

Extra weak bosons in a confining gauge theory with complementarity

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In composite models of weak bosons based on a confining gauge theory, extra weak bosons introduced as composites of scalar constituents are shown to exhibit kinetic mixing interactions describing the vector-meson dominance of the photon and to disguise themselves as massive elementary gauge particles in accord with the notion of complementarity. Detailed discussions are presented for a $U(1)_{em}^{loc}$ model with a confined $SU(2)_L^{loc} \times \mathcal{G}^{loc}$ gauge symmetry, where $\mathcal{G}^{loc} = \mathcal{SU}(2)_L^{loc} \times \mathcal{SU}(2)_R^{loc}$, which is responsible for compositeness of quarks and leptons as well. This phenomenon is also shown to arise in QCD with three flavors, utilizing the diquark degrees of freedom, where the octet baryons and vector mesons are viewed as massive quarks and gluons, respectively.

I. INTRODUCTION

The current interest in physics of electroweak interactions lies in whether new phenomena due to supersymmetry and/or compositeness of quarks and leptons¹ arise above, say, the Fermi mass $G_F^{-1/2}$, of ~ 300 GeV, which is within reach of the high-energy colliders such as CERN LEP II, the CERN Linear Collider, (CLIC), the Japan Linear Collider (JLC), and so on. If quarks and leptons as well as weak bosons are composites generated by a confining force based on a non-Abelian gauge theory, one can argue their compositeness on the basis of the notion of complementarity.² It was first recognized in lattice gauge theories with scalars in the fundamental representation and inferred from the observation of the physical equivalence between the Higgs (or broken) and confining (or unbroken) phase.³ The two phases have no sharp phase boundary and belong to the same phase. As a result, the particle spectra in both phases map into each other as long as the energy scale involved does not exceed the vacuum expectation value (VEV) or the confinement scale. If this physical equivalence between the Higgs and confining phases is still valid in our world, we expect the “duality” of compositeness and “elementariness” of composite particles since in the Higgs phase elementary fields are only involved while in the confining phase composite fields are created. These composites can disguise themselves as elementary particles defined in the Higgs phase below the energies where composites in the confining phase behave as “elementary” particles. We further expect that the composites, which satisfy the “duality,” remain as important *low-energy* degrees of freedom.

Complementarity was applied to weak bosons in the Glashow-Weinberg-Salam (GWS) model based on the spontaneously broken $SU(2)_L^{loc} \times U(1)_Y^{loc}$ symmetry.⁴ The model in fact contains the scalar in the fundamental representation, i.e., the Higgs scalar ϕ , and leads to the (almost) equivalent one based on the $U(1)_{em}^{loc}$ symmetry with the confined “color” $SU(2)_L^{loc}$ symmetry, which dynamically realizes the kinetic γ - Z mixing scheme such as in the Bjorken-Hung-Sakurai (BHS) model.⁵ The weak bo-

sons W^\pm and Z are made as⁶ $W_\mu^\pm \sim \text{Tr}(\tau^{(\pm)} \bar{w}_L^\dagger D_\mu \bar{w}_L)$ and $Z_\mu \sim \text{Tr}(\tau^{(3)} \bar{w}_L^\dagger D_\mu \bar{w}_L)$, where \bar{w}_L is a scalar carrying the weak charge and is represented by the Higgs scalar ϕ as $\bar{w}_L = (\phi^G, \phi)$. This particular compositeness provides the transmutation of the gauge bosons into the composite vector mesons.⁷ At the same time, L -handed quarks (q_{iL}^A for $A=1,2,3$, and $i=1,2$) and leptons (l_{iL}) are regarded as composites described by $q_{iL}^A = \bar{w}_L^a c_{aL}^A$ and $l_{iL} = \bar{w}_L^a c_{aL}^0$, where c_{aL}^α ($a=1,2; \alpha=0,1,2,3$) are the $SU(2)_L^{loc}$ -doublet spinors carrying the Pati-Salam four colors.⁸

Along this line of the compositeness, we study properties of composite extra weak bosons generally introduced in models⁹ through their kinetic mixings of γ and demonstrate the validity of the physical equivalence of the Higgs and confining phases postulated in complementarity. The transmutation of gauge bosons into composite vector mesons is expected to occur at the extra W and Z sector if they are regarded as composites of scalar constituents. The extended gauge group will be $SU(2)_L^{loc} \times U(1)_{em}^{loc} \times \mathcal{G}^{loc}$ with $\mathcal{G}^{loc} = \mathcal{SU}(2)_L \times \mathcal{SU}(2)_R$ for extra W and Z bosons, which is realized in the confining phase of a “color” $SU(2)_L^{loc} \times \mathcal{G}^{loc}$ gauge theory. The kinetic mixing interactions are indeed involved in the confining phase provided that dynamics allows scalar constituents to condense in a gauge-invariant way. Their strengths are controlled by the matter couplings of composite vector mesons. The physical vector mesons with diagonalized kinetic terms are translated into the massive elementary gauge bosons in the Higgs phase. The exact “flavor” symmetry in the both phases turns out to be $U(1)_{em}^{loc}$ that comes from the linear combination of the diagonal charges of each broken group or from $SU(2)_L^{loc} \times \mathcal{G}^{loc}$ being confined. If the “color” gauge bosons themselves are further composite, complementarity can include the notion of a “hidden” symmetry.¹⁰

It is further instructive to note that QCD with three flavors, u , d , and s , can be viewed as the confining theory that exhibits the equivalent Higgs phase.¹¹ We first discuss in Sec. II how complementarity is implemented in QCD. The flavor octet baryons such as p , n , and Λ and

vector mesons such as ρ and K^* in the confining phase are regarded as the massive color-octet gluons and octet constituent quarks giving three colors and three flavors in the Higgs phase. Section III deals with a model of extra weak bosons based on the "color" $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ symmetry, which includes the previously discussed case of $\text{SU}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ as its limiting case.¹² The final section is devoted to summary and discussions.

II. LESSON FROM QCD

Let us outline the discussion of Ref. 11, which is extended to include the Nambu-Goldstone modes for pseudoscalars such as π . The flavor group is taken to be $\text{SU}(3)_f$ for u, d , and s . As is well known, at low energies, QCD exhibits confinement and generates the chiral-symmetry breaking of $\text{SU}(3)_L \times \text{SU}(3)_R$ to $\text{SU}(3)_f$. Complementarity in QCD will state that low-lying baryons and vector mesons, which form octets of $\text{SU}(3)_f$, can be viewed as either massive quarks and gluons in the Higgs phase or color-singlet composites in the confining phase. This phenomenon is simply understood that $\text{SU}(3)_c^{\text{loc}} \times \text{SU}(3)_f$ goes to the flavor- $\text{SU}(3)$ symmetry in both phases, where $\text{SU}(3)_c^{\text{loc}}$ and $\text{SU}(3)_f$ are mixed into the diagonal subgroup $\text{SU}(3)_{c+f}$ in the Higgs phase or reduced to $\text{SU}(3)_f$ with the confined $\text{SU}(3)_c^{\text{loc}}$ in the confining phase. Let us go into detail.

Complementarity is only possible if the scalar degrees of freedom are available. In QCD with the $\text{U}(1)_{\text{em}}^{\text{loc}}$ sym-

metry, we assume two scalars, ξ_{Li}^A ($A, i=1,2,3$): $(3, 1, Q_{\text{em}}^{\xi}; 3^*)$ and ξ_{Ri}^A : $(1, 3, Q_{\text{em}}^{\xi}; 3^*)$, which will be represented by *diquarks*, where the numbers in the parentheses denote the transformation properties of $(\text{SU}(3)_L, \text{SU}(3)_R, \text{U}(1)_{\text{em}}^{\text{loc}}, \text{SU}(3)_c^{\text{loc}})$. The scalars describe chiral-symmetry breaking, which is readily realized by $\xi_V = f_{\Pi}$ (or $\xi_V^{\dagger} \xi_V = \xi_V^{\dagger} \xi_V = f_{\Pi}^2$), where ξ_V is introduced as

$$\xi_{Li}^A = \xi_i^j \xi_{Vj}^A \quad \text{and} \quad \xi_{Ri}^A = \xi_i^{\dagger j} \xi_{Vj}^A. \quad (2.1)$$

Hereafter, we use the scaled $\xi_{L,R}$ by f_{Π} , i.e., $\xi_V = I$ and $\xi_V^{\dagger} \xi_V = \xi_V^{\dagger} \xi_V = I$. The remaining component ξ plays the role of Nambu-Goldstone modes though nonlinear realization: $\xi_i^j \sim \exp(i\Pi/f_{\Pi})_i^j$, where Π represents 8 pseudoscalars. Furthermore, one may regard ξ_{Vi}^A as scalar diquarks, $\xi_{Vi}^A = \epsilon^{ABC} \epsilon_{ijk} q_B^j q_C^k / f_{\Pi}^2$,¹¹ leading to $Q_{\text{em}}^{\xi} = (-\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$ since $Q_{\text{em}}^q = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ for (u, d, s) . Therefore, the charge of $\xi_{L,R}$ is carried by ξ_V and ξ is taken neutral. Our starting Lagrangian except for quark mass terms is given by

$$\mathcal{L} = -\frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + |f_{\Pi} D_{\mu} \xi_L|^2 + |f_{\Pi} D_{\mu} \xi_R|^2 + \bar{q} \gamma^{\mu} (i \partial_{\mu} + g_c G_{\mu} + g' Q_{\text{em}}^q B_{\mu}) q, \quad (2.2)$$

where $G_{\mu\nu} = G_{\mu\nu}^{(n)} T^{(n)} = \partial_{\mu} G_{\nu} - \partial_{\nu} G_{\mu} - ig_c [G_{\mu}, G_{\nu}]$ ($n=1-8$) is the gluon field strength and $B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$ is the primary photon field strength.

Phenomenologically, it is plausible to replace the scalar kinetic terms by the terms¹⁰

$$\begin{aligned} \mathcal{L}_{\text{scalar}} = & -\frac{f_{\Pi}^2}{4} \text{Tr} \{ [(i \partial_{\mu} + g_c G_{\mu}) \xi_L^{\dagger} - \xi_L^{\dagger} g' Q_{\text{em}}^{\xi} A_{\mu}^0] \xi_L + [(i \partial_{\mu} + g_c G_{\mu}) \xi_R^{\dagger} - \xi_R^{\dagger} g' Q_{\text{em}}^{\xi} A_{\mu}^0] \xi_R \}^2 \\ & + \{ [(i \partial_{\mu} + g_c G_{\mu}) \xi_L^{\dagger} - \xi_L^{\dagger} g' Q_{\text{em}}^{\xi} A_{\mu}^0] \xi_L - [(i \partial_{\mu} + g_c G_{\mu}) \xi_R^{\dagger} - \xi_R^{\dagger} g' Q_{\text{em}}^{\xi} A_{\mu}^0] \xi_R \}^2 \}. \end{aligned} \quad (2.3)$$

with $r=2$, where Tr acts on the color indices. This modification has been lately suggested on the basis of the nonlinear σ model with a hidden local symmetry, where gauge bosons are regarded as composites but without the baryons as qqq . Although it was examined in the Higgs (or broken) phase, the confining (or unbroken) phase is also possible.

Now, the Lagrangian (2.2) is examined both in the Higgs phase and in the confining phase. In the *Higgs* phase, $\text{SU}(3)_c^{\text{loc}}$ is broken completely as far as diquarks are condensed to develop $\langle \xi_{Vi}^A \rangle = \delta_i^A$. As a result, low-energy particles consist of (1) massive gluons as 8 vector mesons, G_i^j , and (2) L - (or R -) handed (constituent) quarks as 8 baryons, $\xi_i^k q_k^j$ ($\xi_i^{\dagger k} q_k^j$) with meson clouds Π in ξ . The resulting Lagrangian takes the form of

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & -\frac{1}{2} \text{Tr} (G_{\mu\nu} G^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{f_{\Pi}^2}{4} [r \text{Tr} (2g_c G_{\mu} - 2g' Q_{\text{em}}^{\xi} B_{\mu} + i \partial_{\mu} \xi^{\dagger} \cdot \xi + i \partial_{\mu} \xi \cdot \xi^{\dagger})^2 + \text{Tr} (i \partial_{\mu} \xi^{\dagger} \cdot \xi - i \partial_{\mu} \xi \cdot \xi^{\dagger})^2] \\ & + \bar{q}_L \gamma^{\mu} (i \partial_{\mu} + \xi g_c G_{\mu} \xi^{\dagger} + \xi i \partial_{\mu} \xi^{\dagger} + g' Q_{\text{em}}^q B_{\mu}) q_L + \bar{q}_R \gamma^{\mu} (i \partial_{\mu} + \xi^{\dagger} g_c G_{\mu} \xi + \xi^{\dagger} i \partial_{\mu} \xi + g' Q_{\text{em}}^q B_{\mu}) q_R. \end{aligned} \quad (2.4)$$

In this phase, the mass mixing of the octet G_{μ} with the photon B_{μ} arises. The physical photon A_{μ} is provided by the gauge field orthogonal to $g_c G_{\mu} - g' Q_{\text{em}}^{\xi} B_{\mu}$ appearing in the mass term:

$$A_{\mu} = \left[g_c B_{\mu} - g' \left[\rho_{\mu}^3 + \frac{\rho_{\mu}^8}{\sqrt{3}} \right] \right] / \left[g_c^2 + \frac{4}{3} g'^2 \right]^{1/2}, \quad (2.5a)$$

where ρ_{μ}^3 (ρ_{μ}^8) is the 3rd (8th) component of the octet and the electric charge is

$$\frac{1}{e^2} = \frac{1}{g'^2} + \frac{4}{3g_c^2}. \quad (2.5b)$$

As a result, quarks possess $Q_{\text{em}}^N = -(\tau_c^{(3)}/2 + Y_c/6) + Q_{\text{em}}^{(q)}$ for $\tau_c^{(3)} = (1, -1, 0)$ and $Y_c = (1, 1, -2)$ for $A = (1, 2, 3)$ of q_A^i . More explicitly, their charges coincide with those of the 8 baryons ($p, n, \Sigma^{\pm 0}, \Lambda, \Xi^{-0}$): $q_1^i (i=1, 2, 3) = \Sigma^0$ and/or Λ ; $q_1^1 = \Sigma^+$; $q_2^1 = \Sigma^-$; $q_3^1 = p$; $q_3^2 = n$; $q_3^3 = \Xi^-$; and $q_3^3 = \Xi^0$.

In the *confining* phase, low-energy particles are assumed to be all color singlets. To examine this phase, we

impose the conditions on $\xi_{L(R)} = \xi \xi_V (\xi^\dagger \xi_V)^\dagger$:

$$\begin{aligned} \langle (\xi_V)_i^A (\xi_V^\dagger)_A^j \rangle &= \delta_i^j, \\ \langle (\xi_V^\dagger)_A^i (\xi_V)_i^B \rangle &= \delta_A^B, \\ \xi_i^k (\xi^\dagger)_k^j &= \delta_i^j, \end{aligned} \quad (2.6)$$

where the last condition is usually satisfied by $\xi = \exp(i\Pi/f_\Pi)$ for the nonlinear realization. These conditions preserve $SU(3)_c^{\text{loc}}$. Our low-lying composite particles consist of (1) 8 baryons $N_{iL,R}^j$ and (2) 8 vector mesons V_i^j (as well as 8 pseudoscalars):

$$\begin{aligned} N_{iL}^j &= \sum_A \xi_{Li}^A q_{AL}^j, \quad N_{iR}^j = \sum_A \xi_{Ri}^A q_{AR}^j, \\ f(V_\mu)_i^j &= \sum_{A,B} \xi_{Vi}^A [(i\partial_\mu + g_c G_\mu)^B_A (\xi_V^\dagger)_B^j \\ &\quad - (\xi_V^\dagger)_A^k (Q_{em}^\xi)_k^j A_\mu^0], \end{aligned} \quad (2.7)$$

where A_μ^0 is the scaled field of B_μ : $eA_\mu^0 = g'B_\mu$. For $\xi_V \sim qq$, the composite baryons are in fact regarded as $\exp(\pm i\Pi)qqq$ and the composite vector bosons are as $qq\bar{q}\bar{q}$. Since $g_c G_{\mu\nu} = \xi_V^\dagger v_{\mu\nu} \xi_V$ with $v_\mu = fV_\mu + eQ_{em}^\xi A_\mu^0$, it is not difficult to reach the Lagrangian, $\mathcal{L}_{\text{conf}}$ from Eq. (2.2) with Eq. (2.3):

$$\begin{aligned} \mathcal{L}_{\text{conf}} &= -\frac{1}{2g_c^2} \text{Tr}(v_{\mu\nu} v^{\mu\nu}) - \frac{e^2}{4g_c'^2} A_{\mu\nu}^0 A^{0\mu\nu} + \frac{f_\Pi^2}{4} [r \text{Tr}(2fV_\mu + i\partial_\mu \xi^\dagger \cdot \xi + i\partial_\mu \xi \cdot \xi^\dagger) + \text{Tr}(i\partial_\mu \xi^\dagger \cdot \xi - i\partial_\mu \xi \cdot \xi^\dagger)^2] \\ &\quad + \bar{N}_L \gamma^\mu [i\partial_\mu + \xi(fV_\mu + eQ_{em}^\xi) \xi^\dagger + \xi i\partial_\mu \xi^\dagger + eQ_{em}^q A_\mu^0] N_L \\ &\quad + \bar{N}_R \gamma^\mu [i\partial_\mu + \xi^\dagger(fV_\mu + eQ_{em}^\xi) \xi + \xi^\dagger i\partial_\mu \xi + eQ_{em}^q A_\mu^0] N_R, \end{aligned} \quad (2.8)$$

up to the scalar excitations in $\xi_V^\dagger \xi_V$, where $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$. The composite baryons exhibit $(Q_{em}^N)_i^j = Q_{em}^{\xi_i} + Q_{em}^{q_j}$, which coincides with Q_{em}^N in the Higgs phase owing to $Q_{em}^{\xi_i} = (-\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$. The coupling constants also satisfy the relation (2.5b) and $f = g_c$ for the canonical kinetic terms of V_μ and A_μ^0 .

It turns out that the same baryon assignment as that in the Higgs phase is still correct in the confining phase with the replacement of $q_i^j \leftrightarrow N_i^j$. The bridge connecting the Higgs and confining phase is to choose $\xi_V (\sim qq) = I$ that yields the baryons (massive constituent quarks) and vector mesons (massive gluons) in the Higgs phase from the expressions of (2.7). The apparent difference between the confining and Higgs phases lies in the vector-meson sector. In the confining phase, the kinetic mixing of the 8 meson with the photon is generated

$$\mathcal{L}_{\text{mix}} = \frac{1}{2} (\lambda_{\gamma\rho} \rho_{\mu\nu}^3 + \lambda_{\gamma\rho'} \rho_{\mu\nu}^8) A^{0\mu\nu}, \quad (2.9)$$

where $\lambda_{\gamma\rho} = \sqrt{3} \lambda_{\gamma\rho'} = e/f$. The physical photon A_μ in this case is $A_\mu = A_\mu^0 + \lambda_{\gamma\rho} \rho_\mu^3 + \lambda_{\gamma\rho'} \rho_\mu^8$. On the other hand, in the Higgs phase

$$A_\mu = [g_c B_\mu - g'(\rho_\mu^3 + \rho_\mu^8/\sqrt{3})] / \sqrt{g_c^2 + (4g'^2/3)}$$

remains massless. Although the expression of the photon differs, it can be demonstrated that $\mathcal{L}_{\text{Higgs}}$ and $\mathcal{L}_{\text{conf}}$ are equivalent to each other. It is evident that for the charged-vector-meson sector, both Lagrangians coincide with each other by setting $G_\mu^{(n)} = V_\mu^{(n)}$ for $n \neq 3, 8$ since the charged vector mesons are not mixed. The mixing is induced in the neutral boson sector. The expressions of $\mathcal{L}_{\text{Higgs,conf}}$ in terms of physical fields after the mass, or ki-

netic, mixing is resolved give the direct proof as shown below.

In the Higgs phase, one can define the massive neutral gauge fields

$$\rho_\mu^0 = c_\theta \rho_\mu^3 + s_\theta (c_\phi B_\mu - s_\phi \rho_\mu^8), \quad (2.10a)$$

$$\rho'_\mu = s_\phi B_\mu + c_\phi \rho_\mu^8, \quad (2.10b)$$

together with $A_\mu = c_\theta (c_\phi B_\mu - s_\phi \rho_\mu^8) - s_\theta \rho_\mu^3$. The mixing angles, $s_\theta \equiv \sin\theta$, etc., are given by

$$s_\theta = g' / \left[g_c^2 + \frac{4g'^2}{3} \right]^{1/2}, \quad (2.11a)$$

$$s_\phi = g' / \sqrt{3g_c^2 + g'^2}, \quad (2.11b)$$

satisfying $g_c s_\theta = \sqrt{3} g_c s_\phi c_\theta = g' c_\phi c_\theta = e$. On the other hand, in the confining phase, the fields with the diagonalized kinetic terms are defined to be

$$\rho_\mu^0 = \sqrt{1 - \lambda_{\gamma\rho}^2} \left[\rho_\mu^3 - \frac{\lambda_{\gamma\rho} \lambda_{\gamma\rho'}}{1 - \lambda_{\gamma\rho}^2} \rho_\mu^8 \right], \quad (2.12a)$$

$$\rho'_\mu = \left[1 - \frac{\lambda_{\gamma\rho'}^2}{1 - \lambda_{\gamma\rho}^2} \right]^{1/2} \rho_\mu^8, \quad (2.12b)$$

together with $A_\mu = A_\mu^0 + \lambda_{\gamma\rho} \rho_\mu^3 + \lambda_{\gamma\rho'} \rho_\mu^8$. It is sufficient for the equivalence to demonstrate that the mass terms for vector mesons and the interaction terms for fermions coincide with each other. The massive fields in the mass terms of Eqs. (2.4) and (2.8) can be rewritten in terms of the physical fields, ρ^0 and ρ' , as

$$\begin{aligned}
g_c G_\mu - g' Q_{\text{em}}^\xi B_\mu|_n &= (g_c \rho_\mu^3 + g' B_\mu) \frac{\tau_c^{(3)}}{2} + (\sqrt{3} g_c \rho_\mu^8 + g' B_\mu) \frac{Y_c}{6} \\
&= g_c \left[\frac{\rho_\mu^0}{c_\theta} \frac{\tau_c^{(3)}}{2} + \frac{\rho'_\mu}{\sqrt{3} c_\phi} \left(t_\theta^2 \frac{\tau_c^{(3)}}{2} + \frac{Y_c}{2} \right) \right], \tag{2.13a}
\end{aligned}$$

$$\begin{aligned}
f V_\mu|_n &= f \left[\rho_\mu^3 \frac{\tau_c^{(3)}}{2} + \frac{\rho_\mu^8}{\sqrt{3}} \frac{Y_c}{2} \right] \\
&= f \left[\frac{\rho_\mu^0}{\sqrt{1-\lambda_{\gamma\rho}^2}} \frac{\tau_c^{(3)}}{2} + \frac{\rho'_\mu}{\sqrt{3}\sqrt{1-[\lambda_{\gamma\rho}'/(1-\lambda_{\gamma\rho}^2)]}} \left(\sqrt{3} \frac{\lambda_{\gamma\rho} \lambda_{\gamma\rho}'}{1-\lambda_{\gamma\rho}^2} \frac{\tau_c^{(3)}}{2} + \frac{Y_c}{2} \right) \right], \tag{2.13b}
\end{aligned}$$

where $|_n$ denotes the neutral components. By noticing the relations such as $f = g_c$, $\lambda_{\gamma\rho} = \sqrt{3} \lambda_{\gamma\rho}' = e/f = s_\theta$, and $\lambda_{\gamma\rho}'/\sqrt{1-\lambda_{\gamma\rho}^2} = s_\phi$, one finds that $g_c G_\mu - g' Q_{\text{em}}^\xi B_\mu = f V_\mu$ as expected in Eq. (2.7) with $\xi_V = I$ and $g' B_\mu = e A_\mu^0$, thus giving the same masses for vector mesons. Similarly, $g_c G_\mu = f V_\mu + e Q_{\text{em}}^\xi A_\mu^0$ holds in the fermion sector giving the same interactions. Collecting these results, one finally understands the physical equivalence between $\mathcal{L}_{\text{Higgs}}$ and $\mathcal{L}_{\text{conf}}$ that ensures the ‘‘duality’’ between compositeness and ‘‘elementariness.’’

The composite 8 baryons and vector mesons are described by $N_i^j = \xi^{(+) \dagger} \xi_{V_i} q^j$ and $(\xi_V i D_\mu \xi_V^\dagger)_i^j$. Thanks to complementarity the identification of N and $\xi i D_\mu \xi^\dagger$, respectively, with the massive 8 quarks q and 8 gluons G_μ does not develop any dynamical difference. The essence deduced from our lesson can be summarized as follows:

(1) Confining gauge theory with the scalar ϕ provides, in the confining phase of $\langle \phi^\dagger \phi \rangle \neq 0$ (driven from the confining force), the same physics as the one in the Higgs phase of $\langle \phi \rangle \neq 0$ (driven from the appropriate Higgs potential), which only contains the apparent difference such as the kinetic mixing for composite vector mesons in the confining phase and the well-known orthogonal mixing for massive gauge bosons in the Higgs phase.

(2) The composites corresponding to the particles involved in the Higgs phase (except for scalars associated with VEV’s) satisfy the ‘‘duality’’ of compositeness and ‘‘elementariness’’ and may remain as important *low-energy* degrees of freedom.

(3) The exact ‘‘duality’’ at the Lagrangian level can be achieved by adjusting couplings of composite vector mesons so as to mimic those of massive gauge theory.

Of course, going to higher energies obtains the right answer, the confining phase of QCD. These features can be readily translated into those of electroweak theory based on $\text{SU}(2)_L^{\text{loc}} \times \text{U}(1)_{\text{em}}^{\text{loc}}$, which is discussed in the next section with the emphasis on the inclusion of extra W and Z bosons as composites.

III. EXTRA W AND Z BOSONS

The QCD case is now extended to electroweak dynamics. The simplest extension is to prepare a ‘‘flavor’’- $\text{SU}(2)_L^{\text{loc}} \times \text{U}(1)_Y^{\text{loc}}$ symmetry instead of the flavor- $\text{SU}(3)_L \times \text{SU}(3)_R$ symmetry and a new ‘‘color’’- $\text{SU}(2)_V^{\text{loc}}$ symmetry instead of the color- $\text{SU}(3)_c^{\text{loc}}$ symmetry. Then,

we expect extra W and Z bosons analogous to ρ^\pm and ρ^0 to appear.¹³ Fundamental fermions are $\text{SU}(2)_V^{\text{loc}}$ doublets and will be denoted by $c_{a'}^{\alpha}$ ($a'=1,2$): $(1, Y; 2)$ for $(\text{SU}(2)_L^{\text{loc}}, \text{U}(1)_Y^{\text{loc}}; \text{SU}(2)_V^{\text{loc}})$ with $Y (=B-L) = \frac{1}{3}$ for $\alpha=1,2,3$; $=-1$ for $\alpha=0$. The required scalars for complementarity of $\text{SU}(2)_V^{\text{loc}}$ are \bar{w}_L and \bar{w}_R (corresponding to $\xi_{L,R}$ in the QCD case), $\bar{w}_{La'}^a$ ($a=1,2$): $(2, 0; 2)$ and $\bar{w}_{Ra'}^i$ ($i=1,2$): $(1, -\tau^3; 2)$, which carry the weak charge. The ‘‘color’’-singlet quarks and leptons are described by $q_{aL}^A = \sum_{a'} \bar{w}_{La'}^a c_{a'L}^A$, $q_{iR}^A = \sum_{a'} \bar{w}_{Ra'}^i c_{a'R}^A$, $l_{aL} = \sum_{a'} \bar{w}_{La'}^a c_{aL}^0$, and $l_{iR} = \sum_{a'} \bar{w}_{Ra'}^i c_{a'R}^0$. To generate the consistent couplings of W and Z (as well as γ), composite extra W and Z ($\equiv \rho$) bosons should exhibit the substructure

$$f \rho_\mu = \bar{w}_L^\dagger [(i \partial_\mu + g_V G_{V\mu}) \bar{w}_L - \bar{w}_L g G_\mu], \tag{3.1a}$$

$$f \rho_\mu = \bar{w}_R^\dagger [(i \partial_\mu + g_V G_{V\mu}) \bar{w}_R - \bar{w}_R g' \frac{\tau^{(3)}}{2} B_\mu], \tag{3.1b}$$

where G_μ , B_μ , and $G_{V\mu}$, respectively, denote the gauge bosons of $\text{SU}(2)_L^{\text{loc}}$, $\text{U}(1)_Y^{\text{loc}}$, and $\text{SU}(2)_V^{\text{loc}}$. The dynamical assumption is to form scalar condensations as $\langle \bar{w}_{L(R)}^\dagger \bar{w}_{L(R)} \rangle = \langle \bar{w}_{L(R)} \bar{w}_{L(R)}^\dagger \rangle = I$. Complementarity is easily seen by taking $\bar{w}_{L(R)} = I$, which yields $f \rho_\mu = g_V G_\mu - g G_\mu$ from Eq. (3.1a) and $f \rho_\mu = g_V G_\mu - g' (\tau^{(3)}/2) B_\mu$ from Eq. (3.1b). Since $g G_\mu \neq g' (\tau^{(3)}/2) B_\mu$, these equalities contradict each other.

The consistent result can be obtained by introducing either (1) two kinds of ρ for L - and R -handed states or (2) one kind of ρ for L -handed states but with R -handed states being elementary (‘‘color’’ singlet) or vice versa. In case (1), the ‘‘color’’ gauge group should be $\text{SU}(2)_V^{\text{loc}} \times \text{SU}(2)_A^{\text{loc}}$ for the vector and axial-vector couplings to fermions, where ρ - and A_1 -like mesons are generated as extra W and Z bosons, while in case (2) $\mathcal{S}\mathcal{U}(2)_{L(R)}^{\text{loc}}$ is the ‘‘color’’ gauge group that provides ρ -like mesons with the L (R) coupling. It should be noted that if the extra vector mesons with the R - (or L -) handed couplings to quarks and leptons get very heavy, case (1) reduces to case (2) of $\mathcal{S}\mathcal{U}(2)_{L(R)}^{\text{loc}}$. It is thus sufficient to consider case (1) only.

The gauge group for extra W and Z bosons is $\text{SU}(2)_V^{\text{loc}} \times \text{SU}(2)_A^{\text{loc}}$ or, equivalently, $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$. For the compositeness of W and Z , the $\text{SU}(2)_L^{\text{loc}}$ symmetry is also considered as a confining ‘‘color’’ gauge group. The whole gauge group is thus $\text{SU}(2)_L^{\text{loc}} \times \text{U}(1)_{\text{em}}^{\text{loc}}$

$\times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$, in which all non-Abelian groups are confined at low energies. Since there are two $\text{SU}(2)_L$'s, the particle assignment has the freedom of choosing which $\text{SU}(2)_L^{\text{loc}}$ "color" is carried by c_L and \bar{w}_L . We discuss two cases of (Sec. III A) $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublet c_L and \bar{w}_L and (Sec. III B) $\text{SU}(2)_L^{\text{loc}}$ doublet c_L and $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublet \bar{w}_L . Other cases are simply given by exchanging two "colors," $\text{SU}(2)_L^{\text{loc}}$ and $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$.

A. $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublet c_L and \bar{w}_L

Both c_L and \bar{w}_L are assumed to carry the new $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ "color". The fields contained are as follows.

(1) "Color"- $\mathcal{S}\mathcal{U}(2)_{L,R}$ -doublet fermions with the "flavor" suffix α ($=0, 1, 2, 3$) for the three colors ($\alpha=1, 2, 3$) and $B-L$ number ($\alpha=0$):

$$c_{mL}^\alpha : (1, Y; 2, 1), \quad (3.2a)$$

$$c_{mR}^\alpha : (1Y; 1, 2), \quad (3.2b)$$

for $(\text{SU}(2)_L^{\text{loc}}, \text{U}(1)_{\text{em}}^{\text{loc}}, \mathcal{S}\mathcal{U}(2)_L^{\text{loc}}, \mathcal{S}\mathcal{U}(2)_R^{\text{loc}})$, where $m(=1, 2)$ denotes the $\mathcal{S}\mathcal{U}(2)_{L,R}^{\text{loc}}$ -"color" and Y ($=B-L$) $= -1$ for c^0 ; $=\frac{1}{3}$ for c^A ($A=1, 2, 3$).

(2) Three kinds of "color" scalars:

$$\bar{w}_{Li}^m : (1, \tau^{(3)}; 2, 1), \quad (3.2c)$$

$$\bar{w}_{Ri}^m : (1, \tau^{(3)}; 1, 2), \quad (3.2d)$$

$$\xi_m^a : (2, 0; 2, 1), \quad (3.2e)$$

where $i(=1, 2)$ and $a(=1, 2)$, respectively, denote the "flavor" and $\text{SU}(2)_L^{\text{loc}}$ "color."

(3) "color" gauge bosons, $(G_\mu)_a^b$ of $\text{SU}(2)_L^{\text{loc}}$ with the gauge coupling g , $(G_{L\mu})_m^n$ of $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ with g_L , and $(G_{R\mu})_m^n$ of $\mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ with g_R , and a "flavor" gauge boson, B_μ , of $\text{U}(1)_{\text{em}}^{\text{loc}}$ with g' .

From these constituents, composite quarks and leptons, which are "color" singlets, are made as

$$q_{iL(R)}^A = \bar{w}_{L(R)i} c_{L(R)}^A, \quad (3.3a)$$

$$l_{iL(R)} = \bar{w}_{L(R)i} c_{L(R)}^0, \quad (3.3b)$$

and composite vector mesons are

$$fV_\mu = \bar{w}_L iD_\mu \bar{w}_L^\dagger, \quad (3.4a)$$

$$f_L L_\mu = \bar{w}_L (\xi iD_\mu \xi^\dagger) \bar{w}_L^\dagger, \quad (3.4b)$$

$$f_R R_\mu = \bar{w}_R iD_\mu \bar{w}_R^\dagger. \quad (3.4c)$$

The dynamical constraint on the *confining* phase is to form the scalar condensations (except for the freedom of the radial excitations of composite scalars)

$$\begin{aligned} \langle \bar{w}_{L(R)i} \bar{w}_{L(R)i}^\dagger \rangle &= \langle \bar{w}_{L(R)i} \bar{w}_{L(R)i} \rangle \\ &= \langle \xi \xi^\dagger \rangle = \langle \xi^\dagger \xi \rangle = I. \end{aligned} \quad (3.5)$$

The rest of argument goes through in the same way as in the QCD case. Our Lagrangian for "color"-singlet composites is calculated from the $\text{SU}(2)_L^{\text{loc}} \times \text{U}(1)_{\text{em}}^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ gauge theory¹⁴ as $\mathcal{L}_{\text{conf}}^A = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_0$:

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= -\frac{1}{2g^2} \text{Tr}(v_{1\mu\nu} v_1^{\mu\nu}) - \frac{1}{2g_L^2} \text{Tr}(v_{2\mu\nu} v_2^{\mu\nu}) \\ &\quad - \frac{1}{2g_R^2} \text{Tr}(v_{3\mu\nu} v_3^{\mu\nu}) - \frac{e^2}{4g'^2} A_{\mu\nu}^0 A^{0\mu\nu}, \end{aligned} \quad (3.6a)$$

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= \Lambda^2 \text{Tr}(fV_\mu)^2 + \Lambda_L^2 \text{Tr}(f_L L_\mu)^2 \\ &\quad + \Lambda_R^2 \text{Tr}(f_R R_\mu)^2, \end{aligned} \quad (3.6b)$$

$$\begin{aligned} \mathcal{L}_0 &= \bar{\psi}_L \gamma^\mu (i\partial_\mu + fV_\mu + eQ_{\text{em}} A_\mu^0) \psi_L \\ &\quad + \bar{\psi}_R \gamma^\mu (i\partial_\mu + f_R R_\mu + eQ_{\text{em}} A_\mu^0) \psi_R, \end{aligned} \quad (3.6c)$$

up to the scalar and fermion mass terms, where

$$v_{1\mu} = fV_\mu + f_L L_\mu + e(\tau^{(3)}/2) A_\mu^0, \quad (3.7a)$$

$$v_{2\mu} = fV_\mu + e(\tau^{(3)}/2) A_\mu^0, \quad (3.7b)$$

$$v_{3\mu} = f_R R_\mu + e(\tau^{(3)}/2) A_\mu^0, \quad (3.7c)$$

with $eA_\mu^0 = g'B_\mu$. It should be stressed that the extra L -handed vector meson L_μ does not directly couple to the L -handed quarks and leptons ψ_L , as shown in Eq. (3.6c). The canonical scalar fields with mass dimension = 1 are accompanied by the mass scales Λ for \bar{w}_L , Λ_L for ξ , and Λ_R for \bar{w}_R . The composite quarks and leptons are grouped into ψ_i^α and their electric charge Q_{em} is given by the sum of Q_{em} for the scalars $\tau^{(3)}/2$ and the fermions $Y/2$, which yields the familiar relation of $Q_{\text{em}} = (\tau^{(3)} + Y)/2$. The coupling constants are related to each other:

$$\frac{1}{f^2} = \frac{1}{g^2} + \frac{1}{g_L^2}, \quad (3.8a)$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} + \frac{1}{g_L^2} + \frac{1}{g_R^2}, \quad (3.8b)$$

$$f_L = g, \quad (3.8c)$$

$$f_R = g_R, \quad (3.8d)$$

for canonical kinetic terms of V_μ , L_μ , R_μ , and A_μ^0 .

The vector dominance of the photon is described by the kinetic mixing part (\mathcal{L}_{mix}) of $\mathcal{L}_{\text{conf}}^A$:

$$\begin{aligned} \mathcal{L}_{\text{mix}} &= -\frac{1}{2} (\lambda_{\gamma V} V_{\mu\nu}^{(3)} + \lambda_{\gamma L} L_{\mu\nu}^{(3)} + \lambda_{\gamma R} R_{\mu\nu}^{(3)}) A^{0\mu\nu} \\ &\quad - \frac{\lambda_{VL}}{2} L_{\mu\nu}^{(i)} V^{(i)\mu\nu}, \end{aligned} \quad (3.9)$$

where $\lambda_{\gamma V} = e/f$, $\lambda_{\gamma L(R)} = e/f_{L(R)}$, and $\lambda_{VL} = f/f_L$. The Lagrangian with extra composite weak bosons characterized by vector-meson dominance has been discussed in the literature.⁹ The fields with the kinetic mixings resolved are provided by

$$A_\mu = A_\mu^0 + \lambda_{\gamma V} V_\mu^{(3)} + \lambda_{\gamma L} L_\mu^{(3)} + \lambda_{\gamma R} R_\mu^{(3)}, \quad (3.10a)$$

$$\mathcal{V}_\mu^{(3)} = \sqrt{1 - \lambda_{\gamma V}^2} \left[V_\mu^{(3)} + \lambda_{VL} L_\mu^{(3)} - \frac{\lambda_{\gamma V} \lambda_{\gamma R}}{1 - \lambda_{\gamma V}^2} R_\mu^{(3)} \right], \quad (3.10b)$$

$$\mathcal{V}_\mu^{(\pm)} = V_\mu^{(\pm)} + \lambda_{VL} L_\mu^\pm, \quad (3.10c)$$

$$\mathcal{L}_\mu^{(i)} = \sqrt{1 - \lambda_{VL}^2} L_\mu^{(i)}, \quad (3.10d)$$

$$\mathcal{R}_\mu^{(3)} = \sqrt{(1 - \lambda_{\gamma V}^2 - \lambda_{\gamma R}^2)/(1 - \lambda_{\gamma V}^2)} R_\mu^{(3)}, \quad (3.10e)$$

$$\mathcal{R}_\mu^{(\pm)} = R_\mu^{(\pm)}. \quad (3.10f)$$

The vector fields A_μ , V_μ , L_μ , and R_μ are also expressed in terms of fields with the orthogonal mixings in the Higgs phase of $\langle \bar{w}_{L(R)} \rangle = \langle \xi \rangle \neq 0$, which thus reflect $SU(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \rightarrow SU(2)_D^{\text{loc}}$ with the gauge coupling $g_D = gg_L/(g^2 + g_L^2)^{1/2}$ ($=g \cos\theta_L = g_L \sin\theta_L$), $U(1)_{\text{em}}^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}} \rightarrow U(1)_D^{\text{loc}}$ with $g'_D = g'g_R/(g'^2 + g_R^2)^{1/2}$ ($=g' \cos\theta_R = g_R \sin\theta_R$), and $SU(2)_D^{\text{loc}} \times U(1)_D^{\text{loc}} \rightarrow U(1)_{\text{em}}^{\text{loc}}$ with $e = g_D g'_D/(g_D^2 + g'^2_D)^{1/2}$ ($=g'_D \cos\theta = g_D \sin\theta$):

$$A_\mu = \sin\theta a_\mu^{(3)} + \cos\theta b_\mu, \quad (3.11a)$$

$$V_\mu^{(3)} = \cos\theta a_\mu^{(3)} - \sin\theta b_\mu, \quad (3.11b)$$

$$V_\mu^{(\pm)} = \sin\theta_L G_{L\mu}^{(\pm)} + \cos\theta_L G_{\mu}^{(\pm)}, \quad (3.11c)$$

$$\mathcal{L}_\mu^{(i)} = \cos\theta_L G_{L\mu}^{(i)} - \sin\theta_L G_\mu^{(i)}, \quad (3.11d)$$

$$\mathcal{R}_\mu^{(3)} = \cos\theta_R G_{R\mu}^{(3)} - \sin\theta_R B_\mu, \quad (3.11e)$$

$$\mathcal{R}_\mu^{(\pm)} = G_{R\mu}^{(\pm)}, \quad (3.11f)$$

where $a_\mu^{(3)} = \sin\theta_L G_{L\mu}^{(3)} + \cos\theta_L G_\mu^{(3)}$ and $b_\mu = \sin\theta_R G_{R\mu}^{(3)} + \cos\theta_R B_\mu$. Following these relations together with the identification of $f = g_D$, $f_L = g$, and $f_R = g_R$ leading to $\lambda_{\gamma V} = \sin\theta$, $\lambda_{\gamma L} = \sin\theta \sin\theta_L$, $\lambda_{\gamma R} = \cos\theta \sin\theta_R$, and $\lambda_{VL} = \sin\theta_L$, it is shown that the Lagrangian evaluated in the Higgs phase is exactly the same as the one in the confining phase. Thus, complementarity works.

After including the vector meson dominance of the photon, one can derive the effective four-Fermi interactions for charged and neutral currents as follows:

$$\mathcal{L}_{\text{eff}}^{\text{CC}} = \frac{1}{2} \left[\frac{f^2}{m_V^2} J_{L\mu}^{(-)} J_L^{(+)\mu} + \frac{f_R^2}{m_R^2} J_{R\mu}^{(-)} J_R^{(+)\mu} \right], \quad (3.12a)$$

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = \frac{f^2}{m_V^2} \left[J_L^{(3)} - \frac{e^2}{f^2} J^{\text{em}} \right]^2 + \frac{1}{m_L^2} \left[\frac{e^2}{f_L} J^{\text{em}} \right]^2 + \frac{f_R^2}{m_R^2} \left[J_R^{(3)} - \frac{e^2}{f_R^2} J^{\text{em}} \right]^2, \quad (3.12b)$$

where $m_V = f\Lambda$, $m_L = f_L \Lambda_L$, and $m_R = f_R \Lambda_R$. These forms are easily obtained by noticing that the effective Lagrangians are determined by exchanging the fields with diagonal masses, which are in this case V_μ , L_μ , and R_μ . To be phenomenologically consistent, the dynamics should ensure that the R -handed mass m_R be much heavier than the mass m_V , so that at least right-handed current interactions are suppressed. For $SU(2)_L^{\text{loc}} \times U(1)_{\text{em}}^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ at the absence of R_μ , the phenomenology of W' and Z' has been discussed in the literature.¹⁵ The masses $m_{W'}$ and $m_{Z'}$ are constrained to be $m_{W',Z'} \gtrsim 200$ GeV from low-energy data¹⁶ letting $(e/f)^2 \cong 0.22 - 0.24$, from $m_{W(Z)} = 80.00 \pm 0.56$ (Ref. 17) [91.09 ± 0.06 (Refs. 18 and 19) GeV], and from $p\bar{p} \rightarrow W'Z' + \dots$ followed by $W'(Z') \rightarrow e\nu(e^+e^-)$ at Fermilab:²⁰ $\sigma_{W'} B_{e\nu}(\sigma_{Z'} B_{e^+e^-}) \leq 7.6(1.0)$ pb.

B. $SU(2)_L^{\text{loc}}$ doublet c_L and $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublet \bar{w}_L

One may insist that the fundamental fermions c_L^α , which was taken as the “color”- $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublets, are equally possible to carry the $SU(2)_L^{\text{loc}}$ “color.” In this case, left-handed states of quarks and leptons are provided by

$$q_{iL}^A = \bar{w}_{Li} \xi c_L^A, \quad (3.13a)$$

$$l_{iL} = \bar{w}_{Li} \xi c_L^0, \quad (3.13b)$$

with $q_R^A, l_R \sim \bar{w}_R c_R^{A,0}$ of Eqs. (3.3c) and (3.3d), whose interactions turn out to be $\mathcal{L}_{\text{conf}}^B = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}'_0$ with

$$\mathcal{L}'_0 = \bar{\psi}_L \gamma^\mu (i\partial_\mu + fV_\mu + f_L L_\mu + eQ_{\text{em}} A_\mu^0) \psi_L + \bar{\psi}_R \gamma^\mu (i\partial_\mu + f_R R_\mu + eQ_{\text{em}} A_\mu^0) \psi_R, \quad (3.14)$$

where ψ_L does couple to the extra boson, L_μ . It can be found that $m_{W',Z'} \gtrsim 700$ GeV to be consistent with low-energy data as well as the measured $m_{W,Z}$.

IV. SUMMARY AND DISCUSSIONS

We have discussed properties of composite particles, especially those of vector mesons, which are generated by a confining force based on a non-Abelian gauge theory, and derive their interactions at low energies, where their substructure is hidden and composite particles behave as “elementary” particles. If coupling constants are constrained, low-energy physics for the composite particles realized in the confining phase is identical to that for the elementary particles such as massive gauge bosons in the Higgs phase as long as the scalar degrees of freedom (except for the Nambu-Goldstone bosons) are frozen. It can be directly seen from the Lagrangian of the confining theory with the assumed dynamical behavior of the scalar ϕ : $\langle \phi \rangle \neq 0$ (from the Higgs potential) $\langle \phi^\dagger \phi \rangle \neq 0$ (from the confining force). The characteristics of low-energy physics in these two phases lie in the mixing scheme: in the Higgs phase, the mass mixing gives the physical fields but in the confining phase the kinetic mixing manifests itself. The equivalence generally occurs as a result of complementarity and ensures the “duality” between compositeness and “elementariness” of particularly selected composite particles that have the corresponding degrees of freedom in the Higgs phase. The low-lying composites are expected to satisfy the “duality.”

The examples discussed are the cases of color- $SU(3)_c^{\text{loc}}$ symmetry for QCD and “color”- $SU(2)_L^{\text{loc}}$ symmetry for electroweak dynamics (EWD) that is extended to include extra W and Z bosons as composites. For QCD, massive gluons G_μ act as composite $\mathbf{8}$ vector mesons (such as ρ , etc.), $g_c G_\mu = \xi_V i D_\mu \xi_V^\dagger$ for $\xi_V \sim qq$, while for EWD massive gauge bosons $W_\mu^\pm = G_\mu^{(\pm)}$ and $Z_\mu = \cos\theta G_\mu^{(3)} - \sin\theta B_\mu$ are disguised by composite weak bosons as

$$g W_\mu^\pm = \sqrt{2} \text{Tr}(\tau^{(\pm)} \bar{w}_L i D_\mu \bar{w}_L^\dagger)$$

and

$$g_Z Z_\mu = \sqrt{1 - (e/f)^2} \text{Tr}(\tau^{(3)} \bar{w}_L i D_\mu \bar{w}_L^\dagger)$$

for $\bar{w}_L \sim (\phi^G, \phi)$, where ϕ is the Higgs scalar.

The L - R -asymmetrically realized weak interactions allow the additional W and Z bosons to be associated with $\mathcal{S}\mathcal{U}(2)_{L(R)}^{\text{loc}}$ [for ρ -like mesons with L (R) coupling to fermions] or $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ (for ρ - and A_1 -like mesons with V and A couplings), but not to be associated with the QCD-like $\text{SU}(2)_V$ symmetry as discussed in the Sec. III. The composite vector mesons, V_μ , L_μ , and R_μ , in the confining “color” $\text{SU}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_L^{\text{loc}} \times \mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ gauge theory, are taken to be

$$\begin{aligned} f(V_\mu)_i^j &= (\bar{w}_L iD_\mu \bar{w}_L^\dagger)_i^j, \\ f_L(L_\mu)_i^j &= [\bar{w}_L (\xi iD_\mu \xi^\dagger) \bar{w}_L^\dagger]_i^j, \\ f_R(R_\mu)_i^j &= (\bar{w}_R iD_\mu \bar{w}_R^\dagger)_i^j. \end{aligned}$$

where \bar{w}_L and \bar{w}_R , respectively, carry the “color” $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ and $\mathcal{S}\mathcal{U}(2)_R^{\text{loc}}$ “colors” while ξ is the $\text{SU}(2)_L^{\text{loc}}$ and $\mathcal{S}\mathcal{U}(2)_L^{\text{loc}}$ doublet. Quarks and leptons are composites of the scalar $\bar{w}_{L,Ri}$ ($i=1,2$) and spinor $c_{L,R}^{0,A}$ ($A=1,2,3$) constituents:²¹

$$\begin{aligned} q_{iL(R)}^A &= \bar{w}_{L(R)i} c_{L(R)}^A \quad (\text{or } \bar{w}_{L(R)i} \xi c_{L(R)}^A), \\ l_{iL(R)} &= \bar{w}_{L(R)i} c_{L(R)}^0 \quad (\text{or } \bar{w}_{L(R)i} \xi c_{L(R)}^0), \end{aligned}$$

The interactions of q, l ($=\psi$) with the vector mesons are derived as $\bar{\psi}_L \gamma^\mu (i\partial_\mu + fV_\mu + eQ_{\text{em}} A_\mu^0) \psi_L$ for $\psi = \bar{w}c$ and $\bar{\psi}_L \gamma^\mu (i\partial_\mu + fV_\mu + f_L L_\mu + eQ_{\text{em}} A_\mu^0) \psi_L$ for $\psi = \bar{w}\xi c$ and those of ψ_R are as $\bar{\psi}_R \gamma^\mu (i\partial_\mu + f_R R_\mu + eQ_{\text{em}} A_\mu^0) \psi_R$. The kinetic mixings are specified by $\lambda_{\gamma V} = e/f$ ($=\sin\theta$) for $\gamma - V_\mu$, $\lambda_{\gamma L} = e/f_L$ ($=\sin\theta \sin\theta_L$) for $\gamma - L_\mu$, $\lambda_{\gamma R} = e/f_R$ ($=\cos\theta \sin\theta_R$), and $\lambda_{VL} = f/f_L$ ($=\sin\theta_L$) for $V_\mu - L_\mu$. Then, the composite vector mesons are

shown to simulate massive gauge bosons properly constructed from the fundamental gauge fields G_μ , $G_{L\mu}$, $G_{R\mu}$, and B_μ and then to satisfy the “duality.”

A similar procedure can also be possible for an extra Z boson associated with the unbroken “color”- $\text{U}(1)_C^{\text{loc}}$ symmetry^{22,23} as far as it is hidden inside composites. The weak bosons V_μ and extra Z boson X_μ^0 are $fV_\mu = \bar{w}_L iD_\mu \bar{w}_L^\dagger$ and $f_X X_\mu^0 / 2 = \xi iD_\mu \xi^\dagger$. The $\text{SU}(2)_L^{\text{loc}}$ “colored” \bar{w}_L carries either the $\text{U}(1)_{\text{em}}^{\text{loc}}$ charge $Q_{\text{em}} = \tau^{(3)}$ or the $\text{U}(1)_C^{\text{loc}}$ charge $Q_C = \tau^{(3)}$ while ξ has $Q_{\text{em}} = 1$ and $Q_C = -1$. Composites are arranged to be $\text{SU}(2)_L^{\text{loc}} \times \text{U}(1)_C^{\text{loc}}$ “color” singlets. Quarks ($\alpha=1,2,3$) and leptons ($\alpha=0$) are constructed as $\psi_L^\alpha = \xi^Q \Phi c_L^\alpha$ and $\psi_R^\alpha = \xi^{Q_i} c_{Ri}^\alpha$ with $\Phi_i = \bar{w}_{Li}$ for the $\text{U}(1)_{\text{em}}^{\text{loc}}$ -charged one; $= (\xi \bar{w}_{L1}, \xi \bar{w}_{L2})$ for the $\text{U}(1)_C^{\text{loc}}$ -charged one. The $\text{SU}(2)_L^{\text{loc}}$ “colored” fermions, $c_{L,R}$, carry $Q_{\text{em}} = (B - L - Q, 2Q_{\text{em}}^{q,l} - Q_i)$ and $Q_C = (Q, Q_i)$ for (c_L, c_R) . The interesting examples have been discussed by identifying the $\text{U}(1)_C^{\text{loc}}$ charge, $Q(Q_i)$, with (i) the lepton number for a “leptonic gluon,”²⁴ (ii) the baryon number for a “singlet gluon,”²² (iii) the hypercharge for a “hypercharge boson,”²³ in the case of the $\text{U}(1)_{\text{em}}^{\text{loc}}$ -charged \bar{w}_L , and (iv) the hypercharge for a “heavy photon,”²² in the case of the $\text{U}(1)_C^{\text{loc}}$ -charged \bar{w}_L . The details will be found elsewhere.²⁵

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