Nonperturbative calculations of h_c and h_b masses

A. M. Badalyan*
Theoretical Physics Institute, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455

V. P. Yurov
Institute of Theoretical and Experimental Physics, Moscow, 117259, U.S.S.R.
(Received 14 May 1990)

Using the vacuum correlator method the nonperturbative contribution to the P-wave spin-spin shift $\Delta = M_{\text{c.o.g.}}(n\,^3P_J) - M(n\,^1P_1)$ is calculated. It is shown that the sign and absolute value of Δ are very sensitive to the sign and amplitude σ_1 of the nonconfining vacuum correlator D_1. If $\sigma_1 < 0$ then the shift Δ is also negative, but if $\sigma_1 > 0$ we have obtained a positive $\Delta > 0$ in contrast with the prediction in the framework of perturbative QCD.

I. INTRODUCTION

Until now we had only very preliminary data[1,2] about the masses of the h_c and h_b mesons. These data can only be considered as a hint about the possible positive sign of the mass difference Δ:

$$\Delta(n) = M_{\text{c.o.g.}}(n\,^3P_J) - M(n\,^1P_1)\;, \tag{1}$$

where $M_{\text{c.o.g.}}(n\,^3P_J)$ is the center-of-gravity mass for $n\,^3P_J$ states. In the near future new accurate experimental measurements of h_c and h_b are expected from Fermilab[1] and CLEO Collaborations.[2] These data are very important from theoretical points of view and the main goal of this paper is to show why the exact information about the h_c, h_b masses could represent a unique opportunity to understand the role of nonperturbative (NP) effects in heavy quarkonia.

In the framework of perturbative QCD, masses of $^1P_1$ states for $c\bar{c}$ and $b\bar{b}$ were studied in detail in Refs. 3–5. It was shown that Δ^{pert} in the leading order of α_s has the form

$$\Delta^{\text{pert}}(n) = \frac{8\alpha_s^2}{9\pi m_q^2}\left[\frac{1}{4} - \frac{N_f}{3}\right]\langle r^{-3}\rangle_n\;. \tag{2}$$

Expression (2) is model dependent through matrix element $\langle r^{-3}\rangle$ and for theoretical predictions it is more instructive to use another form for Δ^{pert},

$$\Delta^{\text{pert}}(n) = [M(n\,^3P_2) - M(n\,^3P_0)]\frac{10\alpha_s}{81\pi}\left[\frac{1}{4} - \frac{N_f}{3}\right]\;, \tag{3}$$

which was derived by Pantaleone and Tye[5] in the same leading order of perturbation theory.

From (3) and (2) it is evident that in perturbative QCD Δ^{pert} always has a negative sign, because $M(^3P_2) > M(^3P_0)$ and $\frac{1}{4} - N_f/3 < 0$ for any $N_f \geq 1$. In Ref. 5 the following numerical values of Δ have been predicted:

$$\begin{aligned}
&\Delta^{\text{pert}}(c\bar{c}, 1P) = -1.4\ \text{MeV}\;,\\
&\Delta^{\text{pert}}(b\bar{b}, 1P) = -0.5\ \text{MeV}\;,\\
&\Delta^{\text{pert}}(b\bar{b}, 2P) = -0.4\ \text{MeV}\;,
\end{aligned} \tag{4}$$

which implies that, in (4), $\alpha_s(c\bar{c}) = 0.23 (N_f = 4)$, $\alpha_s(b\bar{b}) \equiv 0.17$.

II. THE NONPERTURBATIVE INTERACTIONS

In this paper we shall calculate the NP contribution to masses of h_c and h_b mesons. For this purpose we will use the new method developed by Simonov and Dosch in Refs. 6–8, the so-called vacuum correlator method (VCM). In Ref. 8 it was shown that starting from QCD and using the VCM one comes to the usual potential picture if a characteristic quark time T_q is assumed to be much larger than a vacuum correlation time $T_g (T_q \gg T_g)$. In the framework of VCM new specific and important features appear: all potentials—static, spin-spin, spin-orbit, tensor—are expressed only through two correlation functions, denoted as D and D_1.[8] In particular, the static NP potential $\epsilon(r)$ and spin-spin NP potential V_{SS} are given by expressions

$$\epsilon(r) = \beta\left\{2r\int_0^r d\lambda \int_0^\infty d\nu\, D(\lambda,\nu) + \int_0^r \lambda\, d\lambda \int_0^\infty d\nu[-2D(\lambda,\nu) + D_1(\lambda,\nu)]\right\}\;, \tag{5}$$

$$V_{\text{SS}}^{NP}(r) = \frac{V_4^{\text{NP}}(r)}{3m_q^2}\mathbf{s}_1\cdot\mathbf{s}_2\;, \tag{6a}$$

$$V_4^{\text{NP}}(r) = 2\beta\int_0^\infty d\nu\left[3D(r,\nu) + 3D_1(r,\nu) + 2r^2\frac{\partial D_1}{\partial r^2}\right]\;. \tag{6b}$$

The expressions for spin-orbit and tensor potentials through D and D_1 are also given in Ref. 8. In (5), and (6)

D and D_1 are the bilocal correlators which are defined through gauge-invariant averages of vacuum fields $F_{\mu\nu}(x)$:

$$g^2\langle F_{\rho\mu}(x)\Phi(x,y)F_{\sigma\nu}(y)\Phi(y,x)\rangle = f_{\rho\mu\sigma\nu}D[(x-y)^2]+f^{(1)}_{\rho\mu\sigma\nu}D_1[(x-y)]^2 \quad (7)$$

In (7) Φ is a vacuum field transporter,

$$\Phi(x,y)=P\exp\left[ig\int_y^x A_\mu(z)dz_\mu\right],$$

and the tensors f and f_1 have the algebraic structure

$$\begin{aligned} f_{\rho\mu\sigma\nu} &= \hat{1}\beta(\delta_{\rho\sigma}\delta_{\mu\nu}-\delta_{\rho\nu}\delta_{\mu\sigma}) \\ f^1_{\rho\mu\sigma\nu} &= \hat{1}\frac{\beta}{2}\left[\frac{\partial}{\partial x_\mu}(h_\nu\delta_{\rho\sigma}-h_\sigma\delta_{\rho\nu}) + \frac{\partial}{\partial x_\rho}(h_\sigma\delta_{\mu\nu}-h_\nu\delta_{\mu\sigma})\right] . \end{aligned} \quad (8)$$

Here $h_\nu = x_\nu - y_\nu$, and the constant β is connected with the gluon condensate:

$$\beta=\frac{g^2\langle \mathrm{tr}F^2(0)\rangle}{12N_c[D(0)+D_1(0)]} . \quad (9)$$

Note that the variables λ,ν in (5) and (6) are simply connected with space-time Euclidean coordinates y and x in Eq. (7):

$$\begin{aligned} (x-y)^2 &= (\bar{x}-\bar{y})^2+\tau^2 , \\ \lambda^2 &\equiv (\bar{x}-\bar{y})^2, \quad \nu^2\equiv\tau^2 . \end{aligned} \quad (10)$$

The very important question is how to choose these vacuum correlators D and D_1. Until now they are still unknown functions in QCD although in principle they can be calculated in lattice QCD. Moreover, there are some preliminary calculations of D (Ref. 9) that indicate that correlator D drops exponentially with the growth of $|x-y|$. Also it was proven[6] that to guarantee the linear confinement the D correlator should be nonzero and decrease more rapidly than $(x-y)^{-2}$.

As to the nonconfining D_1 correlator we do not yet have any information about it. Also $D(0)+D_1(0)$ can be chosen positive to provide positive gluonic condensate [as separated out as in (9), it is a matter of convention]. In another paper[10] we chose the most simple Gaussian form: namely,

$$\begin{aligned} D(x,y) &= D(\lambda,\nu)=ae^{-\gamma(\lambda^2+\nu^2)} , \\ D_1(x,y) &= D_1(\lambda,\nu)=a_1e^{-\gamma_1(\lambda^2+\nu^2)} . \end{aligned} \quad (11)$$

Then for this choice of D and D_1 all NP potentials are given by analytical expressions, in particular for $\epsilon(r)$ and $V_4(r)$ we have obtained

$$\begin{aligned} \epsilon(r) &= V^{\mathrm{NP}}_{\mathrm{static}}(r) \\ &= r\sigma\,\mathrm{erf}(\sqrt{\gamma}r)-\frac{\sigma}{\sqrt{\pi\gamma}}(1-e^{-\gamma r^2}) \\ &\quad +\frac{\sigma_1}{2\sqrt{\pi\gamma_1}}(1-e^{-\gamma_1 r^2}) \end{aligned} \quad (12)$$

$$\begin{aligned} V^{\mathrm{NP}}_4(r) &= 6\sigma\left[\frac{\gamma}{\pi}\right]^{1/2} e^{-\gamma r^2} \\ &\quad +2\sigma_1\left[\frac{\gamma_1}{\pi}\right]^{1/2}(3-2\gamma_1 r^2)e^{-\gamma_1 r^2} . \end{aligned} \quad (13)$$

Here we introduced the string tension σ:

$$\sigma=\frac{\beta\pi a}{2\gamma}, \quad a=D(0) , \quad (14)$$

which is connected with Regge slope α' and usually adopted numerical value 0.18 ± 0.03 GeV^2. The parameter σ_1 was defined in a similar way but it is not necessarily positive:

$$\sigma_1=\frac{\beta\pi a_1}{2\gamma_1}, \quad a_1=D_1(0) . \quad (15)$$

This set of parameters $(\sigma,\gamma,\sigma_1,\gamma_1)$ was fitted in[10] to describe the gross features of $b\bar{b}$ and $c\bar{c}$ spectra, when the hyperfine and fine structure of levels was not taken into account. It was shown in[10] that to describe experimental $c\bar{c}$ and $b\bar{b}$ data we need (i) the correlation length $l=1/\sqrt{\gamma}\cong 0.1$ fm should be smaller than correlation length $l_1=1/\sqrt{\gamma_1}\cong 0.5-0.7$ fm, or $\gamma\gg\gamma_1$ ($\gamma\cong 2-6$ GeV^2, $\gamma_1=0.1-0.5$ GeV^2); (ii) the correct absolute value of mass J/ψ and $\Upsilon(1S)$ can be obtained not introducing any additive constant (usually for the potential of Eichten *et al.*[11] with $\epsilon(r)=\sigma r$, the large flavor-dependent constant is needed); (iii) the gross features of spectra are weakly dependent on the values and even sign of σ_1 parameter, if $|\sigma_1|\le 0.18$ GeV^{-2} or $|\sigma_1|\le\sigma$; (iv) the fine and hyperfine splittings for P-wave states are very sensitive to sign and also absolute value of σ_1; and (v) the best description of $b\bar{b}, c\bar{c}$ spectra was obtained with current quark masses: $m_b\cong 4.8$ GeV, $m_c\cong 1.4$ GeV.

We give below two sets of parameters: set A for which

$$\begin{aligned} &\sigma=0.2\ \mathrm{GeV}^2, \quad \gamma=4\ \mathrm{GeV}^2 \\ &\sigma_1=-0.18\ \mathrm{GeV}^2, \quad \gamma_1=0.08\ \mathrm{GeV}^2 , \end{aligned} \quad (16)$$

and set B for which $\sigma_1>0$ and

$$\sigma=0.16\ \mathrm{GeV}^2, \quad \gamma=4\ \mathrm{GeV}^2, \quad \gamma_1=0.20\ \mathrm{GeV}^2 \quad (17)$$

and σ_1 has two different positive values, e.g., $\sigma_1=0.01$ GeV^2 and $\sigma_1=0.16$ GeV^2.

For both set A and set B we shall use current quark masses: $m_b=4.8$ GeV, $m_c=1.35$ and 1.5 GeV.

III. HEAVY-QUARKONIA SPECTRA

To calculate heavy-quarkonia spectra we should define perturbative or Coulomb part of interaction, which is also very important. For $V_{\mathrm{pert}}(r)\equiv E(r)$ we have used[10] the conventional one-loop expression

$$\begin{aligned} E(r) &= -\frac{4}{3}\frac{\alpha_s(r)}{r} , \\ \alpha_s(r) &= \frac{16\pi}{(33-2N_f)\ln(1/\bar{\Lambda}^2 r^2)} . \end{aligned} \quad (18)$$

TABLE I. Masses $M_{\text{c.o.g.}}(n\,^3L_J)$ (MeV) and leptonic width $\Gamma_{e^+e^-}(1S)$ (keV) for low-lying states in bottomonium for different sets of parameters $\sigma,\gamma,\sigma_1,\gamma_1$ ($m_b=4.8$ GeV, $\alpha_s^{\max}=0.39$, $\tilde{\Lambda}=0.46$ GeV for any set).

Set of parameters σ_1 (GeV^2)	$M(1S)$	$M(2S)-M(1S)$	$M(3S)-M(2S)$	$M(2S)-M(1P)$	$M(2P)-M(1P)$	$\Gamma_{e^+e^-}$
Set A						
$\sigma_1=-0.18$	9456	554	342	112	351	1.22
Set B						
$\sigma_1=-0.01$	9448	558	342	116	357	1.21
Set B						
$\sigma_1=0.16$	9462	570	343	117	359	1.25
Experiment	9460	563	332	123	360	1.34±0.005

We have also imposed an additional condition for $\alpha_s(r)$: $\alpha_s(r)\le\alpha_s^{\max}$, where $\alpha_s^{\max}$ is a fixed constant which is reached at some critical point r_{cr}:

$$\alpha_s(r)=\alpha_s^{\max} \quad \text{for } r\ge r_{\text{cr}} \ . \tag{19}$$

This condition was already used before[12] and now can be understood as a result of an influence of the strong vacuum field (for discussion of this question see Ref. 8).

Note also that the choice of $N_f=3$ or $N_f=4$ does not change practically (with 1% accuracy) the eigenvalues of the Schrödinger equation and we fixed $N_f=3$ both for $c\bar{c}$ and $b\bar{b}$ quarkonia.

Some remarks about $\tilde{\Lambda}$ in (18) follow. The value of $\tilde{\Lambda}$ in (18) should be different from Λ_{QCD} because to get a coordinate-space potential via a Fourier transform we should know $\alpha_s(q^2)$ beyond the asymptotic freedom region (the latter refers to large q^2 or $r\lesssim0.01$ fm). Thus a value of $\tilde{\Lambda}$ depends on the chosen interpolation function $\alpha_s(q^2)$ when we go from large momenta q^2 to small q^2. For some reasonable choice of interpolation function $\tilde{\Lambda}\simeq\Lambda_{\text{QCD}}e^{3C/2}$ where $C=0.5772$ is the Euler constant, or for $\Lambda_{\text{QCD}}=0.1$–0.2 GeV we have $\tilde{\Lambda}=0.24$–0.48 GeV.

Our generic results for $c\bar{c},b\bar{b}$ spectra are presented in Tables I and II and are in rather good agreement with experimental data. Note that a better fit for the ground-state masses can be reached by a small variation of current quark mass, in particular, $m_b=4.77$ GeV and $m_c=1.4$ GeV are preferable.

Having fitted all parameters $\sigma,\gamma,\sigma_1,\gamma_1$ as a next step we have calculated hyperfine and fine structure of S- and P-wave levels. The most striking and transparent result was obtained for hyperfine P-wave splitting which we shall discuss below. As for the fine splitting structure, it turns out to be very sensitive to the choice of Coulomb parameters $\alpha_s^{\max}$ and $\tilde{\Lambda}$ and we shall discuss this problem in another paper.

The masses of h_b and h_c mesons ($^1P_1$ state, $S=0$) are defined as

$$M(n\,^1P_1)=M_{\text{c.g.}}-\Delta \ , \tag{20}$$

where $\Delta=\Delta_{\text{pert}}+\Delta_{\text{NP}}$ was introduced in (1) and Δ_{NP} can be expressed through matrix elements of potential V_{ss} or $V_4(r)$, namely

$$\Delta_{\text{NP}}=\frac{1}{3m_q^2}\langle V_4^{\text{NP}}\rangle \ . \tag{21}$$

For fixed parameters (set A and set B) the calculated values of Δ_{NP} are presented in Table III, where one can see that the sign of Δ_{NP} directly depends on the sign of parameter σ_1.

TABLE II. Masses $M_{\text{c.o.g.}}(n\,^3L_J)$ (MeV) and leptonic width $\Gamma_{e^+e^-}(1S)$ (keV) for low-lying states in charmonium for different sets of parameters $\sigma,\gamma,\sigma_1,\gamma_1$ ($\alpha_s^{\max}=0.39$, $\tilde{\Lambda}=0.46$ GeV for any set).

Set of parameters σ_1 (GeV^2)	$M(1S)$	$M(2S)-M(1S)$	$M(3S)-M(2S)$	$M(2S)-M(1P)$	$M(2P)-M(1P)$	$\Gamma_{e^+e^-}$
Set A						
$m_c=1.35$ GeV	3011	582	451	173	474	4.24
$\sigma_1=-0.18$						
Set $\tilde{\text{B}}$						
$m_c=1.5$ GeV	3251	564	410	155	430	4.46
$\sigma_1=-0.01$	[3080[a]]					
Set B						
$m_c=1.5$ GeV	3294	604	418	168	441	4.48
$\sigma_1=0.16$	[3124[a]]					
Experiment	3098	594	378[b]	161	absent	4.72±0.35

[a]This value of J/ψ mass was calculated with $m_c=1.4$ GeV.
[b]$\Psi(3S)$ lies above the threshold of open charm.

TABLE III. The nonperturbative contribution $\Delta_{\rm NP}$ (MeV) to the full shift $\Delta = M_{\rm c.o.g.}(n\,{}^3P_J) - M(n\,{}^1P_1)$ for different sets of parameters $\sigma, \gamma, \sigma_1, \gamma_1$ ($\alpha_s^{\max}=0.39$, $\tilde{\Lambda}=0.46$ GeV, $m_b=4.8$ GeV, $m_c=1.35$ GeV). The values of $\Delta_{\rm pert}$ are taken from Ref. 5.

Set of parameters σ_1 (GeV^2)	$c\bar{c}$ $\Delta_{\rm NP}(P)$	$c\bar{c}$ $\Delta(1P)$	$c\bar{c}$ $\Delta_{\rm NP}(2P)$	$b\bar{b}$ $\Delta_{\rm NP}(1P)$	$b\bar{b}$ $\Delta(1P)$	$b\bar{b}$ $\Delta_{\rm NP}(2P)$	$b\bar{b}$ $\Delta(2P)$
Set A $\sigma_1=-0.18$	-6.6	-8.0	-3.3	-1.26	-1.76	-1.27	-1.67
Set B $\sigma_1=0.01$	0.27	-1.13	0.34	0.42	-0.08	0.35	-0.05
Set B $\sigma_1=0.18$	3.2	1.8	2.74	0.82	0.32	0.74	0.34

For $\sigma_1<0$, or for negative amplitude of correlator D_1, we have obtained $\Delta_{\rm NP}<0$, i.e., the same sign as in perturbative QCD.[5] It means that the full sum Δ is also negative and has rather large absolute value (for $\sigma_1=-0.18$ GeV^2, $\Delta\cong -8$ MeV for h_c and $\Delta\equiv -1$ MeV for h_b). In this case ${}^1P_1$ states lie above $M_{\rm c.o.g.}(n\,{}^3P_J)$.

For $\sigma_1=0$ the sign of $\Delta_{\rm NP}$ is already changing and becoming positive. In Table III the values of $\Delta_{\rm NP}$ are given for $\sigma_1=0.01$ GeV^2 and as for $\sigma_1=0$ these values are very small, so that the full shift Δ remains negative due to perturbative contribution both for $h_c(\Delta\cong -1.1$ MeV) and for $h_b(1P)$, where $\Delta\cong -0.1$ MeV.

The situation is more interesting for relatively large positive values σ_1, e.g., for $\sigma_1\cong\sigma\cong 0.16$ GeV^2 (see Table III, set B). In this case $\Delta_{\rm NP}$ is positive and larger than $\Delta_{\rm pert}$ so that the full shifts Δ are also positive, e.g., $\Delta=1.8$ MeV for $h_c(1P)$ and $\Delta=0.3$ MeV for h_b meson. It means that for $\sigma_1\cong 0.1$–0.2 GeV^2 $M({}^1P_1)$ state lies below $M_{\rm c.g.}({}^3P_J)$ both for $1P$ and $2P$ levels.

IV. SUMMARY

In conclusion we would like to emphasize that the experimental measurements of h_c, h_b masses are very important not only for the fullness of the spectroscopic picture in heavy quarkonia. Having an opportunity to calculate NP effects in heavy quarkonia with the help of the VC method we have observed that (i) the sign of spin-spin shift $\Delta_{\rm NP}$ for P-wave levels coincides with the sign of parameters σ_1 which enter into the D_1 correlator, if $\sigma_1\neq 0$, (ii) if $\sigma_1<0$, then both NP and perturbative contributions to Δ are negative and the ${}^1P_1$ level lies above $M_{\rm c.o.g.}({}^3P_J)$, (iii) if $\sigma_1=0$ or $\sigma_1\le 0.05$ GeV^2, then the sign of Δ is very sensitive to the choice of different parameters but in any case the absolute value of Δ should be very small ($|\Delta|\le 1$ MeV for h_c), and (iv) if $\sigma_1>0$ or $\sigma_1\ge 0.05$ GeV^2, then the NP shift $\Delta_{\rm NP}$ is becoming positive and larger than $\Delta_{\rm pert}$, so that the ${}^1P_1$ level lies below $M_{\rm c.o.g.}({}^3P_J)$ both for $1P$ and $2P$ levels.

Here it is worthwhile to note that for light mesons (a and f) we have the following situation: $h_1(1170\pm 40$ MeV) has a smaller mass than $M_{\rm c.o.g.}(1\,{}^3P_J$, f mesons$)=1242$ MeV and the mass of $b_1(1235\pm 10$ MeV) is smaller $M_{\rm c.o.g.}(1\,{}^3P_J)=1262$ MeV for a mesons. Unfortunately, ${}^3P_J$ states for light mesons are rather broad, so $\Delta_{\rm expt}$ has a rather large experimental error and we can speak about positive shifts Δ (for a, f mesons) only in a limited sense.

The situation with heavy h_c, h_b mesons could be more interesting and unambiguous. If we know the h_c, h_b masses we shall obtain the unique opportunity to extract useful information about the D_1 vacuum correlation function which is of fundamental importance for nonperturbative QCD.

The author is grateful to Yuri A. Simonov for many useful remarks and to K. Seth and J. Rosen for fruitful discussions.

*Permanent address: ITEP, Moscow, 117259, U.S.S.R.

[1] R704 Collaboration, C. Beglin *et al.*, Phys. Lett. B **171**, 135 (1986).
[2] CLEO Collaboration, T. Bowcock *et al.*, Phys. Rev. Lett. **58**, 307 (1987).
[3] J. Pantaleone, S.-H. H. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986).
[4] K. Igi and S. Ono, Phys. Rev. D **33**, 3349 (1986); **37**, 1338(E) (1988).
[5] J. Pantaleone and S.-H. H. Tye, Phys. Rev. D **37**, 3337 (1988).
[6] H. G. Dosch and Yu. A. Simonov, Phys. Lett. B **205**, 339 (1988).
[7] Yu. A. Simonov, Nucl. Phys. **B307**, 512 (1988).
[8] Yu. A. Simonov, Nucl. Phys. **B324**, 67 (1989).
[9] M. Campostrini, A. DiGiacomo, and G. Mussardo, Z. Phys. C **25**, 173 (1984).
[10] A. M. Badalyan and V. P. Yurov, Yad. Fiz. **51**, 1368 (1990).
[11] E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980).
[12] N. Isgur and S. Godfrey, Phys. Rev. D **32**, 189 (1985).