omega_\pi+\Omega_\eta & \Omega_k\\ \Omega_k^\dagger & -2\Omega_\eta\end{bmatrix}, \tag{4.2}$$

where $\Omega_\pi=\frac{1}{2}\sum_{a=1}^{3}\tau_a\Omega_a$, etc., and the λ_a are the 3×3 Gell-Mann matrices. Then

$$\xi_k\equiv e^{iz}=1+iz-\tfrac{1}{2}z^2+\cdots\ , \qquad (4.3)$$

where the simplest *Ansatz* for z taking into account the pseudoscalar nature of the kaons is

$$z=\begin{bmatrix} 0 & w(r)\tau\cdot\hat{r}\Omega_k \\ \Omega_k^\dagger\tau\cdot\hat{r}w^*(r) & 0 \end{bmatrix} . \qquad (4.4)$$

Notice that $w(r)$ is in general a complex function and that the expansion for ξ_k will be truncated at quadratic order. We have not introduced a possible isoscalar pseudoscalar excitation in (4.4) because it has been shown elsewhere[17] that such a function would vanish in a pure Skyrme model of pseudoscalars (vector mesons are needed for this purpose).

The collective Lagrangian is to be obtained by substituting the above *Ansatz* into the action (2.1). The result is extremely complicated and not tractable without some approximations. First, we assume as usual that only terms up to order Ω^2 are to be kept and that $\dot{\Omega}_a=0$. Second, we wish to find the effect of the relatively large kaon mass on the moment of inertia for rotation into strange directions. This was not previously taken into account. With our *Ansatz* the main effect is related to the large numerical value of m_k, rather than the detailed form of $\mathcal{L}_{SB}$. Hence, for computing the terms in the collective Lagrangian proportional to Ω and Ω^2 we shall simplify $\mathcal{L}_{SB}$ in (2.6) by treating it as an SU(3) singlet. Then we obtain a Lagrangian that is the same as that considered by Yabu and Ando:

$$L=-M_{\rm cl}+\tfrac{1}{2}\alpha^2\sum_{a=1}^{3}\Omega_a^2+\tfrac{1}{2}\beta^2\sum_{a=4}^{7}\Omega_a^2 +\frac{\sqrt{3}}{2}\Omega_8-\tfrac{1}{2}\gamma[1-D_{88}(A)]\ . \qquad (4.5)$$

The classical mass $M_{\rm cl}$ is given in (3.3) and the symmetry-breaking coefficient γ is obtained as

$$\gamma=\frac{32\pi}{3}\int dr\, r^2\left[(\delta''-\delta')(1-\cos F) +(\beta'-\beta'')\left[F'^2+2\frac{\sin^2F}{r^2}\right]\cos F\right] , \qquad (4.6)$$

where $D_{88}(A)=\frac{1}{2}{\rm Tr}(\lambda_8A\lambda_8A^\dagger)$ is an element of the SU(3)-octet representation matrix. The formula for the space rotation moment of inertia α^2 is given in Appendix A and only differs slightly from that of Ref. 9. The main difference is seen in β^2, which is a moment of inertia for rotation into strange directions. This rather lengthy formula is also displayed in Appendix A. In calculating the moments of inertia we have made the simplifying approximation that the terms from $\mathcal{L}_{SB}$ proportional to Ω^2 be considered as flavor singlets, i.e.,

$$\mathcal{L}_{SB}\to{\rm Tr}[y_1(\partial_\mu U\partial_\mu U^\dagger U+U^\dagger\partial_\mu U\partial_\mu U^\dagger) +y_2(U+U^\dagger-2)]\ , \qquad (4.7)$$

where the real coefficients y_1 and y_2 are taken to satisfy

$$\frac{y_2}{y_1}=\frac{\delta'}{\beta'}=\frac{\delta''}{\beta''}\ . \qquad (4.8)$$

A second consistency condition for completely determining y_1 and y_2 will be discussed shortly.

As mentioned before, the main new feature of the present approach is the maximization of the moment of inertia for internal space rotations of nonstrange into strange directions. We thus impose

$$\frac{\delta}{\delta w(r)}\beta^2[w(r)]=0\ . \qquad (4.9)$$

This results in a second-order differential equation for $w(r)$. One boundary condition is obtained by demanding finiteness as $r\to0$. The second boundary condition results from noting that at large distances

$$w\sim\frac{1}{r}e^{-m_kr}\ ,$$

which implies

$$y_2=m_k^2\left[\frac{F_\pi^2}{8}-2y_1\right]\ . \qquad (4.10)$$

This, together with (4.8), serves to fix both y_1 and y_2:

$$y_1=-482\ {\rm MeV}^2,\quad y_2=7.56\times10^{-4}\ {\rm GeV}^4\ . \qquad (4.11)$$

As expected y_1 and y_2 are similar to β'' and δ'', respectively, in (2.9). It should be noted that if the Wess-Zumino term in the action were absent, $w(r)$ would vanish. As can be seen from (4.4) and (4.2), the other terms would just contribute even powers of w to β^2, so $w=0$ would be a solution of (4.9). However, since the Wess-Zumino term (which provides a contribution proportional to $\epsilon_{\mu\nu\alpha\beta}$ in four-dimensional Minkowski space) contains only one time derivative, it produces terms proportional to $[{\rm Re}w(r)]\dot{\Omega}_k^\dagger\Omega_k$. These linear terms act as a *source* for the w field and prevent it from vanishing. Since only ${\rm Re}(w)$ appears, ${\rm Im}(w)=0$ is a solution of (4.9).

As a check on our *Ansatz* as expressed in (4.1)–(4.4) as well as on our collective quantization procedure, including the approximation (4.7), we have verified in Appendix B that the identical expressions for the flavor-symmetry charges Q^a emerge when they are computed directly from the collective Lagrangian (4.5) as

$$Q^a=-\sum_{b=1}^{8}D_{ab}\frac{\partial L}{\partial\Omega_b}$$

and when they are computed by integration of the microscopic Noether currents.

In order to gauge the accuracy of our approximations we have performed some exploratory calculations in Appendix C. The main effects of using (4.7) rather than (2.6) for $\mathcal{L}_{SB}$ can be recaptured by retaining (4.9) as an approximate equation of motion with however a different function β^2. Then it is found that the kaon profile $w(r)$ does in fact fall off as

$$\frac{1}{r}e^{-m_k r}$$

as used in deriving (4.10). Furthermore, the numerical results for mass splittings are substantially unchanged. It is pointed out that an exact solution of the equation of motion would require a much more complicated *Ansatz* for z in (4.4).

Passage from the Lagrangian (4.5) involving the angular velocities (4.2) to the Hamiltonian has been discussed in the literature.[1,9] It results in

$$H=M_{\rm cl}+\frac{J(J+1)}{2\alpha^2}+\frac{1}{2\beta^2}\{C_2[{\rm SU}(3)]-J(J+1)-\tfrac{3}{4}\}+\frac{\gamma}{2}[1-D_{88}(A)]\ . \qquad (4.12)$$

Here J is the angular momentum eigenvalue and $C_2[{\rm SU}(3)]$ is the quadratic Casimir operator for the SU(3) group. Note that the coefficient of $1/2\beta^2$ is positive definite so that maximization of β^2 sensibly decreases the energy. We should stress that, while (4.12) has the same form as in Ref. 9, the dependence of $M_{\rm cl}$, α^2, β^2, and γ on the underlying parameters in (2.1) is different. The only parameter not determined from the meson sector is the Skyrme constant e. Unlike Yabu and Ando,[9] we will always consider $F_{\pi p}$ and m_k to be fixed at their physical values.

V. PREDICTIONS OF THE MODEL

In the absence of the last symmetry-breaking term in (4.12) the eigenfunctions of H are known[1] to be SU(3) D functions corresponding (for baryon number 1) to irreducible representations which contain a state with hypercharge $Y=1$ and $I=J$. The initial treatments[1] of the SU(3) Skyrme model assumed that it was sufficient to use these eigenfunctions even in the presence of symmetry breaking. But Yabu and Ando[9] showed that this is not an adequate approximation and that it is better to diagonalize (4.12) exactly. Acting on a multiplet of definite J, I, and Y it is sufficient to diagonalize the operator

$$C_2[{\rm SU}(3)]+\beta^2\gamma[1-D_{88}(A)]\ . \qquad (5.1)$$

This may be converted into a system of coupled second-order linear differential equations by introducing a "Euler-angle" representation for A and representing the generators appearing in $C_2[{\rm SU}(3)]$ as differential operators. This equation is discussed at length in Ref. 9. However, instead of introducing an approximate basis[9] we have numerically performed a direct integration of these differential equations to obtain the eigenvalues $\epsilon_{\rm SB}$ and the eigenfunctions of (5.1). Evidently $\epsilon_{\rm SB}$ is a function of the product of $\gamma\beta^2$. The energy of each state is obtained as

$$E=\frac{1}{2\beta^2}\epsilon_{\rm SB}+M_{\rm cl}+\frac{J(J+1)}{2\alpha^2}-\frac{1}{2\beta^2}[J(J+1)+\tfrac{3}{4}]\ . \qquad (5.2)$$

One may understand the physical significance of this procedure by treating (4.12) with perturbation theory beyond the leading order. Then it is found,[10] for example, that the proton is not a pure octet but looks like

$$|p\rangle \simeq |p,8\rangle+0.0745\,\gamma\beta^2|p,\overline{10}\rangle+0.0490\,\gamma\beta^2|p,27\rangle+\cdots\ . \qquad (5.3)$$

This illustrates that the product $\gamma\beta^2$ is the *effective* measure of symmetry breaking. The $\overline{\mathbf{10}}$ and **27** states play a very important role since matrix elements of physical quantities depend sensitively on their admixture. For realistic values of $\gamma\beta^2$ the second-order mass corrections are of similar magnitude to the first-order ones. The physical content of the $\overline{\mathbf{10}}$ and 27 is that of 3 constituent quarks plus an extra constituent quark-antiquark pair. In the case of the proton the *pure* octet wave function in the SU(3) Skyrme model contains a relatively large amount of $s\bar{s}$ "current" quark pairs. The effect of increasing symmetry breaking and the consequent mixing of higher SU(3) representations turns out to *decrease* the $s\bar{s}$ content.

As mentioned we will keep $F_{\pi p}$ and m_k at their physical values and vary e to achieve a "best fit." By agreeing to use physical values for well-known parameters it would seem easier for various groups of workers to readily compare their results and determine when improvements have been really made. Now it is known[14,18] that using the experimental $F_{\pi p}$ gives a very large $M_{\rm cl}$ in the SU(2) Skyrme model. In the SU(3) Skyrme model the masses are additionally pushed up by the term $\gamma/2$ in (4.12). Thus we will end up with outlandishly high (but honest) masses and will take the attitude that only predictions for mass *differences* and dynamical properties should be taken seriously. Various subtraction schemes[19] have been proposed to overcome this defect of the model but we shall not consider them here.

Now that we have set up our model let us present the numerical results. The masses and mass differences of the low-lying baryons are given in Table I. The first column in that table gives the eigenvalues of the operator (5.1) for each degenerate isospin multiplet. The second column

TABLE I. The predicted low-lying baryon masses for the best fit, $e=4.0$ in our model and comparison of the mass splittings with experiment. $M_{\rm cl}=1744$ MeV, $\alpha^2=0.00674$ MeV^{-1}, $\beta^2=0.00523$ MeV^{-1}, and $\gamma=1374$ MeV.

	Eigenvalue of (5.1)	Baryon masses (MeV)	Mass differences (MeV) Model	Expt.
N	6.87	2313		
			157	177
Λ	8.51	2470		
			81	77
Σ	9.36	2551		
			133	125
Ξ	10.75	2684		
			−125	−86
Δ	10.12	2559		
			135	153
Σ^*	11.53	2694		
			139	148
Ξ^*	12.99	2833		
			141	139
Ω	14.40	2974		

presents the corresponding energies (5.2), while the mass differences between neighboring multiplets are listed in the third column. We have chosen the single adjustable parameter. e to be 4.0 to obtain a "best fit." These mass differences reproduce the experimental values inside each spin multiplet to within better than about 20 MeV. The difference between the spin-$\frac{1}{2}$ and -$\frac{3}{2}$ multiplets as a whole (measured by Δ-Ξ) is seen to be only about 40 MeV too small. In terms of the average multiplet masses we predict $\overline{m}_{3/2}-\overline{m}_{1/2}=181$ MeV (cf. expt=231 MeV).

The main consequence of introducing $w(r)$ is that α^2 and β^2 now are of the same order of magnitude. While Yabu and Ando[9] have shown for their model that the inequality $\alpha^2>2\beta^2$ is valid this is no longer the case when $w\neq0$. We illustrate in Table II for various values of e the results for β^2 with and without an excited $w(r)$. The values with $w\neq0$ are uniformly larger than in the case $w\equiv0$. As is clear from the discussion of the Hamiltonian in (4.12) this larger β^2 leads to a smaller energy. Thus the fact that a nontrivial w maximizes the corresponding moment of inertia provides the most compelling evidence to include w. Furthermore, we compare in Table III the mass splittings for both models, with and without w. The input parameters are the same as in Table I. It is obvious that the predicted mass differences within each spin multiplet are considerably improved due to the inclusion of w. The difference between the two multiplets, however, suffers to some extent in the case of nontrivial w as can be seen from the Δ-Ξ splitting.

A plot of Re$w(r)$, as determined from the differential equation (4.9) is shown in Fig. 1. Remember that Im$w(r)=0$ is the indicated solution of (4.9). We notice from Fig. 1 that $w(r)$ falls off more quickly than the Skyrme profile $F(r)$, which is also shown.

There are two different factors underlying the improvement of the intramultiplet mass splittings in this model. These splittings are seen from (5.2) to be given by $(1/\beta^2)\epsilon_{\rm SB}(\gamma\beta^2)$ for each particle. Their detailed pattern depends on the product $\gamma\beta^2$ and tends to improve as this product increases. Their overall magnitudes, however, are dominated by the dependence on γ alone. For this last dependence, we profit from the derivative-type symmetry-breaking terms in (2.6) needed to accommodate $F_{\pi p}\neq F_{kp}$. Taking this experimental feature into account increases the contribution of the usual (δ-type) symmetry-breaking terms to γ by about 50% since we have (see Ref. 7)

$$4(\delta''-\delta')=F_{kp}^2m_k^2-F_{\pi p}^2m_\pi^2\approx1.5F_{\pi p}^2(m_k^2-m_\pi^2)\ .$$

TABLE II. The effect of including $w(r)$ on the moment of inertia for "strange direction" rotations β^2 (in MeV^{-1}) for various values of the Skyrme constant e.

e	With w	Without w
3.0	0.006 86	0.005 48
3.5	0.005 65	0.003 73
4.0	0.005 23	0.002 65
4.5	0.005 31	0.001 96
5.0	0.005 74	0.001 49

TABLE III. Comparison to experiment of the predicted mass splittings with and without $w(r)$. The input parameters are the same as in Table I.

	Mass differences (MeV) With w	Without w	Expt.
Λ-N	157	155	177
Σ-Λ	81	111	77
Ξ-Σ	133	106	125
Δ-Ξ	-125	-102	-86
Σ^*-Δ	135	127	153
Ξ^*-Σ^*	139	126	148
Ω-Ξ^*	141	117	139

Furthermore, there is also a direct contribution from the β terms to γ of the order of 150 MeV. Thus we end up with $\gamma=1374$ MeV, which is almost twice as large as a Yabu-Ando-type fit with physical parameters. The larger value of the moment of inertia due to the new function w increases the deviation of the exact eigenvalue of (5.1) from first-order perturbation theory. This deviation gives rise to an improved pattern of the mass splittings. For example in the spin-$\frac{1}{2}$ multiplet we have

$$m(\Lambda)-m(N):m(\Sigma)-m(\Lambda):m(\Xi)-m(\Sigma)$$
$$=1:0.51:0.85\ ,$$

which is in significantly better agreement with the experimental numbers 1:0.44:0.71 than the first-order predictions 1:1:0.5. Hence our improvement of the baryon mass splittings is based on a considerable enlargement of the effective symmetry breaker $\beta^2\gamma$. By noting that $\beta^2\gamma$ very approximately scales as $m_k^2/F_\pi^2e^6$ we can understand why (for experimental $F_{\pi p}$) Yabu and Ando found it necessary to increase m_k by about 40% in order to obtain sufficiently large magnitudes for the mass splittings within an SU(3) multiplet. For comparison with Table I, we show in Table IV the Yabu-Ando[9] results (their Table II) when one uses the experimental value of m_k with $e=3.95$. We also show in Table IV a best fit to the mass

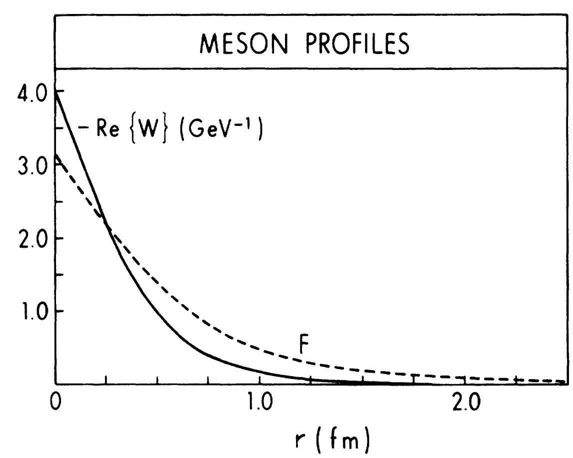

FIG. 1. Re$w(r)$ compared with the "hedgehog" profile $F(r)$. Note that $F(r)$ is dimensionless while Rew is measured in GeV^{-1}.

TABLE IV. Mass splittings in the Yabu-Ando model with physical values of $F_{\pi p}$ and m_k. $m_k=495$ MeV, $F_{\pi p}=132$ MeV.

	$e=3.95$	$e=3.1$
Λ-N	87	162
Σ-Λ	74	93
Ξ-Σ	51	130
Δ-Ξ	75	−227
Σ*-Δ	66	139
Ξ*-Σ*	63	140
Ω-Ξ*	59	138

splittings in the Yabu-Ando model based on keeping $F_{\pi p}$ and m_k physical and varying e. This yields $e=3.1$. They presumably rejected such a fit since they also were interested in lowering the *absolute* masses; from the fact that $M_{\rm cl}$ scales approximately as F_π/e it can be seen that choosing $e\approx 3$ would make the predictions for the absolute masses significantly worse. For example, one would then get a nucleon mass of 2907 MeV (compared to 2318 MeV gotten for the present model in Table I).

There is a more direct effect of nonzero $w(r)$ on the difference between the masses of the low-lying spin-$\frac{1}{2}$ baryons and the spin-$\frac{3}{2}$ baryons. This can be understood from the terms in (5.2):

$$\left[\frac{1}{2\alpha^2}-\frac{1}{2\beta^2}\right]J(J+1)\ . \tag{5.4}$$

One effect of nonzero $w(r)$ is to drive β^2 closer to α^2 and thus to decrease the magnitude of (5.4) (for our choice $e=4$). Hence we predict $\Delta-\Xi=-125$ MeV (expt$=-86$ MeV) which should be compared to the value -227 MeV shown for the best fit in Table IV for the Yabu-Ando model.

To partially summarize, we have seen that the effect of larger β^2 due to nontrivial $w(r)$ is rather beneficial but somewhat complicated for the predicted mass differences. Now we will discuss the fact that it has a clear and substantial effect on the structure of the baryons as well as on various important matrix elements. Reference to the perturbation theory expression for the proton state in (5.3) shows that increasing β^2 increases the admixture of representations other than the pure octet. Hence the effect of nonzero w is to more strongly excite higher representations. It has been noted[10] that representations higher than $\overline{\mathbf{10}}$ and **27** will not arise until fourth order of perturbation theory. Thus to a reasonable approximation the proton contains not only three constituent quarks but a nontrivial amplitude for an extra constituent quark-antiquark pair to be present, as one sees from the way in which the $\overline{\mathbf{10}}$ and **27** representations can be made up out of quarks. It is evidently interesting to consider the properties of the states that start out in perturbation theory as pure $\overline{\mathbf{10}}$ and pure **27**. In the zero symmetry-breaking limit the energy differences between the states $N(\overline{\mathbf{10}})$ and $N(\mathbf{27})$ carrying nucleon quantum numbers and the nucleon are simply given by the differences of the corresponding Casmir operators:

$$N(\overline{\mathbf{10}})-N(\mathbf{8})=\frac{3}{2\beta^2}=287\ \text{MeV}\ ,$$
$$N(\mathbf{27})-N(\mathbf{8})=\frac{5}{2\beta^2}=478\ \text{MeV}\ , \tag{5.5}$$

where we have used $\beta^2=0.005\,23\ \text{MeV}^{-1}$ from our best fit in Table I. Employing the exact diagonalization of operator (5.1) yields significantly larger mass differences:

$$N(\overline{\mathbf{10}})-N(\mathbf{8})=495\ \text{MeV}\ ,$$
$$N(\mathbf{27})-N(\mathbf{8})=608\ \text{MeV}\ . \tag{5.6}$$

The difference between (5.5) and (5.6) shows that the eigenvalues of corresponding members of different representations evolve differently with $\gamma\beta^2$. The relative position of the $N(\overline{\mathbf{10}})$ seems to suggest an identification with the Roper resonance (1440). An identification of the $N(\mathbf{27})$ with the experimentally observed $P11$ resonance at 1710 MeV appears reasonable but less compelling. Having such an interpretation of P11 states would be interesting since at least the Roper is completely absent[20] in the SU(2) Skyrme model approach to pion-nucleon scattering [in pion photoproduction,[20] however, the SU(2) Skyrme model shows some evidence for the presence of the Roper resonance].

It is furthermore clear from (5.3) that matrix elements of various operators sandwiched between the proton states will depend on the admixture of higher representations and hence on $\gamma\beta^2$. It has already been noted[5] in the present model that the effect is to lower the strange-quark axial-vector-current matrix element. The SU(3)-singlet axial-vector-current matrix element needed to understand the EMC experiment[6] is identically zero in this model. This was shown for the case of a nonderivative symmetry breaker in Ref. 4 and both by explicit calculation and general argument for the present case where a derivative-type symmetry breaker also exists in Ref. 5. In the present model the axial-vector renormalization constant in neutron decay g_A comes out to be about 1 where the contributions of the β' and β'' terms in (2.6) have been, for simplicity, neglected. One can understand why this number is somewhat larger than the typical SU(2) Skyrme model value of about 0.6 by noting that g_A roughly scales as $1/e^2$ and e is here reduced. In Ref. 10 it is furthermore explained how the use of a large value of $\gamma\beta^2$ drives the SU(3) prediction for g_A to the SU(2) prediction.

It is also interesting to consider the matrix elements of the scalar densities, which are related to the so-called "flavor content" fractions.[2,3,10] The content fractions for the nucleon are defined as

$$X_a=\frac{\langle p|\bar{q}_a q_a|p\rangle-\langle 0|\bar{q}_a q_a|0\rangle}{\langle p|\bar{u}u+\bar{d}d+\bar{s}s|p\rangle-\langle 0|\bar{u}u+\bar{d}d+\bar{s}s|\rangle}\ . \tag{5.7}$$

For $a=s$ (5.7) yields with a σ model interpretation of the quark bilinears

$$X_s=\tfrac{1}{3}\langle p|1-D_{88}(A)|p\rangle\ , \tag{5.8}$$

which predicts in the zero symmetry-breaking limit $X_s=\frac{7}{30}\approx 0.233$. This value is remarkably reduced to

$X_s=0.142$ when the exact diagonalization of (5.1) with the "best fit" parameters is performed. There is an even greater reduction for the isovector density which now becomes

$$X_u-X_d=0.006\ . \quad (5.9)$$

This should be compared to the value 0.033 when symmetry-breaking corrections are neglected[1] and to the value 0.0133 for a typical Yabu-Ando fit. This has the further consequence that the predicted nonelectromagnetic part of the neutron-proton mass difference in the SU(3) Skyrme model is only about 0.25 MeV (compared to the 2 MeV or so which is required). This strengthens an earlier conclusion[7] that the Skyrme model of pseudoscalars must be extended to include new (short-distance) degrees of freedom in order to explain m_n-m_p.

It is also of interest to define, analogously to (5.7), the strange content fraction X_s for baryons other than the nucleon. This requires us to compute the expectation value of the operator $\frac{1}{3}[1-D_{88}(A)]$ for the baryon under consideration. We have evaluated these expectation values and present the results in Table V for the two choices of effective symmetry breaker $\omega^2\equiv\frac{3}{2}\gamma\beta^2=0$ and $\omega^2=10.78$. The latter corresponds to our "best fit" parameter $e=4$ and the former to the unjustified use of SU(3)-symmetric wave functions. Table V reveals two interesting features. The first is that using a realistic value for ω^2 reduces the strange content fraction of all low-lying baryons considerably. This is expected since increasing the kaon mass decreases the probability for exciting strange degrees of freedom. The second point may be illustrated by considering the Ω baryon; its strange content is reduced from $\frac{5}{12}\approx 0.417$ to 0.360 if realistic symmetry breaking is adopted. For both cases this is surprisingly small since the quark model tells us that the Ω is built out of three strange quarks. One would have naively expected a value not much less than 1. The SU(3) Skyrme model evidently predicts a large amount of $u\bar{u}$ and $d\bar{d}$ pairs in addition to the three strange valence quarks in the Ω baryon.

TABLE V. "Strange content" of the low-lying baryons.

	$\omega^2=0$	$\omega^2=10.78$
N	0.233	0.141
Λ	0.300	0.213
Σ	0.367	0.227
Ξ	0.400	0.308
Δ	0.292	0.137
Σ^*	0.333	0.203
Ξ^*	0.375	0.275
Ω	0.417	0.360

The "content-fractions" discussed above measure various diagonal matrix elements of the operator $U+U^\dagger-2$. These are structurally similar to the so-called σ terms deduced from applying current-algebra approximations to πN and KN scattering amplitudes. For completeness we will give the relevant σ terms here, taking into account also the contributions from the derivative-type symmetry-breaking terms in (2.5). The π and $K\sigma$ terms at zero momentum transfer are conventionally defined[21] by the expectation values of the equal-time double commutators

$$\sigma_\pi(0)=\frac{1}{3}\sum_{a=1}^{3}\langle P(p)|[Q_a^5,[Q_a^5,H]]|P(p)\rangle\ , \quad (5.10a)$$

$$\sigma_K(0)=\frac{1}{2}\sum_{a=4}^{5}\langle P(p)|[Q_a^5,[Q_a^5,H]]|P(p)\rangle\ , \quad (5.10b)$$

where the axial generators are normalized as $[Q_a^5,Q_b^5]=if_{abc}Q_c$, $[Q_a,Q_b]=if_{abc}Q_c$ and $P(p)$ is a proton state of four-momentum p. Intepreting the operators in (5.10) as the quadratic terms in the axial variation of the Hamiltonian H we obtain

$$[Q_a^5,[Q_a^5,H]]=-\frac{1}{4}\mathrm{Tr}[(\partial_\mu U\partial_\mu U^\dagger U+U^\dagger\partial_\mu U\partial_\mu U^\dagger)(\lambda_a^2B+2\lambda_aB\lambda_a+B\lambda_a^2)+(U+U^\dagger-2)(\lambda_a^2D+2\lambda_aD\lambda_a+D\lambda_a^2)]\ , \quad (5.11)$$

where the λ_a are the Gell-Mann matrices, $B=\beta' T+\beta'' S$ and $D=\delta' T+\delta'' S$ (see Sec. II). Setting $U=AU_0A^\dagger$ we get

$$\sigma_\pi(0)=\frac{4}{3}\langle(2+D_{88})\rangle_P\int d^3r\left[-\beta'\cos F\left[F'^2+\frac{2}{r^2}\sin^2F\right]+\delta'(1-\cos F)\right]\ , \quad (5.12a)$$

$$\sigma_K(0)=\frac{1}{3}\langle(4+\sqrt{3}D_{38}-D_{88})\rangle_P\int d^3r\left[-(\beta'+\beta'')\cos F\left[F'^2+\frac{2}{r^2}\sin^2F\right]+(\delta'+\delta'')(1-\cos F)\right]\ . \quad (5.12b)$$

With our best-fit parameter we finally obtain

$$\sigma_\pi(0)=49\ \mathrm{MeV},\quad \sigma_K(0)=624\ \mathrm{MeV}\ . \quad (5.13)$$

The value of $\sigma_\pi(0)$ is in agreement with the "experimental" value 56±8 MeV. The nonderivative symmetry-breaking term contributions to (5.13) are the dominant ones: 43 MeV and 552 MeV for $\sigma_\pi(0)$ and $\sigma_K(0)$, respectively.

VI. SUMMARY AND DISCUSSION

We have stressed that, due to the relatively large K-meson mass, the SU(3) Skyrme model is not a trivial generalization of the SU(2) case. We followed the path of quantizing the collective modes in an SU(3) [rather than in an SU(2)] framework and treating the symmetry breaking exactly (as pioneered by Yabu and Ando).[9] Our approach differs from this by taking into account the contribution of an induced kaon field to the moment of inertia for "strange" flavor rotations. Using physical values for known meson parameters and a best-fit value $e=4.0$ for the Skyrme constant, we found the ordinary and strange flavor rotation moments of inertia, α^2 and β^2, respectively, to be

$$\alpha^2=6.74\ \mathrm{GeV}^{-1}\ ,\qquad \beta^2=5.23\ \mathrm{GeV}^{-1}\ . \tag{6.1}$$

The ratio β^2/α^2 is much larger than in the Yabu-Ando case and this results in an improved description of the mass splittings. It also increases the effective symmetry breaking that yields a decreased strange content for physical baryons. This can be intuitively related to the difficulty of accommodating "heavy" $\bar{s}s$ pairs in the proton and is physically reasonable.

An interesting aspect of the SU(3) Skyrme model with symmetry breaking taken into account is the fact that the low-lying spin-$\frac{1}{2}$ baryons do not belong to a pure octet but contain sizable $\overline{\mathbf{10}}$ and **27** components. The nucleon-like members of the $\overline{\mathbf{10}}$ and **27** were found to have masses that support tentative identification with the P_{11} Roper $N(1440)$ and $N(1710)$ resonances. These states thus have a $qqqq\bar{q}$ (rather than qqq) structure at the *constituent*-quark level and the proton inherits some of this too. Further investigation of this aspect of the model would seem to be very worthwhile.

Our calculations of the "strange content" of those baryons that contain strange valence quarks strongly suggests that they contain many $\bar{u}u$ and $\bar{d}d$ "current"-type quark pairs. For example, the "strange content" of the Ω^- is found to be only about 36%.

While the developing physical implications of the SU(3) Skyrme model are very fascinating it should be borne in mind that the main thrust of the paper here has been related to verifying that the technical refinement introduced of "cranking" the K-meson excitation leads to an improved and more sensible picture. There are clearly many further technical refinements required to perfect the soliton picture of baryons in the SU(3) framework. We conclude with a brief mention of some relevant points for this purpose.

(i) Even though the relatively good agreement with a large and intricate pattern of mass splittings, assuming physical values of meson parameters, gives us confidence that we are on the right track, the need for an overall energy subtraction of about 1400 MeV to obtain agreement with the *absolute* nucleon mass is a disturbing feature. Arguments presented[19] in the literature include motivating an arbitrary subtraction by appealing to operator-ordering ambiguities in going from the classical to the quantum theory and subtracting[9] the eigenvalue of (5.1) associated with the state which develops out of the "bare" SU(3) singlet. One problem with the latter approach is that there is no obvious reason to make this special subtraction. This singlet state is actually ruled out of the allowed spectrum of (5.1) because its baryon number is zero. The justification given[9] for this subtraction is that it reproduces the SU(2) spectrum in the limit of large symmetry breaking. In any event, it should be recognized that the SU(2) Skyrme model suffers from a similar problem[18] (yielding nucleon masses ~ 1.5 GeV) if experimental values of the mesonic parameters are used. The introduction of vector mesons with "realistic" parameters[17,22] at the SU(2) level does not seem to help much in this regard. Perhaps this problem is nature's way of telling us that we cannot completely forget additional "short-range" effects in the "core" of the nucleon. Alternatively, it may indicate the need for a still more sophisticated collective quantization scheme.

(ii) In some sense the present model is intermediate between the "rotation"[1,9] and "bound-state"[8] approaches. The general formalism is that of the former but like the latter, the K meson is singled out for special treatment. In fact, our ansatz for the kaon isospinor in (4.4) $w(r)\tau\cdot\hat{r}\Omega_k$ is comparable to the lowest-lying bound-state kaon wave function[8] in the Skyrmion background. Considering the entire 8 and 10 multiplet particles, our mass splittings come out appreciably better than those in the bound-state approach.[23]

(iii) It seems worthwhile to comment on the connection between the $w(r)$ excitation just mentioned and the "zero mode" for rotations into the strange direction. As discussed in Sec. IV there is only a true zero mode if $\beta'=\beta''$ and $\delta'=\delta''$ (which implies $m_\pi=m_k$ and $F_{\pi p}=F_{kp}$). Note that (4.7) holds exactly rather than as an approximation in this limit. This zero mode can be obtained by an infinitesimal rotation of the classical solution $U_0^{1/2}$ [see (3.11)] with the matrix

$$A(t)\simeq 1+\frac{it}{2}\begin{bmatrix} 0 & \Omega_k \\ \Omega_k^\dagger & 0 \end{bmatrix}+\cdots\ . \tag{6.2}$$

The contribution from this to the kaon isospinor is

$$\Psi_0=\frac{it}{2}\left[\left[1-\cos\frac{F}{2}\right]-i\hat{r}\cdot\tau\sin\frac{F}{2}\right]\Omega_k\ . \tag{6.3}$$

On the other hand, introducing the excitation w as in (4.3) and (4.4) gives the comparable piece from $\xi_k U_0^{1/2}\xi_k$:

$$\Psi=i\left[\left[1+\cos\frac{F}{2}\right]\hat{r}\cdot\tau+i\sin\frac{F}{2}\right]w(r)\Omega_k\ . \tag{6.4}$$

Although the structure of (4.1) suggests that this mode is of axial type it actually differs from an axial mode by a factor of $\hat{r}\cdot\tau$, which converts it into a vector-type mode. Thus the overlap

$$\langle\Psi_0|\Psi\rangle\propto\int r^2dr\,w(r)\sin\frac{F}{2} \tag{6.5}$$

is nonvanishing so Ψ would contain a piece of the zero mode Ψ_0. For the physical values of m_k and m_π this overlap is very small; e.g., going from $m_k=200$ MeV to $m_k=495$ MeV reduces the overlap by a factor of 10 (assuming here $\beta'=\beta''=0$ for simplicity). One might think that Ψ would coincide with the zero mode Ψ_0 if the SU(3)-symmetry breaking were to vanish. A naive calculation results in β^2 diverging as this limit is approached in either direction ($m_k > m_\pi$ or $m_k < m_\pi$). This may be understood because then the equation $\delta\beta^2/\delta w = 0$ has a solution (homogeneous case) without the contribution from the Wess-Zumino term. This is the zero mode and corresponds to

$$w_0 = \frac{1-\cos(F/2)}{\sin(F/2)} .$$

The correct approach in this special case is to project out the zero-mode part with a Lagrange multiplier, replacing

$$\beta^2 \to \beta^2 + \lambda \langle \Psi_0 | \Psi \rangle , \tag{6.6}$$

and fixing λ by requiring $\langle \Psi_0 | \Psi \rangle = 0$. Then it is found that $w(r)$ does not vanish and actually has a significant effect on β^2, increasing it by about a factor of 2 (using $m_\pi = m_k = 138$ MeV, $\beta' = \beta'' = 0$, $e = 4.0$). It would thus seem that the present technique may be useful for other applications where one considers a classical topological solution belonging to a subgroup H of a group G.

(iv) At the two-flavor level it is known that a better description of the nucleon may be achieved by introducing vector mesons.[20,24] Then one also does not require the Skyrme term; the stabilization is achieved by terms proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$ which are anyway required to describe vector-meson decays. One may use, for example, the Lagrangian discussed in Ref. 22. The vector-meson *nonet* field ρ is now given by an *Ansatz* similar to (4.1)–(4.4):

$$\rho_\mu = A \begin{pmatrix} \rho_\mu \cdot \tau + \omega_\mu & K_\mu^* \\ K_\mu^{*\dagger} & \varphi_\mu \end{pmatrix} A^\dagger , \tag{6.7}$$

where in addition to the ρ_μ and ω_μ fields which were given elsewhere,[17] the strange vectors are

$$K_0^* = S(r)\Omega_k ,$$
$$K_i^* = [E(r)\hat{r}_i + \epsilon_{ijk}\hat{r}_j \tau_k D(r)]\Omega_k . \tag{6.8}$$

$S(r)$, $D(r)$, and $E(r)$ are *a priori* complex functions but it turns out (on explicit calculation) that only ReS, ReD, and ImE get excited by extremizing the moment of inertia β^2. This is a lengthy program compared to the present calculation. In general, one might perhaps expect somewhat less effective symmetry breaking than in the present case because the vectors are more nearly degenerate than the pseudoscalars. Masak[25] has recently carried out part of this program. However, he neglected $D(r)$, $E(r)$, and $w(r)$ and also restricted himself to first-order perturbation theory. It thus seems likely that the results of the full program would be rather different from his results.

(v) One might wonder about the possibility of also exciting a pionlike mode (different from the pion zero mode which must be present) in our formalism. A suitable *Ansatz* would be to replace the zero in the upper left 2×2 subblock of (4.4) by $P(r)\boldsymbol{\tau}\cdot(\mathbf{r}\times\boldsymbol{\Omega})$. Unlike the kaon function $w(r)$, $P(r)$ will not get excited in this model. This can be simply understood by noting that the source for $w(r)$ is provided by the Wess-Zumino term. However, the Wess-Zumino term does not provide a source for $P(r)$ since it vanishes in SU(2).

(vi) The $N_c \to \infty$ limit of QCD has provided many helpful suggestions about the interpretation and treatment of the Skyrme model, although for numerical purposes the closeness of 3 to ∞ is hard to establish. One might wonder about the N_c dependence associated with SU(3)-symmetry breaking. In the SU(3) Skyrme model the relevant parameter [see (5.1)] for discussing symmetry breaking is the combination $\gamma\beta^2$, which is immediately seen to scale like N_c^2. If one sets up the conventional perturbation theory as in Ref. 10, the energy splittings are given as power series in $\gamma\beta^2$. Naively, this might suggest that the splittings depend on N_c in a very drastic way. However, it must be remembered that the SU(3) representations of the baryons must contain a state with right hypercharge $Y_R = N_c/3$ so that the representations themselves cannot be held fixed[26] if one makes an N_c expansion. This means that the coefficients multiplying $(\gamma\beta^2)^n$ will also depend on N_c so no conclusions can be immediately drawn. In the Yabu-Ando approach no SU(3) representation is assumed and they obtain a mass formula [(4.5) in Ref. 9] for large $\gamma\beta^2 \equiv \frac{2}{3}\omega^2$ and arbitrary Y_R, which may therefore be regarded as a large-N_c formula. (Note that their eigenvalue $\epsilon_{\rm SB}$ has to be divided by β^2 to obtain the expression for the energy eigenvalue.) Then it is seen by inspection that the mass splittings within a given spin multiplet go as N_c^0 for large N_c as one knows from QCD.

ACKNOWLEDGMENTS

We would like to thank Norbert Kaiser for his collaboration in the early stages of this work. We would also like to thank Hans Walliser for helpful correspondence on the treatment of "zero modes." This work was supported in part by the U.S. Department of Energy under Contracts Nos. DE-FG02-85ER40231 and DE-AC02-76ER03069 and in part by the Deutsche Forschungsgemeinschaft. U.-G.M. acknowledges a Heisenberg grant.

APPENDIX A: SU(3) MOMENTS OF INERTIA

In this appendix we will briefly present the expressions derived for the moments of inertia α^2 and β^2 as defined in (4.5):

$$\alpha^2 = \frac{8\pi}{3}\int dr\, r^2 \left\{ \sin^2 F \left[\tfrac{1}{2}F_\pi^2 + \frac{1}{e^2}\left[F'^2 + \frac{\sin^2 F}{r^2} \right] \right] - 8y_1 \cos F \sin^2 F \right\} . \tag{A1}$$

The expression for β^2 is a bit lengthy; it is therefore convenient to divide it into two parts:

$$\beta^2=\beta^2_{\rm NSB}+\beta^2_{\rm SB}\ . \tag{A2}$$

In (A2) $\beta^2_{\rm NSB}$ represents the moment of inertia originating from the chiral-invariant part of the model Lagrangian while $\beta^2_{\rm SB}$ stems from (4.7). Furthermore, we will split the radial function w, as defined in (4.4), into real and imaginary parts:

$$w=w_1+iw_2\ , \tag{A3}$$

$$\begin{aligned}\beta^2_{\rm NSB}=\pi\int dr\Bigg\{&F_\pi^2r^2(1-\cos F)+\frac{1}{2e^2}(1-\cos F)(F'^2r^2+2\sin^2F)-2\left[F_\pi^2r^2+\frac{1}{e^2}\sin^2F\right]\left[1+\cos\frac{F}{2}\right]^2(w_1'^2+w_2'^2)\\&-2F'\left[F_\pi^2r^2\sin\frac{F}{2}+\frac{4}{e^2}\sin^2F\sin\frac{F}{2}+\frac{6}{e^2}\sin F\cos\frac{F}{2}\left[1+\cos\frac{F}{2}\right]^2\right](w_1w_1'+w_2w_2')\\&+\Bigg[F'^2\left[\tfrac{1}{2}F_\pi^2r^2+\frac{8}{e^2}\sin^2F+\frac{8}{e^2}\sin F\sin\frac{F}{2}\right]+\frac{2}{r^2}\sin F\left[\tfrac{1}{2}F_\pi^2r^2+\frac{\sin^2F}{e^2}\right]\left[3\sin F+4\sin\frac{F}{2}\right]\\&-\frac{1}{r^2}\left[F_\pi^2r^2+\frac{1}{2e^2}F'^2r^2+\frac{2}{e^2}\sin^2F\right]\left[1+2\cos\frac{F}{2}+\cos^2F\right]^2\Bigg](w_1^2+w_2^2)\\&+\frac{1}{\pi^2}\left[3F'\sin\frac{F}{2}\sin^2F+4F'\cos^2\frac{F}{2}\sin F\left[1+\cos\frac{F}{2}\right]\right]w_1+\frac{2}{\pi^2}\cos\frac{F}{2}\sin^2F\left[1+\cos\frac{F}{2}\right]w_1'\Bigg\}\ .\end{aligned} \tag{A4}$$

A prime indicates a derivative with respect to r. Note that the two last terms in (A4) are linear in w_1; thus there are inhomogeneous terms in the differential equation derived from (4.9). Hence this results in a nontrivial solution for the real part of w. The first two terms in (A4) comprise the result of Ref. 9. Finally, for $\beta^2_{\rm SB}$ we find

$$\begin{aligned}\beta^2_{\rm SB}=8\pi\int dr\Bigg[&-y_1r^2\sin^2F-2y_2r^2\left[1+2\cos\frac{F}{2}+\cos F\right](w_1^2+w_2^2)+2y_1r^2(1+\cos F)\left[1+\cos\frac{F}{2}\right]^2(w_1'^2+w_2'^2)\\&+y_1\Bigg\{2F'^2r^2\sin\frac{F}{2}\left[\sin F-\sin\frac{F}{2}\left[1+\cos\frac{F}{2}\right]\right]\\&-2\left[F''r^2\sin\frac{F}{2}+2rF'\sin\frac{F}{2}+\tfrac{1}{2}F'^2r^2\cos\frac{F}{2}\right]\left[\cos F-\left[1+\cos\frac{F}{2}\right]^2\right]\\&-\cos F\left[F'^2r^2+2\sin F\left[3\sin F+4\sin\frac{F}{2}\right]\right]-\tfrac{1}{4}(F'^2r^2+2\sin^2F)\left[4\cos\frac{F}{2}+3\cos F+1\right]\\&+(1+\cos F)\left[1+2\cos\frac{F}{2}+\cos F\right]^2-4\sin F\sin\frac{F}{2}\left[1+\cos\frac{F}{2}\right]\left[1+2\cos\frac{F}{2}+\cos F\right]\Bigg\}(w_1^2+w_2^2)\Bigg]\ .\end{aligned} \tag{A5}$$

In (A5) we have performed an integration by parts to avoid terms such as w_1w_1'.

Insertion of the equation of motion (4.9) for w_1 provides a much simpler expression for β^2 (see, e.g., Refs. 15 and 17):

$$\begin{aligned}\beta^2=\pi\int dr\Bigg\{&F_\pi^2r^2(1-\cos F)+\frac{1}{2e^2}(1-\cos F)(F'^2r^2+2\sin^2F)-8y_1r^2\sin^2F\\&+\frac{1}{2\pi^2}\left[3F'\sin\frac{F}{2}\sin^2F+4F'\cos^2\frac{F}{2}\sin F\left[1+\cos\frac{F}{2}\right]\right]w_1+\frac{1}{\pi^2}\cos\frac{F}{2}\sin^2F\left[1+\cos\frac{F}{2}\right]w_1'\Bigg\}\ .\end{aligned} \tag{A6}$$

APPENDIX B: NORMALIZATION OF THE VECTOR CHARGE

A microscopic expression for the vector current is obtained by gauging the theory with an external vector field (e.g., Ref. 17), i.e., replacing the ordinary derivative by a covariant one:

$$D_\mu U=\partial_\mu U-iA_\mu^LU+iUA_\mu^R \tag{B1}$$

with $A_\mu^L=A_\mu^R=a_\mu^b\lambda_b$. This procedure of gauging only holds for the nonanomalous parts of the Lagrangian. The result for the WZ term has been derived in Eq. (4.18) of Ref. 27. In the next step the current is given as the coefficient of the term linear in a_μ^b:

$$\mathcal{J}^{b\mu}=\frac{\delta\mathcal{L}}{\delta a_\mu^b}\bigg|_{a_\mu^b=0}\ . \tag{B2}$$

Substituting in the *Ansätze* for U_0 and ξ_k results in the following expression for the time component of the vector current:

$$\mathcal{J}^{a0} = -\frac{\sqrt{3}}{4\pi^2} F' \frac{\sin^2 F}{r^2} D_{a8} - \tfrac{2}{3}\sin^2 F \left[\tfrac{1}{2} F_\pi^2 + \frac{1}{e^2}\left[F'^2 + \frac{\sin^2 F}{r^2} \right] - 8 y_1 \cos F \right] \sum_{b=1}^{3} D_{ab}\Omega_b$$

$$- \frac{1}{2}\left[(1-\cos F)\left[\tfrac{1}{2} F_\pi^2 + \frac{1}{4e^2}\left[F'^2 + 2\frac{\sin^2 F}{r^2} \right] - 4 y_1 \sin^2 F \right] \right.$$

$$+ \frac{1}{4\pi^2 r^2}\left\{ \left[3F' \sin\frac{F}{2}\sin^2 F + 2F'\cos\frac{F}{2}\sin F \left[1 + 2\cos\frac{F}{2} + \cos F \right] \right] w_1 \right.$$

$$\left.\left. + 2\cos\frac{F}{2}\sin^2 F \left[1 + \cos\frac{F}{2} \right] w_1' \right\} \right] \sum_{b=4}^{7} D_{ab}\Omega_b \ , \tag{B3}$$

where we again made the approximation (4.7) for the symmetry-breaking terms.

The vector charge is defined as the spatial integral of (B3); together with (A1) and (A6) this leads to

$$Q^a = \int d^3 r\, \mathcal{L}^{a0} = -\frac{\sqrt{3}}{2} D_{a8} - \alpha^2 \sum_{b=1}^{3} D_{ab}\Omega_b - \beta^2 \sum_{b=4}^{7} D_{ab}\Omega_b \ . \tag{B4}$$

Considering Eq. (4.5) this may be expressed as

$$Q^a = -\sum_{b=1}^{8} D_{ab} \frac{\partial L}{\partial \Omega_b} \ , \tag{B5}$$

which is the vector charge on the collective, macroscopic level. Hence the identity of the microscopic and macroscopic expressions for Q^a provides a consistency check for our approach to strangeness in the Skyrme model.

APPENDIX C: DISCUSSION OF THE APPROXIMATION (4.7)

If we do not make the $y_1 - y_2$ approximation (4.7), the collective Lagrangian would become

$$L = -M_H + \tfrac{1}{2}\alpha^2 \sum_{a=1}^{3} \Omega_a^2 + \tfrac{1}{2}\beta^2 \sum_{a=4}^{7} \Omega_a^2 + \frac{\sqrt{3}}{2}\Omega_8 + \tfrac{1}{2}\gamma(1 - D_{88})$$

$$+ \beta_1 \sum_{a=4}^{7} D_{8a}\Omega_a + \tfrac{1}{2}\beta_2 (1 - D_{88}) \sum_{a=4}^{7} \Omega_a^2 + \tfrac{1}{2}\beta_3 \sum_{k=1}^{3} \sum_{a,b=4}^{7} D_{8k} d_{kab} \Omega_a \Omega_b \ , \tag{C1}$$

where d_{abc} are the structure constants for the SU(3) anticommutators.

$\beta_{\rm NSB}^2$ in (A4) is unaffected by this more precise evaluation. $\beta_{\rm SB}^2$ in (A5), however, is replaced by

$$\beta_{\rm SB}^2 = 8\pi \int dr\, r^2 \left[\beta' \left\{ \cos F \left[\cos F - 1 + 2\left[1 + \cos\frac{F}{2} \right]^2 \left[w'^2 + \frac{2}{r^2}\cos^2\frac{F}{2} w^2 \right] \right.\right.\right.$$

$$\left. + 4F'w'w \sin\frac{F}{2} - F'^2 w^2 - \frac{2}{r^2} w^2 \sin F \left[3\sin F + 4\sin\frac{F}{2} \right] \right]$$

$$+ w^2 \left[F'^2 + \frac{2}{r^2}\sin^2 F \right] \left[1 + \cos\frac{F}{2} \right] \left[1 - 2\cos\frac{F}{2} \right]$$

$$\left. - 2w \left[1 + \cos\frac{F}{2} \right] \left[\sin F + \sin\frac{F}{2} \right] \left[F'w' + \frac{2}{r^2} w \sin F \cos\frac{F}{2} \right] \right\}$$

$$+ \beta'' \left[1 + \cos\frac{F}{2} \right] \left[2\left[\cos\frac{F}{2} - 1 \right] + 2\left[1 + \cos\frac{F}{2} \right] \left[w'^2 + \frac{2}{r^2}\cos^2\frac{F}{2} w^2 \right] \right.$$

$$\left. - \cos\frac{F}{2} w^2 \left[F'^2 + \frac{2\sin^2 F}{r^2} \right] - 2w \sin\frac{F}{2} \left[F'w' + \frac{2}{r^2} w \sin F \cos\frac{F}{2} \right] \right]$$

$$\left. - \tfrac{1}{2} w^2 \left[\delta' \left[3\cos F + 4\cos\frac{F}{2} + 1 \right] + \delta'' \left[\cos F + 4\cos\frac{F}{2} + 3 \right] \right] \right] \ . \tag{C2}$$

This, of course, reduces to (A5) if $\beta'=\beta''=y_1$ and $\delta'=\delta''=y_2$.

The additional coefficients β_1, β_2, and β_3 are given by the integrals

$$\beta_1=\frac{16\pi}{\sqrt{3}}(\delta''-\delta')\int dr\, r^2 w \sin\frac{F}{2}\left[1+\cos\frac{F}{2}\right],$$
$$\beta_2=\frac{4\pi}{3}(\delta''-\delta')\int dr\, r^2 w^2\left[5-\cos F+4\cos\frac{F}{2}\right], \quad \text{(C3)}$$
$$\beta_3=\frac{8\pi}{3\sqrt{3}}(\delta''-\delta')\int dr\, r^2 w^2\left[5+\cos F+4\cos\frac{F}{2}\right]$$

where we have neglected the contribution from the β-type symmetry breaker since the δ type has been proven to be dominant in the baryon sector. The w equation of motion is obtained from (C1) as

$$\frac{\delta}{\delta w}\left[\tfrac{1}{2}\beta^2\sum_{a=4}^{7}\Omega_a^2+\beta_1\sum_{a=4}^{7}D_{8a}(A)\Omega_a+\cdots\right]=0\,, \quad \text{(C4)}$$

where the ellipsis indicates terms proportional to β_2 and β_3 which we will later argue are negligible. Keeping only the first term of (C4) gives still (4.9) but with the β^2 involving (C2) instead of (A5). It is straightforward to find the change in our previous results due to the use of the new expression for β^2. The large distance behavior of $w(r)$ is then determined by solving the differential equation

$$w''=\frac{-2}{r}w'+\frac{2}{r^2}w+\frac{4(\delta'+\delta'')}{F_{\pi p}^2+8(\beta'-\beta'')}w\,. \quad \text{(C5)}$$

The r-independent coefficient of w can be seen[7] to be equal to m_k^2; thus the profile function $w(r)$ decays exponentially with m_k as we required in the y_1-y_2 approximation (4.10). Choosing the Skyrme constant $e=4.0$ we find $\beta^2=4.57\ \text{GeV}^{-1}$, which is only slightly different from the value obtained in the y_1-y_2 model (5.23 GeV^{-1}). Of course, we are free to choose a value for e such that the original value for β^2 is restored. Still this value is twice as large as in the model without $w(r)$, which supports our procedure of cranking the kaon fields.

Having found the profile function $w(r)$ we may evaluate the additional integrals β_1, β_2, and β_3:

$$\beta_1=-0.735\,,$$
$$\beta_2=0.115\beta^2\,, \quad \text{(C6)}$$
$$\beta_3=-0.134\beta^2\,.$$

β_2 and β_3 can be seen to be small compared to β^2 and thus may be neglected. However, β_1 is not completely negligible. Including the β_1 term the Hamiltonian is found to be

$$H=\frac{1}{2\beta^2}C_2+\frac{1}{2}\left[\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right]J(J+1)-\frac{3}{8\beta^2}$$
$$-\frac{\beta_1}{2\beta^2}\sum_{a=4}^{7}D_{8a}(R_a-\beta_1 D_{8a})+\tfrac{1}{2}\gamma(1-D_{88})\,, \quad \text{(C7)}$$

where the R_a denote the "right" SU(3) generators.[9] We therefore have to diagonalize instead of (5.1) the operator:

$$C_2+\gamma\beta^2(1-D_{88})-\beta_1\sum_{a=4}^{7}D_{8a}(R_a-\beta_1 D_{8a})\,. \quad \text{(C8)}$$

The results of this calculation are presented in Table VI and found not to deviate significantly from our earlier results in Table V. Furthermore, a fine-tuning of the Skyrme constant e could restore the original results.

Now the nature of our approximation can be more clearly understood. It corresponds to neglecting β_1 in (C4) for the purpose of determining the profile $w(r)$ but not in the Hamiltonian (C7) for determining the energy eigenvalues. A possible motivation for this treatment may be obtained by reviewing the derivation of the equation of motion for the chiral angle $F(r)$. In the limit $\Omega_a=0$ the Lagrangian reads

$$L(\Omega=0)=-M_H+\frac{\gamma}{2}[1-D_{88}(A)]\,.$$

The chiral angle is gotten by extremizing the hedgehog mass M_H, or equivalently L evaluated with $A=1$ since in this limit $D_{ab}=\delta_{ab}$. If we now substitute $A=1$ but allow $\Omega_a\neq 0$ in the Lagrangian, the profile $w(r)$ would be again determined by

$$\frac{\delta\beta^2[w(r)]}{\delta w(r)}=0\,. \quad \text{(C9)}$$

Suppose one wanted to include the β_1 term in (C4). Then it is seen that (C4) is not just a differential equation for w as a function of r but also depends on the collective coordinates A. This would be rather difficult to handle and would imply a somewhat modified *Ansatz* for z in (4.4). For the formal purpose of verifying the conservation of the isospin and hypercharge generators

$$Q_a=\int d^3r\, J_0^a \quad (a=1,2,3,8)\,, \quad \text{(C10)}$$

it is necessary to use the full (C4). This can be understood by remembering that Noether's construction for the conserved charges requires the exact equation of motion. Specifically, if the β_1 term is retained the new relation between angular velocities and "right" SU(3) generators together with the approximate equation (4.9) leads to

$$Q_a=L_a-\beta_1\sum_{b=4}^{7}D_{ab}D_{8b}\,, \quad \text{(C11)}$$

TABLE VI. Mass splittings derived from diagonalizing (C7).

Mass differences ($e=4.0$)	
N-Λ	182 MeV
Λ-Σ	106
Σ-Ξ	138
Ξ-Δ	-156
Δ-Σ^*	165
Σ^*-Ξ^*	139
Ξ^*-Ω	159

where L_a denote the "left," flavor-SU(3) generators. Here a procedure analogous to going from (B3) to (B4) was used. The charges (C11) do not formally commute with the collective Hamiltonian. However, if one uses the full (C4) one finds again $Q_a = L_a$, which does commute with the Hamiltonian. We should stress that the use of the $y_1 - y_2$ approximation circumvents this problem.

[1]E. Guadagnini, Nucl. Phys. **B236**, 15 (1984); L. Biedenharn, Y. Dothan, and A. Stern, Phys. Lett. **146B**, 289 (1984); P. Mazur, M. Novak, and M. Praszalowicz, *ibid.* **147B**, 137 (1984); J. Bijnens, H. Sonoda, and M. Wise, *ibid.* **140B**, 421 (1984); A. Manohar, Nucl. Phys. **B278**, 19 (1984); M. Chemtob, *ibid.* **B256**, 600 (1985); A. P. Balachandran, A. Barducci, F. Lizzi, V. Rodgers, and A. Stern, *ibid.* **B256**, 525 (1985); M. Praszalowicz, Phys. Lett. **158B**, 214 (1985); S. Jain and S. Wadia, Nucl. Phys. **B258**, 713 (1985); G. Adkins and C. Nappi, *ibid.* **B249**, 507 (1985); M. Sriram, H. Mani, and R. Ramachandran, Phys. Rev. D **30**, 1141 (1986); M. Karliner and M. Mattis, *ibid.* **34**, 1991 (1986); A. Kanazawa, Prog. Theor. Phys. **77**, 212 (1987).

[2]J. Donoghue and C. Nappi, Phys. Lett. **168B**, 105 (1986).

[3]H. Yabu, Phys. Lett. B **218**, 124 (1989).

[4]S. Brodsky, J. Ellis, and M. Karliner, Phys. Lett. B **206**, 309 (1988); J. Ellis and M. Karliner, *ibid.* **213**, 73 (1988).

[5]N. W. Park, J. Schechter, and H. Weigel, Phys. Lett. B **228**, 420 (1989).

[6]EM Collaboration, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988).

[7]P. Jain, R. Johnson, W. N. Park, J. Schechter, and H. Weigel, Phys. Rev. D **40**, 855 (1989).

[8]C. Callan and I. Klebanov, Nucl. Phys. **B262**, 365 (1985); C. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B **202**, 269 (1988); J. Blaizot, M. Rho, and N. Scoccola, *ibid.* **209**, 27 (1988); N. Scoccola, H. Nadeau, M. Nowak, and M. Rho, *ibid.* **201**, 425 (1988).

[9]H. Yabu and K. Ando, Nucl. Phys. **B301**, 601 (1988). See also M. Praszalowicz, in *Skyrmions and Anomalies*, proceedings of the workshop, Krakow, Poland, 1987, edited by M. Jezabek and M. Praszalowicz (World Scientific, Singapore, 1987), p. 112; H. K. Lee and D. P. Min, Phys. Lett. B **219**, 1 (1989); J. H. Kim, C. H. Lee, and H. K. Lee, Nucl. Phys. **A501**, 835 (1989).

[10]N. W. Park, J. Schechter, and H. Weigel, Phys. Lett. B **224**, 171 (1989). See also the last references in 9.

[11]T. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. **31**, 556 (1962).

[12]E. Witten, Nucl. Phys. **B223**, 422 (1983).

[13]In addition to Ref. 4, see Chemtob (Ref. 1); T. N. Pham, Phys. Rev. D **30**, 234 (1984); E. Witten, Ann. Phys. (N.Y.) **128**, 363 (1980).

[14]G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[15]Ulf-G. Meissner, N. Kaiser, and W. Weise, Nucl. Phys. **A466**, 685 (1987).

[16]B. Stotzel, G. Holzwarth, and H. Walliser, Nucl. Phys. **A499**, 686 (1989).

[17]Ulf-G. Meissner, N. Kaiser, H. Weigel, and J. Schechter, Phys. Rev. D **39**, 1956 (1989); **40**, 262(E) (1989).

[18]A. Jackson, A. D. Jackson, and V. Pasquier, Nucl. Phys. **A432**, 567 (1985).

[19]See for example, Yabu and Ando (Ref. 9) and Kanazawa (Ref. 1).

[20]See B. Schwesinger, H. Weigel, G. Holzwarth, and A. Hayashi, Phys. Rep. **173**, 173 (1989), and references therein.

[21]See H. Pagels, Phys. Rep. **16**, 219 (1975), and references therein.

[22]P. Jain, R. Johnson, Ulf-G. Meissner, N. W. Park, and J. Schechter, Phys. Rev. D **37**, 3252 (1988).

[23]U. Blom, K. Dannbom, and D. Riska, Nucl. Phys. **A493**, 384 (1989). See, however, N. N. Scoccola, D. P. Min, H. Nadeau, and M. Rho, Regensburg Report TPR-89-10, 1989 (unpublished).

[24]Ulf-G. Meissner, Phys. Rep. **161**, 213 (1988), and references therein.

[25]D. Masak, Phys. Rev. D **39**, 305 (1989).

[26]D. Kaplan and I. Klebanov, UCSD report, 1989 (unpublished). See also Manohar (Ref. 1).

[27]O. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).