Isocurvature baryon perturbations and inflation

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It has been observed that if an extra scalar field (in addition to the inflaton) is present during the inflationary phase, its decay into thermal radiation after baryogenesis gives rise to fluctuations in the initially smooth entropy-per-baryon ratio. There was a hope that these perturbations were of the isocurvature type and that they may help explain several observed features in the large-scale structure of the Universe. We study in detail the generation of perturbations in such a two-field inflationary model. We find that the resulting fluctuations are not of the isocurvature type, but that the entropy perturbation induces a curvature fluctuation which is larger than the entropic one before the wavelengths of the perturbations enter the Hubble radius. Thus, this model is not a good candidate to provide the initial conditions for the baryon isocurvature perturbations.

I. INTRODUCTION

One of the main achievements of the inflationary scenario is that it has provided a natural mechanism to explain the origin of the energy-density fluctuations in the early Universe. The usual picture[1] is that they arise during the inflationary era from quantum fluctuations of the scalar field which drives inflation and that these quantum fluctuations give rise to perturbations in the classical energy density when the wavelength of the perturbations becomes larger than the Hubble radius. When the scalar field decays, the fluctuations in the energy density of its decay products follow the original fluctuations of the scalar field. In the usual model, baryogenesis occurs after the reheating due to the decay of a heavy boson, but it is also possible to produce the baryon asymmetry during the process of decay of another scalar field.[2] In both cases the ratio of the resulting baryon asymmetry to entropy is only dependent on microphysical parameters, such as the coupling constants and the temperature at which B-violating interactions go out of equilibrium, and it is not expected to show any spatial variation. Thus, these fluctuations are of the adiabatic type.

However, it has been noticed that adiabatic fluctuations are not the only possibility, but that fluctuations of the isocurvature or isothermal type may also be produced provided that other scalar fields were present during the inflationary era. This point has first been studied in the case of the axion[3] and has then been extended to more general scalar fields. Since the contribution of these scalar fields to the global energy density during inflation was small, their fluctuations did not affect it too much and so they can be considered to be of the isocurvature type. However, the interest in these models comes from the fact that, if some of these fields interact weakly with the other fields, during the expansion of the Universe, their energy density will decay more slowly than that of the products of the inflaton decay, so that they will eventually dominate and their inhomogeneities will becomes important. The resulting spectrum of density fluctuations in a wide class of models including several interacting scalar fields has been studied in Ref. 6.

Further, it has recently been pointed out by Peebles[7] that if one of these scalar fields decays into radiation after baryogenesis, the fluctuations of this scalar field will give rise to fluctuations in the ratio of baryon asymmetry to entropy, which are absent in other models that do not modify the baryogenesis mechanism. This gives rise to baryon isocurvature fluctuations. (There is another scenario in which baryon isocurvature fluctuations arise, recently proposed by Turner, Cohen, and Kaplan.[8] In this model an alternative baryogenesis scenario[9] is considered, the so-called spontaneous baryogenesis, in which the baryon asymmetry produced is a function of spatial position, but it is not in the scope of this work to analyze it.)

Finally, the interest in these models has increased recently as some problems have been found within the standard cold-dark-matter adiabatic perturbation model.[10] The main points are that the epoch of galaxy formation seems to occur too late and that the fluctuations in the mass distribution are anticorrelated on scales larger than $\sim 50-100$ Mpc, which seems to be inconsistent with observations of large-scale velocity fields and structures in the galaxy distribution.[11] This calls attention to models with baryon isocurvature initial fluctuations, as in this context galaxies can form early ($z \sim 30$), and mass fluctuations on the scale λ_J can drive large-scale velocity fields.[12] On the other hand, detailed studies of the cosmic-microwave-background radiation anisotropies show that this is a crucial point to test the viability of these models.[13,14] This is the reason why it is interesting to develop a model for the origin of this type of fluctuations.

In this paper, we analyze in detail the fluctuations produced in models with an additional scalar field that is present during inflation and that decays into radiation afterwards. The two main steps in this study are the calculation of the spectrum of quantum fluctuations during the inflationary era (this will give the initial conditions for the classical fluctuations), and following the evolution of the fluctuations outside the Hubble radius, for which it is necessary to know the evolution of the background unperturbed variables. This will allow us to know the amplitude of the density fluctuations when they reenter the Hubble radius, and to estimate if they are predominantly of the adiabatic or the isocurvature type. The subsequent evolution of the fluctuations has been studied before in the context of phenomenological models, which assume the isocurvature as an initial condition in the radiation-dominated era.

In order to study the evolution of the fluctuations from the time they leave the Hubble radius during the inflationary era up to the time they reenter the Hubble radius in the radiation- or matter-dominated era and then inside the Hubble radius, it is necessary to follow the evolution of the fluctuations in the multicomponent system composed by the inflaton, the products of its decay, the other scalar field and eventually the products of its decay (for example, in the Peebles model analyzed here, the scalar field decays into radiation). This study is simplified if we consider one component as composed by the inflaton ϕ and the radiation and baryons to which it decays $(\phi+R_{\phi}+B_{\phi})$ and another component by the other scalar field and its decay products $(\chi+R_{\chi})$. With this choice, we can reduce the problem to the study of the evolution of the fluctuations in a system of two uncoupled fluids at least up to the time at which χ decays in radiation. Up to this time, we can assume that the stress tensor of each component is individually conserved $T^{\alpha}{}_{\mu\nu}{}^{;\nu}=0$ (we will use the greek indices α and β for the fluid components and use μ,ν for the tensorial labels, running from 0 to 3). After χ decays in radiation, it is necessary to consider the momentum transfer from one component to the other through electron scattering.

The evolution of the perturbations in a multicomponent system has been studied by Kodama and Sasaki[15] in the linear approximation. As this is the formalism used here, a brief review of the variables used to describe the system and their equations of motion is given in the Appendix. We will consider only the case of a spatially flat-spacetime background.

In addition to being interested in the evolution of the gauge-invariant fluctuations of the energy density and velocity of each component Δ_{α} and V_{α}, and of the total fluid ones Δ and V, we are also interested in the entropy fluctuations, which are given by

$$S_{\alpha\beta}=\frac{\delta_{\alpha}}{1+w_{\alpha}}-\frac{\delta_{\beta}}{1+w_{\beta}}\ , \tag{1.1}$$

where $\delta_{\alpha}\equiv\delta\rho_{\alpha}/\rho_{\alpha}$ and $w_{\alpha}\equiv p_{\alpha}/\rho_{\alpha}$, in terms of the energy density ρ and the pressure p. $S_{\alpha\beta}$ is gauge invariant and measures the relative fluctuations between components.

Adiabatic fluctuations are characterized by having $S_{\alpha\beta}=0$, which means that all the components fluctuate in the same way. Isocurvature fluctuations instead correspond to relative fluctuations between the different components such that the total curvature is left invariant.

In general fluctuations will not be exclusively of the adiabatic or isocurvature type. In order to see which is the dominant mode in a particular problem, the magnitude of the entropy perturbation $S_{\alpha\beta}$ and the total energy perturbation Δ must be compared. If $|S_{\alpha\beta}|\gg|\Delta|$, this means that the fluctuations in the individual components compensate one with another giving a small total energy-density fluctuation, and in this case we can say that the fluctuations are predominantly of the isocurvature type.

On the other hand, entropy and energy-density perturbations are not decoupled, even outside the Hubble radius. In particular, as it has been pointed out in Refs. 16 and 17, entropy perturbations act as a source for density fluctuations. We follow in the particular model considered in this paper the evolution of entropy and energy-density perturbations outside the Hubble radius. The main result obtained is that, in the model studied here, perturbations which were of the isocurvature type at the Hubble radius crossing during inflation generate a large curvature perturbation by the Hubble radius reentrance time in the radiation- or matter-dominated era. The sources of this effect are nonadiabatic pressure perturbations originated by the presence of the entropy perturbation $S_{\alpha\beta}$.

The organization of the paper is as follows. In Sec. II we study the perturbations in the energy density and velocity of a two-component system originated by quantum fluctuations during inflation of the two scalar fields. In Sec. III we specify the model analyzed in this paper in detail and follow the evolution of the unperturbed variables. In Sec. IV we compute the amplitudes of the adiabatic and isocurvature modes when a given wavelength leaves the Hubble radius, and follow the subsequent evolution up to the time it reenters the Hubble radius. Section V is devoted to a discussion of the results.

Units are chosen so that $c=8\pi G=\hbar=1$.

II. QUANTUM FLUCTUATIONS OF TWO UNCOUPLED SCALAR FIELDS

Let ϕ be the inflation field and χ the other scalar field which contributes to the energy density much less than ϕ during inflation: $\rho_{\chi}\ll\rho_{\phi}$. Both fields will have quantum fluctuations during inflation, $\delta\phi^2(x,t)\equiv\langle\phi(x,t)\phi(0,t)\rangle$ and $\delta\chi^2(x,t)\equiv\langle\chi(x,t)\chi(0,t)\rangle$.

We define the Fourier transform of these quantities as

$$\delta\phi^2(x,t)\equiv\frac{1}{(2\pi)^3}\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\delta\phi^2(k,t)\ , \tag{2.1}$$

and similarly for χ. These fluctuations will be computed in the context of generalized inflationary cosmologies,[18,19] making it possible to apply this analysis to a variety of inflationary models, and as a particular case to the usual exponential inflation. The scale factor takes the form

$$a(t)=a_*\left[1+\frac{H_*}{p}(t-t_*)\right]^p\ , \tag{2.2}$$

where a_*, H_*, and t_* are constants and $p=2/3(1+w)$. For $p>1$, it corresponds to "power law" or "subinflation," for $p\to\infty$ it describes exponential inflation and for negative p "pole law" or "superinflation,"[19] Solving the Klein-Gordon equation for a massless scalar field in the expanding background and replacing it in the definition of $\delta\phi^2(x,t)$, the expression obtained for the Fourier transformation is

$$\delta\phi^2(k,t)=\frac{\pi}{4}\frac{1}{a^3H}\frac{p}{p-1}\left|H_\nu\left[\frac{k}{aH}\frac{p}{p-1}\right]\right|^2 , \quad (2.3)$$

where $H\equiv\dot{a}/a$ is the Hubble constant, H_ν are the Hankel functions and $\nu=(1-3p)/2(1-p)$. For wavelengths well outside the Hubble radius, it can be approximated by

$$\delta\phi^2(k,t)\simeq\frac{1}{4\pi}\frac{p}{p-1}\frac{|\Gamma(\nu)|^2}{a^3H}\left|\frac{k}{aH}\frac{p}{2(p-1)}\right|^{-2\nu} . \quad (2.4)$$

Note that for $p\rightarrow\infty,\nu\rightarrow\frac{3}{2}$ and we recover the quantum fluctuations of a scalar field in a de Sitter space,

$$\delta\phi_k^2\rightarrow\frac{H^2}{2k^3} , \quad (2.5)$$

which corresponds to a scale-invariant spectrum of density fluctuations. However, the spectral index is modified if other values of p are considered, as can be seen from Eq. (2.4).

The gauge-invariant fluctuations in the energy density and velocity produced by the fluctuations of the scalar fields can be computed as follows (see the Appendix):

$$\rho_\alpha\Delta_\alpha=\rho_\alpha\delta_\alpha+3\rho_\alpha(1+w_\alpha)\frac{Ha}{k}(v_\alpha-B) . \quad (2.6)$$

Comparing the perturbed stress tensor of a scalar field, given by Eq. (A3), with that of a fluid, given by Eq. (A2), we can identify

$$\rho_\phi\delta_\phi=-A\dot{\phi}^2+\dot{\phi}\delta\dot{\phi}+U_\phi\delta\phi , \quad (2.7a)$$

$$\rho_\phi(1+w_\phi)v_\phi=B\dot{\phi}^2+\frac{k}{a}\dot{\phi}\delta\phi , \quad (2.7b)$$

where U denotes the potential energy of the scalar field and $U_\phi\equiv\partial U/\partial\phi$. So

$$\rho_\phi\Delta_\phi=-A\dot{\phi}^2+\dot{\phi}\delta\dot{\phi}-\ddot{\phi}\delta\phi . \quad (2.8)$$

It is possible to associate a gauge-invariant variable to $\delta\phi$ by[20]

$$D\phi=\delta\phi+\frac{a}{k}\left[B-\frac{a}{k}\dot{H}_T\right]\dot{\phi} , \quad (2.9)$$

in terms of which the gauge-invariant perturbation to the stress tensor of the scalar field can be expressed as

$$\dot{\phi}V_\phi=\frac{k}{a}D\phi , \quad (2.10a)$$

$$\rho_\phi\Delta_\phi=\dot{\phi}^2\Phi+\dot{\phi}(D\phi)^{\cdot}-\ddot{\phi}D\phi , \quad (2.10b)$$

where Φ is a gauge-invariant quantity which characterizes the perturbations in the geometry and is defined in the Appendix. Similar equations hold for the fluctuations corresponding to the scalar field χ.

We are interested in computing the magnitude of the fluctuations in the individual components and total energy density and velocity at Hubble radius crossing $(k/aH=1)$ in terms of he fluctuation in the scalar field $D\phi$. Noting that

$$\Phi=\frac{a^2}{2k^2}(\rho_\phi\Delta_\phi+\rho_\chi\Delta_\chi) \quad (2.11)$$

can be computed from Eq. (2.10b) and its equivalent for χ as

$$\Phi\left[2\frac{k^2}{a^2}-(\dot{\chi}^2+\dot{\phi}^2)\right]=\dot{\phi}(D\phi)^{\cdot}-\ddot{\phi}D\phi+\dot{\chi}(D\chi)^{\cdot}-\ddot{\chi}D\chi , \quad (2.12)$$

we see that at Hubble radius crossing the total density perturbation is

$$\Delta|_H\simeq\frac{1}{3H^2}[\dot{\phi}(D\phi)^{\cdot}-\ddot{\phi}D\phi+\dot{\chi}(D\chi)^{\cdot}-\ddot{\chi}D\chi]\Big|_H , \quad (2.13)$$

where the contribution of the kinetic energy to the total energy during inflation has been neglected $(\dot{\chi}^2,\dot{\phi}^2\ll H^2)$.

In the same way, the total velocity fluctuation can be computed from (2.10a) and its equivalent for χ:

$$V=\frac{\dot{\phi}^2V_\phi+\dot{\chi}^2V_\chi}{\dot{\phi}^2+\dot{\chi}^2}=\frac{k}{a}\frac{\dot{\phi}D\phi+\dot{\chi}D\chi}{\dot{\phi}^2+\dot{\chi}^2} . \quad (2.14)$$

At Hubble radius crossing

$$V|_H=H\frac{\dot{\phi}D\phi+\dot{\chi}D\chi}{\dot{\phi}^2+\dot{\chi}^2}\Big|_H . \quad (2.15)$$

In Eq. (2.13), $\Delta|_H$ is given as a function of $(D\phi)^{\cdot}$ and $(D\chi)^{\cdot}$, so we need their expressions in terms of $D\phi$ and $D\chi$. They can be computed by solving approximately the equations of motion for $D\phi$ and $D\chi$ near the Hubble radius crossing time $(k/a\sim H)$. $D\phi$ satisfies[20]

$$(D\phi)^{\cdot\cdot}+3H(D\phi)^{\cdot}+\left[\frac{k^2}{a^2}+U_{\phi\phi}\right]D\phi=-4\dot{\phi}\dot{\Phi}+2U_\phi\Phi , \quad (2.16)$$

where $U_{\phi\phi}\equiv\partial^2U/\partial\phi^2$.

Using Eq. (2.12), Φ and $\dot{\Phi}$ can be replaced in terms of $D\phi$ and its derivatives. The complicated resulting equation for $D\phi$ can be largely simplified in a period of inflationary expansion (using the slow-rolling approximation, $\dot{\phi}^2\ll U(\phi)$ and $\ddot{\phi}\ll 3H\dot{\phi},U_{\phi\phi}$) and near the Hubble radius crossing time. Changing finally the derivative variable from time to the scale factor a, Eq. (2.16) yields

$$\frac{d^2D\phi}{da^2}+\frac{4}{a}\frac{dD\phi}{da}+\frac{k^2}{H^2a^4}D\phi\simeq0 , \quad (2.17)$$

which has the form of a Bessel equation. Solving it, it can be seen that, for $k/a\sim H$,

$$(D\phi)^{\cdot}\simeq-H\,D\phi . \quad (2.18)$$

Replacing this in Eq. (2.13), we obtain that

$$\Delta|_H\sim-\frac{1}{3H}(\dot{\phi}D\phi+\dot{\chi}D\chi) . \quad (2.19)$$

The quantum fluctuations of the scalar fields given in (2.3) have been computed in the unperturbed metric, so they correspond to the fluctuations $\delta\phi$ and $\delta\chi$ in any gauge in which the fluctuations in the geometry are small.

In the case in which $m_\chi^2, m_\phi^2 \ll H^2$, we have $D\phi \sim D\chi$ and, as can be seen from Eq. (2.19), the major contribution to the total density fluctuation will be given by the scalar field which has larger kinetic energy when a given wavelength leaves the Hubble radius, which corresponds to having the larger potential energy derivative (U_ϕ or U_χ).

The initial condition for the fluctuations in each particular component can also be computed to be

$$\rho_\phi \Delta_\phi|_H \sim -\dot\phi H\, D\phi|_H, \quad \rho_\chi \Delta_\chi|_H \sim -\dot\chi H\, D\chi|_H \ , \tag{2.20}$$

and

$$V_\phi|_H \sim \left.\frac{H\, D\phi}{\dot\phi}\right|_H, \quad V_\chi|_H \sim \left.\frac{H\, D\chi}{\dot\chi}\right|_H . \tag{2.21}$$

These give the initial conditions for the evolution of the classical perturbations outside the Hubble radius.

III. BACKGROUND EVOLUTION

In order to go on with the analysis of the evolution of these fluctuations, it is necessary to specify in more detail the evolution of the background model: during inflaton, in addition to the inflaton field ϕ, we will consider another scalar field χ, whose contribution to the total energy density is much smaller than that of the inflaton, but whose interactions with the rest of the matter are much weaker, so that the mean life of the associated particles is larger. At the reheating time, ϕ decays into radiation and matter, and baryogenesis takes place as usual. The Universe becomes radiation dominated and its energy density decays as a^{-4}. As in this period the interactions of the field χ can be neglected, it behaves as a free massive field, so its energy density decreases as a^{-3} and after some time it becomes the dominant contribution to the total energy density. After this epoch, the decay of χ into radiation begins to be important and finally this radiation becomes the dominant component. This corresponds to the radiation-dominated epoch of the standard model. Meanwhile, the energy density of the matter produced by the inflaton decay is decreasing as a^{-3} and when it becomes dominant we enter the matter-dominated era.

We will study now the evolution of the background variables that will be needed in the next section to solve the fluctuation evolution equations. We consider a component α formed by the inflaton and its decay products, and a component β formed by χ and the radiation in which it decays. During inflation $\alpha = \phi, \beta = \chi$, and the main contribution to the total energy density is given by the potential energy of ϕ. In a flat Friedman universe,

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{3.1}$$

the Einstein equation is

$$H^2(t) = \frac{\rho}{3} \simeq \frac{U(\phi)}{3} . \tag{3.2}$$

The equation of motion for χ is

$$\ddot\chi + 3H\dot\chi + U_\chi = 0 . \tag{3.3}$$

It is useful to change the variable of derivation from t to the scale factor a:

$$\frac{d}{dt} = \dot a \frac{d}{da}, \quad \frac{d^2}{dt^2} = \dot a^2 \frac{d^2}{da^2} + \dot a \frac{d\dot a}{da}\frac{d}{da} . \tag{3.4}$$

It can be seen that

$$\frac{d\dot a}{da} = H - \tfrac{3}{2}(1+w)H \sim H \ , \tag{3.5}$$

where $w = p/\rho$, and in the last term the kinetic energy has been neglected with respect to the total energy ($|1+w| \ll 1$). Then, (3.3) can be written as

$$a^2 \frac{d^2\chi}{da^2} + 4a\frac{d\chi}{da} + \frac{m_\chi^2}{H^2}\chi = 0 \ , \tag{3.6}$$

where χ has been taken as a massive noninteracting field. Changing the variable to $X \equiv a^2\chi$,

$$\frac{d^2X}{da^2} - \frac{1}{a^2}\left[2 - \frac{m_\chi^2}{H^2}\right]X = 0 . \tag{3.7}$$

In the case that $m_\chi^2 \ll H^2$, it gives the following behavior for χ:

$$\chi = A + Ba^{-3} \ , \tag{3.8}$$

where A and B are constants. It has a constant mode and a decaying mode. After a few expansion times, the constant mode will be the dominant one and this shows that, in this regime, the energy density of the component β stays nearly constant: $\rho_\chi \sim \frac{1}{2}m_\chi^2\chi^2$. During this period the evolution of the inflaton is not affected by χ and so $\rho_\phi \simeq U(\phi)$.

For times larger than the reheating time, the energy density of the inflaton field has been converted into radiaation energy and the Universe expands as $a \sim t^{1/2}$, the energy density decreases as

$$\rho \sim \rho_\phi \sim \rho_{\rm rh}\left[\frac{a}{a_{\rm rh}}\right]^{-4} , \tag{3.9}$$

where $\rho_{\rm rh}$ and $a_{\rm rh}$ refer to their values at the end of inflation.

The evolution equation for χ in this case can be written as

$$a^2\frac{d^2\chi}{da^2} + 2a\frac{d\chi}{da} + \frac{m_\chi^2}{H^2}\chi = 0 \ , \tag{3.10}$$

where the relation $d\dot a/da = -H$ has been used. Replacing H^2 in terms of a and changing variables to $Y \equiv a\chi$ there results

$$\frac{d^2Y}{da^2} + \frac{3m_\chi^2}{\rho_{\rm rh}}\frac{a^2}{a_{\rm rh}^4}Y = 0 \ , \tag{3.11}$$

which has the form of a Bessel equation. We obtain for the field χ,

$$\chi = a^{-1/2}\left[CJ_{1/4}\left[\frac{ma^2}{2\epsilon}\right] + DJ_{-1/4}\left[\frac{ma^2}{2\epsilon}\right]\right] , \tag{3.12}$$

where $\epsilon=\sqrt{\rho_{\rm rh}/3}a_{\rm rh}^2$ and C and D are constants which must be fixed from the initial conditions for χ at the end of the inflationary era. The expression for χ can be approximated using the asymptotic form of Bessel functions for small $(m \ll H)$ and large $(m \gg H)$ arguments. If $m \ll H/2=\sqrt{4\rho_{\rm rh}/3}(a_{\rm rh}/a)^2$, then

$$\chi \sim \frac{C}{\Gamma(5/4)}\left[\frac{m}{4\epsilon}\right]^{1/4}+\frac{D}{\Gamma(3/4)}\left[\frac{m}{4\epsilon}\right]^{-1/4} . \quad (3.13)$$

As χ is negligible at the last stages of inflation, the initial condition is that D must be very small. Then, in this regime χ stays approximately constant, and consequently also ρ_χ.

When H becomes smaller than m,

$$\chi \simeq a^{-3/2}\left[\frac{4\epsilon}{m\pi}\right]^{1/2}\left[C\cos\left[\frac{ma^2}{2\epsilon}-\frac{3\pi}{8}\right]+D\cos\left[\frac{ma^2}{2\epsilon}-\frac{\pi}{8}\right]\right], \quad (3.14)$$

which corresponds to an oscillating function of time with frequency $\omega=m$ and a global damping term. The total energy density associated with χ is given by

$$\rho_\chi=\tfrac{1}{2}m_\chi^2\chi^2+\tfrac{1}{2}\dot{\chi}^2 . \quad (3.15)$$

Differentiating Eq. (3.14), $\dot{\chi}$ can be computed and it can be seen from Eq. (3.15) that the potential and kinetic energy contributions are comparable in amplitude and that they oscillate with opposite phases; i.e., the energy is transformed from potential to kinetic with an overall damping

$$\rho_\chi=\rho_\chi^*\left[\frac{a}{\sqrt{2\epsilon/m}}\right]^{-3} , \quad (3.16)$$

where ρ_χ^* denotes the value of ρ_χ at $a=\sqrt{2\epsilon/m}$. The χ density decreases more slowly than the radiation energy and after some time it becomes the dominant component:

$$H^2 \simeq \tfrac{1}{3}\rho_\chi . \quad (3.17)$$

This case has been studied in the regime that the oscillation period is much smaller than the expansion time $(m \gg H)$ (see, e.g., Ref. 21). Under this assumption, averaging the kinetic term over one oscillation period, it can be seen that

$$\rho_\chi=\rho_\chi(t_0)\left[\frac{a}{a_0}\right]^{-3} . \quad (3.18)$$

and $\langle p \rangle=0$.

When the damping of the oscillations due to the decay of χ into light particles (radiation) is taken into account, the evolution is modified to

$$\rho_\chi=\rho_\chi(t_0)\left[\frac{a}{a_0}\right]^{-3} e^{-\Gamma(t-t_0)} . \quad (3.19)$$

For times larger than Γ^{-1} (mean life of the χ particles), the scalar field energy has mainly been converted into radiation. After this time the evolution is identical to that in the standard model.

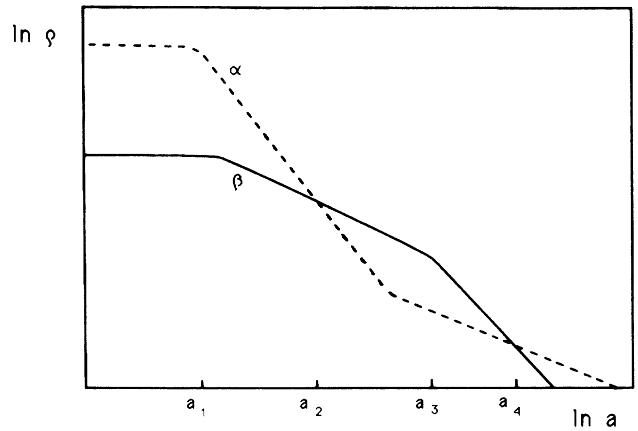

FIG. 1. Evolution of the background model. The dashed line corresponds to ρ_α and the solid line to ρ_β. a_1 corresponds to the scale factor at the end of the inflation, a_2 to that when the χ field becomes dominant, a_3 to that when χ decays into radiation and a_4 to that when the Universe becomes matter dominated.

The general behavior of the energy density of components α and β can be followed in Fig. 1. When the scale factor equals a_2 and a_4, $\rho_\alpha=\rho_\beta$.

IV. EVOLUTION OF THE FLUCTUATIONS

In this section the evolution of the fluctuations in the two-component system is studied. The formalism used[15] is reviewed in the Appendix. The fluctuations are characterized by the gauge-invariant variables Δ_α, Δ_β, and Δ corresponding to the energy-density fluctuations of each component and the total energy, V_α, V_β, and V corresponding to the velocity fluctuation of each component and the total fluid velocity one, the entropy fluctuation $S_{\alpha\beta}$ given by Eq. (1.1) and the relative velocity fluctuation between components $V_{\alpha\beta}$. They are properly defined in the Appendix. Their equations of motion are given by Eqs. (A8), (A11), and (A13). We will refer with α to the component corresponding to the inflaton and its decay products and with β to the χ field and its decay products.

The evolution is divided into different periods according to the changes in the equation of state of the components. The equations of motion are solved in each period with w_α and w_β approximately constant. The initial conditions are taken from Sec. II and the matching between different periods is made by imposing the continuity of all the fluctuation variables. Up to the time in which χ decays into radiation, the two components are decoupled, so $Q_\alpha=E_\alpha=F_\alpha=0$ (and also for β).

A. Inflationary period

The first period to be studied is the inflationary one. In this period $\alpha=\phi$ and $\beta=\chi$ and the energy density of both fields is dominated by the potential term ($|1+w_\alpha| \ll 1$ and $|1+w_\beta| \ll 1$). We study both fields making an analogy with two fluids, so we also need to determine the associated sound velocity to solve the equation of evolution for the fluctuations. It is defined by $c_{s\alpha}^2=\dot{p}_\alpha/\dot{\rho}_\alpha$. In the case of a scalar field ψ, differentiating the associated pressure and energy density we obtain

$$c_{s\psi}^2=\frac{3H\dot\psi+2U_\psi}{3H\dot\psi} . \quad (4.1)$$

In the slow-rolling approximation, we have $3H\dot\psi\simeq -U_\psi$. We see from Eq. (4.1) that with this hypothesis $|c_{s\psi}^2+1|\ll 1$. We will take $c_{s\phi}^2\simeq -1$ and $c_{s\chi}^2\simeq -1$. Another point to be taken into account is that, when dealing with scalar fields, the individual entropy perturbations η_α cannot be neglected; they are given by Eq. (A12). In this case, $w_\alpha\eta_\alpha=2\Delta_\alpha$ and $w_\beta\eta_\beta=2\Delta_\beta$.

We define a_k as the value of the scale factor at the time at which the wavelength associated with k leaves the Hubble radius ($a_kH/k=1$), and a new variable $\xi\equiv a/a_k$.

In order to solve the system of coupled equations in this period, it is convenient to begin by solving the equations for the entropy $S_{\alpha\beta}$ and the relative velocity $V_{\alpha\beta}$ [Eq. (A11)], as they form a system decoupled from the rest of the variables:

$$\xi\frac{dS_{\alpha\beta}}{d\xi}+6S_{\alpha\beta}=-\frac{aH}{k}V_{\alpha\beta}\left[18+\left[\frac{k}{aH}\right]^2\right] ,$$
$$\xi\frac{dV_{\alpha\beta}}{d\xi}-2V_{\alpha\beta}=\frac{k}{aH}S_{\alpha\beta} . \quad (4.2)$$

They can be combined to give a second-order equation for $V_{\alpha\beta}$, and noting that, for constant w, $k/aH=\xi^{-1+3(1+w)/2}$, we obtain

$$\xi\frac{d^2V_{\alpha\beta}}{d\xi^2}+[6-\tfrac{3}{2}(1+w)]\frac{dV_{\alpha\beta}}{d\xi}+\left[4+3(1+w)+\left[\frac{k}{aH}\right]^2\right]\frac{V_{\alpha\beta}}{\xi}=0 . \quad (4.3)$$

For wavelengths much larger than the Hubble radius ($k/aH\ll 1$), Eq. (4.3) admits power-law solutions, $V_{\alpha\beta}\propto\xi^n$. Taking into account that $|1+w|\ll 1$, it follows that

$$V_{\alpha\beta}=A\xi^{-1-3(1+w)/2}+B\xi^{-4+3(1+w)} ,$$
$$S_{\alpha\beta}=-[3+\tfrac{3}{2}(1+w)]A\xi^{-3(1+w)}-[6-3(1+w)]B\xi^{-3+3(1+w)/2} , \quad (4.4)$$

where A and B are constants.

The equations of motion for the total fluid velocity and energy-density fluctuations are also simplified in this case. The system (A13) can be written as

$$\frac{d\Delta}{d\xi}-3w\frac{\Delta}{\xi}=-\frac{k}{aH}(1+w)\frac{V}{\xi} ,$$
$$\frac{dV}{d\xi}+\frac{V}{\xi}=-\frac{3}{2}\frac{aH}{k}\frac{\Delta}{\xi}+\frac{k}{aH}\frac{1}{1+w}\frac{\Delta}{\xi} . \quad (4.5)$$

Then, in this particular case, the global variables behave as the velocity and energy fluctuations of a single fluid, without feeling the individual component fluctuations; and it can be seen that there is a constant of motion for wavelengths larger than the Hubble radius, as is discussed at the end of the Appendix. The solutions are

$$\Delta=C\xi^{-2+3(1+w)}+D\xi^{-3+3(1+w)/2} ,$$
$$V=-\frac{C}{1+w}\xi^{-1+3(1+w)/2}+\tfrac{3}{2}D\xi^{-2} . \quad (4.6)$$

From these solutions for Δ, V, $S_{\alpha\beta}$, and $V_{\alpha\beta}$, we can construct the remaining quantities in which we are interested (Δ_α, Δ_β, V_α, and V_β). The four constants A, B, C, and D can be computed by evaluating Eqs. (4.4) and (4.6) at the Hubble radius crossing time and equating them to the values given by the quantum fluctuations of the fields during inflation. So, let us specify the fluctuations computed in Sec. II for our model. The fluctuations in the total energy density and velocity are given by Eqs. (2.19) and (2.15) in terms of the quantum fluctuations of the fields. We see that the dominant contribution corresponds to the field with larger time derivative. Typically, this will be the inflation.[5] Then

$$\Delta|_H\simeq-\frac{\dot\phi D\phi}{3H}\bigg|_H ,\ V|_H\simeq\frac{H D\phi}{\dot\phi}\bigg|_H . \quad (4.7)$$

We also need the initial conditions for $V_{\alpha\beta}$ and $S_{\alpha\beta}$. They can be computed from (2.20) and (2.21):

$$V_{\alpha\beta}|_H\simeq-\frac{H D\chi}{\dot\chi}\bigg|_H ,$$
$$S_{\alpha\beta}|_H\simeq 2\frac{H D\chi}{\dot\chi}\bigg|_H . \quad (4.8)$$

Note that these expressions imply that the fluctuations are initially of the isocurvature type, as $S_{\alpha\beta}|_H\gg\Delta|_H$ (during inflation $\dot\phi,\dot\chi\ll H$).

The constants A and B in Eq. (4.4) can be computed from Eq. (4.8),

$$A\simeq-\frac{8}{9}\frac{H D\chi}{\dot\chi}\bigg|_H ,\quad B\simeq-\frac{1}{9}\frac{H D\chi}{\dot\chi}\bigg|_H , \quad (4.9a)$$

and C and D in Eq. (4.6) can be computed from Eq. (4.7):

$$D\simeq 0,\quad C\simeq-\frac{\dot\phi D\phi}{3H}\bigg|_H . \quad (4.9b)$$

From these initial conditions and the evolution laws during inflation, the amplitude of the perturbations at the end of inflation (which correspond to a value a_1 for the scale factor in Fig. 1) can be calculated:

$$\Delta\Big|_1 \simeq -\frac{\dot{\phi} D\phi}{3H}\Bigg|_H \left[\frac{a_1}{a_k}\right]^{-2-3(1+w)},$$

$$V|_1 \simeq \frac{H D\phi}{\dot{\phi}}\Bigg|_H \left[\frac{a_1}{a_k}\right]^{-1-3(1+w)/2},$$

$$S_{\alpha\beta}|_1 \simeq \frac{8}{3}\frac{H D\chi}{\dot{\chi}}\Bigg|_H \left[\frac{a_1}{a_k}\right]^{-3(1+w)}, \tag{4.10}$$

$$V_{\alpha\beta}|_1 \simeq -\frac{8}{9}\frac{H D\chi}{\dot{\chi}}\Bigg|_H \left[\frac{a_1}{a_k}\right]^{-1-3(1+w)/2}.$$

B. First radiation-dominated period

After the decay of the inflaton, the component α is mainly made of radiation, so $w_\alpha = \frac{1}{3}$ and $c_{s\alpha}^2 \simeq \frac{1}{3}$. The component β is still given by the field χ, which soon begins to oscillate around the minimum of the potential. In this regime, $\langle p_\beta \rangle = 0$ and this component behaves essentially as dust ($w_\beta \sim c_{s\beta}^2 \sim 0$). However, the fact that it is a scalar field does not allow us to neglect the entropy perturbation η_β, that satisfies $w_\beta \eta_\beta = \Delta_\beta$. For the component α we can take $\eta_\alpha = 0$. This hypothesis holds up to the decay of χ. In this regime there are two periods to be considered, first when the Universe is dominated by radiation (component α) and then when it is dominated by χ (component β). For both of them, the system of equations to be solved is

$$\xi \frac{dS_{\alpha\beta}}{d\xi} = -\frac{k}{aH} V_{\alpha\beta} + 3\Delta_\beta, \tag{4.11a}$$

$$\xi \frac{dV_{\alpha\beta}}{d\xi} = \frac{k}{aH}\frac{1}{3}(S_{\alpha\beta} - 2\Delta_\beta), \tag{4.11b}$$

$$\frac{d\Delta}{d\xi} - 3w\frac{\Delta}{\xi} = -(1+w)\frac{k}{aH}\frac{V}{\xi}, \tag{4.11c}$$

$$\frac{dV}{d\xi} + \frac{V}{\xi} = -\frac{3}{2}\frac{aH}{k}\frac{\Delta}{\xi} + \frac{c_s^2}{1+w}\frac{k}{aH}\frac{\Delta}{\xi} + \frac{1}{3(1+w)}\frac{k}{aH}\frac{1}{\xi}\frac{\rho_\beta}{\rho} \times \left[2\Delta_\beta + \frac{\Delta}{1+w} - 3\frac{aH}{k}V_{\alpha\beta}\right]. \tag{4.11d}$$

Let us first analyze the period dominated by the radiation, which corresponds to the scale factor evolving from a_1 to a_2. In this period it is convenient to normalize the variable ξ with $a_1, \xi \equiv (a/a_1)$ and it is easy to see that $aH/k = (a_1 H_1/k)\xi^{-1}$ and $\rho_\beta/\rho_\alpha = (a_1/a_2)\xi$. With these expressions and the help of Eq. (A8a) for Δ_β, Eqs. (4.11a) and (4.11b) can be combined to give

$$V_{\alpha\beta}''' + \frac{V_{\alpha\beta}''}{\xi} + 4\frac{V_{\alpha\beta}'}{\xi^2} - 10\frac{V_{\alpha\beta}}{\xi^3} = \left[\frac{k}{a_1 H_1}\right]^2 \left(\tfrac{2}{3}\xi V' - \tfrac{1}{3}V\right), \tag{4.12}$$

where $d/d\xi$ has been denoted by a prime. This equation holds for wavelengths much larger than the Hubble radius. On the other hand, from Eqs. (4.11c), (4.11d), and (A8a), we obtain (for $k/aH \ll 1$)

$$V''' + 2\frac{V''}{\xi} - 4\frac{V'}{\xi^2} + 4\frac{V}{\xi^3} = -\frac{3}{4}\frac{a_1}{a_2}\left[V_{\alpha\beta}'' - 3\frac{V_{\alpha\beta}'}{\xi} - 5\frac{V_{\alpha\beta}}{\xi^2}\right]. \tag{4.13}$$

Equations (4.12) and (4.13) form a system of coupled equations for V and $V_{\alpha\beta}$. As initially the amplitude of the relative velocity is much larger than the total velocity, we will solve the system, neglecting the right-hand side of (4.12), solving first for $V_{\alpha\beta}$ and inserting this solution into the source term on the right-hand side of (4.13). This corresponds to studying the effect of the entropy perturbations as a source for curvature perturbations, which can be important in this problem, and not vice versa. The result is

$$V_{\alpha\beta} = E\sin(\sqrt{5}\ln\xi) + F\cos(\sqrt{5}\ln\xi) + G\xi^2,$$
$$V = \frac{a_1}{a_2}\left[-\frac{2\sqrt{5}F + 13E}{28}\xi\sin(\sqrt{5}\ln\xi) - \frac{-2\sqrt{5}E + 13F}{28}\xi\cos(\sqrt{5}\ln\xi) + \frac{27}{40}G\xi^2\right] + I\xi^2 + J\xi + \frac{K}{\xi^2}, \tag{4.14}$$

where E, F, G, I, J, and K are constants. The remaining perturbation variables can be computed with the help of these expressions. In particular,

$$\Delta = \frac{k}{aH}\left[\frac{a_1}{a_2}\left[\frac{5\sqrt{5}F + E}{42}\xi\sin(\sqrt{5}\ln\xi) + \frac{-5\sqrt{5}E + F}{42}\xi\cos(\sqrt{5}\ln\xi) - \tfrac{3}{10}G\xi^3\right] - \tfrac{2}{3}I\xi^2 - \tfrac{4}{3}J\xi + \frac{2}{3}\frac{K}{\xi^2}\right],$$
$$S_{\alpha\beta} = -\frac{aH}{k}[(\sqrt{5}F + 2E)\sin(\sqrt{5}\ln\xi) + (-\sqrt{5}E + 2F)\cos(\sqrt{5}\ln\xi) - 18G\xi^3]. \tag{4.15}$$

Since we have differentiated Eq. (4.11) to derive (4.12) and (4.13), we must check whether these solutions satisfy (4.11). The result is that there are two spurious modes and we must take $G = I = 0$. The remaining constants can be evaluated by matching these solutions with the fluctuation amplitudes at the end of inflation(4.10). The result is

$$F=-\frac{8}{9}\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_k}{a_1},\quad E=\frac{8}{9\sqrt{5}}\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_k}{a_1},$$

$$J=\frac{1}{3}\frac{a_k}{a_1}\left[\frac{HD\phi}{\dot\phi}\bigg|_H-\frac{2}{3}\frac{a_1}{a_2}\frac{HD\chi}{\dot\chi}\bigg|_H\right],\quad K=\frac{1}{3}\frac{a_k}{a_1}\left[\frac{2HD\phi}{\dot\phi}\bigg|_H-\frac{16}{21}\frac{a_1}{a_2}\frac{HD\chi}{\dot\chi}\bigg|_H\right]. \tag{4.16}$$

With these values and (4.14) and (4.15), it is possible to compute the amplitude of the perturbations at the end of this period, just before the component β becomes dominant:

$$S_{\alpha\beta}|_2\simeq\frac{8}{9}\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_1}{a_2}\left[\frac{7}{\sqrt{5}}\sin\gamma+3\cos\gamma\right],\quad V_{\alpha\beta}\Big|_2\simeq\frac{8}{9}\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_k}{a_1}\left[\frac{1}{\sqrt{5}}\sin\gamma-\cos\gamma\right],$$

$$\Delta|_2=\left[\frac{a_k}{a_1}\right]^2\frac{a_2}{a_1}\left[-\frac{4}{9}\frac{a_2}{a_1}\frac{HD\phi}{\dot\phi}\bigg|_H+\frac{8}{9}\left[\frac{1}{3}-\frac{2}{7}\cos\gamma-\frac{4}{7\sqrt{5}}\sin\gamma\right]\frac{HD\chi}{\dot\chi}\bigg|_H\right], \tag{4.17}$$

$$V|_2=\frac{a_k}{a_1}\left[\frac{1}{3}\frac{a_2}{a_1}\frac{HD\phi}{\dot\phi}\bigg|_H-\left[\frac{2}{3}-\frac{10}{21}\cos\gamma+\frac{2}{21\sqrt{5}}\sin\gamma\right]\frac{HD\chi}{\dot\chi}\bigg|_H\right].$$

where $\gamma=\sqrt{5}\ln(a_2/a_1)$.

C. χ-dominated period

The next period begins when the field χ becomes dominant ($\rho_\beta>\rho_\alpha$) and ends when χ decays in radiation (at a_3). The evolution equations for the fluctuations are also given by (4.11). We now normalize the variable ξ with a_2, $\xi\equiv a/a_2$ and we use that $aH/k=(a_2H_2/k)\xi^{-1/2}$ and $\rho_\alpha/\rho_\beta=\xi^{-1}$. Combining Eqs. (4.11) and using (A10), we obtain for $k\ll aH$ the couple of equations

$$S''_{\alpha\beta}+\left[\frac{5}{2}+\frac{4}{3\xi}\right]\frac{S'_{\alpha\beta}}{\xi}+\frac{6}{\xi^3}S_{\alpha\beta}=3\frac{\Delta'}{\xi}+\frac{9}{2}\frac{\Delta}{\xi^2}, \tag{4.18a}$$

$$\Delta''+\frac{3}{2}\frac{\Delta'}{\xi}-\frac{3}{2}\frac{\Delta}{\xi^2}=\frac{8}{9}\left[\frac{k}{aH}\right]^2\frac{S_{\alpha\beta}}{\xi^3}+4\frac{k}{aH}\frac{V_{\alpha\beta}}{\xi^2}. \tag{4.18b}$$

As in the previous period, we solve the system neglecting the right-hand side of Eq. (4.18a), solving for $S_{\alpha\beta}$, then computing $V_{\alpha\beta}$ from Eq. (4.11) and inserting it into the source term on the right-hand side of Eq. (4.18b) (which corresponds to considering the curvature perturbation generated by the entropy perturbation and not vice versa). The result is

$$S_{\alpha\beta}\simeq L\xi^{-3/4}J_{3/2}(2\sqrt{6}\xi^{-1/2})+M\xi^{-3/4}J_{-3/2}(2\sqrt{6}\xi^{-1/2}),$$

$$V_{\alpha\beta}\simeq-\frac{2}{9}\frac{k}{aH}\xi^{-3/4}\left[L\left[J_{3/2}(2\sqrt{6}\xi^{-1/2})-\frac{2\xi^{1/2}}{\sqrt{6}}J_{1/2}(2\sqrt{6}\xi^{-1/2})\right]+M\left[J_{-3/2}(2\sqrt{6}\xi^{-1/2})+\frac{2\xi^{1/2}}{\sqrt{6}}J_{-1/2}(2\sqrt{6}\xi^{-1/2})\right]\right], \tag{4.19}$$

where J_n are the Bessel functions and L and M are constants that can be computed from the initial conditions at a_2:

$$L=\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_1}{a_2}(13\sin\gamma-5.6\cos\gamma),$$

$$M=\frac{HD\chi}{\dot\chi}\bigg|_H\frac{a_1}{a_2}(1.6\sin\gamma+9\cos\gamma). \tag{4.20}$$

Solving for V and Δ is more involved because the source term has a complicated expression. The solutions of the homogeneous equations are

$$V_{\rm hom}=O\xi^{1/2}+P\xi^{-2},$$

$$\Delta_{\rm hom}=-\frac{k}{aH}(O\xi^{1/2}-\tfrac{2}{3}P\xi^{-2}), \tag{4.21}$$

where O and P are constants.

A particular solution of Eq. (4.18b) was obtained, but its expression is too lengthy to be quoted here. The asymptotic behavior for $a \gg a_2$ is

$$V_p \sim -1.3\times 10^{-2}\frac{a_k a_2}{a_1^2} L\xi^{-1} ,$$
$$\Delta_p \sim 1.1\times 10^{-2}\left[\frac{a_k a_2}{a_1^2}\right]^2 M . \quad (4.22)$$

However, to compute the constants O and P, it is necessary to fit the initial conditions at a_2 using the exact solutions. There results

$$O=\frac{6}{15}\frac{a_k a_2}{a_1^2}\frac{H\,D\phi}{\dot\phi}\bigg|_H + \frac{a_k}{a_1}\frac{H\,D\chi}{\dot\chi}\bigg|_H (-0.4+143\sin\gamma-53\cos\gamma) ,$$
$$P=-\frac{1}{15}\frac{a_k a_2}{a_1^2}\frac{H\,D\phi}{\dot\phi}\bigg|_H + \frac{a_k}{a_1}\frac{H\,D\chi}{\dot\chi}\bigg|_H (-0.22+356\sin\gamma-45\cos\gamma) . \quad (4.23)$$

The amplitude of the perturbations at the end of this period can be obtained from these results:

$$S_{\alpha\beta}|_3 \simeq \frac{H\,D\chi}{\dot\chi}\bigg|_H \frac{a_1}{a_2}(0.12\sin\gamma-0.6\cos\gamma) ,$$
$$V_{\alpha\beta}|_3 \simeq \frac{H\,D\chi}{\dot\chi}\bigg|_H \frac{a_1}{a_2}\left[\frac{a_2}{a_3}\right]^{1/2}(0.5\sin\gamma-0.7\cos\gamma) ,$$
$$\Delta|_3 \simeq -\frac{a_k a_3}{a_1^2}\left[\frac{6}{15}\frac{a_k a_2}{a_1^2}\frac{H\,D\phi}{\dot\phi}\bigg|_H + \frac{a_k}{a_1}\frac{H\,D\chi}{\dot\chi}\bigg|_H (-0.4+143\sin\gamma-53\cos\gamma)\right] , \quad (4.24)$$
$$V|_3 \simeq \left[\frac{a_3}{a_2}\right]^{1/2}\left[\frac{6}{15}\frac{a_k a_2}{a_1^2}\frac{H\,D\phi}{\dot\phi}\bigg|_H + \frac{a_k}{a_1}\frac{H\,D\chi}{\dot\chi}\bigg|_H (-0.4+143\sin\gamma-53\cos\gamma)\right] .$$

D. From χ decay to Hubble radius crossing

The last period to be studied before the wavelengths reenter the Hubble radius corresponds to the epoch after the decay of χ in radiation. The situation is quite different in this period as the hypothesis of uncoupled fluids does not hold anymore. The Universe is composed by radiation and baryons tightly coupled through electron scattering. Then, the momentum transfer between components must be taken into account. This means that the source term f_α which appears in (A8) cannot be neglected anymore, but is given by[15]

$$f_r = R_c(v_m - v_r),\quad f_m = \frac{4\rho_r}{3\rho_m}R_c(v_r - v_m) , \quad (4.25)$$

where v_r and v_m are the radiation and matter velocity fluctuation defined in (A2) and R_c is the ratio of the Hubble radius to the mean free path for photons colliding with electrons. The effect of this interaction corresponds to the introduction of an extra source term in the right-hand side of (A11b) given by $F_{\alpha\beta} \equiv f_\alpha - f_\beta = -\gamma_{\alpha\beta}V_{\alpha\beta}$ where $\gamma_{\alpha\beta}$ is proportional to R_c, and is much larger than unity before decoupling. As has been pointed out in Ref. 22, $S_{\alpha\beta}$ stays nearly constant in this regime and the relative velocity between components goes to zero.

On the other hand, after the decay of χ we can neglect both η_α and η_β. In this case, the couple of Eqs. (A13) for the total fluid perturbations Δ and V can be combined to give a second-order equation:

$$\Delta'' - \left(-\tfrac{3}{2}+\tfrac{15}{2}w-3c_s^2\right)\frac{\Delta'}{\xi} + \left[-\tfrac{3}{2}-12w+9c_s^2+\tfrac{9}{2}w^2+\left[\frac{k}{aH}\right]^2 c_s^2\right]\frac{\Delta}{\xi^2} = -\left[\frac{k}{aH}\right]^2\frac{h_\alpha h_\beta}{\rho h}(c_{s\alpha}^2-c_{s\beta}^2)\frac{S_{\alpha\beta}}{\xi^2} . \quad (4.26)$$

The homogeneous equation in a radiation-dominated universe has the solution $\Delta_{\rm hom} = Q\xi^2 + R\xi^{-1}$ for wavelengths much larger than the Hubble radius, with Q and R constants (we normalize here ξ with the scale factor at the radiation and matter equivalence time, a_4). When the component α is mainly made of baryons and β of radiation, the entropy perturbation, acting as a source, gives rise to an extra growing mode given by

$$\Delta_p=\frac{1}{6}\left[\frac{k}{a_4H_4}\right]^2 S_{\alpha\beta}\left[\frac{a}{a_4}\right]^3 . \tag{4.27}$$

The corresponding velocity fluctuation is given by

$$V=-\frac{3}{4\sqrt{2}}\frac{a_4H_4}{k}(Q\xi-2R\xi^{-2})-\frac{\sqrt{2}}{8}\frac{k}{a_4H_4}S_{\alpha\beta}\xi^2 . \tag{4.28}$$

The constants Q and R can be computed fitting the initial conditions at the beginning of this period.

In the matter-dominated era ($\xi>1$) the behavior of Δ and V is given by

$$\Delta=\frac{9}{10}Q\xi+R\xi^{-3/2}+\frac{4}{15}\left[\frac{k}{a_4H_4}\right]^2 S_{\alpha\beta}\xi ,$$
$$V=-\frac{a_4H_4}{k}\frac{1}{\sqrt{2}}(\tfrac{9}{10}Q\xi^{1/2}-\tfrac{3}{2}R\xi^{-2}) -\frac{2\sqrt{2}}{15}\frac{k}{a_4H_4}S_{\alpha\beta}\xi^{1/2} . \tag{4.29}$$

In order to see if the perturbations so obtained are of the isocurvature type, the amplitude of the perturbations Δ and $S_{\alpha\beta}$ must be compared. For wavelengths that reenter the Hubble radius during the radiation- and matter-dominated era, the magnitude of Δ are, respectively,

$$\Delta|_{H(\text{rad})}\simeq-\frac{6+4\sqrt{2}}{9}\frac{a_1}{a_2}\left[\frac{6}{15}\frac{a_2}{a_1}\frac{HD\phi}{\dot\phi}\bigg|_H+\frac{HD\chi}{\dot\chi}\bigg|_H(-0.4+143\sin\gamma-53\cos\gamma)\right] ,$$
$$\Delta|_{H(\text{mat})}\simeq-\frac{6+4\sqrt{2}}{10}\frac{a_1}{a_2}\left[\frac{6}{15}\frac{a_2}{a_1}\frac{HD\phi}{\dot\phi}\bigg|_H+\frac{HD\chi}{\dot\chi}\bigg|_H(-0.4+143\sin\gamma-53\cos\gamma)\right] . \tag{4.30}$$

Comparison of these amplitudes with $S_{\alpha\beta}$ from Eq. (4.24) shows that the perturbations are no longer of the isocurvature type, since the perturbation in the total energy density has gown larger than the entropy perturbation [note that the term proportional to $D\chi$ in Eq. (4.30) is by itself larger than the amplitude of $S_{\alpha\beta}$ given in Eq. (4.24)]. This result does not depend on how small the initial perturbation of Δ during inflation is (given by the fluctuation in ϕ), as the perturbation in the total energy density originated by the original entropy perturbation (given by the fluctuation in χ) has grown larger than $S_{\alpha\beta}$. This means that, in this kind of model, initially isocurvature perturbations develop a large adiabatic mode, and consequently do not provide a good model for the phenomenologically proposed baryon isocurvature perturbations.

Another criterion has been proposed in Ref. 22 to define isocurvature perturbations, which has been used to impose the initial conditions in Ref. 14. It can be seen that the conclusion obtained is the same in this frame. (The term $Q\xi^2$ in Δ corresponds to a growing adiabatic mode in Ref. 14, which is not small compared to the "isocurvature" mode given here by Δ_p.)

V. CONCLUSIONS

The perturbations in the energy density arising from quantum fluctuations during inflation in a model in which there is a second scalar field present besides the inflaton have been studied in detail. In particular, we have considered the case in which this scalar field decays into radiation after baryogenesis producing spatial fluctuations in the baryon number per photon. The perturbations in the energy density and velocity of the individual components and of the total system, originated from the quantum fluctuations of the fields at the Hubble radius crossing, have been determined in the case of generalized inflationary models. We then followed the evolution of the perturbations in the composite system from the time that a given wavelength leaves the Hubble radius up to the time it reenters it. Since the model considered here has been proposed as a way for generating isocurvature baryon perturbations, we have analyzed in detail the evolution of the total energy perturbation Δ and the entropy one $S_{\alpha\beta}$. In particular, the fact that an entropy perturbation acts as a source for density perturbations, even outside the Hubble radius, has been carefully considered. The main result is that this effect is very important indeed, and is responsible for originating, from an initially isocurvature model ($S_{\alpha\beta}\ll\Delta$), a large curvature perturbation during the evolution of the wavelengths outside the Hubble radius, so that the total energy-density perturbation grows to be proportional but approximately 2 orders of magnitude larger than the entropy perturbation at the Hubble radius crossing. This result is not in agreement with a previous claim that the evolution should tend to maintain the initially homogeneous mass distribution on scales larger than the matter-radiation Jeans length.[7] The point here is that the Jeans length does not correspond in this problem to the scale over which pressure gradient effects can be neglected. The reason being that when we deal with a nonadiabatic pressure perturbation (i.e., a pressure perturbation not given by $\delta p=c_s^2\delta\rho$), as in the case considered here, there is an extra source term for the energy-density perturbations which makes fluctuations grow from an initially homogeneous universe, as has been shown in Refs. 16 and 17. This source corresponds to the entropy perturbation defined as $\eta\equiv\pi_L-(c_s^2/w)\delta$.[17] In the case of a two-component fluid, it is given by $\eta=(p_\alpha\eta_\alpha+p_\beta\eta_\beta)/p+h_\alpha h_\beta(c_{s\alpha}^2-c_{s\beta}^2)S_{\alpha\beta}/hp$. So, there are two kinds of contributions, corresponding to a nonadiabatic pressure perturbation of the individual components (as it happens, for example, when one of them is given by a scalar field) and to the relative fluctuation between components. In the model analyzed here, both need to be taken into account. As has been stressed before, the effect is significant and in the model discussed here it is responsible for the growth of the adiabatic mode which prevents the model from being a good candidate for the origin of baryon isocurvature fluctuations.

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APPENDIX: EVOLUTION OF THE FLUCTUATIONS IN A MULTICOMPONENT SYSTEM

The analysis can be done in close analogy with the one-fluid case.[17] We will concentrate only on scalar perturbations as this is the only mode which is excited when dealing with scalar fields. Perturbations in all the variables are expanded in terms of a complete set of scalar harmonics $Y(x)$ (a label k indicating the associated wave number will be everywhere understood). The metric perturbations are described by four functions of time, A, B, H_L, H_T, defined by

$$ds^2 = -(1+2AY)dt^2 - aBY_j dt\, dx^j + a^2(\delta_{ij} + 2H_L \delta_{ij} Y + 2H_T Y_{ij}) dx^i dx^j ,$$

where $Y_i \equiv k^{-1}\nabla_i Y$ and $Y_{ij} \equiv k^{-2}\nabla_i \nabla_j Y + \frac{1}{3}\delta_{ij} Y$ (latin indices denote spatial labels running from 1 to 3). We can define the gauge-invariant variables

$$\Phi \equiv H_L + \frac{H_T}{3} + \frac{aH}{k}\left[B - \frac{a}{k}\dot{H}_T\right] ,$$
$$\Psi \equiv A + \frac{a}{k}\dot{B} + a\frac{H}{k}B - \frac{1}{k^2}(a^2 \dot{H}_T)^{\cdot} . \tag{A1}$$

The matter perturbations must be studied more carefully because when dealing with a many-component system, the stress of each one is not conserved individually; we define $T^{\alpha}{}_{\mu\nu}{}^{;\nu} = Q^{\alpha}_{\mu}$. The source terms are constrained by the total energy-momentum conservation $T_{\mu\nu}{}^{;\nu} = 0$, $\sum_\alpha Q^\alpha_\mu = 0$.

Each component is described by a perfect-fluid stress-energy tensor. Denoting by ρ_α and p_α the background energy and pressure density, perturbations are defined by

$$T^{\alpha 0}_{0} = -\rho_\alpha(1+\delta_\alpha Y) ,$$
$$T^{\alpha 0}_{j} = (\rho_\alpha + p_\alpha)(v_\alpha - B)Y_j ,$$
$$T^{\alpha j}_{0} = -(\rho_\alpha + p_\alpha) v_\alpha Y^j ,$$
$$T^{\alpha i}_{j} = p_\alpha(\delta^i_j + \Pi_{L\alpha}\delta^i_j + \Pi_{T\alpha} Y^i_j) . \tag{A2}$$

If the component α is given by a scalar field, which can be decomposed as $\phi(\mathbf{x},t) \equiv \phi(t) + \delta\phi Y(\mathbf{x})$, the perturbed energy-momentum tensor is

$$T^{\phi 0}_{0} = -\tfrac{1}{2}\dot{\phi}^2 - U + (A\dot{\phi}^2 - \dot{\phi}\delta\dot{\phi} - U_\phi \delta\phi)Y ,$$
$$T^{\phi 0}_{j} = \frac{k}{a}\dot{\phi}\delta\phi Y_j ,$$
$$T^{\phi j}_{0} = -(B\dot{\phi}^2 + \frac{k}{a}\dot{\phi}\delta\phi)Y^j ,$$
$$T^{\phi i}_{j} = [\tfrac{1}{2}\dot{\phi}^2 - U - (A\dot{\phi}^2 - \dot{\phi}\delta\dot{\phi} + U_\phi\delta\phi)Y]\delta^i_j . \tag{A3}$$

Writing the source term in the background as $Q^\alpha_\nu = (-aQ^\alpha, 0)$, the unperturbed continuity equation for a given component is given by

$$\dot{\rho}_\alpha = -3Hh_\alpha + Q_\alpha ,$$

where $h_\alpha \equiv \rho_\alpha + p_\alpha$.

In addition, it is necessary to consider perturbations associated to the energy-momentum source term. They are characterized by two new variables ϵ_α and f_α given by

$$\tilde{Q}^{\alpha}_{o} = -aQ^\alpha[1 + (A - \epsilon_\alpha)Y] ,$$
$$\tilde{Q}^{\alpha}_{j} = a[Q^\alpha(v - B) + Hh_\alpha f_\alpha]Y_j , \tag{A4}$$

where v corresponds to the total fluid velocity perturbation.

Gauge-invariant variables can be defined from the gauge-dependent ones as

$$V_\alpha \equiv v_\alpha - \frac{a}{k}\dot{H}_T ,$$
$$\Delta_\alpha \equiv \delta_\alpha + 3(1+w_\alpha)\left[1 - \frac{Q^\alpha}{3Hh_\alpha}\right]\frac{Ha}{k}(v_\alpha - B) , \tag{A5}$$
$$\eta_\alpha \equiv \Pi_{L\alpha} - \frac{c^2_{s\alpha}}{w_\alpha}\delta_\alpha .$$

and $\Pi_{T\alpha}$ is gauge invariant by itself.

Analogously, for the energy-momentum source perturbations

$$E_\alpha \equiv \epsilon_\alpha - \frac{aH}{k}\frac{\dot{Q}^\alpha}{Q^\alpha}(v_\alpha - B), \quad F_\alpha \equiv f_\alpha - \frac{Q^\alpha}{Hh_\alpha}(V_\alpha - V) .$$

Δ_α corresponds to the perturbation in the energy density of the component α with respect to its own rest frame. It is useful, when comparing the fluctuations in different components, to use, instead of Δ_α, the perturbation relative to the total matter rest frame $\Delta_{c\alpha}$, which is given by

$$\Delta_{c\alpha} \equiv \delta_\alpha + 3(1+w_\alpha)\left[1 - \frac{Q^\alpha}{3Hh_\alpha}\right]\frac{Ha}{k}(v - B) . \tag{A6}$$

These variables are related to the total fluid perturbation variables Δ, V, Π_L, and Π_T, by

$$\rho\Delta = \sum_\alpha \rho_\alpha \Delta_{c\alpha}, \quad hV = \sum_\alpha h_\alpha V_\alpha ,$$
$$p\Pi_L = \sum_\alpha p_\alpha \Pi_{L\alpha}, \quad p\Pi_T = \sum_\alpha p_\alpha \Pi_{T\alpha} . \tag{A7}$$

The equations of motion for the perturbation variables Δ_α and V_α, neglecting the anisotropic stress perturbations $\Pi_{T\alpha}$ and in a flat universe, are

$$\frac{d\Delta_\alpha}{da}-3w_\alpha\frac{\Delta_\alpha}{a}=-\frac{3}{2}\frac{aH}{k}(1+w_\alpha)\frac{h_\beta}{H^2}\frac{V_{\alpha\beta}}{a}-(1+w_\alpha)\frac{k}{aH}\frac{V_\alpha}{a}+\frac{1}{aH}\frac{Q_\alpha E_\alpha}{\rho_\alpha}+(1+w_\alpha)\frac{F_\alpha}{a}\ , \tag{A8a}$$

$$\frac{dV_\alpha}{da}+\frac{V_\alpha}{a}=-\frac{3}{2}\frac{aH}{k}\frac{\Delta}{a}+\frac{k}{aH}\left[\frac{c_{s\alpha}^2}{1+w_\alpha}\frac{\Delta_\alpha}{a}+\frac{w_\alpha}{1+w_\alpha}\frac{\eta_\alpha}{a}\right]+\frac{F_\alpha}{a}\ , \tag{A8b}$$

where $V_{\alpha\beta}\equiv V_\alpha-V_\beta$. Another variable of interest is the entropy perturbation already introduced in (1.1):

$$S_{\alpha\beta}\equiv\frac{\Delta_{c\alpha}}{1+w_\alpha}-\frac{\Delta_{c\beta}}{1+w_\beta}\ .$$

The interpretation of this variable is quite clear when the component α describes matter and the component β radiation, then $S_{\alpha\beta}$ reduces to

$$S_{\alpha\beta}=\frac{\delta\rho_m}{\rho_m}-\frac{3}{4}\frac{\delta\rho_r}{\rho_r}=\frac{\delta n}{n}-\frac{\delta s}{s}=\frac{\delta(n/s)}{n/s}\ , \tag{A9}$$

where n is baryon number density and s the entropy density of radiation. A useful relation is

$$\frac{\Delta_\alpha}{1+w_\alpha}=\frac{\Delta}{1+w}+\frac{h_\beta}{h}S_{\alpha\beta}+3\frac{H_a}{k}\frac{h_\beta}{h}V_{\alpha\beta}\ . \tag{A10}$$

If the fluids are uncoupled $Q_\alpha=F_\alpha=E_\alpha=0$.

The equations of motion for $S_{\alpha\beta}$ and $V_{\alpha\beta}$ for a two-component system when the interactions between components can be neglected are given by

$$\frac{dS_{\alpha\beta}}{da}=-\frac{k}{aH}\frac{V_{\alpha\beta}}{a}-3\left[\frac{w_\alpha}{1+w_\alpha}\frac{\eta_\alpha}{a}-\frac{w_\beta}{1+w_\beta}\frac{\eta_\beta}{a}\right]\ ,$$
$$\frac{dV_{\alpha\beta}}{da}+[1-\tfrac{3}{2}(c_{s\alpha}^2+c_{s\beta}^2]\frac{V_{\alpha\beta}}{a}-\tfrac{3}{2}(c_{s\alpha}^2-c_{s\beta}^2)\left[\frac{-h_\alpha+h_\beta}{h}\right]\frac{V_{\alpha\beta}}{a}$$
$$=\frac{k}{aH}\frac{(c_{s\alpha}^2-c_{s\beta}^2)}{1+w}\frac{\Delta}{a}+\frac{1}{2}\frac{k}{aH}(c_{s\alpha}^2+c_{s\beta}^2)\frac{S_{\alpha\beta}}{a}+\frac{1}{2}\frac{k}{aH}(c_{s\alpha}^2-c_{s\beta}^2)\left[\frac{-h_\alpha+h_\beta}{h}\right]\frac{S_{\alpha\beta}}{a}+\frac{k}{aH}\frac{1}{a}\left[\frac{w_\alpha\eta_\alpha}{1+w_\alpha}-\frac{w_\beta\eta_\beta}{1+w_\beta}\right]\ . \tag{A11}$$

For the interacting case, see Ref. 15.

We will assume that $\eta_\alpha=0$ except when the component α is a scalar field. In this case, it can be seen that

$$p_\phi\eta_\phi=(1-c_{s\phi}^2)\rho_\phi\Delta_\phi\ , \tag{A12}$$

On the other hand, the equations for the total fluid perturbations are given by

$$\frac{d\Delta}{da}-3w\frac{\Delta}{a}=-(1+w)\frac{k}{aH}\frac{V}{a}\ ,$$
$$\frac{dV}{da}+\frac{V}{a}=-\frac{3}{2}\frac{aH}{k}\frac{\Delta}{a}+\frac{k}{aH}\frac{1}{ah}\sum_\gamma(p_\gamma\eta_\gamma+c_{s\gamma}^2\rho_\gamma\Delta_{c\gamma})\ . \tag{A13}$$

This couple of equations is equivalent to the second-order equation for the Bardeen variable Φ, related to Δ by $\Phi\equiv\frac{3}{2}(aH/k)^2\Delta$. In the one-component case, this equation has a first integral for wavelengths larger than the Hubble radius;[23] this can be easily seen by writing (A13) in terms of Φ and $\Gamma\equiv(aH/k)V$:

$$\frac{d\Phi}{da}+\frac{\Phi}{a}=-\tfrac{3}{2}(1+w)\frac{\Gamma}{a}\ , \tag{A14a}$$

$$\frac{d\Gamma}{da}+\tfrac{3}{2}(1+w)\frac{\Gamma}{a}=-\frac{\Phi}{a}+3\frac{aH}{k}\sum_\gamma\left[c_{s\gamma}^2\frac{h_\gamma}{h}\frac{1}{a}(V_\gamma-V)\right]+\frac{1}{ah}\sum_\gamma(p_\gamma\eta_\gamma+c_{s\gamma}^2\rho_\gamma\Delta_\gamma)\ , \tag{A14b}$$

When there is only one component, the second term on the right-hand side of (A14b) vanishes and the third one is much smaller than the first for wavelengths larger than the Hubble radius, so there is an approximate constant of motion given by $\mathcal{R}\equiv\Phi-\Gamma$. The physical meaning of this quantity can be understood by noting that

$$\Phi-\Gamma=H_L+\frac{H_T}{3}+\frac{aH}{k}(V-B)\ . \tag{A15}$$

Then, in the comoving gauge what is conserved is the spatial curvature of hypersurfaces orthogonal to the total fluid flow [it can be seen that $\delta(^3R)=4(k/a)^2\mathcal{R}Y$, with $\mathcal{R}=H_L+H_T/3$].

However, when dealing with a multicomponent system, this conservation law need not necessarily hold. In the first place, the second term in the right-hand side of (A14b) only vanishes in the case that all the components have the same sound velocity or when the perturbations in the velocity of all the components are equal. And second, the third term can only be neglected in the case that the perturbations in the energy density of the individual components are comparable to (or smaller than) the perturbation in the total energy density. This is actually true for adiabatic perturbations, but not for isocurvature ones. So, in general, the spatial curvature of hypersurfaces orthogonal to the total fluid flow is not a constant of motion outside the Hubble radius.

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